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Hybrid Loewner Data Driven Control

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Abstract: This article describes how the Loewner framework can be exploited to create a discrete-time control-law from input-output frequency-data of a continuous-time plant so that their hybrid interconnection matches a given continuous-time reference model up to the Nyquist frequency. The resulting Hybrid Loewner Data Driven Control scheme is illustrated on two numerical examples.

Keywords: Loewner approximation, data-driven control, sampled-data system

1. INTRODUCTION

Data-Driven Control (DDC) methods (see e.g. Hou and Wang (2013) for an overview) enable to design a control-law solely based on input-output data from the system to be controlled. It does not require to explicitly identify a model of the plant first, contrary to model-based control techniques. Data-driven techniques can be classified according to their type of problem formulation (model-reference control, robust control or predictive and learning control).

In the present paper, the focus is put on the data-driven model-reference problem. In this case, the control objective is to design a controller such that the closed-loop matches a given reference model. This formulation allows to switch the identification problem from the plant to the controller by allowing to express the input-output data that the ideal controller should produce. Among these techniques, the CbT (Karimi et al. (2002)), the IFT (Hjalmarsson et al. (1994)) or the VRFT (Campi et al. (2002)) enforce a controller structure specified by the user, and then solve an optimization problem to get its parameters. Another approach consists in using approximation techniques directly on the data of the ideal controller, as it is the case in the LDDC approach, proposed in Kergus (2019). This last approach is based on the Loewner framework, see Mayo and Antoulas (2007).

In all the above mentioned data-driven model-reference techniques, the problem is generally considered either completely in continuous-time or completely in discrete-time. However, it is quite common in real-world control applications that the phenomenon to be controlled is known through continuous-time data while the control-law will eventually be implemented digitally on a computer. This is generally dealt with in a second step by discretising the control-law.

The objective of this paper is to show how the LDDC algorithm can be adapted to account for the hybrid nature of the interconnection and to embed directly the discretisation step. For that purpose, the LDDC method is coupled with the idea presented in Vuillemin and Poussot-Vassal (2019) for the discretisation of a LTI dynamical model. In particular, the frequency-domain representation of the ideal controller is adjusted to reflect the effect of the analog/digital converters leading to an irrational model that should be interpolated onto the unit disk.

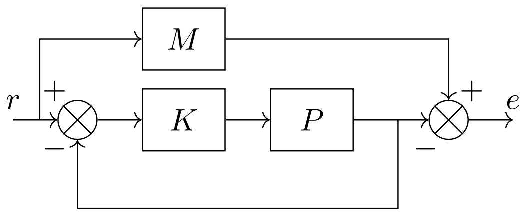

Fig. 1. Classical model-reference problem.

This article is organized in five sections. In Section 2, the LDDC technique and the discretisation proposed in Vuillemin and Poussot-Vassal (2019) are recalled. In Section 3, the hybrid control problem is formulated and an enhanced algorithm called HLDDC (Hybrid LDDC) is then introduced. The proposed approach is detailed and its differences with the original continuous-time only method are highlighted. The HLDDC method is then applied on two numerical examples in Section 4 before concluding remarks in Section 5.

2. PRELIMINARIES

2.1 The original LDDC approach

As said earlier, the LDDC approach proposed in Kergus (2019) is a model-reference technique. The problem is recalled on Figure 1: in established techniques, the triplet $\{P, M, K\}$ is considered exclusively either in continuous-time (as in the LDDC approach) or in discrete-time.

The pivotal concept in model-reference control is the ideal controller, which is the one rational LTI controller that would have given the desired reference-model behaviour if inserted in the closed-loop:

$$K^\star = P^{-1}M(I-M)^{-1}. \quad (1)$$

However when P is solely known through input-output data, $K^\star$ cannot be obtained as in (1). Instead, the idea in LDDC is to identify it from its input-output data.

Assume P is known through frequency-domain data $\{\omega_i, \Phi_i\}_{i=1}^N$ with $\Phi_i = P(j\omega_i)$. The LDDC method in Kergus (2019) exploits the Loewner approach Mayo and Antoulas (2007) to build a control-law K that matches the frequency response of the ideal one at ω_i, i.e.

$$K(j\omega_i) = K^\star(j\omega_i) = \Phi_i^{-1}M(j\omega_i)(I - M(j\omega_i))^{-1}. \quad (2)$$

Therefore, the LDDC approach can be summarized very simply as in Algorithm 1.

