

Parameter Varying Approach For A Combined (Kinematic + Dynamic) Model Of Autonomous Vehicles^{*}

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Abstract: This paper proposes a solution for the integrated longitudinal and lateral control problem of autonomous vehicles. A mixed model including kinematic and dynamic behaviour of the vehicle is used to design a single controller to achieve stability and tracking performances. The proposed solution is based on the Linear Parameter Varying (LPV) control approach, where an output-feedback dynamical controller is designed based on Linear Matrix Inequalities (LMIs). The control synthesis is carried on using the gridded-based approach to reduce the conservatism. Simulation results show the stabilization of the vehicle with robustness in tracking performances, in the presence of the road friction coefficient disturbances.

Keywords: Autonomous vehicles, Time varying systems, \mathcal{H}_∞ Control, Linear multivariable systems, Disturbance rejection.

1 INTRODUCTION AND MOTIVATION

The field of autonomous vehicles has been expanding rapidly in the last decade, in order to meet road safety and environmental objectives. Autonomous driving aims to avoid accidents, reduce fuel consumption, improve traffic flow (Németh et al. (2017)). It also provides passenger comfort in critical situations and make it possible to car travelling for everyone regardless of their abilities or conditions. However, control of autonomous vehicles is not trivial since they are equipped with many sensors and actuators (Navas (2018)).

Several models (linear or not) can be considered for control design objectives. For example, in Paden et al. (2016), the authors neglect the dynamics of the vehicle for path-planning design. In Navas (2018), an LTI model is used including the dynamics of the vehicle, whereas in Alcala et al. (2018) and Alcala et al. (2019), the authors prefer to use a Linear Parameter Varying (LPV) bicycle model to track normal and racing car references, respectively. However, in the field of heavy vehicles, the vertical dynamics cannot be neglected, as observed in Vu et al. (2016). In our work, a more complete model, considering kinematic and dynamic equations, rewritten in the LPV form, is proposed. In addition to increasing accuracy, the LPV approach represents the nonlinear model without using linearization, allowing to use linear-like control design tools (see Sename et al. (2013)). In Wit et al. (2004) and Alcala

et al. (2018), the authors design a cascaded control scheme for the kinematic and the dynamic controls. This strategy adds some limitations and constraints into the control design. In this paper, the model is improved by mixing the kinematic and the dynamic bicycle models which will simplify the control scheme.

Vehicle control methodologies that include the vehicle, the path, and the driver are currently being developed at several research centers and automotive suppliers. The importance of the used control strategy lies in its simplicity and the achieved performance. The main objective in this work is to track an offline trajectory safely by minimizing the external disturbances and noises (from the driver, sensors, etc.), and dealing with uncertainties due to some neglected (or unmodeled) dynamics or fixed parameters. Referring to these demanded performances, robust control is a suitable technique to achieve our goal where it is able to deal with model uncertainties and disturbance rejection, see Scherer (2001). In Alcala et al. (2018), the authors use the LMI-based LPV-LQR/ \mathcal{H}_2 cascaded controllers with an LPV UIO observer to estimate the road friction force and achieve less tracking error. The controller is computed offline using the polytopic approach. Moreover, in Alcala et al. (2019), the authors apply the LPV-MPC approach, for a racing car, where the controller is computed online with reducing the computational cost. The main contributions of this study are as follows:

- Extend the work done in Alcala et al. (2018) by combining the kinematic and the dynamic bicycle

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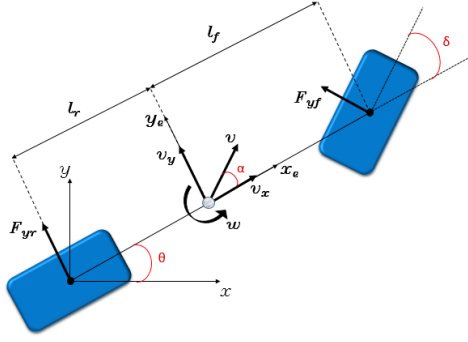


Fig. 1. Vehicle bicycle model

models which helps in simplifying the control scheme compared to the cascade form.

- Formulate a quasi-LPV model of the combined vehicle model with a parameter-varying input matrix and show why the gridded-based LPV approach is more preferable to be used than of the polytopic one.
- Design an LPV/ \mathcal{H}_∞ control for the combined model to achieve tracking and disturbance/noise rejection performances. Unlike the existing works, the gridded-based approach, is considered using a parameter-dependent Lyapunov function that decreases the possibility of conservatism.
- Simulation results, in normal and critical situations, are shown to ensure the vehicle stability.

