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Output Feedback Based Iterative Learning Control with Finite Frequency Range Specifications via a Heuristic Approach for Batch Processes with Polytopic Uncertainties ⋆

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Abstract: For robust control and iterative optimization of industrial batch processes with polytopic uncertainties, this paper proposes a robust output feedback based iterative learning control (ILC) design in terms of finite frequency range stability specifications. Robust stability conditions for the closed-loop ILC system along both time and batch directions are first established based on the generalized Kalman-Yakubovich-Popov lemma and linear repetitive system theory. To facilitate the ILC controller design with respect to process uncertainties described in a polytopic form, extended sufficient conditions for the system stability are then derived in terms of matrix inequalities. Correspondingly, a two-stage heuristic approach is developed to iteratively compute feasible ILC controller gains for implementation. An illustrative example is given to demonstrate the effectiveness of the proposed control design.

Keywords: Batch processes, polytopic uncertainties, output feedback, iterative learning control, finite frequency range design.

1. INTRODUCTION

Batch processes have been widely built up in industrial applications, such as industrial injection molding (Gao et al., 2001; Hao et al., 2016) and pharmaceutical crystallization (Nagy, 2009). Over the past decades, many control methods have been well explored for such processes, see the survey papers, e.g., Wang et al. (2009) and Ahn et al. (2007). Among these methods, iterative learning control (ILC) has attracted considerable attentions since it can gradually improve the system performance with respect to trajectory tracking and disturbance rejection by making use of the historical data.

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It is well known that frequency-domain specifications are widely adopted to assess system performance in practical applications. However, frequency-domain specifications are difficult to be converted into tractable conditions for controller synthesis. In the modern control theory, this obstacle was overcome by the celebrated Kalman-Yakubovich-Popov (KYP) lemma (Rantzer, 1996), which bridged frequency-domain inequalities and tractable linear matrix inequality (LMI) conditions in the entire frequency range. It is noted, however, that frequency-domain specifications in many practical applications are limited to finite or semi-finite ranges. For example, trial-to-trial error convergence for ILC system is typically in the 'low' frequency range. To extend the applicability of the standard KYP lemma to a finite frequency range, Iwasaki and Hara (2005) proposed a generalized KYP lemma. Since then, a number of finite frequency range control designs have been explored for continuous or batch operation processes. Li and Gao (2017) proposed a two-stage heuristic approach for finite frequency range control design based on output feedback. For batch operation processes, finite frequency range robust ILC designs were proposed for delay-free systems (Paszke et al., 2013) and state-delay systems (Tao et al., 2019). However, accurate state measurement must be available for these ILC designs, which is usually expensive or unavailable in practice. Although output feedback (OF) based ILC was addressed in Hładowski et al. (2012), the established matrix inequality conditions were quite conservative due to the introduction of additional equality constraints. Besides, uncertainties widely exist in industrial processes due to modeling error or parameter drifting etc. To the best of our knowledge, robust finite frequency range OF based ILC design for batch processes with uncertainties remains open as yet, which motivates this study.

In this paper, a novel robust OF based ILC design in terms of finite frequency range specifications is proposed for batch processes with polytopic uncertainties. Based on the generalized KYP lemma, together with the matrix dilatation technique, sufficient conditions are established to guarantee robust stability of the resulting ILC system along the pass in a repetitive system setting. The corresponding ILC controller gains are iteratively solved by a two-stage heuristic approach. The effectiveness of the proposed design is validated by an illustrative example.

Notations: $\mathbb{Z} \triangleq \{1, 2, 3, \ldots\}$. The superscripts "$-1$", "$\perp$", "$*$" and "$\top$" stand for inverse, null space, conjugate transpose and transpose of a matrix, respectively. $\mathbb{R}^n$, $\mathbb{H}_n$, $\mathbb{R}^{n\times m}$ and $\mathbb{C}^{n\times m}$ denote n-dimensional Euclidean space, Hermitian matrix space, $n \times m$ real matrix space and complex matrix space, respectively. I or 0 indicates the identity or zero matrix (vector) with appropriate dimensions. For any symmetric matrix $P \in \mathbb{R}^{n\times n}$, $P \succ 0$ ($\prec 0$) indicates a positive (negative) definite matrix, where "$(\star)$" indicates the symmetric elements. For two integers a and b satisfying $a \leq b$, denote $\mathbf{I}[a, b] \triangleq \{a, a+1, \ldots, b\}$. For matrices Φ and P, $\Phi \otimes P$ is the Kronecker product. For a square matrix A, $\mathrm{sym}\{A\}$ indicates $A^* + A$. For $G \in \mathbb{C}^{n\times m}$ and $\Pi \in \mathbb{H}_{n+m}$, a function $\sigma : \mathbb{C}^{n\times m} \times \mathbb{H}_{n+m} \to \mathbb{H}_m$ is defined by $\sigma(G, \Pi) \triangleq \begin{bmatrix} G \\ I \end{bmatrix}^* \Pi \begin{bmatrix} G \\ I \end{bmatrix}$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following batch process with polytopic uncertainties

$$\begin{cases} x_k(t+1) = A(\theta)x_k(t) + B(\theta)u_k(t), \\ y_k(t) = Cx_k(t), \\ x_k(0) = x_0,\ t \in \mathbf{I}[0, T-1], k \in \mathbb{Z}, \end{cases} \tag{1}$$

where t and k represent the time and batch indices, respectively; T is time period of each batch; $x_k(t) \in \mathbb{R}^n, y_k(t) \in \mathbb{R}^p$ and $u_k(t) \in \mathbb{R}^m$ are the state, output and input of the process; x_0 is the identical initial condition for each batch. Matrices $G(\theta) \triangleq (A(\theta), B(\theta))$ are real-valued, time-invariant, and assumed to belong to a polytopic parametric domain defined as $\mathbb{G} \triangleq \{G(\theta)|G(\theta) = \sum_{i=1}^{s} \theta_i G_i,\ \theta \in \Delta\}$, where $G_i \triangleq (A_i, B_i)$ and $\Delta \triangleq \{\theta \in \mathbb{R}^s | \sum_{i=1}^{s} \theta_i = 1, \theta_i \geq 0,\ i = 1, \ldots, s\}$. Constant matrices G_i, $i = 1, \ldots, s$ denote $G(\theta)$ at the s vertices of $\mathbb{G}$, which are assumed to be known.

