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# Data-Driven Stochastic Distribution Network Reconfiguration

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**Abstract:** Distribution network reconfiguration (DNR) is indispensable for the operation of active distribution networks. To address the uncertainties of renewables and variables loads, a data-driven stochastic DNR model is proposed for day-ahead DNR of three-phase unbalanced distribution networks. The switching cost and expected costs resulted from power losses and load balance are minimized. Based on the analysis of historical data, the probability distribution of DG output and load demand is derived using a data-driven method. To improve computation efficiency, a mixed-integer linear programming problem is formulated using linearization techniques. Numerical tests are carried out in an IEEE unbalanced benchmark. The comparison with the conventional deterministic method verifies the effectiveness of the proposed method.

*Keywords:* Stochastic optimization, data-driven, renewable energy, unbalanced distribution network, distribution network reconfiguration.

## NOMENCLATURE

*Indices and Sets*

| | |
|---|---|
| *i, j, k* | Indices of buses, from 1 to *N*. |
| $\mathcal{N}$, *N* | Set of all buses and its cardinality. |
| $\mathcal{E}$, *E* | Set of all branches and its cardinality. |
| $\mathcal{S}$, *S* | Set of scenarios and its cardinality. |
| $\varphi$, $\Phi_i$ | Index of phases and set of phases at bus *i*. |

*Input Parameters*

| | |
|---|---|
| $\bar{S}_{ij}$ | Branch transmission capacity for branch *ij*. |
| $\underline{V}^{\varphi},\bar{V}^{\varphi}$ | Lower and upper bounds for voltage magnitude in phase $\varphi$ of bus *i*. |
| $l_{ij}^{0}$ | Initial network configuration indicator; equals 1 if branch *ij* was initially connected, and 0, otherwise. |
| $\bar{L}$ | The maximum number of lines to reconfigure. |
| $\underline{q}_{gi}^{\varphi},\bar{q}_{gi}^{\varphi}$ | Lower and upper bounds for reactive generation in phase $\varphi$ of bus *i*. |
| $R_{ij}^{\varphi\phi},X_{ij}^{\varphi\phi}$ | Resistance and reactance of branch *ij* between phases $\varphi$ and $\phi$. |
| $w_{ij}^{con},w_{ij}^{dis}$ | Cost to connect/disconnect branch *ij*. |
| $w_{loss}, w_{LBI}$ | Coefficients that transform losses and load balance index into costs. |
| M | A large positive constant in big-M method. |

*Uncertain Parameters*

| | |
|---|---|
| $p_{gi}^{\varphi,s}$ | Active power generation in phase $\varphi$ of bus *i* in scenario *s*. |
| $p_{dj}^{\varphi,s},q_{dj}^{\varphi,s}$ | Active and reactive load demands in phase $\varphi$ of bus *i* in scenario *s*. |

*Variables*

| | |
|---|---|
| $\boldsymbol{\pi}$ | Empirical probability distribution. |
| $l_{ij}$ | Binary for network configuration; equals 1 if branch *ij* is connected, and 0, otherwise. |
| $p_{ij}^{\phi,s},q_{ij}^{\phi,s}$ | Active/reactive power flow of branch *ij* in phase $\phi$. |
| $q_{gi}^{\varphi,s}$ | Reactive power generation in phase $\varphi$ of bus *i* in scenario *s*. |
| $v_{i}^{\varphi,s}$ | Squared voltage magnitude in phase $\varphi$ at bus *i* in scenario *s*. |

## 1. INTRODUCTION

Distribution network reconfiguration (DNR) is an effective method for power loss minimization (Rao et al, 2012) and load balance (Kashem et al, 1999), which is conducted by changing the open/closed states of line switches while maintaining the radial topology of the network. Nevertheless, with the increasing penetration of renewables, such as wind power and solar power, there are more uncertainties of power flows in distribution networks, which brings new challenges for the reconfiguration optimal process.

