A Modular Feedback Approach
for Distributed Control [⋆]

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Abstract: This work presents a method based on linear matrix inequalities (LMIs) to design a feedback controller that guarantees stability regardless of the network topology considered. The proposed controller has a modular structure, that is, it is composed of blocks associated with communication links in a control network, obtaining the feedback of each topology by inserting zeros in the blocks related to disabled links. Conditions to obtain the desired modular structure are provided, as well as a numerical example to assess the feasibility of the design method.

Keywords: Coalitional control, clustering, modular feedback control, Lyapunov function, linear matrix inequalities (LMIs)

1. INTRODUCTION

The key idea of distributed control is to work with a group of independent subsystems, whose actions are co-ordinated by controllers that may exchange information to increase the performance of the global system. This way, distributed approaches are a valuable tool to deal with control problems that may not be easily solved in a centralized manner, e.g., in large-scale systems, among others (Fele et al., 2017; Bakule, 2008).

In recent years, several works have studied the design of distributed controllers that fulfill constraints associated with the *network topology*, i.e., the configuration of the control network depending on the enabled/disabled communication links. For instance, Lian et al. (2017) propose a gradient-based method for defining sparsity constraints in a centralized and decentralized manner by applying game theory principles. In (Lin et al., 2013), communication constraints are implemented by sparsity-promoting penalty functions on the number of links involved in the control scheme. In (Lidström and Rantzer, 2016), a control law based on a H_∞ approach is applied to linear time-invariant (LTI) systems represented by symmetric and Hurwitz state matrices. In (Tanaka and Langbort, 2011), a version of the bounded real lemma for internally positive systems jointly with H_∞ techniques are used to design structured feedback controllers. Also, in (Rotkowitz and Lall, 2002), the concept of quadratic invariance (QI) is applied to a convex problem where a set of sparse constraints to design a feedback controller are defined. Likewise, a similar approach is used in (Furieri and Kamgarpour, 2018).

Another approach to deal with topologies and sparsity constraints is the use of linear matrix inequalities (LMIs) for the design of linear feedback controllers as in *coali-*

tional control (Maestre, 2010; Maestre et al., 2014). These schemes commute between different network topologies to cluster controllers in cooperating sets referred to as *coalitions*, thereby the feedback controllers need to be computed for each topology. Another LMI method to design stable column sparse feedback controllers is proposed in (Polyak et al., 2013). Also, Blanchini et al. (2013, 2015) study a set of decoupled subsystems interconnected through a block-structured feedback matrix, whose stability is guaranteed via LMIs.

This work proposes an LMI-based common feedback controller for a set of topologies, which is composed of *blocks* associated with the different communication links in the network. Therefore, when a link is disabled, the corresponding block is replaced by zeros, while the rest of the elements remain unaltered. Moreover, the design method is characterized by the existence of a common Lyapunov function for all network topologies, guaranteeing the stability of the closed-loop system independently of any link switching sequence and with no need of a supervisory layer to select the switchings. While this choice is a well-known source of conservativeness, it is a necessary condition to obtain a controller with the desired *modular* structure.

The proposed controller might be of interest in applications where the information structure plays an important role (Maestre et al., 2014; Fele et al., 2017; Marzband et al., 2017). Likewise, this approach can be combined with other LMI-based coalitional methods described in (Muros et al., 2017a,b), which are focused on cooperative game theory. Furthermore, other potential applications are plug & play control methods (Stoustrup, 2009; Bendtsen et al., 2013; Riverso et al., 2013), as well as applications where communications issues, such as information losses and jamming attacks, occur (Cetinkaya et al., 2017; Azimi-Sadjadi, 2003; Yu et al., 2004). In this context, the missing information from a neighbor can be modeled as a disabled link, and the modular feedback controller presented here can be applied. Similarly, the results of this paper can be

extended to a model predictive control (MPC) framework, especially to works related to coupling dependent clustering, e.g., (Zheng et al., 2018; Jain et al., 2018; Fele et al., 2018; Barreiro-Gomez et al., 2019).

The remaining of the paper is structured as follows. Section 2 presents the problem setting. Section 3 introduces the concept of modular feedback. Section 4 provides an LMI-based design method for the controller together with some properties of interest. Section 5 illustrates the proposed method through an academic example. Finally, concluding remarks are given in Section 6.