2 MODEL FORMULATION

This section focuses on vehicle modelling for control design. The main dynamics which are usually considered in the vehicle model are the longitudinal, lateral and vertical dynamics which are well described in Pacejka (2005) and Rajamani (2011). Simplified models are used for control design such as the bicycle model (two-wheels model). However, to achieve the tracking performance at high speeds, the kinematic and the dynamic equations are required.

From Fig. 1 and referring to Pacejka (2005), Rajamani (2011), and Alcalá et al. (2019), the kinematic and dynamic models are derived below.

2.1 Kinematic Model

The kinematic model assumes that the tire-slip of the vehicle with the ground is null. This kind of model is mainly used at low speed systems such as mobile-robots as well as for the vehicles in automatic parking tasks, also it is very useful for motion planning (Paden et al. (2016)). It is derived geometrically where no forces are considered, as if the vehicle is a point-object at the vehicle's center of mass. It is based on the velocity vector movement in order to compute longitudinal and lateral velocities referenced to the global inertial frame. The kinematic equations are expressed as:

$$\begin{cases} \dot{x} = v_x \cos \theta \\ \dot{y} = v_x \sin \theta \\ \dot{\theta} = w, \end{cases} \quad (1)$$

where x , y and θ specifies the position in meters (m) and the orientation in radians (rad) respectively, with respect to the global frame (x , y). v_x and w represents

the longitudinal velocity in $\frac{m}{s}$ and the yaw rate in $\frac{rad}{s}$, respectively.

2.2 Dynamic Model

The consideration of the tire-slip in the model is vital for high speed vehicles and especially in the racing one. Thus, in such vehicles, the dynamic model is used to introduce the side-slip angle with the applied forces on the wheels (see Fig. 1). This model can better use the vehicle's capabilities for executing aggressive maneuvers that will be significant in planning motions with high accelerations and jerks. The dynamic model is derived using physical concepts by applying Newton's second law to the longitudinal and lateral motions of the vehicle (Pacejka (2005)), leading to:

$$\begin{cases} \dot{v}_x = \frac{-F_{yf} \sin \delta - \mu mg}{m} + wv_y + a \\ \dot{v}_y = \frac{F_{yf} \cos \delta + F_{yr}}{m} - wv_x \\ \dot{w} = \frac{F_{yf} l_f \cos \delta - F_{yr} l_r}{I}, \end{cases} \quad (2)$$

where v_x , v_y and w are the longitudinal, lateral and rotational velocities in the vehicle's frame. δ and a are the control inputs, the steering angle of the front tire and the longitudinal acceleration respectively. F_{yf} and F_{yr} are the lateral forces applied to the front and rear tires, respectively. I , m , l_f and l_r are the vehicle's inertia, mass and the distance from the center of gravity to the front and rear wheel axes respectively. μ and g are the friction coefficient and the gravity acceleration constant respectively.

F_{yf} and F_{yr} can be modelled using Pacejka's tire model (Pacejka (2005)) as follows:

$$\begin{aligned} F_{yf} &= c_3 \sin(c_2 \tan^{-1}(c_1 \alpha_f)); \quad \alpha_f = \delta - \tan^{-1}\left(\frac{v_y}{v_x} + \frac{l_f w}{v_x}\right), \\ F_{yr} &= c_3 \sin(c_2 \tan^{-1}(c_1 \alpha_r)); \quad \alpha_r = -\tan^{-1}\left(\frac{v_y}{v_x} - \frac{l_r w}{v_x}\right), \end{aligned} \quad (3)$$

where c_1 , c_2 and c_3 are constants that should be empirically determined. It is worth mentioning that the above tire model shows a nonlinear-dependency of F_{yf} and F_{yr} with respect to α . Thus, assuming small α , the equations can be reduced to:

$$\begin{aligned} F_{yf} &= C_f \left(\delta - \frac{v_y}{v_x} - \frac{l_f w}{v_x} \right), \\ F_{yr} &= C_r \left(-\frac{v_y}{v_x} + \frac{l_r w}{v_x} \right), \end{aligned} \quad (4)$$

where C_f and C_r represent the stiffness of the front and rear wheel-tires respectively.