The DNR is typically studied by the deterministic method which assumes the renewables and loads keep constant throughout the reconfiguration process. There are only a few research on DNR considering the uncertainties of distribution generators (DGs) and loads. A probabilistic generation-load model is proposed to combine all possible operating conditions of renewables and loads for each season (Zidan & El-Saadany, 2013). Monte Carlo simulation is also used to consider uncertainties in renewables and load (Gangwar et al, 2019). The wind speed, solar irradiance and load demand are

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assumed to follow Weibull, Beta and normal distributions, respectively (Gangwar et al, 2019; Zidan & El-Saadany, 2013). However, there are no justified common distributions of renewables (Arthur et al, 2013; Carta et al, 2009) and loads (Singh et al, 2009). For instance, the Weibull distribution is the most widely used distribution for wind speed, but it is not appropriate for wind regimes presenting bimodality or with a high percentage of null wind speeds which should be fitted by mixture distributions (Carta et al, 2009). Therefore, there is no guarantee that the renewables or load demand will follow the assumed distribution types.

With the recent development of monitoring, sensing and communication, it is possible to collect and analyze data from millions of various distributed endpoints such as smart meters, phasor measurement units (Depuru et al, 2011). Hence, the uncertainties of DG output and load demand can be captured by analysis of historical data and the probability distribution can be derived using a data-driven method without assumption on the distribution types.

On the other hand, the distribution system is usually simplified as a balanced single-phase system in most existing studies (Zheng & Wu, 2018). Nevertheless, due to the non-uniform load distribution, asymmetrical network structures and renewables, the distribution network is intrinsically unbalanced, which can introduce additional power losses and limit the loading capability of distribution transformers (Bina & Kashefi, 2011). The consideration of the unbalanced nature of distribution networks makes the DNR problem more complicated.

The main contribution of this study is summarized as follows. First, the probability distribution of DG output and load demand is derived using a data-driven method. Second, a stochastic model is established for the day-ahead three-phase unbalanced DNR with the minimization of switching cost and the costs resulting from power losses and load imbalance. The performance of the proposed method is verified by comparison with the conventional deterministic method.

## 2. SCENARIO GENERATION AND PROBABILITY DISTRIBUTION

Given a set of historical data, we can get $H$ observations of DG active power generations and active/reactive load demands of all buses with each observation denoted as $\boldsymbol{p} = \left\{ p_{gi,h}^{\varphi}, p_{di,h}^{\varphi}, q_{di,h}^{\varphi} \middle| h=1,2,...,H, \forall i \in \mathcal{N}, \forall \varphi \in \Phi_i \right\}$ . Then the observations can be clustered into $S$ categories using K-medoids clustering algorithm (Park & Jun, 2009), which can be summarized as follows:

Step 1: Choose $S$ samples at random to be the initial cluster medoids. Assign each sample to the cluster associated with the closest medoid and calculate the sum of distances from all samples to their medoids;

Step 2: Find a new medoid of each cluster, which is the sample minimizing the total distance to other samples in its cluster. Update the current medoid in each cluster by replacing with the new medoid;

Step 3: Assign each sample to the nearest medoid and obtain the clustering result. Calculate the sum of the distance from all samples to their medoids. If the sum is equal to the previous one, stop the algorithm. Otherwise, go back to Step 2.

Each cluster is a typical type of scenario, and the cluster medoid is regarded as the representative of the corresponding scenario. Therefore, there are $S$ scenarios, and the scenario $s$ is represented by $\boldsymbol{\xi}^s = \left\{ p_{gi}^{\varphi,s}, p_{di}^{\varphi,s}, q_{di}^{\varphi,s} \middle| \forall i \in \mathcal{N}, \forall \varphi \in \Phi_i \right\}$ . Then the empirical distribution $\boldsymbol{\pi} = [\pi_1, ..., \pi_S]$ is obtained by $\pi_s = H_s / H$ where $H_s$ is the number of observations clustered in scenario $s$. The number of clusters is determined by a trade-off between clustering accuracy and computational efficiency

## 3. STOCHASTIC DISTRIBUTION NETWORK RECONFIGURATION MODEL

To deal with the uncertainties of DGs and loads, a stochastic DNR model is established with the objective to minimize expected power losses and switch costs and balance load demand. A distribution network with $N$ buses and $E$ branches is considered. The point of common coupling is indexed as bus 1 without loss of generality. The DGs are assumed to work at the maximum power point tracking mode to make full utilization of renewable energy sources.