Remark 1. This very basic version of the LDDC algorithm only aims at recalling the general idea and does not tackle the challenges of model-reference control. Regarding how to handle noisy-data, the choice of the specifications and closed-loop stability enforcement, the reader can refer to Kergus (2019). It is also worth mentioning that the LDDC approach is suitable for SISO and MIMO systems, and that the order of the identified controller K is a tunable parameter and can be reduced during step 2 of Algorithm 1.

Algorithm 1 LDDC algorithm

Require: Data of the plant $\{\omega_i, \Phi_i\}_{i=1}^N$ and a reference model M
1: Compute the frequency response $K^\star(j\omega_i)$ of the ideal controller trough equation (1).
2: Apply the Loewner approach to the data set $\{j\omega_i, K^\star(j\omega_i)\}_{i=1}^N$ to obtain K.
3: **return** The continuous-time control law K.

2.2 Loewner-based discretisation

In Vuillemin and Poussot-Vassal (2019), a Loewner-based discretisation technique is proposed to improve the matching between a continuous-time LTI model and its discretised counterpart in comparison with standard discretisation methods like ZOH or Tustin.

Considering a continuous-time LTI model H, the approach consists in creating its discrete-time counterpart H_d by interpolating the continuous-time frequency-domain response filtered by the transfer function of the holder. More specifically, let $\{\omega_i\}_{i=1}^N$ be a set of frequencies below the Nyquist frequency associated with the sample time. In addition, let also suppose that $R(s)$ is the transfer function associated with the digital to analog converter (e.g. the ideal holder as in the sequel of the article). Then, based on the Loewner interpolating framework developed in Mayo and Antoulas (2007), H_d is built so that

$$H_d(e^{j\omega_i T}) = R(j\omega_i)H(j\omega_i). \quad (3)$$

Note that H_d is interpolating on the unit circle the filtered frequency response of H along the imaginary axis, thus accounting for the different natures of both systems.

The order of H_d is, to some extent, a free parameter that can be increased above the order of H to improve the matching (or conversely, decreased). Note however that undesired instabilities may appear and it may be necessary to project H_d onto a stable subspace to ensure its stability.

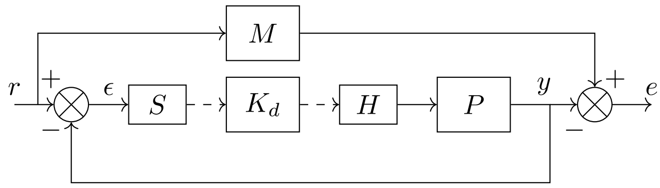

Fig. 2. Hybrid model-reference problem

3. HYBRID LOEWNER DATA DRIVEN CONTROL

The process used in the Loewner-based discretisation mentioned above is carried out in the present work and combined with the LDDC framework summarised in Algorithm 1 in order to obtain a data-driven direct hybrid design procedure.

3.1 Problem formulation

Let us consider a Single Input Single Output (SISO) LTI plant P and a given reference model M. The considered hybrid problem is shown on Figure 2. In comparison with the classical model-reference problem of Figure 1, a discrete-time controller K_d with a sampling period T is sought, instead of a continuous-time one K. The scheme is therefore completed by analog/digital converters that must be taken into account during control design: S and H are the ideal sampler and holder (see e.g. (Chen and Francis, 1995, chap.3)).

Such a mixed discrete/continuous interconnection is called a *sampled-data system* (see Chen and Francis (1995) and references therein for an overview) and requires dedicated tools to be studied. In particular, even if P, M and K_d are LTI, the overall interconnection is not. It is instead a T-periodic system [1] and has no transfer function thus preventing from using the equation (2) as in standard DDC.