The specifications of the racing car and the road used in this work are shown in the following table:

$l_f = 0.902 \text{ m}$	$C_f = 17974 \frac{N}{rad}$
$l_r = 0.638 \text{ m}$	$C_r = 24181 \frac{N}{rad}$
$m = 196 \text{ Kg}$	$\mu = 0.5$
$I = 93 \text{ Kg.m}^2$	$g = 9.81 \frac{m}{s^2}$

2.3 LPV Model Formulation

There are many possibilities to formulate a nonlinear system as an LPV state-space representation, where one should first choose the parameters of the system. The parameters can be state-dependent (that results in quasi-LPV system), or they can be time-dependent (Boyd et al. (1994)).

In this paper, the kinematic-error model is used for the control design to achieve tracking performance. Since the equations in (1) were derived in the global frame, the following rotation matrix is used to write them in terms of the vehicle's frame as follows:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix} \quad (5)$$

where x_d , y_d and θ_d specify the position and the yaw angle of the plan to be tracked, respectively. x_e , y_e and θ_e represent the longitudinal, lateral and rotational errors, respectively, in the vehicle's frame. From (1), and using the non-holonomic constraint of the rear wheels of the form: $\dot{x} \sin(\theta) = \dot{y} \cos(\theta)$, the derivative of (5) gives the kinematic-error model differential equations as follows:

$$\begin{cases} \dot{x}_e = w y_e + v_{x_d} \cos \theta_e - v_x \\ \dot{y}_e = -w x_e + v_{x_d} \sin \theta_e \\ \dot{\theta}_e = w_d - w, \end{cases} \quad (6)$$

where v_{x_d} and w_d are the desired longitudinal and rotational velocities to be reached by the vehicle, respectively.

To avoid a cascaded kinematic and dynamic controls scheme, both models are mixed to formulate the following quasi-LPV state-space representation, considering $\rho(t) = [v_x \ w \ \delta \ \theta_e \ v_{x_d}]^T$:

$$G(\rho) \begin{cases} \dot{x}(t) = A_{lpv}(\rho)x(t) + B_{lpv}(\rho)u(t) + E v(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (7)$$

where:

$$x(t) = \begin{bmatrix} v_x \\ v_y \\ w \\ x_e \\ y_e \\ \theta_e \end{bmatrix}, u(t) = \begin{bmatrix} \delta \\ a \end{bmatrix}, B_{lpv}(\rho) = \begin{bmatrix} B_{11} & 1 \\ B_{21} & 0 \\ B_{31} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_{lpv}(\rho) = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & 0 & 0 & 0 \\ 0 & A_{32} & A_{33} & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & w & 0 \\ 0 & 0 & 0 & -w & 0 & v_{x_d} \frac{\sin(\theta_e)}{\theta_e} \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$v(t) = \begin{bmatrix} v_{x_d} \cos(\theta_e) \\ w_d \end{bmatrix}, \quad (8)$$

and:

$$\begin{aligned} A_{11} &= \frac{-\mu g}{v_x}, \quad A_{12} = \frac{C_f \sin \delta}{m v_x} + w, \quad A_{13} = \frac{C_f l_f \sin \delta}{m v_x}, \\ A_{22} &= -\frac{C_r + C_f \cos \delta}{m v_x}, \quad A_{23} = -\frac{C_f l_f \cos \delta - C_r l_r}{m v_x} - v_x, \\ A_{32} &= -\frac{C_f l_f \cos \delta - l_r C_r}{I v_x}, \quad A_{33} = -\frac{C_f l_f^2 \cos \delta + l_r^2 C_r}{I v_x}, \\ B_{11} &= \frac{-1}{m} C_f \sin(\delta), \quad B_{21} = \frac{1}{m} C_f \cos(\delta), \\ B_{31} &= \frac{1}{m} C_f l_f \cos(\delta). \end{aligned} \quad (9)$$

Notice that the system is not affine with respect to the varying parameters. An affine model could be obtained but with the price of reaching 11 varying parameters. The choice of parameter's extremums is done depending on the