In general, the output tracking error in the current batch is defined as

$$e_k(t) \triangleq y_k(t) - y_r(t), \tag{2}$$

where $y_r(t)$ represents the desired reference trajectory.

For the batch process in (1), a commonly used ILC updating law takes the following form

$$u_k(t) = u_{k-1}(t) + r_k(t), \tag{3}$$

where $r_k(t)$ is referred to as the control update to be determined, the initial value of $u_k(t)$, i.e., $u_0(t)$ is usually reset to zero for batch operation.

It follows from (1)-(3) that

$$\begin{aligned} e_k(t+1) &= e_{k-1}(t+1) + C\delta x_k(t+1), \\ \delta x_k(t+1) &= A(\theta)\delta x_k(t) + B(\theta)r_k(t), \end{aligned} \tag{4}$$

where $\delta x_k(t) = x_k(t) - x_{k-1}(t)$. To facilitate practical applications, the following output feedback (OF) based iterative learning control (ILC) law is adopted

$$r_k(t) = K_1\delta y_k(t) + K_2 e_{k-1}(t+1), \tag{5}$$

where $K_1 \in \mathbb{R}^{m\times p}$ and $K_2 \in \mathbb{R}^{m\times p}$ are learning gains to be designed. Therefore, an equivalent description of the discrete linear repetitive system (DLRS) is formulated as

$$\begin{cases} \mathcal{X}_{k+1}(t+1) = \mathcal{A}(\theta)\mathcal{X}_{k+1}(t) + \mathcal{B}(\theta)e_k(t), \\ e_{k+1}(t) = \mathcal{C}(\theta)\mathcal{X}_{k+1}(t) + \mathcal{D}(\theta)e_k(t), \end{cases} \tag{6}$$

where $\mathcal{X}_k(t) \triangleq \delta x_k(t-1)$, and

$$\mathcal{A}(\theta) = A(\theta) + B(\theta)K_1C,\ \mathcal{B}(\theta) = B(\theta)K_2,$$
$$\mathcal{C}(\theta) = C(A(\theta) + B(\theta)K_1C),\ \mathcal{D}(\theta) = I + CB(\theta)K_2.$$

For any fixed $\theta \in \Delta$, the transfer function from e_k to e_{k+1} is given by

$$\mathcal{G}(e^{j\omega}, \theta) \triangleq \mathcal{C}(\theta)(e^{j\omega}I - \mathcal{A}(\theta))^{-1}\mathcal{B}(\theta) + \mathcal{D}(\theta),\ \omega \in [-\pi, \pi]. \tag{7}$$

Define $\Omega \triangleq [-\omega_l, \omega_l]$ for a low frequency (LF) range, $[\omega_1, \omega_2]$ for a middle frequency (MF) range, and $[\omega_h, \pi]$ for a high frequency (HF) range, respectively, with $\omega_1, \omega_2, \omega_l$ and ω_h in $[-\pi, \pi]$. For the uncertain batch process in (1), our aim is to design a robust OF based ILC scheme such that the system output can track the desired reference trajectory as close as possible over a finite frequency range Ω against polytopic uncertainties.

To this end, we present the following technical lemmas.

Lemma 1. (Rogers et al., 2005) The uncertain DLRS described by (6) is robustly stable for all $\theta \in \Delta$ along the pass if and only if

i) $\rho(\mathcal{D}(\theta)) < 1$,
ii) $\rho(\mathcal{A}(\theta)) < 1$,
iii) all eigenvalues of $\mathcal{G}(e^{j\omega}, \theta)$, $\forall\omega \in [-\pi, \pi]$ have modulus strictly less than unity.

Lemma 2. (Iwasaki and Hara, 2005) Let $\Pi \in \mathbb{H}_{2p}$, $\Phi \in \mathbb{H}_2$, $\Psi \in \mathbb{H}_2$, and the state-space realization of stable $\mathcal{G}(e^{j\omega}, \theta)$ in (7) be given. For arbitrarily fixed $\theta \in \Delta$, the following statements are equivalent:

(i) $\sigma(\mathcal{G}(e^{j\omega}, \theta), \Pi) < 0$ holds for all $\omega \in \Omega$.
(ii) There exist matrices $P = P^\top, Q \succ 0$ such that

$$\begin{bmatrix} \mathcal{A}(\theta) & \mathcal{B}(\theta) \\ I & 0 \end{bmatrix}^\top (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} \mathcal{A}(\theta) & \mathcal{B}(\theta) \\ I & 0 \end{bmatrix} + \begin{bmatrix} \mathcal{C}(\theta) & \mathcal{D}(\theta) \\ 0 & I \end{bmatrix}^\top \Pi \begin{bmatrix} \mathcal{C}(\theta) & \mathcal{D}(\theta) \\ 0 & I \end{bmatrix} \prec 0, \tag{8}$$

where $\Phi = [1\ 0; 0\ -1]$ and Ψ is given in the following table with $\omega_c = (\omega_1 + \omega_2)/2$ and $\omega_m = (\omega_2 - \omega_1)/2$.

Ω	$[-\omega_l,\omega_l]$(LF)	$[\omega_1,\omega_2]$(MF)	$[\omega_h,\pi]$(HF)
Ψ	$\begin{bmatrix}0 & 1\\ 1 & -2\cos(\omega_l)\end{bmatrix}$	$\begin{bmatrix}0 & e^{\mathrm{j}\omega_c}\\ e^{-\mathrm{j}\omega_c} & -2\cos(\omega_m)\end{bmatrix}$	$\begin{bmatrix}0 & -1\\ -1 & 2\cos(\omega_h)\end{bmatrix}$

3. A HEURISTIC APPROACH FOR OUTPUT FEEDBACK BASED ILC DESIGN

For ease of presentation, the parameter θ is omitted in the following analysis. Note that the matrix inequality in (8) can be rewritten as

$$\tilde{E}^\top \tilde{\Xi}\tilde{E} \prec 0, \tag{9}$$

where

$$\tilde{E} \triangleq \begin{bmatrix}\mathcal{A}^\top & I & \mathcal{C}^\top & 0\\ \mathcal{B}^\top & 0 & \mathcal{D}^\top & I\end{bmatrix}^\top, \ \tilde{\Xi} \triangleq \begin{bmatrix}\Phi\otimes P+\Psi\otimes Q & 0\\ 0 & \Pi\end{bmatrix}.$$

