

# Fuzzy-based Dissipative Consensus for Multi-Agent Systems with Markov Switching Topologies

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**Abstract:** In this paper, the dissipativity-based consensus problem for polynomial fuzzy multi-agent systems is considered. First, a novel fuzzy modeling method is proposed to describe the error dynamics. By establishing the changing topologies by Markov process, a new consensus protocols are designed. By polynomial Lyapunov function and sum of squares, sufficient condition is given to assure even-square consensus with dissipative performance. Finally, an illustrative example is employed to verify the proposed dissipativity-based even-square consensus design schemes.

*Keywords:* Fuzzy system, dissipativity, multi-agent system, switching topology, Markov process.

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## 1. INTRODUCTION

The T-S fuzzy model can efficiently approximate the smooth nonlinear system at any precision, and has been widely concerned in the past decades. For example, see Tanaka and Wang (2001); Wang et al. (2019); Li et al. (2019); Fei et al. (2018); Shi et al. (2016a,b); Zhou et al. (2017); Sun et al. (2019); Qiu et al. (2019) and the references therein.

Recently, Tanaka et al. (2009) presents the polynomial fuzzy model to approximate the nonlinear system using polynomial expression. Fruitful achievement have been yielded. One may refer to Lam and Tsai (2014); Tanaka et al. (2016); Shi and Yu (2020).

The consensus of multi-agent systems (MASs) has attracted a lot of attentions because of its wide application,

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including flocking, formation control, and synchronization of dynamical networks, for example, see Tanner et al. (2007), Dong and Hu (2016), Liu et al. (2014) and the references therein. All kinds of control strategies are adopted to reach agreement, such as  $H_\infty$  control in Zhao et al. (2013); Tabarisaadi et al. (2017) and adaptive control in Deng and Yang (2019); Shi and Shen (2015, 2017). For example, Tabarisaadi et al. (2017) addresses a consensus control problem for the nonlinear MASs with fixed topology in a polynomial fuzzy framework. On the other hand, the communication topologies among agents often change partly caused by communication equipment failures and disturbances. The switching topology is usually modeled by Markov process. For example, see Ding and Guo (2015).

On another research front line, Willems (1972) On another research frontier, [1] introduced the dissipativity theory based on input-output energy correlation. It plays an important role in the analysis and synthesis of control systems. Dissipativity is described by the supply rate and the storage function, which represent the energy provided by the external systems and internal storage, respectively. Dissipativity has been applied to various kinds of systems. For instance, Shi et al. (2016a) investigates the stabilization problem with strictly dissipative performance for T-S fuzzy systems. Liu et al. (2019) concerns with the problem

of event-triggering sliding mode control with strictly dissipative performance for the switched stochastic systems.

Motivated by the above discussion, this paper study the strictly dissipative consensus issue of polynomial fuzzy MASs. The main contributions of this paper are summarized as follows:

- (i) A new fuzzy modeling method is presented for first-order nonlinear MASs.
- (ii) Distributed consensus protocols are designed to ensure that the states of all followers converge to those of the leader.
- (iii) New relaxed sufficient conditions in the form of SOS are proposed to ensure the even-square consensus.

*Notation:* Symbol  $\otimes$  stands for the Kronecker product.  $\|\cdot\|$  denotes the Euclidean norm.  $(\Omega, \mathcal{F}, \mathcal{P})$  stands for a probability space.  $E\{\cdot\}$  represents the expectation operator.  $I$  refers to the identity matrix with appropriate dimensions.  $X > 0$  means that  $X$  is a symmetric and positive definite matrix.  $He(A)$  is defined as  $A + A^T$ .

## 2. PRELIMINARIES AND PROBLEM FORMULATION

### 2.1 Graph Theory

Define  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$  as a directed graph, where  $\mathcal{V} = \{1, 2, \dots, N\}$  denotes the node set,  $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\}$  is the edge set, and  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$  denotes a weighted adjacency matrix.  $a_{ij} = 1$  if  $(j, i) \in \mathcal{E}$ , otherwise  $a_{ij} = 0$ . Denote  $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$  as the neighboring set of node  $i$ . The Laplacian matrix  $\mathcal{L} = [\mathcal{L}_{i,j}] \in \mathbb{R}^{N \times N}$  of  $\mathcal{G}$  is defined as  $\mathcal{L}_{i,j} = \sum_{k=1, k \neq i}^N a_{ik}$ , if  $i = j$ , otherwise  $\mathcal{L}_{i,j} = -a_{ij}$ .