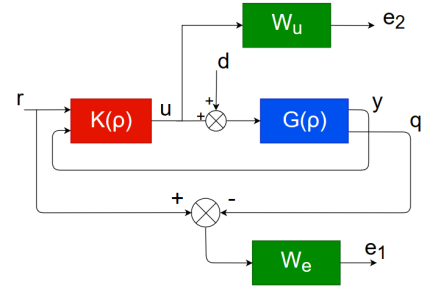


Fig. 2. Control scheme

specifications of the vehicle and the objectives in terms of trajectory tracking. Here, they are selected as follows:

$$\begin{aligned} v_x, v_{x_d} &\in [1, 18], \quad w \in [-2.5, 2.5], \\ \delta &\in [-0.3, 0.3], \quad \theta_e \in [-0.06, 0.06]. \end{aligned} \quad (10)$$

3 LPV/ \mathcal{H}_∞ CONTROL DESIGN

The LPV/robust control approach is well known in the literature and it is used in several systems, as seen in Mohammadpour and Scherer (2012) and Sename et al. (2013). Fig. 2 introduces a two-degree-of-freedom control scheme which presents an output-feedback parameter varying controller $K(\rho)$ designed by solving an LPV/ \mathcal{H}_∞ control problem of the LPV model $G(\rho)$. The first step is to choose the required performances using frequency domain weighting functions. For tracking, W_e is usually introduced on the tracking error. Another weight W_u is used to minimize the control effort (see Fig. 2). The signals in y are used to be available measurements in vehicles by making use of sensors like GPS, IMU and wheel encoders. However, the lateral velocity v_y is not directly available by a sensor. Notice that:

$$\begin{aligned} r &= [v_{x_d} \ w_d]^T, \quad u = [\delta \ a]^T, \quad q = [v_x \ w]^T, \\ y &= [v_x \ w \ x_e \ y_e \ \theta_e]^T, \quad d = [d_1 \ d_2]^T, \end{aligned} \quad (11)$$

3.1 Tracking specification (using W_e)

Considering that the variables to be controlled are the longitudinal velocity v_{x_d} and the yaw rate w_d , two weighting transfer functions are chosen of the form:

$$W_e(s) = \frac{\frac{s}{M_s} + w_b}{s + w_b \epsilon} \quad (12)$$

where the parameters M_s , w_b and ϵ are tuned as follows:

- $M_s = 2$, to ensure robustness at any frequency.
- $w_b \geq 10$, to get fast tracking (short rise-time).
- $\epsilon \leq 10^{-4}$, it represents the steady-state tracking error.

3.2 Specification on the control input limitations (W_u)

Additionally two weighting transfer functions are included for the control inputs δ and a to minimize the actuator control effort. A second order weighting function W_u is used to achieve the trade-off between control action and sensitivity to noises:

$$W_u(s) = \left(\frac{s + \frac{w_{b_u}}{\sqrt{M_u}}}{\sqrt{\epsilon_u} s + w_{b_u}} \right)^2. \quad (13)$$

The parameters M_u , w_{b_u} and ϵ_u are chosen as follows:

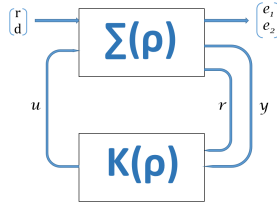


Fig. 3. General control configuration

- $M_u = \frac{\Delta u_{max}}{\Delta r_{max}}$, represents the limitations on the maximum allowed effort of the actuators.
- w_{b_u} , is related to the actuator bandwidth.
- $\epsilon_u \leq 10^{-2}$, it establishes the noise rejection from the control inputs at high frequencies.

3.3 Generalized Plant

From the previous steps, the general configuration is obtained as shown in Fig. 3. The state vector of $\Sigma(\rho)$ is $x_\Sigma = [x \ x_{W_e} \ x_{W_u}]^T$, and the controlled output $z = [e_1 \ e_2]^T$ represents the objective function to be optimized when designing the controller. The state-space representation of the parameter-varying generalized plant $\Sigma(\rho)$ is structured as follows:

$$\begin{bmatrix} \dot{x}_\Sigma \\ e_1 \\ e_2 \\ y \\ r \end{bmatrix} = \begin{bmatrix} A_\Sigma(\rho) & B_{\Sigma_{11}}(\rho) & B_{\Sigma_{12}}(\rho) & B_{\Sigma_2}(\rho) \\ 0 & W_e & 0 & -W_e G(\rho) \\ 0 & 0 & 0 & W_u \\ 0 & 0 & 0 & G(\rho) \\ 0 & I_{2 \times 2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_\Sigma \\ r \\ d \\ u \end{bmatrix}. \quad (14)$$

The controller $K(\rho)$ chosen here is an LPV Dynamic Output-Feedback Controller.