2. PROBLEM SETTING

We consider a discrete linear time-invariant system, which is composed of a set of subsystems $\mathcal{N} = \{1, 2, \ldots, N\}$, being each agent $i \in \mathcal{N}$ characterized as

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + d_i(k),$$
$$d_i(k) = \sum_{j \neq i} A_{ij}x_j(k) + \sum_{j \neq i} B_{ij}u_j(k), \quad (1)$$

where $x_i \in \mathbb{R}^{q_i}$ and $u_i \in \mathbb{R}^{r_i}$ are respectively the states and inputs of each subsystem, with $A_{ii} \in \mathbb{R}^{q_i \times q_i}$ and $B_{ii} \in \mathbb{R}^{q_i \times r_i}$ being the state and input-to-state matrices, and where $d_i \in \mathbb{R}^{q_i}$ is the influence of the neighbors' states and inputs in the update of x_i. Notice that the global system model does not include external disturbances. Likewise, matrices $A_{ij} \in \mathbb{R}^{q_i \times q_j}$ and $B_{ij} \in \mathbb{R}^{r_i \times r_j}$ map respectively the states and inputs of subsystem $j \in \mathcal{N}$ into the state of subsystem i. Finally, $\mathcal{N}_i$ denotes the set of neighbors of agent i, i.e., $\mathcal{N}_i = \{j \in \mathcal{N} : A_{ij} \neq 0 \vee B_{ij} \neq 0\}$.

The goal of the different subsystems in $\mathcal{N}$ is to minimize their stage costs, which are defined as

$$\ell_i(k) = x_i^{\mathrm{T}}(k)Q_i x_i(k) + u_i^{\mathrm{T}}(k)R_i u_i(k), \quad (2)$$

where $Q_i \in \mathbb{R}^{q_i \times q_i}$ and $R_i \in \mathbb{R}^{r_i \times r_i}$ are respectively positive semi-definite and definite constant weighting matrices. From a global viewpoint, the overall dynamics become

$$x_{\mathcal{N}}(k+1) = A_{\mathcal{N}}x_{\mathcal{N}}(k) + B_{\mathcal{N}}u_{\mathcal{N}}(k), \quad (3)$$

where subscript $\mathcal{N}$ emphasizes that all system vectors and matrices come from the aggregation of local subsystems, i.e., $x_{\mathcal{N}} = [x_i]_{i \in \mathcal{N} \in R^q}$, $u_{\mathcal{N}} = [u_i]_{i \in \mathcal{N} \in R^r}$, $A_{\mathcal{N}} = [A_{ij}]_{i,j \in \mathcal{N}}$, and $B_{\mathcal{N}} = [B_{ij}]_{i,j \in \mathcal{N}}$, with $q = \sum_{i \in \mathcal{N}} q_i$ and $r = \sum_{i \in \mathcal{N}} r_i$. Notice that mutual interactions are implicitly considered in (3). Likewise, the stage cost of the overall system can be expressed as the aggregation of that of the corresponding subsystems, i.e.,

$$\ell_{\mathcal{N}}(k) = x_{\mathcal{N}}^{\mathrm{T}}(k)Q_{\mathcal{N}}x_{\mathcal{N}}(k) + u_{\mathcal{N}}^{\mathrm{T}}(k)R_{\mathcal{N}}u_{\mathcal{N}}(k), \quad (4)$$

where $Q_{\mathcal{N}} = \mathrm{diag}(Q_i)_{i \in \mathcal{N}}$ and $R_{\mathcal{N}} = \mathrm{diag}(R_i)_{i \in \mathcal{N}}$.

