

# Adaptive strategies to platoon merging with vehicle engine uncertainty<sup>★</sup>

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**Abstract:** While several synchronization-based protocols have been provided for formation-keeping of cooperative vehicles, the problem of synchronized merging is more challenging. Challenges associated to the merging scenario include the need for establishing bidirectional interaction (in place of unidirectional look-ahead interaction), and the need for considering different engine dynamics (in place of homogeneous engine dynamics). This work shows how such challenges can be tackled via a newly proposed strategy based on adaptive control with bidirectional error: the adaptive control framework autonomously adapts to different engine dynamics, while the bidirectional error seamlessly allows the vehicle that wants to merge to interact with both the front and the rear vehicles, in a similar way as humans do.

*Keywords:* Autonomous vehicles, platoon merging maneuvers, cooperative adaptive cruise control, adaptive control.

## 1. INTRODUCTION

Cooperative automated driving is a recognized idea for improving road throughput, as it can group vehicles into platoons via Cooperative Adaptive Cruise Control (CACC). CACC uses the feedback from inter-vehicle communication and on-board sensors to control acceleration and braking (Günther et al. (2016)) and make the platoon behave in a synchronized way. The most typical synchronized behavior studied in CACC is the *formation keeping* task (Ploeg et al. (2014); Kianfar et al. (2015); Tao et al. (2019)). Recent surveys on CACC (Dey et al. (2016); Larsson et al. (2015)) show that a more challenging task is *synchronized merging*, crucial for forming or grouping platoons.

Currently, ad-hoc and sequential protocols are used to implement merging maneuvers. Such protocols, usually based on state machines, define the procedure for opening a gap between two vehicles: examples are Amoozadeh et al. (2015); Maiti et al. (2017) (vehicle entry and leaving); Scarinci et al. (2017) (creating gaps for on-ramp vehicles); Chien et al. (1995) (merge and split); Rai et al. (2015); Bengtsson et al. (2015); Baldi et al. (2018b) (lane changing, merging and overtaking). Two observations follow:

- *Engine:* the state machine does not define the CACC command (acceleration/braking) to bring the vehicle in each state. Defining such command is complex, especially with engine uncertainty. The limitation of

most CACC protocols is the imposed homogeneity of vehicle engines and inability to tackle uncertainties (Acciani et al. (2018); Harfouch et al. (2017a)).

- *Sequentiality:* Phases in state machines occur sequentially rather than synchronously (Taş et al. (2018)): a vehicle makes a gap; once the gap is open, the state machine goes into **Wait For Merge** state; the vehicle that opened the gap sends a **Safe To Merge** flag; when the merging vehicle indicates that it has finished merging the state changes to **Pace Making**. However, humans perform merging looking at each other, i.e. with bidirectional interaction. It was recently shown that handling CACC bidirectional interaction is difficult, because the input of a vehicle turns out to depend on the input of the neighbors (Baldi and Frasca (2018)): this creates algebraic loops that can make the input not well posed (Baldi et al. (2018a)).

These observations open the problem of embedding synchronized merging in a seamless way, despite the presence of engine uncertainty during the maneuvers.

### 1.1 Motivational scenario and contributions

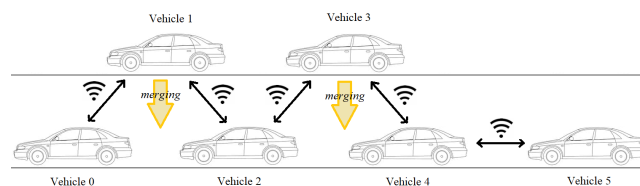


Fig. 1. The motivational merging scenario.

<sup>★</sup> This work was partly supported by the Fundamental Research Funds for the Central Universities grant no. 4007019109 (RECONSTRUCT), and the special guiding funds “double first-class” grant no. 4007019201 (corresponding author: S. Baldi)

We will focus on a lane reduction benchmark scenario in which two platoons formed in different lanes are required to merge (cf. Fig. 1). This benchmark scenario has become popular thanks to the activities of the Grand Cooperative Driving Challenge (GCDC 2011 and 2016), a competition aiming at testing in real-life implementation of communication-based automated driving (Ploeg et al. (2018); Englund et al. (2016)). All teams in the most recent GCDC 2016 adopted ad-hoc merging protocols that do not address synchronization, bidirectionality and uncertainty issues, cf. the designs published in (Taş et al. (2018); Aramrattana et al. (2018); Alonso et al. (2018); Dolk et al. (2018)). It is worth mentioning that in this work we will focus on the longitudinal dynamics only (gap creation and gap closing), as synchronization for lateral dynamics is to a great extent still an unsolved problem: note that team Halmstad, winner of GCDC 2016, had no support for lateral control of the vehicle (Aramrattana et al. (2018)).