Three popular LPV control approaches can be found in the literature which are: 1) Linear Fractional Transformation (LFT) approach (see Packard (1994)); 2) Polytopic approach which is defined in Apkarian et al. (1995) as a convex combination of the LTI systems defined at the vertices of the polytope obtained by the varying parameter bounds; and 3) Grid-based approach which characterizes a set of LTI models that are linearized at different operating points. Although the polytopic approach is considered to be more popular than the others, however the grid-based approach is used instead because of the following points:

- As can be seen in (8), the control matrix B_{lpv} is parameter-dependent that can not be handled when using the polytopic approach. A fast pre-filter must be used to shift the input parameter varying functions to the state matrix as explained in Apkarian et al. (1995)
- The usage of the polytopic approach shall require the affinity of the system with respect to the parameters, which can be obtained at the cost of 11 varying parameters.
- Singularities are present in the polytopic domain at $\rho = [v_x, w = 0, \delta, \theta_e = 0, v_{x_d}]^T$, which prevent finding a constant Lyapunov function, that solves the LPV/ \mathcal{H}_∞ LMIs.

Finally, as explained in (Wu (1995)) the gridding approach considers a parameter dependent Lyapunov function, which is less conservative.

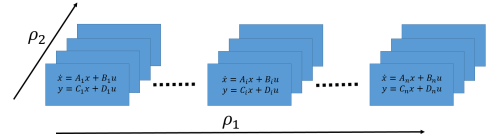


Fig. 4. 2-D grid space of two parameters

3.4 Implementation of the Gridded-based LPV Approach

The grid-based LPV model of a system is a group of linearized systems along the grid-points (see Fig. 4), i.e. at each grid-point ρ_i , there exist a corresponding LTI system $\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$.

The implementation of this approach is done either by using *LPVToolsTM* or by solving the LMI equations in *Theorem 4.3.1* in Wu (1995). *LPVToolsTM* is a *MATLAB* toolbox dedicated to LPV systems. The toolbox contains data structures that can represent both LFT and gridded-based approaches (Hjartarson et al. (2015)). For this approach, the rate-bounds of the parameters are considered to be used as shown below:

$$\begin{aligned} \dot{v}_x, \dot{v}_{x_d} &\in [-4, 4], \quad \dot{w} \in [-3, 3], \\ \dot{\delta} &\in [-0.6, 0.6], \quad \dot{\theta}_e \in [-0.1, 0.1] \end{aligned} \quad (15)$$

Moreover, a basis function is chosen that describes the dependency of the Lyapunov function on the parameters (linear, polynomial, exponential,...). A linear dependency is chosen to limit the problem complexity, i.e. the parameter-varying Lyapunov function is introduced as:

$$P(\rho) = P_0 + v_x P_1 + w P_2 + \delta P_3 + \theta_e P_4 + v_{x_d} P_5 \quad (16)$$

where P_0, P_1, P_2, P_3, P_4 and P_5 are constant matrices to be obtained from the LMI equations.

As a result, the controller $K(\rho)$ is designed offline and an array of controllers is obtained from the *lpvsyn* function from *LPVToolsTM*. At each control iteration, the instantaneous dynamic controller is linearly interpolated in the gridded space domain of parameters.

4 SIMULATION RESULTS

Simulations have been performed using the nonlinear vehicle model (1)-(2)-(3). Indeed, the nonlinear tire model (3) considers a more accurate model, where parameters c_1, c_2 , and c_3 define the shape of the semi-empirical curve. Moreover, a more accurate computation of the tire-slip angles is given. Fig. 5 shows the chosen scenario, this plan is integrated to a racing vehicle and is obtained by integrating the longitudinal and lateral accelerations. In Fig. 6, the steady-state tracking errors are observed to be very small which ensures a perfect tracking performance. Notice that the noise in the longitudinal velocity error is caused by a noisy longitudinal velocity reference. Moreover, the control inputs δ and a are represented in Fig. 7 and Fig. 8. It is observed that both control actions are smooth and limited to be applied for real actuators.

