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Application of a nonovershooting tracking control method for the Double Buck Converter

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Abstract: The problem of avoiding overshoot in tracking control problems has an extensive history, but only a few works have offered methods that are applicable to a nonlinear plant. The recent paper (Schmid, 2019) adapted some methods from the linear control systems literature to offer state feedback laws to deliver a nonovershooting response in all outputs of a multi-input multi-output feedback linearisable plant.
A double-buck converter consists of cascaded buck converters connected to a single voltage supply. Taking the system outputs as the voltages across the two load resistors, we use dynamic averaging to obtain a nonlinear state model for the converter. We introduce suitable coordinates to show it has a well-defined vector relative degree and show the system is feedback linearisable. The methods of (Schmid, 2019) are then employed to obtain a state feedback law that ensures both outputs track arbitrary time-varying reference trajectories without overshoot.

Keywords: Nonovershooting tracking control, feedback linearisation, buck-buck converter

1. INTRODUCTION

In tracking control problems, the twin performance objectives of obtaining a rapid response while minimising overshoot have traditionally been viewed as competing objectives, with most control methods seeking a suitable trade-off between the two (Chen et al, 2003). Most works on controller synthesis for avoiding overshoot entirely have considered linear time-invariant plants (Darbha and Battacharyya, 2003; Bement and Jayasuriya, 2004; Schmid and Ntogramatzidis, 2010). Only a very few papers, notably (Krstić and Bement, 2006) and (Zhu and Zhao, 2013) have given nonovershooting control methods for nonlinear single-input single output (SISO) plants.

In the recent paper (Schmid, 2019), a design method was offered to avoid overshoot in all outputs of a feedback linearisable multiple-input multiple output (MIMO) system. It was claimed that the paper addressed a wider class of systems than were considered in (Krstić and Bement, 2006), and the controller synthesis method was rather simpler than the one given in (Zhu and Zhao, 2013). However, none of these three papers considered any application of their proposed methods to a plant model derived from real-world engineering practice, and hence the practical utility of these methods is not known. In this paper we seek to demonstrate the utility of the method proposed in (Schmid, 2019) by considering its application to a voltage converter circuit.

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The double buck converter circuit, with outputs taken to be the voltages on the two load resistors and the control inputs given by the pulse width modulations (PWM) of the transistors, was considered in (Bustamante, 2018). The double buck converter presents a suitably challenging real-world application to investigate the effectiveness of the nonovershooting tracking control methods proposed for feedback linearisable systems in (Schmid, 2019).

The paper is organised as follows. In Section 2 we briefly revisit the controller synthesis methods of (Schmid, 2019). In Section 3 we develop the plant model for the double buck converter circuit and introduce the state coordinates that yield a well-defined vector relative degree. A coordinate transformation and feedback linearising control law are given to render the system as two decoupled chain-of-integrator systems. Section 4 provides simulation results of the proposed control methods. Finally Section 5 offers some concluding thoughts on possible further investigations.

2. NONOVERSHOOTING TRACKING CONTROL

In this section we present a summary of the controller synthesis methods given in (Schmid, 2019) for feedback linearisable nonlinear plants. We consider an affine nonlinear square MIMO system Σ_{nonlin} in the form

$$\Sigma_{nonlin}: \begin{cases} \dot{x} = f(x) + g(x)u, \ x_0 = x(0) \\ y = h(x), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, $y(t) \in \mathbb{R}^p$, and f, g and h are smooth vector fields. The problem of *nonovershooting* *output regulation* is to find a state feedback control law $u = u(x)$ that stabilizes the closed-loop system and ensures the system output tracks a reference signal $r(t) \in \mathbb{R}^p$ without overshoot; thus the tracking error signal $e(t) = r(t) - y(t) \to 0$ without changing sign in all components. We assume the reference signal is obtained as the output of a linear exosystem

$$\Sigma_{exo} : \begin{cases} \dot{w} = S\,w, \quad w_0 = w(0) \\ r = Hw, \end{cases} \tag{2}$$

where $w(t) \in \mathbb{R}^m$ is the state of the exosystem and w_0 is an arbitrary known initial condition.

2.1 Normal forms for feedback linearisable systems

We assume (1) is feedback linearisable by state feedback. The following standard assumptions ensure the existence of suitable linearising state feedback law (Sastry, 1999):

Assumption 2.1. The origin is an equilibrium point of (1).