To ensure that conditions (i) and (ii) of Lemma 1 hold, the following inequalities are introduced

$$\mathcal{A}^\top P_s\mathcal{A}-P_s \prec 0,\ P_s\succ 0, \tag{10}$$

$$\mathcal{D}^\top P_e\mathcal{D}-P_e \prec 0,\ P_e\succ 0, \tag{11}$$

which can be further formulated, respectively, as

$$\tilde{E}_s^\top(\Phi\otimes P_s)\tilde{E}_s \prec 0, \tag{12}$$

$$\tilde{E}_e^\top(\Phi\otimes P_e)\tilde{E}_e \prec 0, \tag{13}$$

where $\tilde{E}_s=[\mathcal{A}^\top \ \ I]^\top$, $\tilde{E}_e=[\mathcal{D}^\top \ \ I]^\top$, and Φ is the same as that in Lemma 2. Moreover, by taking $\Pi=\mathrm{diag}\{I,-\mu^2 I\}$ with $\mu\in(0,1]$ in (8), condition (iii) of Lemma 1 can be guaranteed over a finite frequency range Ω using Lemma 2 . Inspired by the work in Li and Gao (2017), it follows that inequalities in (9), (12) and (13) are, respectively, equivalent to the dilated matrix inequality conditions below

$$\left[\begin{array}{c}\tilde{E}\\ \hline K_1C-L_1 \ \ K_2-L_2\end{array}\right]^\top \Xi \left[\begin{array}{c}\tilde{E}\\ \hline K_1C-L_1 \ \ K_2-L_2\end{array}\right] \prec 0, \tag{14}$$

$$\left[\begin{array}{c}\tilde{E}_s\\ \hline K_1C-L_1\end{array}\right]^\top \Xi_s \left[\begin{array}{c}\tilde{E}_s\\ \hline K_1C-L_1\end{array}\right] \prec 0, \tag{15}$$

$$\left[\begin{array}{c}\tilde{E}_e\\ \hline K_2-L_2\end{array}\right]^\top \Xi_e \left[\begin{array}{c}\tilde{E}_e\\ \hline K_2-L_2\end{array}\right] \prec 0, \tag{16}$$

where $L_1\in\mathbb{R}^{m\times n}$, $L_2\in\mathbb{R}^{m\times p}$, and

$$\Xi \triangleq \begin{bmatrix}\tilde{\Xi} & 0\\ 0 & 0\end{bmatrix}, \ \Xi_s \triangleq \begin{bmatrix}\Phi\otimes P_s & 0\\ 0 & 0\end{bmatrix}, \ \Xi_e \triangleq \begin{bmatrix}\Phi\otimes P_e & 0\\ 0 & 0\end{bmatrix}. \tag{17}$$

Next, we give the following condition characterizing the robust stability of the resulting DLRS along the pass by using the dilated conditions in (14)-(16).

Theorem 3. Let matrices $\Pi=\mathrm{diag}\{I,-\mu^2 I\}$, K_1, K_2, and the state-space realization of $\mathcal{G}(e^{\mathrm{j}\omega},\theta)$ be given with $\mu\in(0,1]$. For any fixed $\theta\in\Delta$, the DLRS in (6) is robustly stable along the pass over the finite frequency range Ω if there exist matrices $P=P^\top$, $Q\succ 0$, $P_s\succ 0$, $P_e\succ 0$, X_{11}, X_{12}, X_{21}, X_{22}, F_s, F_e, L_1, L_2 and R such that

$$\Xi+\mathrm{sym}\{X\Sigma\}\prec 0, \tag{18}$$

$$\Xi_s+\mathrm{sym}\{X_s\Sigma_s\}\prec 0, \tag{19}$$

$$\Xi_e+\mathrm{sym}\{X_e\Sigma_e\}\prec 0, \tag{20}$$

where Ξ, Ξ_s and Ξ_e are defined in (17), and

$$X\triangleq\begin{bmatrix}X_{11} & X_{12} & 0\\ X_{21} & X_{22} & 0\\ 0 & 0 & R\end{bmatrix}, \ X_s\triangleq\begin{bmatrix}F_s & 0\\ 0 & R\end{bmatrix}, \ X_e\triangleq\begin{bmatrix}F_e & 0\\ 0 & R\end{bmatrix},$$

$$\Sigma\triangleq\left[\begin{array}{cc|cc|c}-I & A+BL_1 & 0 & BL_2 & B\\ \hline 0 & CA+CBL_1 & -I & I+CBL_2 & CB\\ \hline 0 & K_1C-L_1 & 0 & K_2-L_2 & -I\end{array}\right],$$

$$\Sigma_s\triangleq\left[\begin{array}{cc|c}-I & A+BL_1 & B\\ \hline 0 & K_1C-L_1 & -I\end{array}\right], \ \Sigma_e\triangleq\left[\begin{array}{cc|c}-I & I+CBL_2 & CB\\ \hline 0 & K_2-L_2 & -I\end{array}\right].$$

Proof. Note that the DLRS in (6) is robustly stable along the pass over a finite frequency range Ω if matrix inequalities in (14)-(16) are satisfied with $\Pi=\mathrm{diag}\{I,-\mu^2 I\}$ and $\mu\in(0,1]$. Thus, it suffices to prove that conditions in (14)-(16) are equivalent to those in (18)-(20), respectively.

(*Sufficiency*) By multiplying $\Sigma^{\perp\top}$, $\Sigma_s^{\perp\top}$, $\Sigma_e^{\perp\top}$ and their transposes on both sides of (18)-(20), it follows that

$$\Sigma^{\perp\top}\Xi\Sigma^\perp\prec 0,\ \Sigma_s^{\perp\top}\Xi_s\Sigma_s^\perp\prec 0,\ \Sigma_e^{\perp\top}\Xi_e\Sigma_e^\perp\prec 0. \tag{21}$$

Moreover, it is noted that $\Sigma^\perp$, $\Sigma_s^\perp$ and $\Sigma_e^\perp$ can be chosen, respectively, as

$$\begin{aligned}\Sigma^\perp &= \left[\begin{array}{c}\tilde{E}\\ \hline K_1C-L_1 \ \ K_2-L_2\end{array}\right],\\ \Sigma_s^\perp &= \left[\begin{array}{c}\tilde{E}_s\\ \hline K_1C-L_1\end{array}\right],\ \Sigma_e^\perp = \left[\begin{array}{c}\tilde{E}_e\\ \hline K_2-L_2\end{array}\right].\end{aligned} \tag{22}$$

Therefore, it is not difficult to find that inequalities in (21) are exactly those in (14)-(16), respectively.