4.1 Comparison Between LTI and LPV Controls

The trade-off between the accuracy of LPV systems and the simplicity of using LTI systems encouraged us

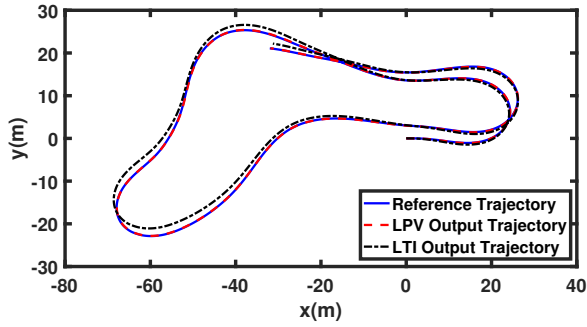


Fig. 5. Reference and controlled trajectories

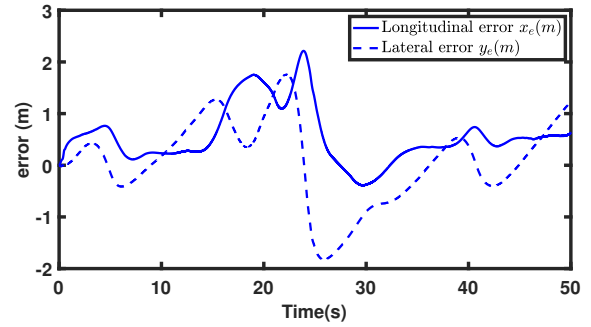


Fig. 9. LTI longitudinal and lateral errors

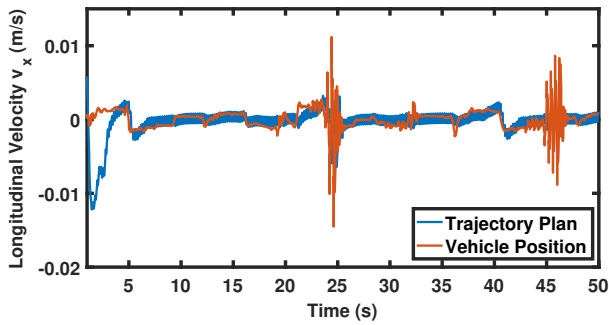


Fig. 6. Steady-state tracking errors

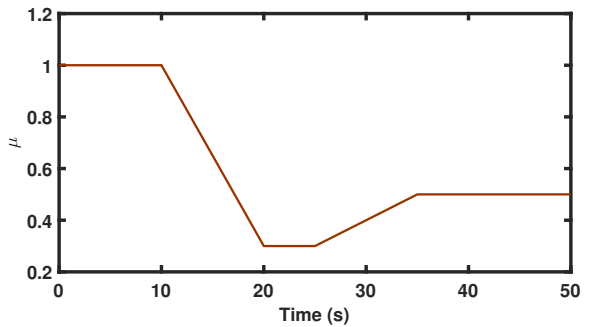


Fig. 10. Varying friction road coefficient μ

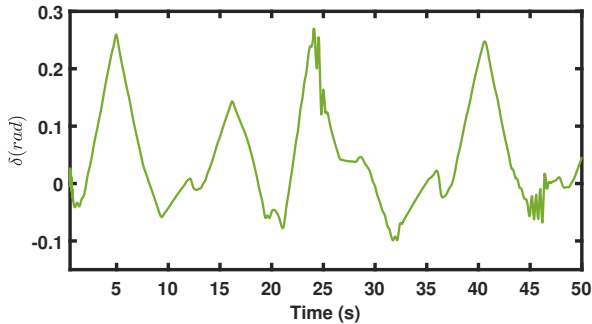


Fig. 7. Input steering angle

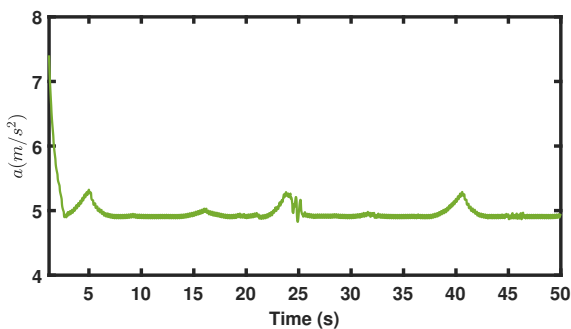


Fig. 8. Input acceleration

to compare both results. An LTI controller is designed at a specific grid point of the gridded space $\rho_{LTI} = [7, 0.5, 0.1, 0, 7]^T$. The chosen values of the parameters are considered to be the approximation mean, which results the most reliable LTI controller. The error of the LTI control scheme is clearly observed in Fig. 5, unlike

the LPV system which is exactly covering the reference. Fig. 9 shows the longitudinal and the lateral errors of the simulated LTI system (at ρ_{LTI}) to prove how significant the errors are when using LTI control.