However it is possible to express the relation between the Fourier transforms of r and e. In particular, using the frequency-domain relations for S and H detailed in (Chen and Francis, 1995, chap.3), the relation between the input ϵ of the sampler and the output y of the plant is

$$\hat{y}(j\omega) = \underbrace{P(j\omega)R(j\omega)K_d(e^{j\omega T})}_{F(j\omega)} \sum_{k\in\mathbb{Z}} \hat{\epsilon}(j\omega + jk\omega_s) \quad (4)$$

where $R(s) = \frac{1-e^{-sh}}{sh}$ and $\omega_s = 2\pi/T$ is the sampling frequency. Assuming that $\hat{\epsilon}$ is bandlimited [2] on $[-\omega_N, \omega_N]$, $\omega_N = \omega_s/2$, then for $|\omega| < \omega_N$, the usual feedback relation between the reference signal r and the output y is retrieved,

$$\hat{y}(j\omega) = (I + F(j\omega))^{-1}F(j\omega)\hat{r}(j\omega). \quad (5)$$

[1] A system G is T-periodic if $D_T G = G D_T$ where D_T is time-delay of length T. A LTI model is T-periodic for all T.

[2] The hypothesis is reasonable as in practice, a sampler is generally preceded by an anti-aliasing filter aimed at cutting the frequency contributions above the Nyquist frequency.

Therefore, the mismatch error between the closed-loop and the reference model satisfies, for $|\omega| < \omega_N$,

$$\hat{e}(j\omega) = \left(M(j\omega) - (I + F(j\omega))^{-1}F(j\omega)\right)\hat{r}(j\omega). \quad (6)$$

To minimise this mismatch, the ideal discrete-time control-law $K_d^\star$ should be such that for $|\omega| < \omega_N$,

$$M(j\omega) = (I + F(j\omega))^{-1}F(j\omega), \quad (7)$$

or equivalently,

$$K_d^\star(e^{j\omega T}) = (P(j\omega)R(j\omega))^{-1}M(j\omega)(I - M(j\omega))^{-1}. \quad (8)$$

Note that the ideal control-law (8) is irrational due to the transfer function of the ideal holder. The infinite number of interpolation conditions (8) is restricted to a finite number of interpolations conditions by sampling the frequency interval $[0, \omega_N]$. The sought discrete-time control-law K_d should therefore satisfies, for $i = 1, \ldots, N$,

$$K_d(e^{j\omega_i T}) = (\Phi_i R(j\omega_i))^{-1}M(j\omega_i)(I - M(j\omega_i))^{-1}. \quad (9)$$

A rational interpolant satisfying (9) can readily be created with the Loewner approach as illustrated in Vuillemin and Poussot-Vassal (2019) for the discretisation objective.

The two main differences with the LDDC framework of Kergus (2019) lie in the fact that the frequency response of the plant is filtered by the transfer function of the holder and that the control-law must match the data on the unit circle instead of the imaginary axis. The overall approach is summarised in Algorithm 2 in its simplest form. The following remarks can be made:

- The Multiple Input Multiple Output (MIMO) case can be handled as in the continuous-time LDDC approach by completing the interpolation conditions (9) with tangential directions to fit the Loewner framework.
- It should be noted that it is still possible to choose an achievable reference model as done in Kergus (2019), ensuring that the ideal controller stabilizes the plant internally.
- For practical implementation, the stability of the control-law is generally preferred. Yet, the Loewner framework does not ensure the stability of the resulting interpolating model. Therefore, as suggested in Gosea and Antoulas (2016), the resulting controller K_d may be projected onto a stable subspace (see e.g. Mari (2000) for the discrete-time Nehari problem). During this step, the error induced to the interpolation conditions (9) should be monitored as it gives insights concerning the mismatch between the closed-loop and the reference model.
- In Algorithm 2, the order of the resulting control-law K_d is determined by the Loewner approach and may thus results in a large dimension. For practical implementation, this may be an issue. In that case, an additional reduction step can be used. As in the previous point, the interpolation error must also be monitored during this step.
- Considering noisy data, it is possible to use alternative implementations of the Loewner framework that are more robust to noisy data, such as the one presented in Lefteriu et al. (2010) and used in Kergus (2019). More recent work on using the Loewner framework with noisy data can be found in Drmač and Peherstorfer (2019).