To solve the problem of Fig. 1, we consider a bidirectional CACC in which each vehicle ‘looks’ at the vehicle in front and also at the vehicle in the back: by doing this, vehicles that merge from a different lane can interact with both the front and the rear vehicle during the maneuver. The main contribution of this work is tackling merging maneuvers with engine uncertainty, while analyzing scalability to long platoons in terms of string stability.

The rest of the paper is organized as follows: Sect. 2 gives the CACC structure, and Sect. 3 describes the merging maneuver. The simulations with the benchmark scenario are in Sect. 4, with conclusions in Sect. 5.

## 2. CACC SYSTEM STRUCTURE

To handle the scenario of Fig. 1 we must consider a platoon as in Fig. 2, where  $v_i$  and  $d_{i-1,i}$  represent the velocity (m/s) of vehicle  $i$ , and the spacing (m) between vehicle  $i$  and its preceding vehicle  $i-1$ . Because of bidirectional communication with preceding and succeeding vehicle, the goal of each vehicle in the platoon is to maintain a desired distance considering both the preceding and the succeeding vehicle. A bidirectional version of the constant time headway policy (Ploeg et al. (2014)) regulates the spacing between vehicles, implemented by defining the look-ahead desired spacing  $d_{des,f,i}$  and look-back desired spacing  $d_{des,b,i}$ :

$$\begin{aligned} d_{des,f,i}(t) &= r_i + hv_i(t) \\ d_{des,b,i}(t) &= r_i + hv_{i+1}(t), \quad i \in S_M \end{aligned}$$

where  $r_i$  is the standstill distance (m),  $h$  the time headway (s), and  $S_M = \{i \in \mathbb{N} | 1 \leq i \leq M\}$ , being  $M$  the number of vehicles and  $i=0$  reserved for the leading vehicle.

Due to the presence of bidirectionality, errors in both the look-ahead and look-back direction are considered, the look-ahead error being

$$\begin{aligned} e_{f,i}(t) &= d_{i-1,i}(t) - d_{des,f,i}(t) \\ &= (q_{i-1}(t) - q_i(t) - L_i) - (r_i + hv_i(t)) \end{aligned} \quad (1)$$

and the look-back error being

$$\begin{aligned} e_{b,i}(t) &= -(d_{i,i+1}(t) - d_{des,b,i}(t)) \\ &= -((q_i(t) - q_{i+1}(t) - L_{i+1}) - (r_i + hv_{i+1}(t))) \end{aligned} \quad (2)$$

with  $q_i$  and  $L_i$  representing vehicle  $i$ 's rear-bumper position (m) and length (m), and  $d_{i-1,i}$  and  $d_{i,i+1}$  representing

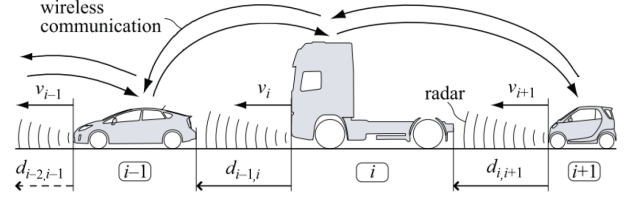


Fig. 2. CACC platoon with bidirectional communication (edited from Naus et al. (2010)).

the intervehicle distances. Finally, the total spacing error is taken as the convex combination of  $e_{f,i}$  and  $e_{b,i}$

$$e_i(t) = c_1 e_{f,i}(t) + c_2 e_{b,i}(t), \quad 1 \leq i < M \quad (3)$$

with  $c_1 \in (0, 1]$  and  $c_2 = 1 - c_1$ . To explain the meaning of the convex combination, let us consider two cases: for  $c_1 = 1$  and  $c_2 = 0$  one gets the standard CACC unidirectional situation in which only the look-ahead spacing error is considered; for  $c_1 = c_2 = 0.5$  one gets a bidirectional situation in which look-ahead and look-back errors are equally weighted. As the leading and the last vehicle can only measure look-back and look-ahead error respectively, their error is simply

$$\begin{aligned} e_0(t) &= e_{b,0}(t) = q_1(t) - q_0(t) + L_1 + r + hv_1(t) \\ e_M(t) &= e_{f,M}(t) = q_{M-1}(t) - q_M(t) - L_M - r - hv_M(t). \end{aligned}$$