(*Necessity*) Suppose that inequalities in (14)-(16) hold true, then inequalities in (9), (12) and (13) also hold. Let

$$\tilde{\Sigma}=\begin{bmatrix}-I & \mathcal{A} & 0 & \mathcal{B}\\ 0 & \mathcal{C} & -I & \mathcal{D}\end{bmatrix}, \ \tilde{\Sigma}_s=[-I \ \ \mathcal{A}], \ \tilde{\Sigma}_e=[-I \ \ \mathcal{D}], \tag{23}$$

it is easy to get $\tilde{\Sigma}^\perp=\tilde{E}$, $\tilde{\Sigma}_s^\perp=\tilde{E}_s$, $\tilde{\Sigma}_e^\perp=\tilde{E}_e$. Thus, (9), (12) and (13) can be rewritten as

$$\tilde{\Sigma}^{\perp\top}\tilde{\Xi}\tilde{\Sigma}^\perp\prec 0,\ \tilde{\Sigma}_s^{\perp\top}(\Phi\otimes P_s)\tilde{\Sigma}_s^\perp\prec 0,\ \tilde{\Sigma}_e^{\perp\top}(\Phi\otimes P_e)\tilde{\Sigma}_e^\perp\prec 0.$$

With the help of the Finsler's Lemma in de Oliveira and Skelton (2001), it follows that

$$\exists\tilde{X},\ \tilde{\Xi}+\mathrm{sym}\{\tilde{X}\tilde{\Sigma}\}\prec 0, \tag{24}$$

$$\exists\tilde{X}_s,\ \Phi\otimes P_s+\mathrm{sym}\{\tilde{X}_s\tilde{\Sigma}_s\}\prec 0, \tag{25}$$

$$\exists\tilde{X}_e,\ \Phi\otimes P_e+\mathrm{sym}\{\tilde{X}_e\tilde{\Sigma}_e\}\prec 0, \tag{26}$$

then, for a sufficiently large $\varepsilon>0$, there always hold that

$$\begin{aligned}&\begin{bmatrix}\tilde{\Xi}+\mathrm{sym}\{\tilde{X}\tilde{\Sigma}\} & (\star)\\ [B^\top \ \ B^\top C^\top]\tilde{X}^\top & -2\varepsilon I\end{bmatrix}\prec 0,\\ &\begin{bmatrix}\Phi\otimes P_s+\mathrm{sym}\{\tilde{X}_s\tilde{\Sigma}_s\} & (\star)\\ B^\top\tilde{X}_s^\top & -2\varepsilon I\end{bmatrix}\prec 0,\\ &\begin{bmatrix}\Phi\otimes P_e+\mathrm{sym}\{\tilde{X}_e\tilde{\Sigma}_e\} & (\star)\\ B^\top C^\top\tilde{X}_e^\top & -2\varepsilon I\end{bmatrix}\prec 0,\end{aligned} \tag{27}$$

which are further equivalent to

$$\Xi+\mathrm{sym}\left\{\begin{bmatrix}\tilde{X} & 0\\ 0 & \varepsilon I\end{bmatrix}\begin{bmatrix}\tilde{\Sigma} & \begin{bmatrix}B\\ CB\end{bmatrix}\\ 0 & -I\end{bmatrix}\right\}\prec 0, \tag{28}$$

$$\Xi_s+\mathrm{sym}\left\{\begin{bmatrix}\tilde{X}_s & 0\\ 0 & \varepsilon I\end{bmatrix}\begin{bmatrix}\tilde{\Sigma}_s & B\\ 0 & -I\end{bmatrix}\right\}\prec 0, \tag{29}$$

$$\Xi_e+\mathrm{sym}\left\{\begin{bmatrix}\tilde{X}_e & 0\\ 0 & \varepsilon I\end{bmatrix}\begin{bmatrix}\tilde{\Sigma}_e & CB\\ 0 & -I\end{bmatrix}\right\}\prec 0, \tag{30}$$

where Ξ and Ξ_s are defined in (17). By taking

$$\tilde{X} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix},\ \tilde{X}_s = F_s,\ \tilde{X}_e = F_e, \\ R = \varepsilon I,\ L_1 = K_1 C,\ L_2 = K_2, \tag{31}$$

it is easy to find that inequalities in (28)-(30) are exactly those in (18)-(20), respectively. The proof is complete. ■

Remark 4. It is not difficult to find from the proof of Theorem 3 that L_1 and L_2 can be viewed as learning gains of a state feedback (SF) based ILC law, i.e.,

$$r_k(t) = L_1 \delta x_k(t) + L_2 e_{k-1}(t+1), \tag{32}$$

for the considered batch process in (1), which has been well developed and also recognized as the base of the two-stage heuristic algorithm for the OF based ILC design.

Remark 5. The results in Theorem 3 can be easily extended to parameter-dependent matrix inequalities by replacing X_{11}, X_{12}, X_{21}, X_{22}, F_s, F_e, P, Q, P_s and P_e as $X_{11}(\theta)$, $X_{12}(\theta)$, $X_{21}(\theta)$, $X_{22}(\theta)$, $F_s(\theta)$, $F_e(\theta)$, $P(\theta)$, $Q(\theta)$, $P_s(\theta)$ and $P_e(\theta)$, respectively. Besides, it is noted that R, L_1 and L_2 are set to be parameter-independent due to the independence of ε, K_1, K_2 and C in (31) with respect to θ.

The following result gives a parametrization of the desired OF based ILC gains.