Assumption 2.2. The system (1) has a well-defined vector relative degree $(\gamma_1, \ldots, \gamma_p)$ at $x_0 \in \mathbb{R}^n$, if (i) for all x in a neighbourhood U of x_0 the Lie derivatives satisfy

$$L_{g_j} L_f^k h_j(x) \equiv 0, \quad k \in \{0, 1, \ldots, \gamma_j - 2\}, \tag{3}$$

where g_i and h_i are the j-th components of the vector fields g and h, for $j \in \{1, \ldots, p\}$, and (ii) the matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{\gamma_1 - 1} h_1 & \ldots & L_{g_p} L_f^{\gamma_1 - 1} h_1 \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{\gamma_p - 1} h_p & \ldots & L_{g_p} L_f^{\gamma_p - 1} h_p \end{bmatrix} \tag{4}$$

is nonsingular at $x = x_0$.

Under these assumptions, in the neighbourhood U of x_0 there exists a change of coordinates

$$\left[\begin{array}{c} T_1(x) \\ \hline T_2(x) \end{array}\right] = \left[\begin{array}{c} \eta_1 \\ \vdots \\ \eta_{n-\gamma} \\ \hline \xi^1 \\ \vdots \\ \xi^p \end{array}\right] = \left[\begin{array}{c} \eta \\ \hline \xi \end{array}\right], \tag{5}$$

where $\gamma = \gamma_1 + \cdots + \gamma_p$ and for each $j \in \{1, \ldots, p\}$,

$$\xi^j = (\xi_1^j, \xi_2^j, \ldots, \xi_{\gamma_j}^j)^\top = (h_j(x), L_f h_j(x), \ldots, L_f^{\gamma_j - 1} h_j(x))^\top. \tag{6}$$

Applying the feedback linearising control law

$$u(x) = -A^{-1}(x) \begin{bmatrix} L_f^{\gamma_1} h_1 \\ \vdots \\ L_f^{\gamma_p} h_p \end{bmatrix} + A^{-1}(x)\nu \tag{7}$$

to (1) yields the linear closed-loop system in chain-of-integrator normal form coordinates

$$\Sigma_{normal} : \begin{cases} \dot{\eta} = f_o(\eta, \xi) \\ \dot{\xi} = A_c \xi + B_c \nu, \quad \xi_0 = \xi(0) \\ y = C_c \xi, \end{cases} \tag{8}$$

where $\xi(t) \in \mathbb{R}^\gamma$, $\eta(t) \in \mathbb{R}^{n-\gamma}$, and (A_c, B_c, C_c) is a decoupled MIMO chain-of-integrator system with

$$A_c = \text{blkdiag}(A_1, \ldots, A_p), \tag{9}$$

$$B_c = \text{blkdiag}(B_1, \ldots, B_p), \tag{10}$$

$$C_c = \text{blkdiag}(C_1, \ldots, C_p), \tag{11}$$

where, for each $j \in \{1, \ldots, p\}$, each system (A_j, B_j, C_j) is a SISO chain-of-integrator system of order γ_j:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \; C = [\,1\ 0\ 0 \cdots 0\,]. \tag{12}$$

Our final assumption is

Assumption 2.3. The zero dynamics $\dot{\eta} = f_0(\eta, 0)$ is stable.

Control of (1) may then be achieved by designing the feedback ν for (8) and including it in (7).

2.2 Nonovershooting output regulation for linear systems

For the case where (1) is an LTI system, we have

$$\Sigma_{lin} : \begin{cases} \dot{x} = A\,x + B\,u, \quad x(0) = x_0 \\ y = C\,x \\ \dot{w} = S\,w, \quad w(0) = w_0 \\ r = Hw, \end{cases} \tag{13}$$

for suitably dimensional state matrices A, B and C. The problem of *output regulation by linear state feedback* (Saberi, 2000) can be solved by a control input of the form

$$u = F\,x + G\,w. \tag{14}$$

Here F can be any matrix such that $A + B\,F$ is Hurwitz-stable, and the feedforward matrix $G = \Gamma - F\,\Pi$, where Γ and Π are obtained by solving the Sylvester equations

$$\Pi\,S = A\,\Pi + B\,\Gamma \tag{15}$$

$$0 = C\,\Pi - H. \tag{16}$$

In (Schmid and Ntogramatzidis, 2014), it was shown that the control input (14) achieves nonovershooting output regulation from (x_0, w_0) provided the closed-loop system Σ_{nom}, defined by

$$\Sigma_{nom} : \begin{cases} \dot{\tilde{x}} = (A + BF)\,\tilde{x}, \; \tilde{x}(0) = \tilde{x}_0 \\ \tilde{e} = C\,\tilde{x}, \end{cases} \tag{17}$$

has a nonovershooting natural response from initial condition $\tilde{x}_0 = x_0 - \Pi w_0$. (Schmid, 2019) gave a method for obtaining a nonovershooting response for a linear chain-of-integrator system that we describe in the next section.