Algorithm 2 Hybrid LDDC (full-order SISO case)

Require: A sampling period $T > 0$, data of the plant $\{\omega_i, \Phi_i\}_{i=1}^N$ sampled within $[0, \omega_N]$, a reference model M

1: Compute the frequency response $\Psi_i = K_d^\star(e^{j\omega_i T})$ of the ideal discrete-time control-law based on equation (8).
2: Apply the Loewner approach to the data set $\{e^{j\omega_i T}, \Psi_i\}_{i=1}^N$ to obtain K_d
3: **return** the discrete-time control law K_d.

- The problematic of internal stability of the closed-loop is central in data-driven control. It can be addressed within the DDC process in continuous-time (see e.g. chap. 6-7 in Kergus (2019)). However here, due to the hybrid nature of the overall interconnection 2, the internal stability is not guaranteed and should be checked a posteriori with dedicated sampled-data systems theory tools (see e.g. Chen and Francis (1995)).

4. NUMERICAL ILLUSTRATION

This section shows the performances that may be achieved with the HLDDC approach on a DC motor model and a flexible transmission in comparison to the discretisation of the continuous-time controller obtained with the LDDC method.

4.1 DC motor

Let us consider the following plant model,

$$P(s) = \frac{0.01}{0.005s^2 + 0.06s + 0.1001} \quad (10)$$

for which we would like to design a control-law so that the closed-loop behaves as a fully damped second-order model with unitary static gain,

$$M(s) = \frac{1}{s^2 + 2s + 1} \quad (11)$$

In this simple case, the closed-loop is achievable and with 100 samples logarithmically spaced between 0.1 and 10^3, the LDDC approach Kergus (2019) enables to retrieve the ideal control-law $K^\star$ exactly,

$$K^\star(s) = \frac{0.5s^2 + 6s + 10.01}{s^2 + 2s} \quad (12)$$

The control-law $K^\star$ is discretised with the Tustin approach for the sampling period $T = 0.9s$ leading to K_d^{tus},

$$K_d^{tus}(z) = \frac{2.751z^2 + 1.607z - 0.09104}{z^2 - 1.053z + 0.05263}. \quad (13)$$

The latter is compared to the control-law K_d obtained with Algorithm (2) applied with 50 frequency samples logarithmically spaced between 0.1 and $0.95\omega_N$, completed with a projection onto a stable subspace and a balanced truncation to the same order as K_d^{tus}. The resulting controller is,

$$K_d(z) = \frac{4.244z^2 + 3.945z + 0.2366}{z^2 - 0.1295z - 0.8705} \quad (14)$$

The step responses of the reference model and the closed-loops obtained with K_d^{tus} and K_d are plotted in Figure

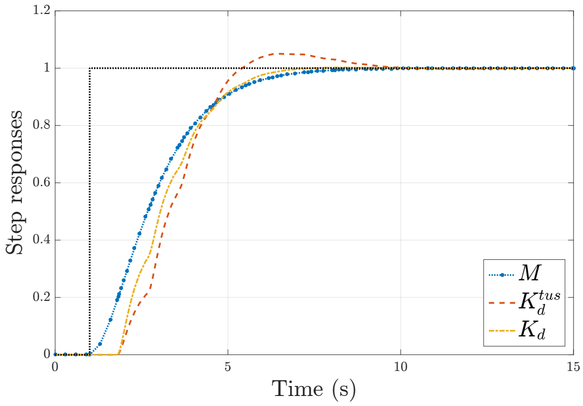

Fig. 3. Step responses of the reference model and the closed-loops obtained with the direct approach and the separated discretisation of the control-law for the DC-motor.

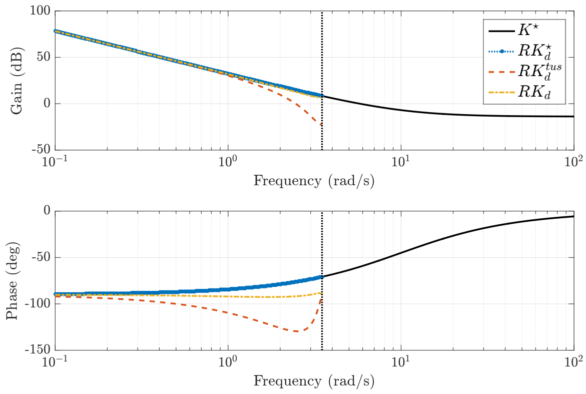

Fig. 4. Frequency responses of the ideal continuous-time control-law $K^\star$ and its discrete-time counterparts completed with the response of the ideal holder for the DC-motor.