Theorem 6. Let the matrix $\Pi = \text{diag}\{I, -\mu^2 I\}$ and the state-space realization of $\mathcal{G}(e^{j\omega}, \theta)$ be given with $\mu \in (0, 1]$. The uncertain DLRS in (6) is robustly stable along the pass over the finite frequency range Ω for any $\theta \in \Delta$ if there exist matrices $P(\theta) = P^\top(\theta)$, $Q(\theta) \succ 0$, $P_s(\theta) \succ 0$, $P_e(\theta) \succ 0$, $X_{11}(\theta)$, $X_{12}(\theta)$, $X_{21}(\theta)$, $X_{22}(\theta)$, $F_s(\theta)$, $F_e(\theta)$, L_1, L_2, R, $\hat{K}_1$ and $\hat{K}_2$ such that the following inequalities are satisfied

$$\Xi(\theta) + \text{sym}\{\Lambda(\theta)\} \prec 0, \tag{33}$$

$$\Xi_s(\theta) + \text{sym}\{\Lambda_s(\theta)\} \prec 0, \tag{34}$$

$$\Xi_e(\theta) + \text{sym}\{\Lambda_e(\theta)\} \prec 0, \tag{35}$$

where $\Gamma_7 \triangleq \hat{K}_1 C - R L_1$, $\Gamma_8 \triangleq \hat{K}_2 - R L_2$, and

$$\Lambda(\theta) \triangleq \left[\begin{array}{cc|cc|c} -X_{11}(\theta) & \Gamma_1(\theta) & -X_{12}(\theta) & \Gamma_2(\theta) & \Gamma_3(\theta) \\ \hline -X_{21}(\theta) & \Gamma_4(\theta) & -X_{22}(\theta) & \Gamma_5(\theta) & \Gamma_6(\theta) \\ \hline 0 & \Gamma_7 & 0 & \Gamma_8 & -R \end{array}\right],$$

$$\Lambda_s(\theta) \triangleq \left[\begin{array}{cc|c} -F_s(\theta) & \Gamma_9(\theta) & F_s(\theta)B(\theta) \\ \hline 0 & \Gamma_7 & -R \end{array}\right],$$

$$\Lambda_e(\theta) \triangleq \left[\begin{array}{cc|c} -F_e(\theta) & \Gamma_{10}(\theta) & F_e(\theta)CB(\theta) \\ \hline 0 & \Gamma_8 & -R \end{array}\right],$$

$$\begin{aligned}
\Gamma_1(\theta) &\triangleq X_{11}(\theta)(A(\theta) + B(\theta)L_1) \\
&\quad + X_{12}(\theta)C(A(\theta) + B(\theta)L_1), \\
\Gamma_2(\theta) &\triangleq X_{11}(\theta)B(\theta)L_2 + X_{12}(\theta) + X_{12}(\theta)CB(\theta)L_2, \\
\Gamma_3(\theta) &\triangleq X_{11}(\theta)B(\theta) + X_{12}(\theta)CB(\theta), \\
\Gamma_4(\theta) &\triangleq X_{21}(\theta)(A(\theta) + B(\theta)L_1) \\
&\quad + X_{22}(\theta)C(A(\theta) + B(\theta)L_1), \\
\Gamma_5(\theta) &\triangleq X_{21}(\theta)B(\theta)L_2 + X_{22}(\theta) + X_{22}(\theta)CB(\theta)L_2, \\
\Gamma_6(\theta) &\triangleq X_{21}(\theta)B(\theta) + X_{22}(\theta)CB(\theta), \\
\Gamma_9(\theta) &\triangleq F_s(\theta)A(\theta) + F_s(\theta)B(\theta)L_1, \\
\Gamma_{10}(\theta) &\triangleq F_e(\theta) + F_e(\theta)CB(\theta)L_2.
\end{aligned}$$

Moreover, if the above matrix inequalities are feasible, then the OF based ILC gains can be computed by

$$K_1 = R^{-1}\hat{K}_1,\ K_2 = R^{-1}\hat{K}_2. \tag{36}$$

Proof. By replacing X_{11}, X_{12}, X_{21}, X_{22}, F_s, F_e, P, Q, P_s and P_e as $X_{11}(\theta)$, $X_{12}(\theta)$, $X_{21}(\theta)$, $X_{22}(\theta)$, $F_s(\theta)$, $F_e(\theta)$, $P(\theta)$, $Q(\theta)$, $P_s(\theta)$, $P_e(\theta)$ and letting $\hat{K}_1 = RK_1$, $\hat{K}_2 = RK_2$, we have $X(\theta)\Sigma(\theta) = \Lambda(\theta)$, $X_s(\theta)\Sigma_s(\theta) = \Lambda_s(\theta)$ and $X_e(\theta)\Sigma_e(\theta) = \Lambda_e(\theta)$. Besides, it follows from (34) that $-R - R^\top \prec 0$, which implies that R is invertible. Based on Theorem 3, the OF based ILC law with $K_1 = R^{-1}\hat{K}_1$, $K_2 = R^{-1}\hat{K}_2$ ensures the robust stability of the resulting DLRP in (6) along the pass over the finite frequency range Ω. The proof is complete. ■

It is observed that conditions in (33)-(35) are obviously infinitely dimensional with respect to θ. To circumvent this issue, the following extended sufficient condition is given in terms of a finite number of matrix inequality conditions.

Corollary 7. Let the matrix $\Pi = \text{diag}\{I, -\mu^2 I\}$ and the state-space realization of $\mathcal{G}(e^{j\omega}, \theta)$ be given with $\mu \in (0, 1]$. The uncertain DLRS in (6) is robustly stable along the pass over the finite frequency range Ω for any $\theta \in \Delta$ if there exist matrices $P_i = P_i^\top$, $Q_i \succ 0$, $P_{s,i} \succ 0$, $P_{e,i} \succ 0$, $X_{11,i}$, $X_{12,i}$, $X_{21,i}$, $X_{22,i}$, $F_{s,i}$, $F_{e,i}$, $i = 1, \ldots, s$, L_1, L_2, R, $\hat{K}_1$ and $\hat{K}_2$ such that the following inequalities hold for $1 \le i \le j \le s$

$$\Theta_{i,j} \triangleq \Xi_i + \Xi_j + \text{sym}\{\Lambda_{i,j} + \Lambda_{j,i}\} \prec 0, \tag{37}$$

$$\Theta_{s,i,j} \triangleq \Xi_{s,i} + \Xi_{s,j} + \text{sym}\{\Lambda_{s,i,j} + \Lambda_{s,j,i}\} \prec 0, \tag{38}$$

$$\Theta_{e,i,j} \triangleq \Xi_{e,i} + \Xi_{e,j} + \text{sym}\{\Lambda_{e,i,j} + \Lambda_{e,j,i}\} \prec 0, \tag{39}$$

where Ξ_i, $\Xi_{s,i}$ and $\Xi_{e,i}$ are Ξ, Ξ_s and Ξ_e in Theorem 3 with P, Q, P_s and P_e replaced by P_i, Q_i, $P_{s,i}$ and $P_{e,i}$, respectively, $\Lambda_{i,j}$, $\Lambda_{s,i,j}$ and $\Lambda_{e,i,j}$ are Λ, Λ_s and Λ_e in Theorem 6 with $X_{11}(\theta)$, $X_{12}(\theta)$, $X_{21}(\theta)$, $X_{22}(\theta)$, $F_s(\theta)$, $F_e(\theta)$ replaced by $X_{11,i}$, $X_{12,i}$, $X_{21,i}$, $X_{22,i}$, $F_{s,i}$, $F_{e,i}$ and $A(\theta)$, $B(\theta)$ replaced by A_j, B_j, respectively.