2.1 Communication network and constraints

The control strategy is based on linear feedbacks. In this context, the controller shall be designed considering the information flow constraints imposed by the network that connects the different subsystems. Consequently, let us assume a communication network represented by a graph $(\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is a set of subsystems and $\mathcal{L}$ is a set
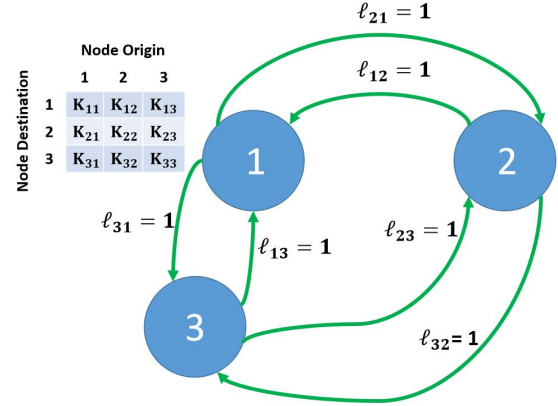

Fig. 1. Nondiagonal entries of a modular feedback controller for a control system composed of three agents connected by six enabled links (in green).

of unidirectional links given by $\mathcal{L} \subseteq \mathcal{L}^{\mathcal{N}} = \{\{i,j\} | \{i,j\} \subseteq \mathcal{N}, i \neq j\}$. For convenience, we will define link $\{i,j\}$ as an edge that goes from j to i, with $i, j \in \mathcal{N}$, to stress that i receives information from j. Note that the status of a link $\{i,j\}$ is denoted as l_{ij}. In this way, it is said that a link is activated/enabled when $l_{ij} = 1$, and deactivated/disabled when $l_{ij} = 0$.

Let us define network topology Λ as the set of enabled links in time step k, and $\mathcal{T}$ as the set of possible topologies, i.e., $\mathcal{T} = \{\Lambda_{\mathrm{DC}}, \Lambda_1, \ldots, \Lambda_{\mathcal{L}}\}$. For convenience, we respectively denote by Λ_{DC} and $\Lambda_{\mathcal{L}}$ the decentralized topology (all links are disabled) and the full communication topology (all links are enabled, i.e., the grand coalition). Notice that the cardinality of Λ is $|\Lambda| = \sum_{\{i,j\} \in \Lambda} l_{ij}$. In the same way, the cardinality of the topologies set is $|\mathcal{T}| = 2^{|\mathcal{L}|}$.

Since the feedback controller design needs to take into account the information flow constraints imposed by network topology Λ, a subscript is added to stress this fact, so that the control law is defined as

$$u_{\mathcal{N}} = K_{\Lambda}x_{\mathcal{N}}. \quad (5)$$

3. MODULAR FEEDBACK

This work proposes a controller composed of *blocks*, where each block corresponds to a link of the communication network and is *common* for every topology where the link is enabled. To illustrate this idea, Fig. 1 shows a system composed of three agents that communicate by means of six directed links, which are associated with the nondiagonal elements of the corresponding modular feedback controller (presented in the top-left side of the figure).

The controller for topology $\Lambda_{\mathcal{L}}$, i.e., when all links are activated (see Fig. 1), is given by

$$K_{\Lambda_{\mathcal{L}}} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}.$$

However, if the topology changes for some reason and becomes that of Fig. 2, say Λ_{ex}, where some links have been deactivated, the control law becomes
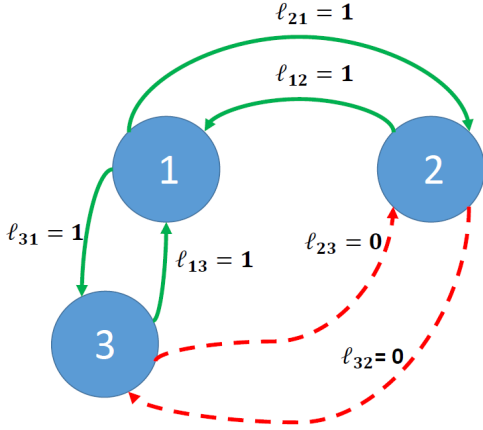
Fig. 2. Example topology Λ_{ex} where two links of Fig. 1 have been disconnected (dashed red links).

$$K_{\Lambda_{\text{ex}}} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & 0 \\ K_{31} & 0 & K_{33} \end{bmatrix}.$$

Note that all nonzero matrix elements have the *same value* that they had in $K_{\Lambda_{\mathcal{L}}}$. To this end, the variables corresponding to activated links are forced to have the same value for all $\Lambda \in \mathcal{T}$, as detailed in Section 4. Thus, modular feedback controller $K_{\Lambda_{\mathcal{L}}}$ provides us with a *family of control laws* that can be adapted depending on the topology by simply making zero the elements that correspond to the disabled links.