2.3 Nonovershooting natural response for linear systems in chain-of-integrator normal form

We consider a n-th order LTI SISO system

$$\begin{cases} \dot{x} = A\,x + B\,u, \quad x(0) = x_0 \in \mathbb{R}^n, \\ y = C\,x, \end{cases} \tag{18}$$

whose input-output map is a chain of integrators. Our aim is to obtain a feedback matrix F such that the state feedback control law $u = Fx$ will ensure that the natural response of the closed-loop system converges to 0 without overshoot. To do this we firstly let $\mathcal{L} = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ be a set of desired real stable closed-loop poles with $\lambda_1 < \lambda_2 < \cdots < \lambda_n < 0$. For each $i \in \{1, \ldots, n\}$, we solve the Rosenbrock equation

$$\begin{bmatrix} A - \lambda_i I & B \\ C & 0 \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{19}$$

and obtain vectors $\mathcal{V} = \{v_1, v_2, \ldots, v_n\} \subset \mathbb{R}^n$ and $\mathcal{W} = \{w_1, w_2, \ldots, w_n\} \subset \mathbb{R}$ given by

$$v_i = \begin{pmatrix} 1 \\ \lambda_i \\ \lambda_i^2 \\ \vdots \\ \lambda_i^{n-1} \end{pmatrix}, \quad w_i = \lambda_i^n. \tag{20}$$

We let $V = [v_1\ v_2\ \ldots\ v_n]$ and $W = [w_1\ w_2\ \ldots\ w_n]$ and use the method of (Moore, 1976) to obtain the matrix

$$F = WV^{-1} \tag{21}$$

that ensures $A + BF$ has eigenvalues $\mathcal{L}$ and eigenvectors $\mathcal{V}$. Introducing $\alpha = (\alpha_1, \ldots, \alpha_n)^\top = V^{-1}x_0$, the natural response of the closed-loop system is given by

$$y(t) = \sum_{i=1}^{n} \alpha_i e^{\lambda_i t}. \tag{22}$$

The following lemma from (Schmid, 2019) provides a sufficient condition for y to not change sign, for $t \geq 0$.

Lemma 2.1. Let (A, B, C) be an n-th order chain-of-integrators SISO system as in (12). Let $\mathcal{L}$ be a desired set of real stable poles with $\lambda_1 < \lambda_2 < \cdots < \lambda_n < 0$, and let F be obtained from (19)-(21). For the initial condition x_0, let $\alpha \in \mathbb{R}^n$ be such that the natural response y of the closed-loop system is given by (22). We introduce

$$c_k := \begin{cases} 1, & \text{if } \alpha_k \alpha_n < 0 \\ 0, & \text{otherwise} \end{cases}, \text{ for } k \in \{1, \ldots, n-1\} \tag{23}$$

$$p(x_0, \mathcal{L}) := |\alpha_n| + (1 - 2c_{n-1})|\alpha_{n-1}| - \sum_{k=1}^{n-2} c_k |\alpha_k|. \tag{24}$$

Then $y(t)$ does not change sign for $t \geq 0$ if $p(x_0, \mathcal{L}) > 0$.

(Schmid, 2019) gave the following algorithm for a nonovershooting state feedback linearising control law for (1):

Algorithm 2.1.

(1) For each chain-of-integrators subsystem (A_j, B_j, C_j) of order γ_j in (12), obtain matrices Γ_j, Π_j satisfying

$$\Pi_j\, S = A_j\, \Pi_j + B_j\, \Gamma_j \tag{25}$$

$$0 = C_j\, \Pi_j - H_j, \tag{26}$$

where H_j denotes the j-th row of H in (2). Let $\xi_0 = T_2(x_0)$ be decomposed as