### *3.1 Objective Function*

In power systems, the voltage magnitude is around 1.0 p.u. Thus, the total power losses in scenario s can be approximated as

$$p_{loss}^s = \sum_{(i,j)\in\mathcal{E}} \sum_{\varphi\in\Phi_i} \left[ \frac{\left(p_{ij}^{\varphi,s}\right)^2 + \left(q_{ij}^{\varphi,s}\right)^2}{v_i^{\varphi,s}} R_{ij}^{\varphi\varphi} \right] \approx \sum_{(i,j)\in\mathcal{E}} \sum_{\varphi\in\Phi_i} \left[ \left(p_{ij}^{\varphi,s}\right)^2 + \left(q_{ij}^{\varphi,s}\right)^2 R_{ij}^{\varphi\varphi} \right] \tag{1}$$

According to (Siti et al, 2007), the load balance index (LBI) of scenario $s$ can be defined as

$$LBI^s = \sum_{(i,j)\in\mathcal{E}} \sum_{\varphi\in\Phi_i} \left[ \frac{\left(p_{ij}^{\varphi,s}\right)^2 + \left(q_{ij}^{\varphi,s}\right)^2}{\bar{S}_{ij}^{\varphi 2}} \right] \tag{2}$$

A larger value of LBI refers to the better performance of load balancing. Therefore, the objective function of the DNR can be expressed as

$$\min_{\{l_{ij}\}} \sum_{(i,j)\in\mathcal{E}} \left[ w_{ij}^{con}(1-l_{ij}^0)l_{ij} + w_{ij}^{dis} l_{ij}^0 (1-l_{ij}) + \sum_{s=1}^{S} \pi_s (w_{loss} p_{loss}^s + w_{LBI} LBI^s) \right] \tag{3}$$

The objective in (3) minimizes the switching cost and the expected costs resulted from power losses and unbalanced load over branches.

### *3.2 Topology Constraints*

The radial topology of the distribution network is guaranteed by constraint (4). Constraint (5) limits the maximum allowable number of switches to change their status. Constraint (6) is the binary constraint for the status of switches.

$$\sum_{(i,j)\in\mathcal{E}} l_{ij} = N-1+E_C-E \qquad (4)$$

$$\sum_{(i,j)\in\mathcal{E}} \left[l_{ij}^0(1-l_{ij})+(1-l_{ij}^0)l_{ij}\right]/2 \le \bar{L} \qquad (5)$$

$$l_{ij}\in\{0,1\}, \qquad \forall (i,j)\in\mathcal{E} \qquad (6)$$

### *3.3 Three-Phase Power Flow Constraints*

The active and reactive power flow balance of each bus are ensured by constraints (7) and (8), respectively. The voltage drops of branches are described by constraints (9).

$$\sum_{i:i\to j} p_{ij}^{\varphi,s} + p_{gj}^{\varphi,s} - p_{dj}^{\varphi,s} = \sum_{k:j\to k} p_{jk}^{\varphi,s}, \qquad \forall j\in\mathcal{N},\forall s\in\mathcal{S},\ \forall\varphi\in\Phi_j \qquad (7)$$

$$\sum_{i:i\to j} q_{ij}^{\varphi,s} + q_{gj}^{\varphi,s} - q_{dj}^{\varphi,s} = \sum_{k:j\to k} q_{jk}^{\varphi,s}, \qquad \forall j\in\mathcal{N},\forall s\in\mathcal{S},\ \forall\varphi\in\Phi_j \qquad (8)$$

$$v_i^{\varphi,s} - v_j^{\varphi,s} = 2\sum_{\phi\in\Phi_j}\left(R_{ij}^{\varphi\phi}p_{ij}^{\phi,s}+X_{ij}^{\varphi\phi}q_{ij}^{\phi,s}\right) \qquad \forall (i,j)\in\mathcal{E},\forall s\in\mathcal{S} \qquad (9)$$