3.1 Stability

It is important to guarantee the closed-loop stability when the network topology changes, which implies switchings of control law K_Λ, for all $\Lambda \in \mathcal{T}$. In this sense, a common Lyapunov function $f(x_{\mathcal{N}}(k)) = x_{\mathcal{N}}^{\text{T}}(k) P x_{\mathcal{N}}(k)$ shall be defined for all feedback controllers, being $P \in \mathbb{R}^{q \times q}$ a positive definite matrix that provides a bound on cost-to-go $\ell_{\mathcal{N}}(k)$ of the closed-loop system, i.e.,

$$x_{\mathcal{N}}^{\text{T}}(k) P x_{\mathcal{N}}(k) \geq \sum_{n=k}^{\infty} \ell_{\mathcal{N}}(x_{\mathcal{N}}(n)). \quad (6)$$

In particular, the decrease in the value of the Lyapunov function and the bound on the cost-to-go of the closed-loop system are properties that hold if

$$x_{\mathcal{N}}^{\text{T}}(k) P x_{\mathcal{N}}(k) \geq \ell_{\mathcal{N}}(k) + x_{\mathcal{N}}^{\text{T}}(k+1) P x_{\mathcal{N}}(k+1). \quad (7)$$

4. MODULAR CONTROLLER DESIGN

In this section, the design of the proposed modular controller is introduced by means of the following theorem:

Theorem 1. Let us assume a system defined by discrete-time linear matrices $A_{\mathcal{N}}$ and $B_{\mathcal{N}}$, and with a stage cost defined by $Q_{\mathcal{N}}$ and $R_{\mathcal{N}}$. Let us consider a graph described by a set $\mathcal{N}$ of subsystems connected by means of a set $\mathcal{L}$ of communication links that leads us to a set of $\mathcal{T}$ topologies. If there exist matrices $W = W^{\text{T}} = \text{diag}(W_i)_{i \in \mathcal{N}}$, where $W_i \in \mathbb{R}^{q_i \times q_i}$, and $Y_\Lambda = [Y_{ij}]_{i,j \in \mathcal{N}} \in \mathbb{R}^{r \times q}$, with $Y_{ij} \in \mathbb{R}^{r_i \times q_j}$, such that the following LMI,

$$\begin{bmatrix} W & WA_{\mathcal{N}}^{\text{T}} + Y_\Lambda^{\text{T}} B_{\mathcal{N}}^{\text{T}} & WQ_{\mathcal{N}}^{1/2} & Y_\Lambda^{\text{T}} R_{\mathcal{N}}^{1/2} \\ A_{\mathcal{N}} W + B_{\mathcal{N}} Y_\Lambda & W & 0 & 0 \\ Q_{\mathcal{N}}^{1/2} W & 0 & I & 0 \\ R_{\mathcal{N}}^{1/2} Y_\Lambda & 0 & 0 & I \end{bmatrix} > 0, \quad (8)$$

is satisfied for all $\Lambda \in \mathcal{T}$, with $Y_{\Lambda,ij} = Y_{ij}$ if $l_{ij} = 1$ and $Y_{\Lambda,ij} = 0$ otherwise, then there exists a modular controller that provides a set of feedback control laws $K_\Lambda = Y_\Lambda W^{-1}$, which stabilize the closed-loop system despite topology switchings. Also, a common Lyapunov function $f(x_{\mathcal{N}}(k)) = x_{\mathcal{N}}^{\text{T}}(k) P x_{\mathcal{N}}(k)$ that provides a bound on the cost-to-go is obtained with $P = W^{-1}$.