$$\xi_0 = (\xi_0^1, \xi_0^2, \ldots,\ \xi_0^p)^\top \tag{27}$$

and compute, for each $j \in \{1, \ldots, p\}$,

$$\tilde{\xi}_0^j = \xi_0^j - \Pi_j w_0. \tag{28}$$

(2) Let Σ_{nom}^j be the nominal system in (17) with respect to (A_j, B_j, C_j), with initial condition $\tilde{\xi}_0^j$. Select candidate sets of desired negative closed-loop poles $\mathcal{L}_j = \{\lambda_1^j, \ldots, \lambda_{\gamma_j}^j\}$ and use (20) to obtain $\mathcal{V}_j = \{v_1^j, v_2^j, \ldots, v_n^j\}$ and $\mathcal{W}_j = \{w_1^j, w_2^j, \ldots, w_n^j\}$. Compute $\alpha^j = V_j^{-1}\tilde{\xi}_0^j$ and use Lemma 2.1 to test $p(\tilde{\xi}_0^j, \mathcal{L}_j) > 0$. If the test fails, select alternative poles.

(3) Obtain F_j from (21), and compute $G_j = \Gamma_j - F_j\, \Pi_j$.

(4) Combine $F = \mathrm{blkdiag}(F_1, \ldots, F_p)$ and $G = (G_1^\top, \ldots, G_p^\top)^\top$ to obtain the control law

$$\nu = F\xi + Gw, \tag{29}$$

and include ν in (7) to obtain the feedback linearising controller u.

3. THE DOUBLE BUCK CONVERTER CIRCUIT

The multivariable average model for the double buck converter of Figure 1 is (Bustamante, 2018)

$$L_1 \frac{di_1}{dt} = -v_1 + E\, u_1 \tag{30}$$

$$C_1 \frac{dv_1}{dt} = i_1 - \frac{v_1}{R_1} - i_2\, u_2 \tag{31}$$

$$L_2 \frac{di_2}{dt} = -v_2 + v_1\, u_2 \tag{32}$$

$$C_2 \frac{dv_2}{dt} = i_2 - \frac{v_2}{R_2} \tag{33}$$

$$y_1 = v_1 \tag{34}$$

$$y_2 = v_2. \tag{35}$$

The outputs are the resistor voltages v_1 and v_2, and the controls are the switching functions u_1 and u_2 for the pulse width modulation (PWM) of the transistors.

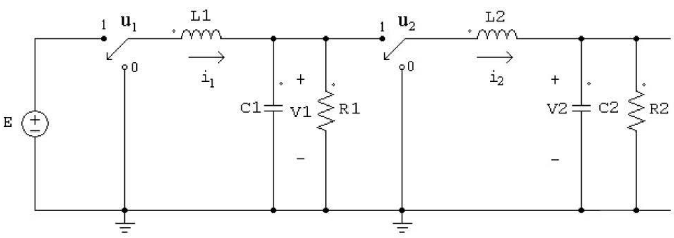

Figure 1. Cascaded double buck converter with load resistors R_1 and R_2.

We collect the state as $x^\top = (i_1,\, v_1,\, i_2,\, v_2)$ and rewrite

$$\dot{x}_1 = -\frac{x_2}{L_1} + \frac{Eu_1}{L_1} \tag{36}$$

$$\dot{x}_2 = \frac{x_1}{C_1} - \frac{x_2}{R_1 C_1} - \frac{x_3 u_2}{C_1} \tag{37}$$

$$\dot{x}_3 = -\frac{x_4}{L_2} + \frac{x_2 u_2}{L_2} \tag{38}$$

$$\dot{x}_4 = \frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \tag{39}$$

$$y_1 = x_2 \tag{40}$$

$$y_2 = x_4. \tag{41}$$

The goal is to regulate the output voltages y to desired setpoint values. Problems associated to this system are that only inputs $u_i(t) \in [0, 1]$ are admissible and that output y has no vector relative degree (singular decoupling matrix).

The latter problem may be solved by introducing a dynamic extension $\bar{u}_2$ to the system, such that $u_2(t) := \int_0^t \bar{u}_2(\tau)d\tau$. Augmenting the state vector with $x_5 = u_2$, we obtain the extended system

$$\dot{x}_1 = -\frac{x_2}{L_1} + \frac{Eu_1}{L_1} \tag{42}$$

$$\dot{x}_2 = \frac{x_1}{C_1} - \frac{x_2}{R_1 C_1} - \frac{x_3 x_5}{C_1} \tag{43}$$

$$\dot{x}_3 = -\frac{x_4}{L_2} + \frac{x_2 x_5}{L_2} \tag{44}$$

$$\dot{x}_4 = \frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \tag{45}$$

$$\dot{x}_5 = \bar{u}_2. \tag{46}$$

The vector relative degree γ can now be found by differentiating $(y_1, y_2) = (x_2, x_4)$ with respect to time until the first appearance of u_1 or $\bar{u}_2$:

$$\begin{aligned}
\dot{y}_1 &= \frac{x_1}{C_1} - \frac{x_2}{R_1 C_1} - \frac{x_3 x_5}{C_1} \\
\ddot{y}_1 &= -\frac{x_2}{C_1 L_1} x_2 + \frac{E u_1}{C_1 L_1} - \frac{x_1}{R_1 C_1^2} + \frac{x_2}{R_1^2 C_1^2} \\
&\quad + \frac{x_3 x_5}{R_1 C_1^2} - \frac{1}{C_1}\left(x_3 \bar{u}_2 - x_5 \left(-\frac{x_4}{L_2} + \frac{x_2 x_5}{L_2} \right)\right) \\
&\triangleq f_1(x) + \frac{E u_1}{C_1 L_1} - \frac{x_3 \bar{u}_2}{C_1} \\
\dot{y}_2 &= \frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \\
\ddot{y}_2 &= -\frac{x_4}{C_2 L_2} + \frac{x_2 x_5}{C_2 L_2} - \frac{x_3}{R_2 C_2^2} + \frac{x_4}{(R_2 C_2)^2} \\
\dddot{y}_2 &= -\frac{1}{C_2 L_2}\left(\frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \right) + \frac{x_2 \bar{u}_2}{C_2 L_2} \\
&\quad + \frac{x_5}{C_2 L_2}\left(\frac{x_1}{C_1} - \frac{x_2}{R_1 C_1} - \frac{x_3 x_5}{C_1} \right) \\
&\quad + \frac{1}{(R_2 C_2)^2}\left(\frac{R_2 x_4}{L_2} - \frac{R_2 x_2 x_5}{L_2} + \frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \right) \\
&\triangleq f_2(x) + \frac{x_2 \bar{u}_2}{C_2 L_2}, \ \text{ where} \\
f_1(x) &= -\frac{x_2}{C_1 L_1} - \frac{x_1}{R_1 C_1^2} + \frac{x_2}{R_1^2 C_1^2} \\
&\quad + \frac{x_3 x_5}{R_1 C_1^2} - \frac{x_5}{C_1}\left(-\frac{x_4}{L_2} + \frac{x_2 x_5}{L_2} \right) \\
f_2(x) &= -\frac{1}{C_2 L_2}\left(\frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \right) \\
&\quad + \frac{x_5}{C_2 L_2}\left(\frac{x_1}{C_1} - \frac{x_2}{R_1 C_1} - \frac{x_3 x_5}{C_1} \right) \\
&\quad + \frac{1}{(R_2 C_2)^2}\left(\frac{R_2 x_4}{L_2} - \frac{R_2 x_2 x_5}{L_2} + \frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \right).
\end{aligned}$$

Hence the extended system has vector relative degree $\gamma = (2, 3)$, and the decoupling matrix has full rank whenever $x_2 \neq 0$. We introduce the diffeomorphism T as:

$$T(x) = \begin{bmatrix} x_2 \\ \frac{x_1}{C_1} - \frac{x_2}{R_1 C_1} - \frac{x_3 x_5}{C_1} \\ x_4 \\ \frac{x_3}{C_2} - \frac{x_4}{R_2 C_2} \\ -\frac{x_4}{C_2 L_2} + \frac{x_2 x_5}{C_2 L_2} - \frac{x_3}{R_2 C_2^2} + \frac{x_4}{(R_2 C_2)^2} \end{bmatrix}. \tag{47}$$

The chain of integrators form of the double buck converter system can be represented in coordinates $\xi = T(x)$ as:

$$\dot{\xi}_1 = \xi_2 \tag{48}$$

$$\dot{\xi}_2 = f_1(x) + \frac{E u_1}{C_1 L_1} - \frac{x_3 \bar{u}_2}{C_1} \tag{49}$$

$$\dot{\xi}_3 = \xi_4 \tag{50}$$

$$\dot{\xi}_4 = \xi_5 \tag{51}$$

$$\dot{\xi}_5 = f_2(x) + \frac{x_2 \bar{u}_2}{C_2 L_2}. \tag{52}$$

Note that for brevity we represent the transformed system in the original x coordinates. From (52) we compute the linearising

$$\bar{u}_2 = \frac{C_2 L_2}{x_2}(-f_2(x) + \nu_2),$$

which results in (49) becoming:

$$\dot{\xi}_2 = f_1(x) + \frac{E u_1}{C_1 L_1} - \frac{C_2 L_2 x_3}{C_1 x_2}(-f_2(x) + \nu_2). \tag{53}$$