The control objective is to regulate  $e_i$  to zero  $\forall i \in S_M \cup \{0\}$ , while ensuring string stability of the platoon. The following model is standard (Ploeg et al. (2014)) to represent the vehicles in the platoon

$$\begin{pmatrix} \dot{d}_{i-1,i} \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i \\ a_i \\ -\frac{1}{\tau_i} a_i + \frac{1}{\tau_i} u_i \end{pmatrix}, \quad i \in S_M \cup \{0\} \quad (4)$$

with  $a_i$  and  $u_i$  being the acceleration ( $\text{m/s}^2$ ) and input ( $\text{m/s}^2$ ), and  $\tau_i$  (s) being the engine time constant of the  $i^{\text{th}}$  vehicle. It is worth mentioning that in some approaches, second-order models are considered, e.g. (Paoletti and Innocenti (2015)): in such second-order models, vehicles mass (or inertia) will appear, whereas in third-order models as the one above, the effect of vehicle mass (or inertia) is captured by the engine time constant.

### 2.1 The CACC control structure

The control action can be designed by formulating the error dynamics. Define the error states as

$$\begin{pmatrix} e_{1,i} \\ e_{2,i} \\ e_{3,i} \end{pmatrix} = \begin{pmatrix} e_i \\ \dot{e}_i \\ \ddot{e}_i \end{pmatrix}, \quad 0 \leq i \leq M. \quad (5)$$

State-of-the-art CACC protocols design the control action assuming identical  $\tau_i$  (baseline homogeneous condition) (Ploeg et al. (2014)), so that the baseline control input (indicated with the subscript  $bl$ ) can be derived from the dynamics of  $e_{3,i}$ , via (3) and (4)

$$\begin{aligned} \dot{e}_{3,i} &= -\frac{1}{\tau_i} e_{3,i} - \frac{1}{\tau_i} p_i \\ &\quad + \frac{c_1}{\tau_i} u_{i-1,bl} + \frac{c_2}{\tau_i} u_{i+1,bl} + \frac{hc_2}{\tau_i} \dot{u}_{i+1,bl} \end{aligned} \quad (6)$$

with  $p_i = u_{i,bl} + hc_1 \dot{u}_{i,bl}$ . From (6) it is clear that  $p_i$  should stabilize the error dynamics (5) while compensating for

the terms  $u_{i-1,bl}$ ,  $u_{i+1,bl}$  and  $\dot{u}_{i+1,bl}$ . Hence, the controller dynamics is given by

$$\begin{aligned} \dot{u}_{i,bl} = & -\frac{1}{hc_1}u_{i,bl} + \frac{1}{hc_1}(k_p e_{1,i} + k_d e_{2,i} + k_{dd} e_{3,i}) \\ & + \frac{1}{h}u_{i-1,bl} + \frac{c_2}{hc_1}u_{i+1,bl} + \frac{c_2}{c_1}\dot{u}_{i+1,bl} \end{aligned} \quad (7)$$

with  $k_p$ ,  $k_d$  and  $k_{dd}$  being gains to be designed in order to have stability/string stability specifications. The feedforward terms  $u_{i-1,bl}$ ,  $u_{i+1,bl}$  and  $\dot{u}_{i+1,bl}$  can be obtained via wireless communication with the preceding and succeeding vehicle. It is well known in literature that  $k_{dd}$  can be set to be zero to avoid feedback from the relative acceleration, which is very difficult to get in practice (Ploeg et al. (2014)). This results in