Proof: The application in an iterative way of the Schur's complement (Zhang, 2005) in a backward manner in combination with the proposed variable changes allow us to derive LMI (8) from (7), which guarantees the stability of the system and the decrease of the cost-to-go at each time step. Then, a telescope summation of this inequality from $t = k$ to infinity leads us to (6). Hence, the Lyapunov function can be used to get a bound on the cost-to-go. The set of LMIs, where Y_Λ and W are the variables, guarantees that K_Λ satisfies the communication constraints imposed by topology Λ, which require $K_{\Lambda,ij} = 0$ if $l_{ij} = 0$. To prove this fact, let us recall that $K_\Lambda = Y_\Lambda W^{-1}$. As W is a block diagonal matrix, $W_{ij}^{-1} = 0$ for $i \neq j$. Also, $Y_{\Lambda,ij} = 0$ if $l_{ij} = 0$. Hence, by the properties of matrix product multiplication, it holds $K_{\Lambda,ij} = 0$ if link $l_{ij} = 0$. Moreover, as $Y_{\Lambda,ij}$ always has the same value for all the possible topologies $\Lambda \in \mathcal{T}$ when $l_{ij} = 1$, otherwise being zero, this forces element $K_{\Lambda,ij}$ to be the same for all topologies $\Lambda \in \mathcal{T}$, hence providing the desired modular structure. Finally, since $P = W^{-1}$ is the same for every K_Λ and following (Maestre et al., 2014), it can be guaranteed that the closed-loop stability is not compromised due to topology switchings. $\blacksquare$

Remark 1. As we stated before, W needs to be a block diagonal matrix to preserve the modular structure of the controller. This can be easily seen with an example. For instance, let us assume that block $K_{\Lambda,23}$ is associated with link $l_{23} = 1$, and note that $K_{\Lambda,23}$ is the product of the 2nd-row of Y_Λ and the 3rd-column of W^{-1}. As Y_Λ changes with the topology, the only way to guarantee that $K_{\Lambda,23}$ remains constant when $l_{23} = 1$, is by forcing W to be block diagonal, i.e., $W = W^{\text{T}} = \text{diag}(W_i)_{i \in \mathcal{N}}$.

Solving the LMIs considered in Theorem 1, a matrix $P = W^{-1}$ and matrices $K_\Lambda = Y_\Lambda W^{-1}$ are obtained for all $\Lambda \in \mathcal{T}$. Element $K_{\Lambda,ij} \neq 0$ defines the contribution of the state of agent j in the action of agent i when $l_{ij} = 1$. Otherwise, state x_j does not affect the computation of u_i.

A necessary and sufficient condition for the fulfillment of the conditions of Theorem 1 is provided by the following lemma:

Lemma 1. Given a set of topologies $\mathcal{S}_\Lambda \subseteq \mathcal{T}$, a sufficient condition to apply Theorem 1 is to have a feasible solution for (8) for a common *descendant* of the topologies considered, i.e., a topology Λ with at most the enabled links of any topology in $\mathcal{S}_\Lambda$. The condition also becomes necessary if the common descendant is added to set $\mathcal{S}_\Lambda$.

Table 1. Network topologies and components
involved in Example 1

Network topologies		Components
Λ_{DC}	$\emptyset$	$\{1\}, \{2\}, \{3\}$
Λ_1	$\{\text{I}\}$	$\{1,2\}, \{3\}$
Λ_2	$\{\text{II}\}$	$\{1,3\}, \{2\}$
Λ_3	$\{\text{III}\}$	$\{1\}, \{2,3\}$
Λ_4	$\{\text{I}, \text{II}\}$	$\mathcal{N}$
Λ_5	$\{\text{I}, \text{III}\}$	$\mathcal{N}$
Λ_6	$\{\text{II}, \text{III}\}$	$\mathcal{N}$
$\Lambda_{\mathcal{L}}$	$\{\text{I}, \text{II}, \text{III}\}$	$\mathcal{N}$

Fig. 3. Topology of the case study with bidirectional links

The result of Lemma 1 is particularized in the following corollary:

Corollary 1. If $\Lambda_{\text{DC}} \in \mathcal{T}$, then a necessary and sufficient condition that establishes that the family of LMIs defined by Theorem 1 exists, is that W and $Y_{\Lambda_{\text{DC}}}$ can be found for Λ_{DC}, so that (8) holds.