Graph  $\mathcal{G}$  refers to the fixed topology formed by followers. Graph  $\bar{\mathcal{G}}$  contains one leader,  $\mathcal{G}$  and all edges between the leader and suitable follower.

Assume that the switching topology is modeled by Markov process. Let  $\sigma(t) : [0, +\infty) \rightarrow S = \{1, 2, \dots, s\}$  be a switching signal.  $\mathcal{G}^{\sigma(t)}$  denotes the interaction topology at  $\sigma(t)$ .

### 2.2 Polynomial Fuzzy Model

For nonlinear MASs composed of  $N$  followers and a leader, we develop a polynomial fuzzy model. Each agent is described by

$$\dot{x}_i = f(x_i) + u_i, \quad (1)$$

$$\dot{x}_0 = f(x_0), \quad (2)$$

where  $x_i \in \mathbb{R}^n$  is the state of the  $i$ th agent,  $i = 1, 2, \dots, N$ .  $x_0 \in \mathbb{R}^n$  denotes the state of the leader. In this paper,  $x_0$  refers to a chaotic orbit.  $u_i \in \mathbb{R}^n$  denotes the control input.  $f(x_i) \in \mathbb{R}^n$  and  $f(x_0)$  represent the polynomial vector function, respectively.

Define the error state as  $e_i = x_i - x_0$ . Then, the error dynamic system is generated as

$$\dot{e}_i = f(x_i) - f(x_0) + u_i. \quad (3)$$

To model (3), a polynomial fuzzy model is established by:

$\mathcal{R}^p$ : If  $\phi_{i,1}(t)$  is  $M_1^p$  and  $\dots$  and  $\phi_{i,q}(t)$  is  $M_q^p$ , then

$$\dot{e}_i = a_p(e_i)e_i + u_i, \quad (4)$$

where  $\phi_i(t) = [\phi_{i,1}^T(t) \ \phi_{i,2}^T(t) \ \dots \ \phi_{i,q}^T(t)]^T$  denotes the premise variable vector.  $M_1^p, \dots, M_q^p$  are the fuzzy sets.  $r$  stands for IF-THEN rules' number.  $a_p(e_i)e_i + u_i$  is a polynomial vector.

The compact form of (3) is

$$\dot{e}_i = \sum_{p=1}^r h_p(\phi_i(t)) \{a_p(e_i)e_i + u_i\}, \quad (5)$$

where

$$h_p(\phi_i(t)) = \frac{\omega_p(\phi_i(t))}{\sum_{p=1}^r \omega_p(\phi_i(t))}, \quad \omega_p(\phi_i(t)) = \prod_{j=1}^q M_j^p(\phi_{i,j}(t)),$$

and  $M_j^p(\phi_{i,j}(t))$  represents the grade of membership of  $\phi_{i,j}(t)$  in  $M_j^p$ . The function  $h_p(\phi_i(t))$  has the properties of

$$h_p(\phi_i(t)) \geq 0, \quad \sum_{p=1}^r h_p(\phi_i(t)) = 1.$$

### 2.3 Markov Switching Topologies

The evolution of the Markov process  $\sigma(t)$  is governed by the following probability transitions:

$$P_r\{\sigma(t+\theta) = \beta | \sigma(t) = \alpha\} = \begin{cases} \lambda_{\alpha\beta}(\theta)\theta + o(\theta), & \alpha \neq \beta, \\ 1 + \lambda_{\alpha\alpha}(\theta)\theta + o(\theta), & \alpha = \beta, \end{cases} \quad (6)$$

where  $\theta$  denotes the dwell time,  $o(\theta)$  is defined by  $\lim_{\theta \rightarrow 0} o(\theta)/\theta = 0$ .  $\lambda_{\alpha\beta}(\theta) \geq 0$  represents the transition rate, and  $\lambda_{\alpha\alpha}(\theta) = \sum_{\beta=1, \alpha \neq \beta}^s \lambda_{\alpha\beta}(\theta)$ . In this paper, consider  $\lambda_{\alpha\beta}(\theta) = \lambda_{\alpha\beta}$ .