$$\begin{aligned} \begin{pmatrix} \dot{e}_{1,i} \\ \dot{e}_{2,i} \\ \dot{e}_{3,i} \\ \dot{u}_{i,bl} \end{pmatrix} = & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{k_p}{\tau_i} & -\frac{k_d}{\tau_i} & -\frac{1}{\tau_i} & 0 \\ \frac{k_p}{hc_1} & \frac{k_d}{hc_1} & 0 & -\frac{1}{hc_1} \end{pmatrix} \begin{pmatrix} e_{1,i} \\ e_{2,i} \\ e_{3,i} \\ u_{i,bl} \end{pmatrix} \\ & + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{h} & \frac{c_2}{hc_1} & \frac{c_2}{c_1} \end{pmatrix} \begin{pmatrix} u_{i-1,bl} \\ u_{i+1,bl} \\ \dot{u}_{i+1,bl} \end{pmatrix} \quad \forall i \in S_M \setminus \{M\}. \end{aligned} \quad (8)$$

If the errors are written in terms of velocity and acceleration, (8) can be equivalently written,  $\forall i \in S_M \setminus \{M\}$ , as

$$\begin{aligned} \begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \\ \dot{u}_{i,bl} \end{pmatrix} = & \begin{pmatrix} 0 & -1 & -hc_1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_i} & \frac{1}{\tau_i} \\ \frac{k_p}{hc_1} & -\frac{k_d}{hc_1} & -k_d & -\frac{1}{hc_1} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \\ u_{i,bl} \end{pmatrix} \\ & + \begin{pmatrix} c_1 & c_2 & hc_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_d}{h} & \frac{k_d c_2}{hc_1} & \frac{k_d c_2}{c_1} & \frac{1}{h} & \frac{c_2}{hc_1} & \frac{c_2}{c_1} \end{pmatrix} \begin{pmatrix} v_{i-1} \\ v_{i+1} \\ a_{i+1} \\ u_{i-1,bl} \\ u_{i+1,bl} \\ \dot{u}_{i+1,bl} \end{pmatrix} \end{aligned} \quad (9)$$

which represents the dynamics of a vehicle equipped with baseline CACC protocol. Notice that (8) (or (9)) are valid for  $i \in S_M \setminus \{M\}$ , i.e. only for those vehicles with both a front and a rear vehicle. The leading vehicles and the last vehicle obey slightly different dynamics.

*Remark 1.* Given the interconnected dynamics (8) (or (9)), a complete analysis of bidirectional string stability for the resulting platoon can be obtained along similar lines as Baldi et al. (2020). String stability refers to the capability of a platoon to avoid amplification of disturbances along the platoon itself: this is particularly crucial in a merging scenario as in Fig. 1, where disturbances arise from the gap creation phase.

### 3. THE MERGING MANUEVER

Having defined string stability for a bidirectional homogeneous platoon, let us see how to handle heterogeneity in  $\tau_i$ . Removing the homogeneous assumption implies considering that  $\forall i \in S_M$ ,  $\tau_i$  can be represented as the sum of two terms

$$\tau_i = \tau_0 + \Delta\tau_i \quad (10)$$

where  $\Delta\tau_i$  is a perturbation with respect to  $\tau_0$ . Two approaches can be used to handle  $\Delta\tau_i$ , i.e. treating  $\Delta\tau_i$  as known (robust control approach (Zegers et al. (2018); Gao et al. (2017))) or treating it as unknown (adaptive control approach (Harfouch et al. (2017b); Guo et al. (2018b,a))). With the intent of pursuing an adaptive approach, the model of a heterogeneous vehicle is obtained using (10) in the third equation of (4)

$$\dot{a}_i = -\frac{1}{\tau_0}a_i + \frac{1}{\tau_0}[u_i + \Omega_i^* \phi_i], \quad \forall i \in S_M \quad (11)$$

where  $\Omega_i^* = -\frac{\Delta\tau_i}{\tau_i}$  is an unknown scalar, and  $\phi_i = (u_i - a_i)$  is the known scalar regressor. What distinguishes adaptive approaches from other approaches (e.g. robust approaches) is that the ideal  $\Omega_i^*$  is unknown and estimated through an additional differential equation in the control law. In the following, the main steps to derive such adaptation mechanism are provided. Using (11) in (4), we get

$$\begin{aligned} \begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = & \begin{pmatrix} 0 & -1 & -hc_1 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_0} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} c_1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} \\ & + \begin{pmatrix} c_2 & hc_2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_{i+1} \\ a_{i+1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \end{pmatrix} [u_i + \Omega_i^* \phi_i]. \end{aligned} \quad (12)$$