Proof. By taking $P(\theta)$, $Q(\theta)$, $P_s(\theta)$, $P_e(\theta)$, $X_{11}(\theta)$, $X_{12}(\theta)$, $X_{21}(\theta)$, $X_{22}(\theta)$, $F_s(\theta)$, $F_e(\theta)$ as matrix functions linearly dependent on θ, we have

$$\begin{aligned}
\Xi(\theta) + \text{sym}\{\Lambda(\theta)\} &= \sum_{i=1}^{s} \theta_i^2 \big(\Xi_i + \text{sym}\{\Lambda_{i,i}\}\big) \\
&\quad + \sum_{i=1}^{s-1} \sum_{j=i+1}^{s} \theta_i \theta_j \big(\Xi_i + \Xi_j + \text{sym}\{\Lambda_{i,j} + \Lambda_{j,i}\}\big), \\
\Xi_s(\theta) + \text{sym}\{\Lambda_s(\theta)\} &= \sum_{i=1}^{s} \theta_i^2 \big(\Xi_{s,i} + \text{sym}\{\Lambda_{s,i,i}\}\big) \\
&\quad + \sum_{i=1}^{s-1} \sum_{j=i+1}^{s} \theta_i \theta_j \big(\Xi_{s,i} + \Xi_{s,j} + \text{sym}\{\Lambda_{s,i,j} + \Lambda_{s,j,i}\}\big), \\
\Xi_e(\theta) + \text{sym}\{\Lambda_e(\theta)\} &= \sum_{i=1}^{s} \theta_i^2 \big(\Xi_{e,i} + \text{sym}\{\Lambda_{e,i,i}\}\big) \\
&\quad + \sum_{i=1}^{s-1} \sum_{j=i+1}^{s} \theta_i \theta_j \big(\Xi_{e,i} + \Xi_{e,j} + \text{sym}\{\Lambda_{e,i,j} + \Lambda_{e,j,i}\}\big).
\end{aligned}$$

Therefore, conditions in (33)-(35) hold true if conditions in (37)-(39) are satisfied, which ensure the robust stability of the resulting ILC system over the finite frequency range Ω by Theorem 6. The proof is complete. ■

Remark 8. Differing from the work in Li and Gao (2017), where only Lyapunov matrices are parameter-dependent, additional free-weighting matrices, e.g., X_{11}, X_{12}, X_{21},

X_{22}, F_s, are also considered to be parameter-dependent due to the independence of C with respect to θ.

With given L_1 and L_2, conditions in (37)-(39) are obviously linear with respect to P_i, Q_i, $P_{s,i}$, $P_{e,i}$, $X_{11,i}$, $X_{12,i}$, $X_{21,i}$, $X_{22,i}$, $F_{s,i}$, $F_{e,i}$, $i = 1, \ldots, s$, R, $\hat{K}_1$ and $\hat{K}_2$. The following heuristic algorithm is therefore summarized to design the OF based ILC law by Corollary 7, where $\mathbb{I} \triangleq \mathrm{diag}\{I_{2n+2p}, 0_m\}$, $\mathbb{I}_s \triangleq \mathrm{diag}\{I_{2n}, 0_m\}$ and $\mathbb{I}_e \triangleq \mathrm{diag}\{I_{2p}, 0_m\}$.

Algorithm 1 (Design of OF based ILC law)

1. Given a specified tolerance $\delta > 0$, the initial SF based ILC gains L_1^1 and L_2^1 are chosen such that the resulting DLRS is robustly stable over the finite frequency range Ω for all $\theta \in \Delta$. Set $k = 1$.

2-1. Solve the following LMI problem to obtain ϵ_1^k

$$\begin{aligned} &\min \ \ \epsilon_1^k = \epsilon \\ &\text{s.t.} \ \ \begin{cases} \Theta_{i,j} \prec \epsilon\mathbb{I}, \ \Theta_{s,i,j} \prec \epsilon\mathbb{I}_s, \\ \Theta_{e,i,j} \prec \epsilon\mathbb{I}_e, \ 1 \le i \le j \le s, \end{cases} \\ &\text{for} \ \ P_i = P_i^\top, \ Q_i \succ 0, \ P_{s,i} \succ 0, \ P_{e,i} \succ 0, \\ &\qquad X_{11,i}, \ X_{12,i}, \ X_{21,i}, \ X_{22,i}, \ F_{s,i}, \ F_{e,i}, \\ &\qquad R, \hat{K}_1, \ \hat{K}_2, \ \epsilon, \\ &\text{with fixed } L_1 = L_1^k \text{ and } L_2 = L_2^k. \end{aligned} \tag{40}$$

If $\epsilon_1^k \le 0$, then $K_1 = R^{-1}\hat{K}_1$ and $K_2 = R^{-1}\hat{K}_2$ are the desired OF based ILC gains, and exit; otherwise, denote $X_{11,i}^k = X_{11,i}$, $X_{12,i}^k = X_{12,i}$, $X_{21,i}^k = X_{21,i}$, $X_{22,i}^k = X_{22,i}$, $F_{s,i}^k = F_{s,i}$, $F_{e,i}^k = F_{e,i}$, $i = 1, \ldots, s$ and $R^k = R$, go to the next step;