### 2.4 Problem Formulation

Here, we construct a distributed consensus protocol.

Considering external disturbance, the augmented system is

$$\begin{aligned} \dot{e}_i &= \sum_{p=1}^r h_p(\phi_i(t)) \{a_p(e_i)e_i + u_i + d_p(e_i)w_i(t)\}, \\ z_i &= \sum_{p=1}^r h_p(\phi_i(t)) (c_{zp}(e_i)e_i + d_{zp}(e_i)w_i(t)), \end{aligned} \quad (7)$$

where  $d_p(e_i) \in \mathbb{R}^{n \times m}$ ,  $c_{zp}(e_i) \in \mathbb{R}^{l \times n}$  and  $d_{zp}(e_i) \in \mathbb{R}^{l \times m}$  denote polynomial matrices,  $i = 1, 2, \dots, N$ .  $z_i \in \mathbb{R}^l$  is the controlled output.  $w_i \in \mathbb{R}^m$  denotes the external disturbance.

Design the following distributed consensus scheme:

$$\begin{aligned} u_i &= -c \sum_{j \in \mathcal{N}_i^{\sigma(t)}} a_{ij}^{\sigma(t)} \Gamma^{\sigma(t)}(e)(x_i - x_j) \\ &\quad - cd_i^{\sigma(t)} \Gamma^{\sigma(t)}(e)(x_i - x_0), \end{aligned} \quad (8)$$

where Coefficient  $c$  stands for the coupling strength.  $\Gamma^{\sigma(t)}(e) \in \mathbb{R}^{n \times n} > 0$  is polynomial matrix.  $d_i^{\sigma(t)} = 1$  if

and only if there exists a directed path from the leader to node  $i$ .

By substituting (8) into (7), the closed-loop error dynamic system is

$$\begin{aligned} \dot{e} &= \sum_{p=1}^r h_p(\phi(t)) \{ (A_p(e) + \mathcal{T}^{\sigma(t)})e + D_p w(t) \}, \\ z &= \sum_{p=1}^r h_p(\phi(t)) (C_{zp}e + D_{zp}w(t)), \end{aligned} \quad (9)$$

where

$$\begin{aligned} h_p(\phi(t)) &= \text{diag}\{h_p(\phi_1(t)), h_p(\phi_2(t)), \dots, h_p(\phi_N(t))\}, \\ A_p(e) &= \text{diag}\{a_p(e_1), a_p(e_2), \dots, a_p(e_N)\}, \\ \mathcal{T}^{\sigma(t)} &= c(\bar{\mathcal{L}}^{\sigma(t)} \otimes \Gamma^{\sigma(t)}(e)) \\ \bar{\mathcal{L}}^{\sigma(t)} &= -\mathcal{L}^{\sigma(t)} - \mathcal{D}^{\sigma(t)}, \\ \mathcal{D}^{\sigma(t)} &= \text{diag}\{d_1^{\sigma(t)}, d_2^{\sigma(t)}, \dots, d_N^{\sigma(t)}\}, \\ D_p &= \text{diag}\{d_p(e_1), d_p(e_2), \dots, d_p(e_N)\}. \end{aligned}$$

Before proceeding further, we give the following assumption and concepts for obtaining the main result.

*Assumption 1.* All  $\bar{\mathcal{G}}^{\sigma(t)}$  have one directed spanning tree.

*Definition 1.* (Shi and Yu (2020) Strictly Dissipative): Given matrices  $\mathcal{Y}^{l \times m}$ ,  $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{l \times l} \leq 0$ ,  $\mathcal{R} = \mathcal{R}^T \in \mathbb{R}^{m \times m} > 0$ , and  $\Phi = \Phi^T \in \mathbb{R}^{l \times l} \geq 0$ , the system (9) is said to be  $(\mathcal{Q}, \mathcal{Y}, \mathcal{R})$ -dissipative if

$$\begin{aligned} & \int_0^{T^*} \begin{bmatrix} z \\ w \end{bmatrix}^T \begin{bmatrix} I_N \otimes \mathcal{Q} & I_N \otimes \mathcal{Y} \\ * & I_N \otimes \mathcal{R} \end{bmatrix} \begin{bmatrix} z \\ w \end{bmatrix} dt \\ & \geq \sup_{0 \leq t \leq T^*} z^T (I_N \otimes \Phi) z, \end{aligned}$$

holds for any  $T^* \geq 0$ .