resulting in the control input  $u_i = u_{i,bl} + u_{i,ad}$ , with

$$hc_1 \dot{u}_{i,bl} = -u_{i,bl} + \xi_{i,bl}, \quad \forall i \in S_M \cup \{0\} \quad (13)$$

$$\xi_{i,bl} = \begin{cases} c_1 u_r + k_p e_0 + k_d \dot{e}_0 & i = 0. \\ +c_2 u_{1,bl} + hc_2 \dot{u}_{1,bl} \\ k_p e_i + k_d \dot{e}_i + c_1 u_{i-1,bl} & i \in S_M \setminus \{M\} \\ +c_2 u_{i+1,bl} + hc_2 \dot{u}_{i+1,bl} \\ k_p e_M + k_d \dot{e}_M + u_{M-1,bl} & i = M. \end{cases}$$

and  $u_{i,ad}$  is the adaptive augmentation controller to handle uncertainty

$$u_{i,ad} = -\hat{\Omega}_i \phi_i \quad \dot{\hat{\Omega}}_i = \Gamma_\Omega \phi_i \tilde{x}_i P_m B_u \quad (14)$$

which is similar to the unidirectional case (cf. details in Harfouch et al. (2017b)). In (14),  $\Gamma_\Omega$  is an adaptation gain,  $\hat{\Omega}_i$  is the estimate of  $\Omega_i^*$ ,  $P_m$  results from an appropriately defined Lyapunov equation,  $B_u$  results from an appropriately defined reference model, and  $\tilde{x}_i$  denotes the deviation of the vehicle's state from the state of the reference model: all these choices are detailed in (Harfouch et al. (2017b)).

*Remark 2.* The importance of (14) is in handling engine uncertainty by adapting to unknown parameters as in (10).

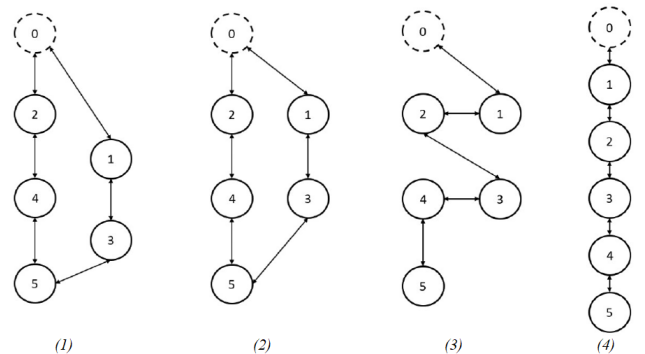


Fig. 3. The phases of the merging maneuver

### 3.1 The merging phases

The merging maneuver is described along 4 phases, depicted in Fig. 3:

- *Phase 1* – The vehicles belong to two distinct platoons. The first platoon has vehicles 0, 2, 4 and 5, while the second platoon has vehicles 1 and 3. Vehicle 1 wants to merge between vehicles 0 and 2, whereas vehicle 3 wants to merge between vehicles 2 and 4. Note that vehicle 5 is not necessary to the purpose of the merging, but it is introduced to show that the proposed framework can handle bidirectional communication with platoons of arbitrary length.
- *Phase 2* – This is the alignment phase, in which vehicle 1 aligns with vehicle 2 and vehicle 3 aligns with vehicle 4. It is worth noting that, due to bidirectional communication, vehicle 1 can interact with the preceding vehicle 0 in such a way to align with vehicle 2, whereas vehicle 3 can interact with the succeeding vehicle 5 in such a way to align with vehicle 4. This is just one of many possible choices, and it shows that the proposed framework is flexible to different communication links. For our choice, for both vehicle 2 and 1, vehicle 0 becomes the predecessor and for both vehicle 4 and 3, vehicle 5 becomes the successor.
- *Phase 3* – This is the phase of rearrangement of the communication links in the platoon. The links are selected taking into account the positioning of the vehicles at the end of the maneuver: vehicle 0 becomes predecessor for vehicle 1, which becomes a predecessor for vehicle 2, and so on (from vehicle 0 till vehicle 5). The error definitions for each vehicle are bidirectional

$$\begin{aligned}
 e_1 &= c_1 e_{f,1} + c_2 (q_2 - q_1) \\
 e_2 &= c_1 (q_1 - q_2) + c_2 e_{b,2} \\
 e_3 &= c_1 e_{f,3} + c_2 (q_4 - q_3) \\
 e_4 &= c_1 (q_3 - q_4) + c_2 e_{b,4}
 \end{aligned} \tag{15}$$

where such definitions take into account the zero gaps, i.e. the successor vehicle 2 should be in the same position as vehicle 1, the predecessor vehicle 1 should be in the same position as vehicle 2, and so on.