2-2. Solve the following LMI problem to obtain ϵ_2^k

$$\begin{aligned} &\min \ \ \epsilon_2^k = \epsilon \\ &\text{s.t.} \ \ \begin{cases} \Theta_{i,j} \prec \epsilon\mathbb{I}, \ \Theta_{s,i,j} \prec \epsilon\mathbb{I}_s, \\ \Theta_{e,i,j} \prec \epsilon\mathbb{I}_e, \ 1 \le i \le j \le s, \end{cases} \\ &\text{for} \ \ P_i = P_i^\top, \ Q_i \succ 0, \ P_{s,i} \succ 0, \ P_{e,i} \succ 0, \\ &\qquad L_1, \ L_2, \ \hat{K}_1, \ \hat{K}_2, \ \epsilon, \\ &\text{with fixed } X_{11,i} = X_{11,i}^k, \ X_{12,i} = X_{12,i}^k, \\ &\qquad X_{22,i} = X_{22,i}^k, \ X_{21,i} = X_{21,i}^k, \ R = R^k, \\ &\qquad F_{s,i} = F_{s,i}^k, \ F_{s,i} = F_{e,i}^k, \ i = 1, \ldots, s. \end{aligned} \tag{41}$$

If $\epsilon_2^k \le 0$, then $K_1 = R^{-1}\hat{K}_1$ and $K_2 = R^{-1}\hat{K}_2$ are the desired OF based ILC gains, and exit; otherwise, set $L_1^{k+1} = L_1$ and $L_2^{k+1} = L_2$, go to the next step;

2-3. If $|\epsilon_1^k - \epsilon_2^k|/\epsilon_2^k < \delta$, then no desired OF based ILC gains can be found, and exit; else, set $k \leftarrow k+1$, and go back to Step **2-1**.

Remark 9. It is easy to find from Algorithm 1 that ϵ_1^k and ϵ_2^k satisfy $\epsilon_1^k \ge \epsilon_2^k \ge \epsilon_1^{k+1}$ for all $k = 1, 2, \ldots$, which means that both ϵ_1^k and ϵ_2^k are non-increasing.

To find the initial SF based ILC law in Algorithm 1, the following theorem is established.

Theorem 10. Let the matrix $\Pi = \mathrm{diag}\{I, -I\}$ and the state-space realization of $\mathcal{G}(e^{\mathrm{j}\omega}, \theta)$ be given. The uncertain DLRS in (6) is robustly stable along the pass over the finite frequency range Ω for any $\theta \in \Delta$ under the SF based ILC law in (32) if there exist matrices $\bar{S}_i \succ 0$, $\bar{P}_i \succ 0$, $\bar{Q}_i \succ 0$, $i = 1, \ldots, s$, $\bar{W}$, $\bar{L}_1$ and $\bar{L}_2$ such that for all $i = 1, \ldots, s$

$$\begin{bmatrix} \bar{S}_i - \bar{W} - \bar{W}^\top & \varphi_i \\ (\star) & -\bar{S}_i \end{bmatrix} \prec 0, \tag{42}$$

$$\begin{bmatrix} \bar{\Xi}_{11,i} & \bar{\Xi}_{12,i} - \bar{W}^\top & 0 & 0 \\ (\star) & \bar{\Xi}_{22,i} + \mathrm{sym}\{\varphi_i\} & B_i\bar{L}_2 & \varphi_i^\top C^\top \\ (\star) & (\star) & -I & I + CB_i\bar{L}_2 \\ (\star) & (\star) & (\star) & -I \end{bmatrix} \prec 0, \tag{43}$$

where $\varphi_i = A_i\bar{W} + B_i\bar{L}_1$, $\bar{\Xi}_{11,i} = -\bar{P}_i$,

$$\bar{\Xi}_{12,i} = \begin{cases} \bar{Q}_i, & \text{(LF)}, \\ e^{\mathrm{j}\omega_c}\bar{Q}_i, & \text{(MF)}, \\ -\bar{Q}_i, & \text{(HF)}, \end{cases} \ \bar{\Xi}_{22,i} = \begin{cases} \bar{P}_i - 2\cos(\omega_l)\bar{Q}_i, & \text{(LF)}, \\ \bar{P}_i - 2\cos(\omega_m)\bar{Q}_i, & \text{(MF)}, \\ \bar{P}_i + 2\cos(\omega_h)\bar{Q}_i, & \text{(HF)}. \end{cases}$$

Moreover, if the above LMIs are feasible, then the SF based ILC gains can be computed by

$$L_1 = \bar{L}_1\bar{W}^{-1}, \ L_2 = \bar{L}_2. \tag{44}$$

Proof. The proof of this theorem follows immediately from that of Theorem 5 in Paszke et al. (2013) and therefore is omitted here. ■

4. CASE STUDY

Consider an injection molding process with the following parameters (Shi et al., 2006; Hao et al., 2019)

$$A = \begin{bmatrix} 1.607 + \lambda_1 & 1 \\ -0.6086 + \lambda_2 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix}, \ C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^\top,$$

where the uncertain parameters λ_1 and λ_2 belong to the intervals $[-0.0804, 0.0804]$ and $[-0.0304, 0.0304]$, respectively. Therefore, there are four vertices for this case. For illustration, the desired output trajectory

$$y_r(t) = \begin{cases} 200, & 0 \le t \le 100, \\ 200 + 5(t - 100), & 100 < t \le 120, \\ 300, & 120 < t \le 200, \end{cases} \tag{45}$$

together with its frequency spectrum are plotted in Fig. 1. Note that the initial part of $y_r(t)$ is smoothed by a user-specified prefilter $G_f(z) = (z^{-1} + z^{-2})/(3 - z^{-1})$ for practical implementation. It is seen from Fig. 1 that the important frequency range is from 0 to 0.1 Hz. So Ψ in Algorithm 1 can be taken as that in Lemma 2 with $\omega_l = 0.6283$. By solving the LMI conditions in Theorem 10, the SF based ILC gains can be obtained as $L_1 = [-1.2725 \ -$