Furthermore, for a given number  $\delta > 0$ , if

$$\begin{aligned} & \int_0^{T^*} \begin{bmatrix} z \\ w \end{bmatrix}^T \begin{bmatrix} I_N \otimes \mathcal{Q} & I_N \otimes \mathcal{Y} \\ * & I_N \otimes \mathcal{R} \end{bmatrix} \begin{bmatrix} z \\ w \end{bmatrix} dt \\ & \geq \delta \int_0^{T^*} w^T w dt, \end{aligned} \quad (10)$$

then (9) is said to be strictly  $(\mathcal{Q}, \mathcal{Y}, \mathcal{R})$ - $\delta$ -dissipative.

*Definition 2.* Under Markov switching topologies and the consensus protocol (8), (1) is called consensus if for  $i = 1, 2, \dots, N$ ,

$$\lim_{t \rightarrow \infty} E \|x_i - x_0\| = 0$$

holds.

### 3. DISSIPATIVITY-BASED CONSENSUS DESIGN

Now, we give our main result.

*Theorem 1.* Given a positive number  $\delta$ , matrices  $\mathcal{Y}^{l \times m}$ ,  $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{l \times l}$  and  $\mathcal{R} = \mathcal{R}^T \in \mathbb{R}^{m \times m}$  with  $\mathcal{Q} \leq 0$  and  $\mathcal{R} > 0$ , suppose that the switching topologies modeling by Markov process, under Assumption 1, there exist polynomial matrices  $p^\alpha(e) \in \mathbb{R}^{n \times n} > 0$  and  $\hat{\Gamma}^\alpha(e) \in \mathbb{R}^{n \times n} > 0$ , arbitrary

vector  $\eta_1$ , nonnegative polynomial  $\varepsilon_1(e)$ , such that for  $\alpha = 1, 2, \dots, s$ ,  $p = 1, 2, \dots, r$ ,

$$-\eta_1(\Xi + \varepsilon_1(e)I)\eta_1 \text{ is SOS}, \quad (11)$$

then (9) is asymptotically even-square stable with a strictly dissipative performance, where

$$\Xi \triangleq \begin{bmatrix} \Psi_1 + \Pi & \Psi_2 \\ * & \Psi_3 \end{bmatrix},$$

$$\Pi = -C_{zp}^T (I_N \otimes \mathcal{Q}) C_{zp},$$

$$\begin{aligned} \Psi_1 &\triangleq \sum_{k=1}^n \frac{\partial P^\alpha(e(t))}{\partial e_k} (A_p^k(e) + (\mathcal{T}^{\sigma(t)})^k) e \\ &\quad + He(P^\alpha(e)A_p(e)) \\ &\quad + \sum_{\beta=1}^s \lambda_{\alpha\beta} (I_N \otimes p^\beta(e)), \end{aligned}$$

$$\Psi_2 \triangleq -C_{zp}^T (I_N \otimes \mathcal{Q}) D_{zp} - C_{zp}^T (I_N \otimes \mathcal{Y}) + P^\alpha(e) D_p,$$

$$\begin{aligned} \Psi_3 &\triangleq -D_{zp}^T (I_N \otimes \mathcal{Q}) D_{zp} - (I_N \otimes \mathcal{R}) \\ &\quad - D_{zp}^T (I_N \otimes \mathcal{Y}) - (I_N \otimes \mathcal{Y})^T D_{zp} + \delta I, \end{aligned}$$

$$P^\alpha(e) \triangleq I_N \otimes p^\alpha(e), \quad \hat{\Gamma}^\alpha(e) = cp^\alpha(e)\Gamma^\alpha(e).$$

$(\mathcal{T}^{\sigma(t)})^k$  and  $A_p^k(e)$  denote the  $k$ th row of  $(\mathcal{T}^{\sigma(t)})^k$  and  $A_p(e)$ , respectively.