- *Phase 4* – This is the phase where the gaps are created and the steering controller (not shown in this work) closes the gap in the lateral direction. Hence the errors go from (15) to

$$\begin{aligned}
 e_1 &= c_1 e_{f,1} + c_2 [(q_2 - q_1 + L) + (r + hv_2)] \\
 e_2 &= c_1 [(q_1 - q_2 - L) - (r + hv_2)] + c_2 e_{b,2} \\
 e_3 &= c_1 e_{f,3} + c_2 [(q_4 - q_3 + L) + (r + hv_2)] \\
 e_4 &= c_1 [(q_3 - q_4 - L) - (r + hv_2)] + c_2 e_{b,4}
 \end{aligned} \tag{16}$$

where  $r$  is increased to create the gap.

Due to space limitations, we cannot provide the stability analysis for the proposed protocol. Nevertheless, the interested reader can verify that the stability analysis is similar to the unidirectional case studied by some of the authors in (Harfouch et al. (2017b)), while the presence of cycles in the communication requires some analysis with respect to well-posedness of the input, as studied by some of the authors in (Baldi and Frasca (2018)). It is important to notice that different from (Baldi et al. (2018b)), string stability of the platoon is addressed here, since in a merging scenario disturbances arise from the creation of new errors (i.e. new communication links) at

Table 1. Vehicles parameters and initial conditions.

	$\tau_i$	$x_i(0)$
Vehicle 0	0.6	[15,0,0]
Vehicle 1	0.5	[9,0,0]
Vehicle 2	0.7	[10,0,0]
Vehicle 3	0.45	[2,0,0]
Vehicle 4	0.7	[5,0,0]
Vehicle 5	0.8	[0,0,0]

the beginning of phases 1 and 3, whose effect should not be amplified along the platoon itself: this string stability capability will be further shown in the simulations of the next section.

## 4. SIMULATIONS

Consider two platoons as in Fig. 3 that should merge into one platoon with six vehicles. Table 1 shows the different engine constants  $\tau_i$  together with the initial states of the vehicles (the engine constants are used only for simulation and are *unknown for control design*). The reference signal given to the leading vehicle is an increasing acceleration till acceleration limit is reached. The gains used for the simulations are  $k_p = 0.2$ ,  $k_d = 0.7$ ,  $c_1 = c_2 = 0.5$ . The maneuver is organized along the phases 1-4 of Fig. 3:

- 0 s (phase 1): this is the set up phase in which vehicles 0, 2, 4 and 5 achieve the initial formation.
- 0-20 s (phase 2): vehicle 1 aligns with vehicle 2 and vehicle 3 aligns with vehicle 4, while vehicles 0, 2, 4 and 5 keep the formation.
- 20-50s (phase 3): vehicle 2 increases its gap with vehicle 0 and simultaneously increases its gap with vehicle 1; also, vehicle 4 increases its gap with vehicle 2 and vehicle 3 simultaneously.
- 50-60s (phase 4): the final formation for the platoon is achieved and kept.

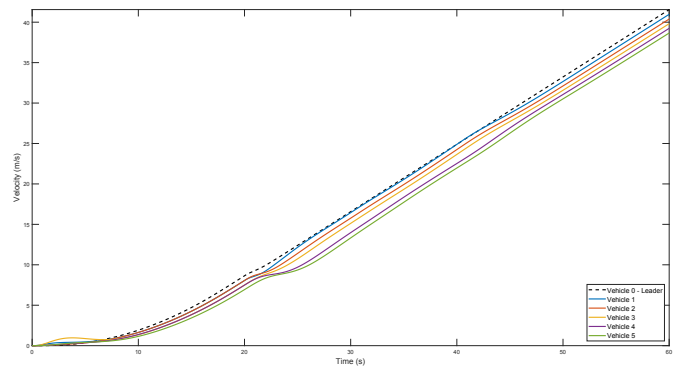


Fig. 4. Merging: velocity response. The maneuver is performed at increasing velocity to make it more challenging.