### *3.4 Branch Thermal Limits*

Constraint (10) limits the apparent power over each branch. The branch flow limit is automatically set as 0 if branch $(i, j)$ is opened.

$$\left(p_{ij}^{\varphi,s}\right)^2+\left(q_{ij}^{\varphi,s}\right)^2 \le l_{ij}\bar{S}_{ij}^2, \qquad \forall (i,j)\in\mathcal{E},\forall s\in\mathcal{S},\forall\varphi\in\Phi_j \qquad (10)$$

### *3.5 Voltage Limits*

$$\underline{V}_i^{\varphi 2} \le v_i^{\varphi,s} \le \bar{V}_i^{\varphi 2}, \qquad \forall i\in\mathcal{N},\forall s\in\mathcal{S},\forall\varphi\in\Phi_i \qquad (11)$$

$$V_{ref}^2 = v_1^{\varphi,s}, \qquad \forall s\in\mathcal{S},\forall\varphi\in\Phi_1 \qquad (12)$$

Constraint (11) specifies the upper and lower bounds for voltage magnitude of each bus. The voltage at the point of common coupling is set as the reference value $V_{ref}$ in constraint (12).

### *3.5 Reactive Power Limits for Distributed Generators*

$$\underline{q}_{gi}^{\varphi} \le q_{gi}^{\varphi,s} \le \bar{q}_{gi}^{\varphi}, \qquad \forall i\in\mathcal{N},\forall s\in\mathcal{S},\forall\varphi\in\Phi_i \qquad (13)$$

Constraint (13) sets the upper and lower bound limits on the reactive power output of DGs.

### *3.6 Reactive Power Limits for Distributed Generators*

Since the constraint in (9) depends on the status of switches, it is replaced by the following constraint in (14) using the big-M method.

$$\begin{aligned}&-(1-l_{ij})M+2\sum_{\phi\in\Phi_j}\left(R_{ij}^{\varphi\phi}p_{ij}^{\phi,s}+X_{ij}^{\varphi\phi}q_{ij}^{\phi,s}\right)\le v_i^{\varphi,s}-v_j^{\varphi,s}\\ &\le(1-l_{ij})M+2\sum_{\phi\in\Phi_j}\left(R_{ij}^{\varphi\phi}p_{ij}^{\phi,s}+X_{ij}^{\varphi\phi}q_{ij}^{\phi,s}\right),\quad \forall (i,j)\in\mathcal{E},\forall s\in\mathcal{S}\end{aligned} \qquad (14)$$

In addition, it is noticed that the constraint (10) is a nonlinear constraint with binary variables, which makes the problem more complicated. In this study, we use the circular constraint linearization technique (Zheng et al, 2018) and extend it to the DNR problem with discrete variables. Intuitively, this procedure uses an octagon to approximate the original circle feasible region. Generally, the accuracy of the approximation increases with the number of square constraints. To balance between the accuracy requirement of industrial applications and the computation burden, we approximate and replace the nonlinear constraint (10) with the following square constraints:

$$\begin{cases}-\bar{S}_{ij}^{\varphi}\le p_{ij}^{\varphi,s}\le\bar{S}_{ij}^{\varphi}\\ -\bar{S}_{ij}^{\varphi}\le q_{ij}^{\varphi,s}\le\bar{S}_{ij}^{\varphi}\\ -\sqrt{2}\bar{S}_{ij}^{\varphi}\le p_{ij}^{\varphi,s}+q_{ij}^{\varphi,s}\le\sqrt{2}\bar{S}_{ij}^{\varphi}\\ -\sqrt{2}\bar{S}_{ij}^{\varphi}\le p_{ij}^{\varphi,s}-q_{ij}^{\varphi,s}\le\sqrt{2}\bar{S}_{ij}^{\varphi}\end{cases} \qquad (15)$$

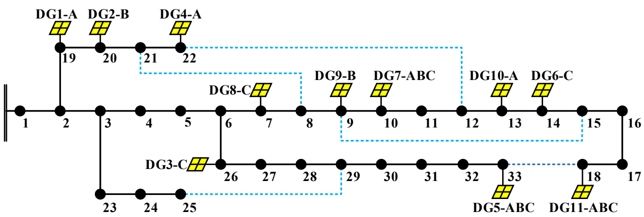



Fig. 1. The modified IEEE 33-bus distribution network with 11 DGs.