3. Due to the large sampling period T, both discrete-time control-law achieve a degraded behaviour in comparison to M. While K_d^{tus} leads to overshoot in the response and a higher settling time, the controller K_d manages to maintain a closer response to the reference model without overshoot and with a comparable settling time.

From a frequency-domain perspective, the frequency responses of the different control-laws, completed with the transfer function R of the holder for the discret-time ones, are reported in Figure 4. First, one can notice that the ideal discrete-time control-law $K_d^\star$ matches exactly the continuous-time one up to the Nyquist frequency (vertical dashed bar). The HLDDC controller K_d matches almost exactly $K_d^\star$ in terms of gain but an error is noticeable in the phase. This mainly comes from the stable projection. Still, the discrepancy is smaller than with the Tustin discretised control-law K_d^{tus} which does not match accurately the gain of $K_d^\star$ either.

Note that for small enough sampling period T, both approaches leads to equivalent results. In that case, the advantage of the HLDDC mainly lies in the direct embedding of the discretisation step.

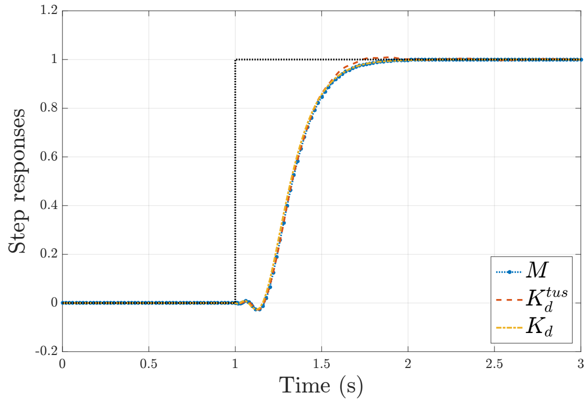

Fig. 5. Step responses of the reference model and the closed-loops obtained with the direct approach and the separated discretisation of the control-law for the flexible transmission.

4.2 Flexible transmission

Here one considers the following plant,

$$P(s) = \frac{0.03616(s-140.5)(s-40)^3}{(s^2+1.071s+157.9)(s^2+3.172s+1936)}. \quad (15)$$

As the plant is non-minimum phase, its closed-loop performances are limited and an arbitrary reference model may not be achievable while maintaining internal stability. In fact, the reference model must be selected with care so that the ideal controller $K^\star$ is stable and leads to internal stability of the closed-loop as detailed in (Kergus, 2019, chap. 4). The latter suggests a pre-treatment process to account for the performances limitations of the plant within the reference model, leading here to

$$M(s) = \frac{100(s-140.6)(s-37.39)(s^2-82.6s+1710)}{(s+10)^2(s+37.39)(s+140.6)(s^2+82.6s+1710)}. \quad (16)$$

The same comparison as before is performed for the sampling period $T = 0.02s$ leading to the following numerator/denominator coefficients for K_d^{tus},

$$num(K_d^{tus}) : \begin{bmatrix} 0.0436 \\ -0.0949 \\ 0.0163 \\ 0.1412 \\ -0.1515 \\ 0.0049 \\ 0.0837 \\ -0.0504 \\ 0.0091 \end{bmatrix}, \; den(K_d^{tus}) : \begin{bmatrix} 1.0000 \\ -3.6971 \\ 5.6519 \\ -4.5649 \\ 2.0349 \\ -0.4463 \\ 0.0039 \\ 0.0216 \\ -0.0039 \end{bmatrix} \quad (17)$$

and for K_d,

$$num(K_d) : \begin{bmatrix} 0.1035 \\ -0.0949 \\ -0.1132 \\ 0.1837 \\ -0.0076 \\ -0.1134 \\ 0.0292 \\ 0.0339 \\ 0.0033 \end{bmatrix}, \; den(K_d) : \begin{bmatrix} 1.0000 \\ -0.8853 \\ -1.4072 \\ 1.4678 \\ 0.4101 \\ -0.6735 \\ 0.0455 \\ 0.0637 \\ -0.0212 \end{bmatrix} \quad (18)$$