The condition in Lemma 1 is necessary since the solution for the decentralized topology, that is, Λ_{DC}, is included in the set of LMIs of Theorem 1. In the same way, it is sufficient given that this decentralized solution is also a feasible solution for any other topology $\Lambda \in \mathcal{T}$. In particular, note that the decentralized case is the more constrained one since some decision variables of other topologies are forced to be zero. In other words, if (8) has solution for a network topology $\Lambda_i \in \mathcal{T}$, any *ascendant* topology $\Lambda_j \in \mathcal{S}_{\Lambda_i}$, i.e., any topology which at least the links enabled of Λ_i, has a feasible solution. Thus, as indicated in Corollary 1, if (8) has a solution for topology Λ_{DC}, a solution for (8) exists $\forall \Lambda \in \mathcal{T}$.

To finish this section, the design method for the controller is introduced below:

Design Method 1

Considering Theorem 1, the controller is designed by solving

$$\max_{W, Y_\Lambda} \; \text{tr}(W), \qquad (9)$$

subject to (8) for all $\Lambda \in \mathcal{T}$. Then, it is enough to take $K_\Lambda = Y_\Lambda W^{-1}$ for each $\Lambda \in \mathcal{T}$. Note that by maximizing the trace of W, the cost-to-go of the closed-loop system is indirectly minimized by minimizing $P = W^{-1}$.

Remark 2. The computation burden of LMIs does not scale linearly with the number of constraints (Vandenberghe and Boyd, 1996), which in this problem are associated with topologies. Hence, the growth in the computation requirements might render the computation unfeasible. Several strategies could be considered to overcome this issue: 1) convex combination of solutions, given that constraint (9) is convex; 2) exploiting the fact that any topology provides a feasible solution for all its ascendants topologies; 3) *branch-and-bound-like* approaches.

5. AN ACADEMIC EXAMPLE

In this section, the modular controller is assessed by means of the simple network illustrated in Fig. 3, being its network topologies shown in Table 1. For simplicity, the links in the communication graph, namely I, II, and III, are
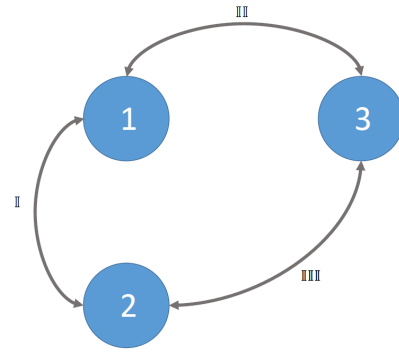
nondirected links. Nevertheless, let us recall that the proposed design method can handle asymmetric information flows defined by unidirectional links as well. The system dynamics in this example are represented as follows:

$$A_{11} = \begin{bmatrix} 1 & 0.8 \\ 0 & 1 \end{bmatrix}, \; A_{22} = \begin{bmatrix} 1 & 0.6 \\ 0 & 0.7 \end{bmatrix}, \; A_{33} = \begin{bmatrix} 1 & 1.1 \\ 0 & 0.8 \end{bmatrix},$$

$$A_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; i \neq j,$$

$$B_{ii} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad B_{ij} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \; i \neq j,$$

$$x_0^{\text{T}} = [2 \; 2 \; 0.5 \; 1 \; 2 \; 0], \qquad (10)$$

where $x_i \in \mathbb{R}^2$, $u_i \in \mathbb{R}$, and x_0 represent the states and inputs of each subsystem, and the initial state of the global system, respectively. Stage costs ℓ_i of all subsystems are defined by matrices $Q_i = \text{diag}(1,1)$ and $R_i = 5$ with $i \in \mathcal{N}$.

The application of the design method results in the following overall Lyapunov matrix:

$$P = \begin{bmatrix} 4.4424 & 4.9893 & 0 & 0 & 0 & 0 \\ 4.9893 & 11.6601 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.0653 & 4.1509 & 0 & 0 \\ 0 & 0 & 4.1509 & 6.2703 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.8375 & 4.8850 \\ 0 & 0 & 0 & 0 & 4.8850 & 11.4150 \end{bmatrix}, \qquad (11)$$

and the feedback control matrix corresponding to topology $\Lambda_{\mathcal{L}}$ (i.e., when all communication links are enabled) is provided by

$$K_{\Lambda_{\mathcal{L}}} = \begin{bmatrix} -0.3151 & -0.9848 & 0.0119 & 0.0195 & 0.0106 & 0.0312 \\ 0.0076 & 0.0235 & -0.3657 & -0.6045 & 0.0069 & 0.0202 \\ 0.0102 & 0.0317 & 0.0105 & 0.0172 & -0.3133 & -0.9274 \end{bmatrix}. \quad (12)$$