## 4. SIMULATION RESULTS

### *4.1 Simulation Configurations and Data Description*

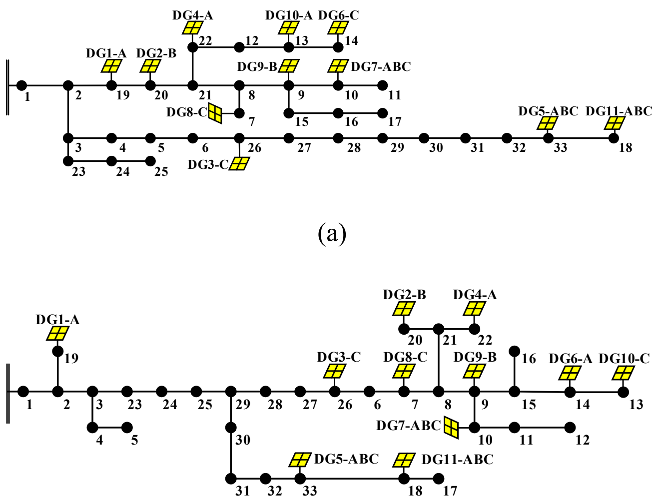
A modified three-phase unbalanced IEEE 33-bus distribution system is considered for numerical tests. There are 11 DGs connected randomly to the distribution systems, as is shown in Fig. 1. For example, DG-1 is connected at bus 10 in phase A, while DG-5 is connected at bus 33 in three phases. For the convenience of analysis, the initial values of $w_{LBI}$, $w_{ij}^{con}$ and $w_{ij}^{dis}$ are all set as 0. The five lines marked by blue dash lines in Fig. 1 are initially open. The real data from the Data platform (Open Power Data platform, 2019) is applied to generate the historical data of DG output and load demand in the test system. The data in Germany from Jan 1, 2015 to Dec 31, 2017 are used for there are no missing values. Other main parameters are set as follows: $\overline{S}_{ij}$ =1.5 p.u., $V_{ref}$ =1.0 p.u., $\underline{V}_i^{\varphi}$ =0.95 p.u., $\overline{V}_i^{\varphi}$ =1.05 p.u., $\overline{L}$ =5, $\underline{q}_{gi}^{\varphi}$ =-0.3 p.u., $\overline{q}_{gi}^{\varphi}$ =0.3 p.u., $w_{loss}$ = 1.0, $S$ = 10.

### *4.2 Effectiveness of the Data-Driven Stochastic Model*

To show the effectiveness of the proposed approach, we compare it with the conventional deterministic DNR method which ignores the uncertainties of DG output and load demand. As is shown in Table 1, the average line losses of the proposed method are much smaller than that of the deterministic method. Besides, the bus voltages are all within the constraints in the proposed method, while the voltage magnitudes of 12 buses are below their lower bounds in the deterministic method. Therefore, the proposed method has better performance than the deterministic DNR method. The reconfiguration results are displayed in Fig. 2.

**Table 1. Comparison results of the proposed and conventional deterministic methods**

| Method | Average Line Losses (kW) | Number of undervoltage buses |
|---|---|---|
| Proposed | 0.090 | 0 |
| D-DNR | 0.173 | 12 |




(a)

(b)

Fig. 2. Configuration results obtained by the (a) proposed method and (b) deterministic DNR methods.

## 5. CONCLUSIONS

In this study, we propose a data-driven stochastic distribution network reconfiguration model for three-phase unbalanced distribution networks considering the uncertainties of DG output and load demand. The numerical test results show that the proposed method has better performance than the deterministic method by much less cost and more securer system under uncertainties. How to handle the errors of probability distribution will be considered in the future.

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