The step responses are reported in Figure 5. In that case, the difference between the two approaches is barely noticeable. This could be expected as a step mainly excites low frequencies and the frequency-domain responses of

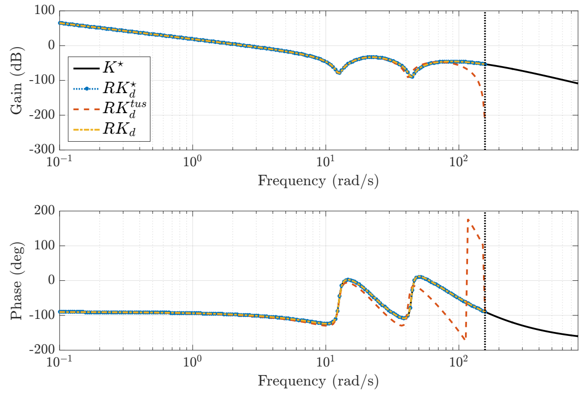

Fig. 6. Frequency responses of the ideal continuous-time control-law $K^\star$ and its discrete-time counterparts completed with the response of the ideal holder for the flexible transmission.

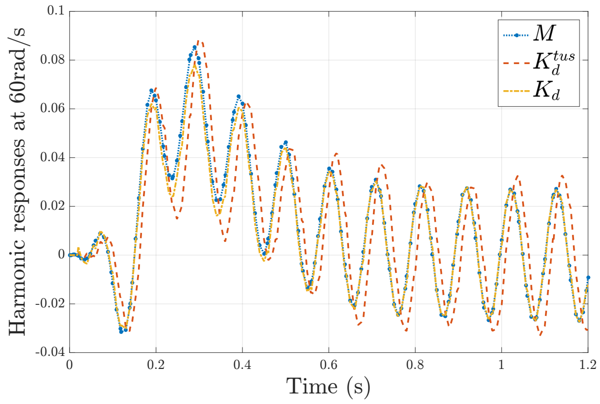

Fig. 7. Harmonic responses at 60rad/s of the reference model and the closed-loops obtained with K_d^{tus} and K_d for the flexible transmission.

the control-laws plotted in Figure 6 are indistinguishable below 20rad/s. For higher frequencies, one can observe that K_d still matches accurately the ideal controller while K_d^{tus} leads to a noticeable error, especially in terms of the phase. This is confirmed by looking at the harmonic responses of the closed-loops at 60rad/s in Figure 7 where the phase lag induced by K_d^{tus} appears clearly.

5. CONCLUSION

The Loewner Data-Driven Control approach presented in Kergus (2019) has been modified by *(i)* taking into account the transfer function of the (ideal) holder and by *(ii)* interpolating the frequency data onto the unit circle so that it can directly create a discrete-time control-law from continuous-time frequency data thus embedding the usual a posteriori discretisation step. From a reduction point of view, the approach consists in the approximation of an infinite-dimensional model with a finite-dimensional rational model on the unit disk.

As illustrated on two numerical examples, the resulting hybrid approach can be more effective than the standard a posteriori step consisting in discretising a continuous-time controller. This highlights the versatility of the Loewner approach which can be exploited to address problems that are not a priori linked to model approximation.

Still, the hybrid method remains quite sensitive to the projection onto a stable subspace. The latter step can have a detrimental effect on the accuracy of the interpolation. Indeed, the approach may have to cope with an unreachable reference model and/or spurious unstable poles may be present in the interpolating model. For instance, the projection may modify the steady state response which is an issue in tracking problems. This point may be addressed by using frequency weightings during the projection step to preserve some frequency bandwidth of interest.

Assessing the stability of the hybrid interconnection also remains a major issue and it is still not clear whether input-output frequency data are sufficient to estimate it as the inter-samples behaviour is discarded.

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