We can now compute u_1 to linearize (53):

$$\begin{aligned}
u_1 &= \frac{C_1 L_1}{E}\left(-f_1(x) + \frac{C_2 L_2 x_3}{C_1 x_2}(-f_2(x) + \nu_2) + \nu_1 \right) \\
&= \frac{C_1 L_1}{E}(-f_1(x) + \nu_1) + \frac{C_2 L_1 L_2 x_3}{E x_2}(-f_2(x) + \nu_2).
\end{aligned}$$

Setting $f(x) = (f_1(x), f_2(x))^\top$, and $\nu = (\nu_1, \nu_2)^\top$, the exactly linearising control reads

$$\begin{pmatrix} u_1 \\ \bar{u}_2 \end{pmatrix} = \frac{C_1 C_2 L_1 L_2}{E x_2}\begin{pmatrix} \frac{x_2}{C_2 L_2} & \frac{x_3}{C_1} \\ 0 & \frac{E}{C_1 L_1} \end{pmatrix}(-f(x) + \nu), \tag{54}$$

and results in two decoupled integrator chains

$$\dot{\zeta}_1 = A_{c,1}\zeta_1 + B_{c,1}\nu_1 \tag{55}$$

$$y_1 = C_{c,1}\zeta_1 \tag{56}$$

$$\dot{\zeta}_2 = A_{c,2}\zeta_1 + B_{c,2}\nu_2 \tag{57}$$

$$y_2 = C_{c,2}\zeta_2, \tag{58}$$

where $\zeta_1 = (\xi_1, \xi_2)^\top, \zeta_2 = (\xi_3, \xi_4, \xi_5)^\top$ and $(A_{c,1}, B_{c,1}, C_{c,1})$ and $(A_{c,2}, B_{c,2}, C_{c,2})$ are chain-of-integrator systems of order 2 and 3, respectively. Combining the controls inputs gives $\nu = [\nu_1, \nu_2]^\top$ for (54). Design of ν_1 and ν_2 must ensure u_1 and u_2 remain within $[0, 1]$ at all times.

4. NONOVERSHOOTING TRACKING FOR THE DOUBLE BUCK CONVERTER

In this Section we demonstrate how the controller design methods from Section 2 can be applied to the double buck converter circuit of Figure 1. We adopt the system parameters used in (Linares-Flores et al, 2006):

E	L_1	C_1	R_1	L_2	C_2	R_2
55 V	12 mH	470 μF	100 Ω	16 mH	470 μF	10 kΩ

Assuming the switch $u_2(0) = 0$ and the switch $u_1(0) = 1$, and assuming the circuit initially starts in steady state, then we can calculate the values of the currents and voltages using circuit laws as $v_1 = 55\,\text{V}$, $i_1 = 0.55\,\text{A}$, $v_2 = 0\,\text{V}$ and $i_2 = 0\,\text{A}$. Hence a valid initial condition for the system is $x_0 = (0.55, 55, 0, 0, 0)^\top$.

4.1 Constant step references

We want to obtain output voltages $v_1 = 40\,V$ and $v_2 = 20\,V$. For (2), we choose the constant dynamics exosystem

$$S = 0, w_0 = 1, \quad H = [40\ 20]^\top, \tag{59}$$

yielding the constant reference signal of $r(t) = [40\ 20]^\top$. We assume the initial condition of $x_0 = (0.55, 55, 0, 0, 0)^\top$ for (42)-(46) and employ Algorithm 2.1 to design feedback and feedforward matrices for control law $\nu_j = F_j\zeta_j + G_j w$, $j = 1, 2$, for the chain-of-integrator systems (55)-(58). In Step 1, we solve (25)-(26) for $j = 1, 2$ and obtain

$$\Pi_1 = \begin{bmatrix} 40 \\ 0 \end{bmatrix}, \ \Gamma_1 = 0, \ \Pi_2 = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}, \ \Gamma_2 = 0. \tag{60}$$