*Proof 1.* Design polynomial Lyapunov function:

$$V(t) = e^T P^\alpha(e) e, \quad (12)$$

where  $P^\alpha(e) > 0$  is a polynomial matrix.

From  $p^\alpha(e) \in \mathbb{R}^{n \times n} > 0$  and  $\hat{\Gamma}^\alpha(e) \in \mathbb{R}^{n \times n} > 0$ , it follows that  $V(t) > 0$  and  $\Gamma(e) > 0$  at  $e \neq 0$ , respectively.

Weak infinitesimal operator  $\mathcal{F}$  of  $V(t)$  is

$$\mathcal{F}V(t) = \sum_{p=1}^r h_p(\phi(t)) \begin{pmatrix} 2e^T(t)P^\alpha(e)(A_p(e)e + \mathcal{T}^\alpha e + D_p w(t)) \\ + e^T \left( \sum_{k=1}^n \frac{\partial P^\alpha(e)}{\partial e_k} (A_p^k(e) + (\mathcal{T}^{\sigma(t)})^k) e \right) \\ + \sum_{\beta=1}^s \lambda_{\alpha\beta}(\theta) e^T(t) (I_N \otimes p^\beta(e)) e(t) \end{pmatrix} e \quad (13)$$

By (11) and SOS Lemma presented by Prajna et al. (2004), then we obtain  $\Xi < 0$  and  $\Psi_1 < 0$ , Hence,  $E\{\mathcal{F}V(t)\} < 0$ .

We will demonstrate strictly dissipative performance of (9). Under zero initial condition, for any  $T^* > 0$ , we denote

$$\begin{aligned} \mathcal{J}(T^*) &= \int_0^{T^*} \begin{bmatrix} z \\ w \end{bmatrix}^T \begin{bmatrix} I_N \otimes \mathcal{Q} & I_N \otimes \mathcal{Y} \\ * & I_N \otimes \mathcal{R} \end{bmatrix} \begin{bmatrix} z \\ w \end{bmatrix} dt \\ &\quad - \delta \int_0^{T^*} w^T w dt. \end{aligned} \quad (14)$$

For any nonzero  $w \in l_2[0, \infty)$ , we show

$$\begin{aligned} & EV(T^*) - V(0) - \mathcal{J}(T^*) \\ &= E \left\{ \sum_{p=1}^r h_p(\phi(t)) \int_0^{T^*} \zeta^T \Pi \zeta dt \right\}, \end{aligned} \quad (15)$$

where  $\zeta = [e^T \ w^T]^T$ . It follows from condition (11) that

$$E \left\{ \sum_{p=1}^r h_p(\phi(t)) \int_0^{T^*} \zeta^T \Xi \zeta dt \right\} < 0.$$

From  $EV(T^*) > 0$  and (15), we obtain

$$\mathcal{J}(T^*) > 0.$$

By Definition 1, (9) is strictly dissipative.

From (11), we have

$$\mathcal{FV}(t) < 0. \quad (16)$$

Therefore, there exist a scalar  $\epsilon > 0$ , such that

$$\mathcal{FV}(t) < -\epsilon e^T e. \quad (17)$$

Employing Dynkins formula, we get

$$E\{V(t)\} - V(0) < -\epsilon E \left\{ \int_0^t \|e(s)\|^2 ds \right\}. \quad (18)$$

That is

$$\lim_{T \rightarrow \infty} E \left\{ \int_0^T \|e(s)\|^2 ds \right\} < \infty.$$

Which means that  $\lim_{T \rightarrow \infty} E \|e(s)\|^2 = 0$ . By Definition 2, (1) reach consensus. This proof is complete.

*Remark 1.* In Theorem 1, for the sake of computational convenience, let  $P^\alpha(e) = P$ , where  $P$  denote the constant matrix. (9) is asymptotically even-square stable, that is, all agents subject to disturbance can reach consensus in even-square sense.