In the case a link is disabled, the corresponding feedback matrix is obtained by introducing zeros in the corresponding elements of $K_{\Lambda_{\mathcal{L}}}$. For example, let us assume that agent 2 becomes isolated (links I and III disabled) as in topology Λ_2 (see Table 1), thereby the feedback that stabilizes the system would be given by

$$K_{\Lambda_2} = \begin{bmatrix} -0.3151 & -0.9848 & 0 & 0 & 0.0106 & 0.0312 \\ 0 & 0 & -0.3657 & -0.6045 & 0 & 0 \\ 0.0102 & 0.0317 & 0 & 0 & -0.3133 & -0.9274 \end{bmatrix}. \quad (13)$$

Notice that local controllers do not require to know the complete centralized feedback matrix $K_{\Lambda_{\mathcal{L}}}$, but only their corresponding rows.
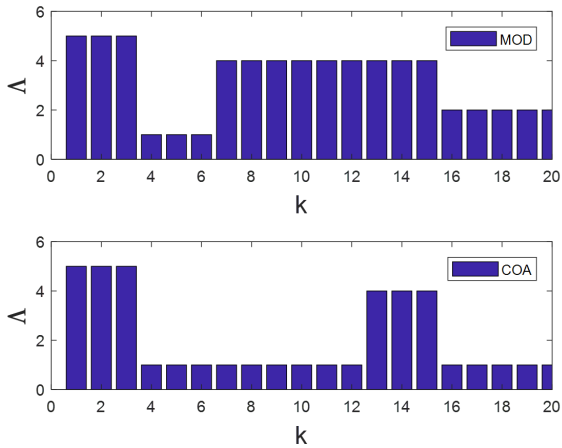
Fig. 4. Topology changes of the MOD and COA control strategies. The topology evaluation has been performed every $T_{\text{top}} = 3$ time steps simultaneously in both schemes.

5.1 Performance assessment

The modular feedback controller (MOD) has been implemented for the network in Fig. 3. This computation is performed in two main steps: an offline procedure where the LMI variables are defined and LMI (8) is evaluated; and an online computation iterating over time step k, and assessing the cost function every $T_{\text{top}} = 3$ time instants.

The results have been compared with the coalitional control scheme (COA) (Maestre et al., 2014), assuming that each link involves a communication cost c_l, which changes in $k = 10$ and $k = 15$, as indicated in Table 2. In general, the controller improves its control performance as more links are activated. Hence, there is a tradeoff between communication and control costs that leads to the dynamic selection of the most appropriate topology.

The topology evolution for both schemes is drawn in Fig. 4, where both the MOD and COA approaches choose the topology that minimizes the overall cost, defined as

$$J(x(k), \Lambda(k)) = \ell_{\mathcal{N}}(k) + \sum_{l \in \Lambda(k)} c_l, \tag{14}$$

with $\ell_{\mathcal{N}}(k)$ defined by (4).

In this way, the aforementioned tradeoff is achieved, being lower for the MOD scheme ($J^{\text{MOD}} = 9.97 \times 10^{-7}$, $J^{\text{COA}} = 4.32 \times 10^{-6}$), as shown in Fig. 5. Moreover, note that by utilizing the MOD scheme, only matrices $K_{\mathcal{L}}$ and P need to be computed, which in turn guarantees stability in the topology switchings, as commented before. In contrast, the COA approach requires computing as many matrices K_{Λ} and P_{Λ} as topologies are considered, and it may be also necessary to introduce an additional hierarchical upper-layer to guarantee stability in the switchings.