From (47) we compute $\xi_0 = (55, 0, 0, 0, 0)^\top$ and from (28) we obtain $\hat{\xi}_0 = (15, 0, -20, 0, 0)^\top$. In Step 2, we select intervals in the negative real line within which the desired closed-loop poles should lie. For $(A_{c,1}, B_{c,1}, C_{c,1})$ we select from the interval $(-4, -1)$ and apply procedure (19)-(21) with $\hat{\xi}_0^1 = (15, 0)^\top$. The choice $\mathcal{L}_1 = \{-1.6256, \ -1.4204\}$ yields the feedback matrix $F_1 = -[2.309, 3.046]$ and the coordinate vector $\alpha = (-103.8, \ 118.8)^\top$ satisfying Lemma 2.1. For $(A_{c,2}, B_{c,2}, C_{c,2})$ we select from the interval $(-4, -1)$ and apply procedure (19)-(21) with $\hat{\xi}_0^2 = (-20, 0, 0)^\top$. The choice $\mathcal{L}_2 = \{-3.9772, -2.0972, -1.0321\}$ yields the feedback matrix $F_2 = -[8.608, 14.61, 7.107]$ and the coordinate vector $\alpha = (-11.63, 89.19, -97.56)^\top$, again satisfying Lemma 2.1. In Step 3, we use $G_j = \Gamma_j - F_j\Pi_j$ to compute $G_1 = 92.361$ and $G_2 = 172.17$. In Step 4 we combine $F = \text{blkdiag}(F_1, \ F_2)$ and $G = (G_1^\top, G_2^\top)^\top$ to finally obtain

$$\nu = F\xi + Gw \tag{61}$$

for (54). Figures 2a-d show the outputs track the reference values without overshooting, and the control inputs satisfy the saturation condition.

4.2 Tracking of time-varying reference signals

The time-varying voltage outputs

$$r(t) = \begin{bmatrix} 40 - \sin(t) \\ 20 - \sin(t) \end{bmatrix} \tag{62}$$

are generated by the exosystem (2) with

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ w_0 = \begin{bmatrix} 0.5 \\ -0.5 \\ 0.5 \end{bmatrix}, \ H = \begin{bmatrix} 1 & 1 & 80 \\ 1 & 1 & 40 \end{bmatrix}.$$

Solving (25)-(26) for $j = 1, 2$ yields

$$\Pi_1 = \begin{bmatrix} 1 & 1 & 80 \\ -1 & 1 & 0 \end{bmatrix}, \ \Gamma_1 = [-1 \ -1 \ 0], \tag{63}$$

$$\Pi_2 = \begin{bmatrix} 1 & 1 & 40 \\ -1 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}, \ \Gamma_2 = [1 \ -1 \ 0]. \tag{64}$$

From (28) we obtain $\hat{\xi}_0 = (15, 1, -20, 1, 0)^\top$. In Step 2, we apply procedure (19)-(21) for $(A_{c,1}, B_{c,1}, C_{c,1})$ with $\hat{\xi}_0^1 = (15, 1)^\top$, and for $(A_{c,2}, B_{c,2}, C_{c,2})$ with $\hat{\xi}_0^2 = (-20, 1, 0)^\top$. Choice $F_1 = -[7.98, 5.77]$ and $F_2 = -[6.25, 11.63, 6.42]$ yield the coordinate vectors $\alpha_1 = (-30.29, \ 45.29)^\top$ and $\alpha_2 = (-5.09, \ 56.64, \ -71.55)^\top$, both of which satisfy Lemma 2.1. Step 3 yields

$$G_1 = [1.21 \ 12.75 \ 638.4], \ G_2 = [-10.81 \ 10.46 \ 249.86]. \tag{65}$$

In Step 4 we again combine matrices to obtain F and G for ν in (54). Figures 3a-d show the outputs track the sinusoidal reference voltages without overshooting, and the control inputs satisfy the saturation condition.

5. CONCLUSION

We investigated the application of the nonovershooting tracking control methods of (Schmid, 2019) for the control of a double buck voltage converter circuit. Our simulation study demonstrated that these methods could deliver the desired nonovershooting tracking response for both voltage outputs. Future work will investigate the effectiveness of the proposed methods in a hardware implementation.