Figs. 4 and 5 show the response of velocities and accelerations during the merging. Fig. 6 shows the relative distances of the vehicles from vehicle 0: initially there are two platoons (phase 1), then vehicles 1 and 2 align with each other, maintaining a distance from vehicle 0 (phase 2). During this time, the velocity of vehicle 1 is greater than the velocity of vehicles 2 and 0 because it aligns with vehicle 2 which was initially ahead. Similarly, vehicles



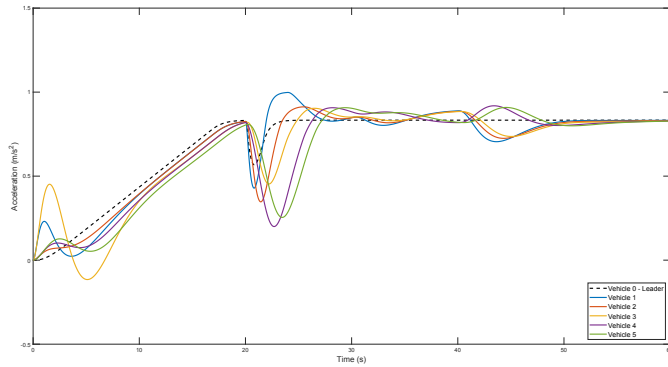


Fig. 5. Merging: acceleration response. Accelerations are adjusted smoothly during all phases.

3 and 4 are maintaining a distance from vehicle 2, and vehicle 3 aligns itself to vehicle 4 and thus, has the highest velocity at that time. Then, at 20 seconds rearrangement of communication takes place (phase 3), after which in the interval 20-50 vehicle 2 creates a gap with vehicle 1 extending its gap with vehicle 0, in order to let vehicle 1 merge in between vehicle 0 and vehicle 2. Similarly, vehicle 4 creates a gap with vehicle 3 and extends its gap with vehicle 2 (phase 4), in order to let vehicle 3 merge in.

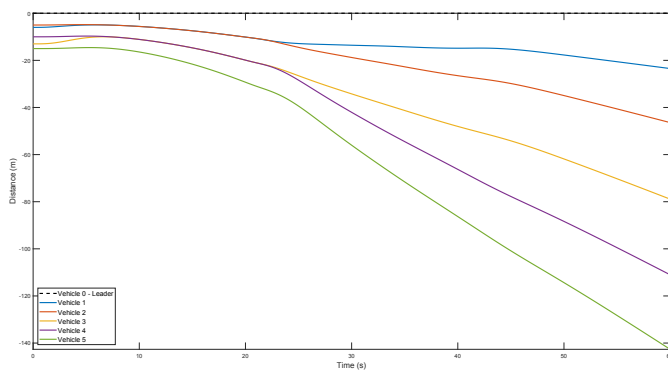


Fig. 6. Merging: relative distances with respect to leading vehicle. Note the alignment followed by gap creation.

*Remark 3. The interest of studying bidirectional interaction is also to tackle engine limits. Bidirectional interaction can allow the leading vehicle to brake if a following vehicle cannot reach its acceleration, so that merging can be performed: in this scenario, braking would not be possible with unidirectional look-ahead interaction.*

*Remark 4. The proposed approach can be applied not only to merging, but also to splitting, which is the dual maneuver. Most derivations follow accordingly and are not shown for space limitations.*

## 5. CONCLUSIONS

The problem of synchronized merging of cooperative vehicles is more challenging than the one of formation-keeping, due to some challenges associated to the merging scenario, such as the need to establish bidirectional interaction, and the need to consider merging among vehicles with

different engine dynamics. In this work we have shown how such challenges can be tackled via a newly proposed adaptive strategy (in the sense of adaptive control) with bidirectional error. Simulations have been presented to show the effectiveness of the proposed strategy, based on a benchmark scenario in which two platoons formed in different lanes are required to merge in a unique lane.

An interesting future work is the study of the impact of merging on traffic conditions. This high-level task possibly requires to embed the proposed merging framework in a traffic simulator (e.g. SUMO, AIMSUN), and they open the problem of mathematically analyze stability/robustness to uncertainty.

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