4.2 Disturbance Rejection Analysis

This analysis observes the behaviour of the vehicle when being disturbed by road friction coefficient (μ) uncertainty. To do this, a simulation is done with a varying μ , then compare it with the constant μ case, following the scenario in Fig. 10.

As shown in Fig. 11 and 12, the controller achieves successfully the demanded tracking performance. The small longitudinal error represents how the controlled vehicle is respecting the demanded longitudinal distance, whereas the negligible lateral error ensures that the vehicle is always on the demanded lane especially at the strict maneuvers. As a result, the controller perfectly rejects the disturbance caused by different road conditions. It preserves vehicle stability with good lateral performance.

4.3 Stability Relative To Side-Slip Motion

This part of the section discusses the stability of the vehicle relative to its side-slip motion. As usual in automotive applications, the analysis of a phase-plane ($\alpha - \dot{\alpha}$) is performed to observe the stability of the vehicle. The stability criterion is formulated as:

$$\mathcal{X} < 1, \quad \text{Where } \mathcal{X} = |2.49\dot{\alpha} + 9.55\alpha| \quad (17)$$

\mathcal{X} is called the "Stability Index", see (Doumiati et al. (2013)). Notice that α is hard to be measured by sensors, so it is usually estimated. This work doesn't focus on the

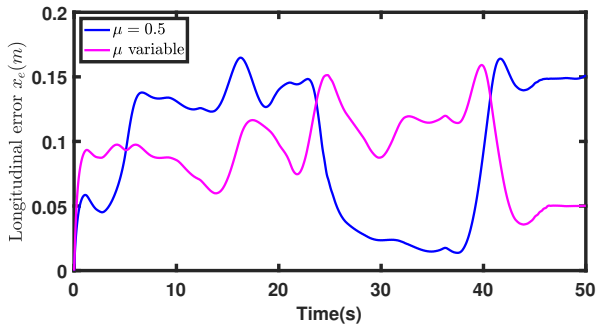


Fig. 11. Longitudinal error

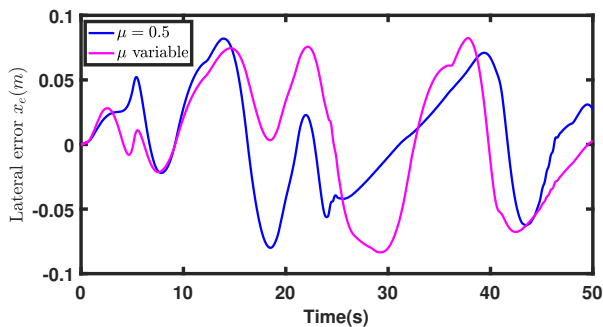


Fig. 12. Lateral error

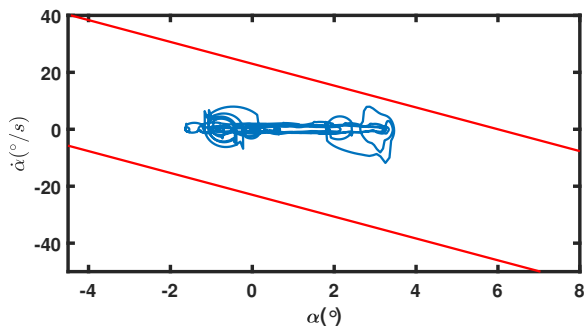


Fig. 13. phase-plane ($\alpha - \dot{\alpha}$)

estimation, it uses \mathcal{X} only to analyse the vehicle's stability from the resulting side-slip motions. Regarding Fig. 13, it is clear that the side-slip motion of the vehicle is always within the stability boundaries, which ensures that there is no unstable slipping of the vehicle during the whole trajectory. Notice that, for the LTI controller, the side-slip motion does not respect the stability limits.

5 CONCLUSIONS

In this paper, an LPV controller has been designed for an autonomous vehicle task, considering a mixed kinematic and dynamic model, and using the grid-based design method. The frequency domain analysis and time-domain simulations on a non-linear model have shown the efficiency and interest of the proposed approach. The results are very attractive comparing them with the results shown in the literature when using a cascaded polytopic approach controller or even MPC (Alcala et al. (2019)).

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