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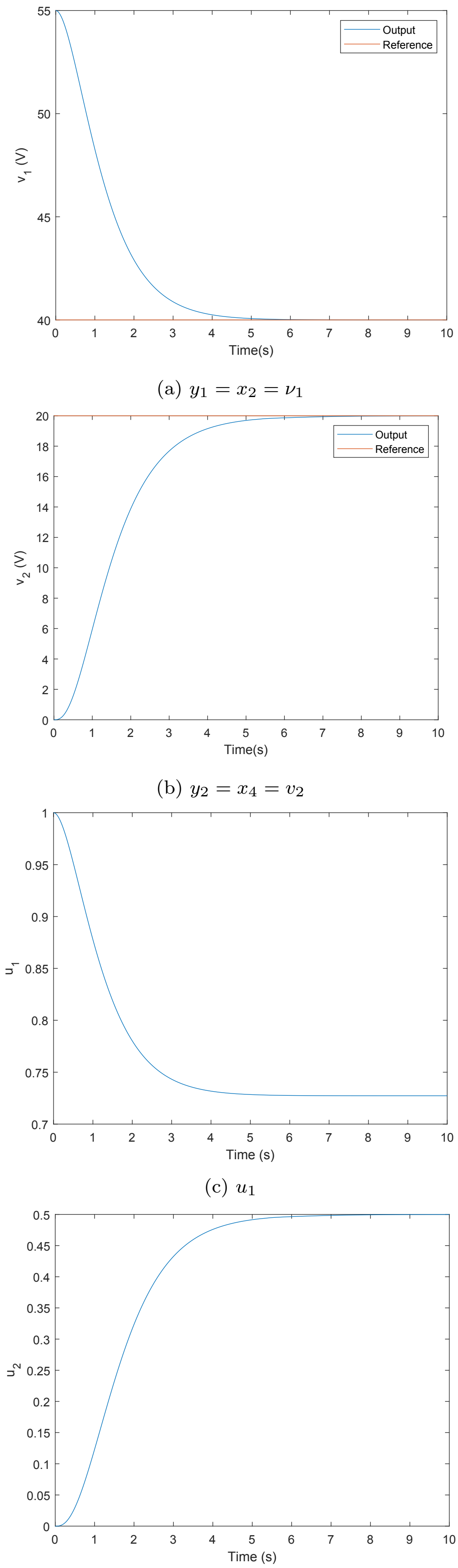


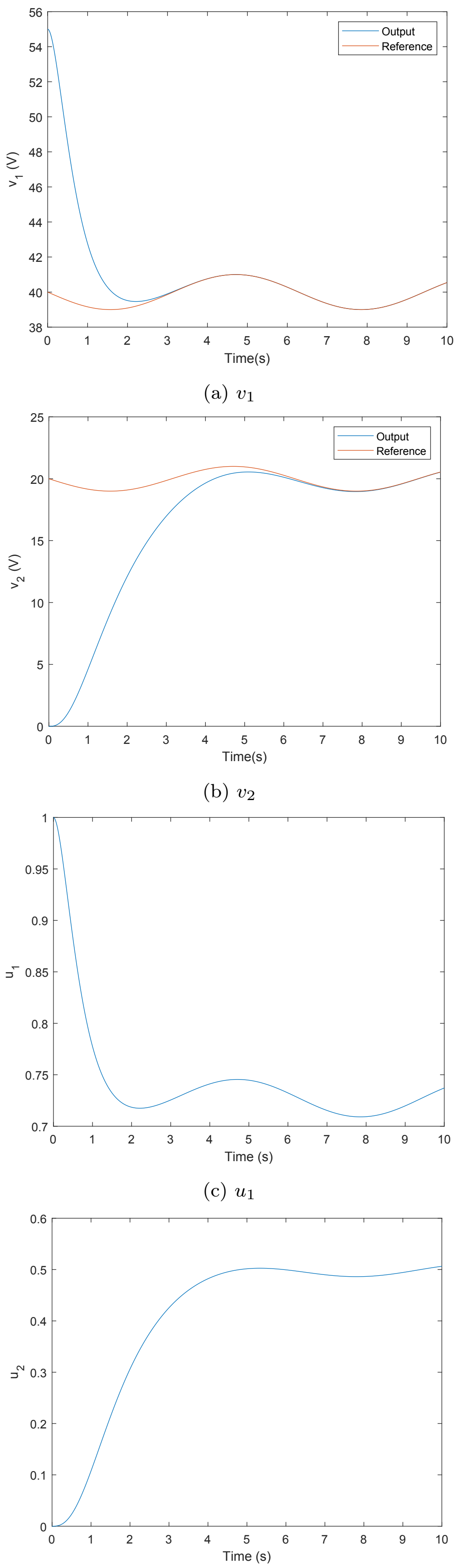
(a) $y_1 = x_2 = \nu_1$

(b) $y_2 = x_4 = v_2$

(c) u_1

(d) u_2

Figure 2. Simulation of Double Buck Converter Circuit with a Constant Reference.



(a) v_1

(b) v_2

(c) u_1

(d) u_2

Figure 3. Simulation of Double Buck Converter Circuit with a Time-Varying Reference.