Table 2. Change of cost associated with the communication links in different time steps

Links	Communication costs c_l		
	$k = 0$	$k = 10$	$k = 15$
I	0.20	0.40	0.00
II	0.20	0.01	0.30
III	0.20	0.00	0.30

Additionally, the cumulated costs of both control strategies are analyzed jointly with the classical centralized (CEN) and decentralized (DEC) configurations, in order to determine which strategy is more cost-efficient. In this comparison, which is provided in Fig. 6, a constant cost per link $c = 0.2$ has been assumed for all control approaches. Notice that the cost in the MOD scheme ($J_{\text{cum}}^{\text{MOD}} = 95.61$) outperforms the ones in the centralized and decentralized configurations ($J_{\text{cum}}^{\text{CEN}} = 98.31$, $J_{\text{cum}}^{\text{DEC}} = 101.2$), and is not so far from the cost of the COA approach ($J_{\text{cum}}^{\text{COA}} = 91.98$). This fact shows that the proposed modular approach is an effective alternative that balances the overall cumulated costs with computation and implementation simplicity.

6. CONCLUSIONS

This paper proposes a methodology based on LMIs to obtain a family of feedback controllers that guarantees stability despite topology switchings in the control network. To this end, a modular controller is designed and associated with a single Lyapunov function, obtaining an overall matrix P for all network topologies. This implies computing a common controller so that the feedback for each topology is determined by simply inserting zeros in the blocks corresponding to disabled links. Moreover, since the Lyapunov function is common to all network configurations, it is not necessary a supervisory control layer to guarantee the stability and to select topology switchings. This fact represents a clear advantage with respect to methods as the coalitional control where $2^{|\mathcal{L}|}$ matrices P_{Λ} might need to be computed, in addition to the implementation of a hierarchical control layer. The aforementioned features make the modular approach stand out in terms of implementation simplicity.

Future work should include the development of distributed modular design methodologies and the application of the proposed approach to more realistic examples such as networks with delays and information dropouts. Finally, the use of strategies in the line of Remark 2 will be addressed in depth to reduce the number of constraints that must be imposed in the design of the modular controller.
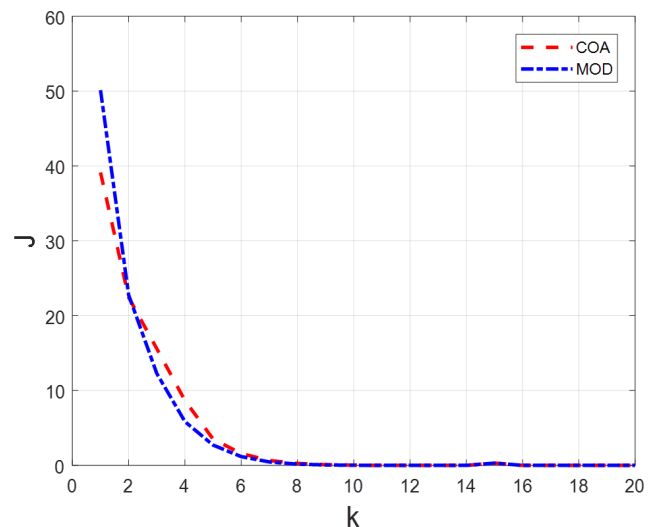

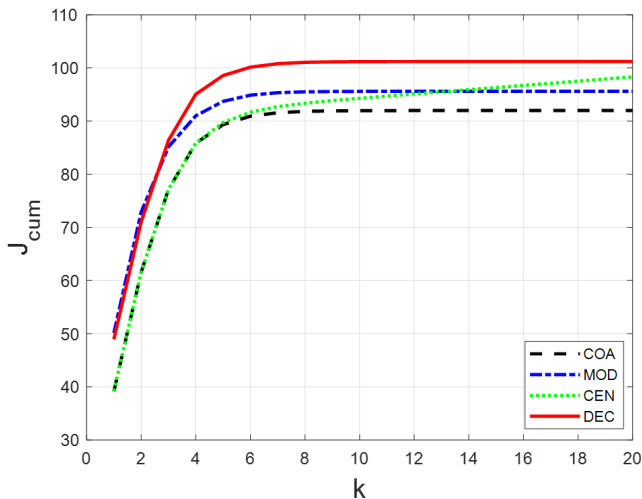Fig. 5. Overall costs for the MOD and COA schemes

Fig. 6. Cumulated costs of the MOD, COA, CEN and DEN approaches when $c = 0.2$

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