#### 4. ILLUSTRATIVE EXAMPLE

Here, considering a nonlinear multi-agent system, the switching topologies are depicted as Fig. 1. The  $i$ th agent is represented from Zhao et al. (2013)

$$\dot{x}_i(t) = f(x_i) + u_i, \quad (19)$$

where

$$f(x_i) = \begin{bmatrix} \mu(x_{i2} - x_{i1}) \\ \vartheta x_{i1} - x_{i1}x_{i3} - x_{i2} \\ x_{i1}x_{i2} - \lambda x_{i3} \end{bmatrix}. \quad (20)$$

The error system is

$$\dot{e}_i = \begin{bmatrix} -\mu & \mu & 0 \\ \vartheta - x_{i3} & -1 & e_{i1} - x_{i1} \\ x_{i2} & x_{i1} - e_{i1} & -\lambda \end{bmatrix} e + u_i \quad (21)$$

where  $\mu = 10$ ,  $\vartheta = 28$ ,  $\lambda = \frac{8}{3}$ . The fuzzy model of (21) is

established by

$$R^{qkm}: \text{If } x_{i1} \text{ is } M_1^q \text{ and } x_{i2} \text{ is } M_2^k \text{ and } x_{i3} \text{ is } M_3^m,$$

$$\text{then } \dot{e}_i = a_{qkm}(e_i)e_i + u_i,$$

where  $q, k, m = 1, 2$ .  $x_{i1}, x_{i2}$  and  $x_{i3}$  are the premise variables. Suppose  $\phi_1(t) \in [M_1^1, M_1^2]$ ,  $\phi_2(t) \in [M_2^1, M_2^2]$ , and  $\phi_3(t) \in [M_3^1, M_3^2]$ , where  $M_1^1 = -24$ ,  $M_1^2 = 24$ ,  $M_2^1 = -32$ ,  $M_2^2 = 32$ ,  $M_3^1 = -56$ ,  $M_3^2 = 56$ .

Suppose that the weight of each edge is 1. The Markov switching signal is described in Fig.2.

Let  $(\mathcal{Q}, \mathcal{Y}, \mathcal{R}) = (-0.8, -0.5, 1.2)$ ,  $w = 1.2e^{-0.1t} \cos t$ ,  $c = 4$ . Choose the transition rates as  $\lambda_{11} = -0.65$ ,  $\lambda_{12} = 0.65$ ,  $\lambda_{21} = 0.35$ ,  $\lambda_{22} = -0.35$ . From Theorem 1, for solving the SOS conditions by the SOSTOOLS in Prajna et al. (2004), we obtain the controller gains.

The error states are depicted in Fig.3. From the Fig.3, it can be seen that all agents achieve agreement.

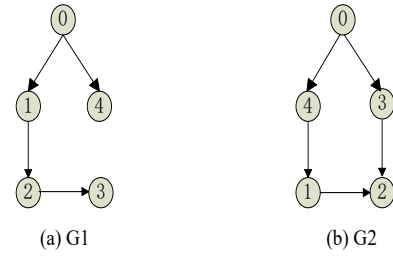


Fig. 1. Switching directed topologies.

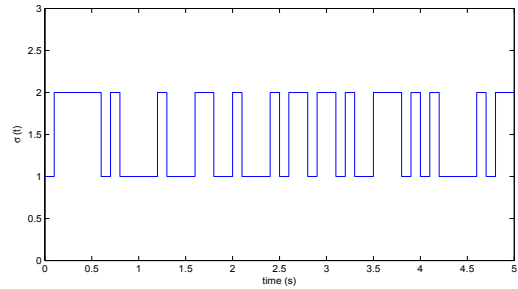


Fig. 2. Switching signal with two models.

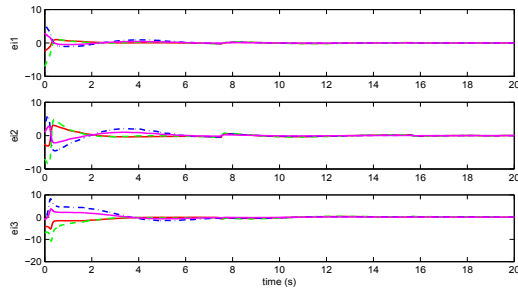


Fig. 3. The error states trajectories.

#### 5. CONCLUSION

This paper investigates strictly dissipative consensus issue of nonlinear MASs under Markov changing topologies. A novel fuzzy modeling method is presented. A new consensus scheme is proposed to assure that MASs can reach a even-square agreement. Simulation results have validated the effectiveness of presented design method.

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