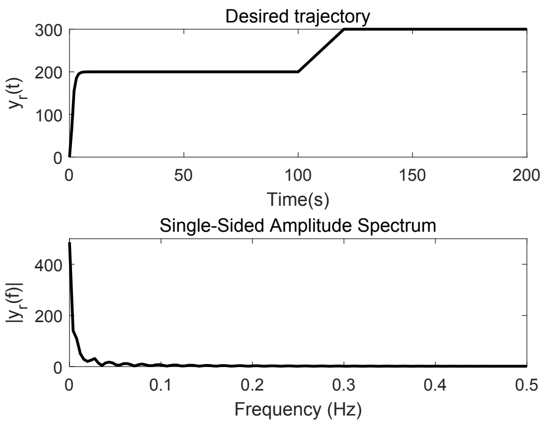

Fig. 1. Desired trajectory and its frequency spectrum

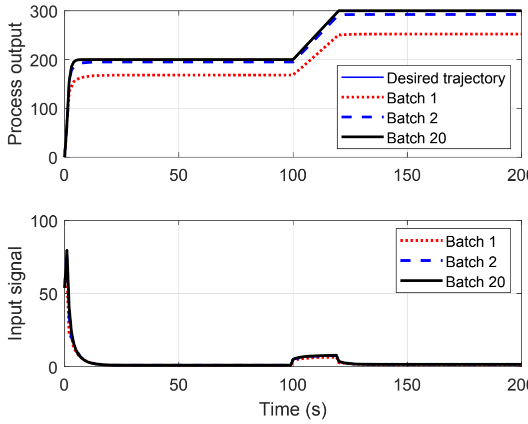

Fig. 2. Tracking results for the nominal case

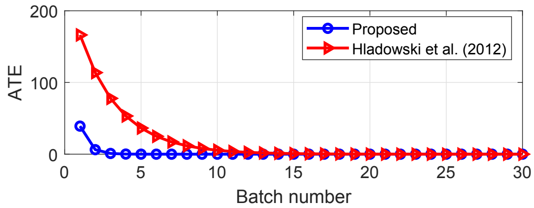

Fig. 3. ATE performance index for the nominal case

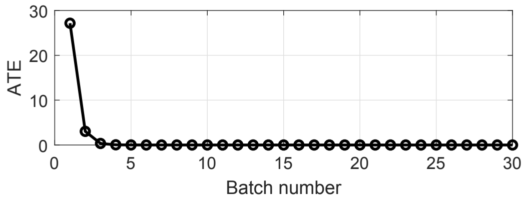

Fig. 4. ATE performance index for the uncertain case

0.7982] and $L_2 = -0.4184$. Moreover, parameters σ and μ in Algorithm 1 are taken as $\sigma = 10^{-4}$ and $\mu = 0.5$. By running the Algorithm 1 with the computed L_1 and L_2 as initial values, a feasible OF based ILC law can be computed as $K_1 = -0.9699$, $K_2 = -0.8195$. For the nominal case, the output feedback based ILC proposed in Hładowski et al. (2012) is also performed for comparison, where the learning gains are obtained as $K_1 = -0.9257$, $K_2 = 1.3335 \times 10^{-7}$ and $K_3 = 0.2946$ by solving the condition given therein. The tracking results are shown in Fig. 2, while the output tracking error in terms of the averaged tracking error $(\text{ATE}(k) = \sum_{t=1}^{T} |e_k(t)|/T)$ is plotted in Fig. 3. It is seen that perfect tracking is achieved by almost 3 cycles, while almost 15 cycles are needed to achieve the same performance by the ILC method given in Hładowski et al. (2012). For the presence of the above polytopic uncertainties, the corresponding ATE is recorded in Fig. 4. It is shown that the tracking performance is improved gradually from batch to batch by the proposed design. Note that the ILC method in Hładowski et al. (2012) cannot be applied any more under such uncertainties.

5. CONCLUSION

In this paper, a novel OF based ILC scheme has been proposed for batch processes with polytopic uncertainties. A two-stage heuristic approach is developed to iteratively compute the ILC gains based on a pre-designed state feedback based ILC gains. Robust stability conditions are established for the resulting closed-loop ILC system, along with an extended sufficient condition with respect to a finite number of matrix inequalities to facilitate the ILC design. An illustrative example of injection molding process has well demonstrated the effectiveness and advantage of the proposed design.

REFERENCES

Ahn, H.S., Chen, Y., and Moore, K.L. (2007). Iterative learning control: Brief survey and categorization. *IEEE Trans Syst. Man Cybern. Part C: Appl. Rev.*, 37(6), 1099–1121.

de Oliveira, M.C. and Skelton, R.E. (2001). Stability tests for constrained linear systems. *Perspectives in Robust Control*, 268, 241–257.

Gao, F., Yang, Y., and Shao, C. (2001). Robust iterative learning control with applications to injection molding process. *Chem. Eng. Sci.*, 56(24), 7025–7034.

Hao, S., Liu, T., and Gao, F. (2019). PI based indirect-type iterative learning control for batch processes with time-varying uncertainties: A 2D FM model based approach. *J. Process Control*, 78, 57–67.

Hao, S., Liu, T., Paszke, W., and Gałkowski, K. (2016). Robust iterative learning control for batch processes with input delay subject to time-varying uncertainties. *IET Control Theor. Appl.*, 10(15), 1904–1915.

Hładowski, L., Gałkowski, K., Cai, Z., Rogers, E., Freeman, C., and Lewin, P.L. (2012). Output information based iterative learning control law design with experimental verification. *ASME Journal of Dynamic Systems Measurement and Control*, 134(2), 021012.

Iwasaki, T. and Hara, S. (2005). Generalized KYP lemma: unified frequency domain inequalities with design applications. *IEEE Trans. Autom. Control*, 50(1), 41–59.

Li, X. and Gao, H. (2017). Robust frequency-domain constrained feedback design via a two-stage heuristic approach. *IEEE Trans. Cybern.*, 45(10), 2065–2075.

Nagy, Z.K. (2009). Model based robust control approach for batch crystallization product design. *Comput. Chem. Eng.*, 33(10), 1685–1691.

Paszke, W., Rogers, E., Gałkowski, K., and Cai, Z. (2013). Robust finite frequency range iterative learning control design and experimental verification. *Control Eng. Pract.*, 21(10), 1310–1320.

Rantzer, A. (1996). On the Kalman-Yakubovich-Popov lemma. *Syst. Control Lett.*, 28(1), 7–10.

Rogers, E., Gałkowski, K., and Owens, D.H. (2005). Control systems theory and applications for linear repetitive processes. *Advances in Control*, 349(5), 327–333.

Shi, J., Gao, F., and Wu, T. (2006). Robust iterative learning control design for batch processes with uncertain perturbations and initialization. *AIChE Journal*, 52(6), 2171–2187.

Tao, H., Paszke, W., Yang, H., and Gałkowski, K. (2019). Finite frequency range robust iterative learning control of linear discrete system with multiple time-delays. *J. Frankl. Inst.*, 356(5), 2690–2708.

Wang, Y., Gao, F., and Doyle III, F.J. (2009). Survey on iterative learning control, repetitive control, and run-to-run control. *J. Process Control*, 19(10), 1589–1600.