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Model Predictive Control of Wave Energy Converters With Prediction Error Tolerance ⋆

Yao Zhang * **Siyuan Zhan** ** **Guang Li** ***

* *School of Engineering and Materials Science, Queen Mary University of London (e-mail: yao.zhang@qmul.ac.uk)*
** *School of Aeronautical and Automotive Engineering, Loughborough University (e-mail: s.zhan@lboro.ac.uk)*
*** *School of Engineering and Materials Science, Queen Mary University of London (e-mail: g.li@qmul.ac.uk)*

Abstract: Sea wave energy converter (WEC) control is a non-causal optimal control problem, and the control performance relies on the accuracy of the prediction of incoming wave profile and the fidelity of the control-oriented model. To maximize energy conversion in real scenario, three issues should be fully considered: (a) the existing wave prediction methods inevitably introduce prediction errors, which degrades the control performance; (b) the model mismatch between the linearized state-space model and the hydrodynamic model also affects the control performance; (c) safe operations with limited power take-off (PTO) should be ensured to rule out the possibility of device damages. To explicitly deal with these problems, this paper proposes a novel control scheme to maximize the energy output subject to inaccurate predictions, model mismatch and multiple constraints. This is achieved by applying a feedback model predictive control (MPC) to handle the constraints and a compensator to cope with the prediction error and model mismatch. Due to the extra compensation input, the state and input constraints of MPC subsystem are further tightened to ensure constraints on both the states and the control input to be satisfied. Theoretical proof and simulation results show that the proposed controller is robust to achieve the maximal energy output subject to inaccurate prediction and inaccurate control-oriented model.

Keywords: Wave Energy Converters (WECs), Model Predictive Control (MPC), Compensation, Wave Prediction, Model Mismatch.

1. INTRODUCTION

Sea waves provide untapped renewable energy with high energy density. It is reported that there are roughly 7∼10 gigawatts (GWs) of power in the ocean waves within the UK and roughly 25 trillion watts (TWs) of power in ocean waves worldwide (Thorpe et al. (1999)). Many types of wave energy converters (WECs) have been developed to harness wave energy, such as point absorbers, overtopping WECs and attenuators, etc.

It has been long recognized that control plays an important role in maximizing energy conversion efficiency. WEC control is essentially a non-causal control problem (Falnes (2002); Ringwood et al. (2014)), in which the control input is determined by not only the current states of a WEC but also the future information of the wave profile. To achieve non-causal control of WECs, several prediction methods have been proposed to provide incoming wave profile for the controller. The existing wave prediction methods are mainly divided into two categories. The first category of prediction methods are based on the statistical methods, e.g. the Auto-Regressive (AR) (Garcia-Abril et al. (2017))and the extended Kalman Filter (EKF) (Fusco and Ringwood (2010)). The second category of prediction methods are accomplished by wave elevation measurements with certain distances away from the WEC, e.g. the deterministic sea wave prediction (DSWP) (Abusedra and Belmont (2011)), which provides more reliable wave prediction but introduces extra hardware for measurements. Although these prediction methods have been verified to be effective, the prediction error is introduced inevitably (Fusco and Ringwood (2011)) due to the measurement noise, etc. and needs to be properly coped with.

For the non-causal control method, it has been developed with a variety of approaches (Faedo et al. (2017)), such as adaptive control (Davidson et al. (2018); Zhan et al. (2018)), pseudo-spectral control (Li (2017); Mérigaud and Ringwood (2017)), constrained optimal control (Zhan and Li (2018)), etc. Due to its unique ability to handle multiple constraints, model predictive control (MPC) has also been widely studied, e.g. Hals et al. (2011); Brekken (2011); Li and Belmont (2014). Since the control-oriented model is normally obtained by model order reduction techniques

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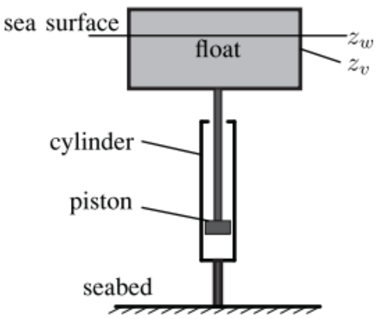

Fig. 1. Schematic diagram of the point absorber

and wave force approximations (Yu and Falnes (1995)), the model mismatch between the control-oriented model and the hydrodynamic model is unavailable and needs to be fully considered.

The novelties of the proposed control scheme are as follows:

- The prediction error is explicitly coped with to maintain the maximal energy output subject to inaccurate predictions;
- The model mismatch is handled in a straightforward manner so that the MPC can be designed based on a simplified model, which reduces the complexity of controller design and online computation load;
- Both the state constraints, including the heave position and velocity of the float, and the input constraint are satisfied with guaranteed recursive feasibility;
- The proposed control scheme has low computational burden so that it can be efficiently implemented in real-time.

The remaining of this paper is organized as follows. The WEC dynamic model and a simplified second-order model are introduced in Section 2, where physical constraints for WEC optimization problem are stated and a feedback MPC is briefly revisited. The compensator based MPC control scheme is proposed in Section 3 with theoretical proof of stability. Section 4 shows simulation results. Section 5 concludes this paper.

2. PROBLEM PRELIMINARY

In this section, a state-space model of the point absorber is introduced. Physical constraints of the point absorber are presented. The optimization problem of energy maximization is formulated. An existing feedback MPC with perfect prediction and accurate model is briefly introduced, which solves the optimization problem subject to multiple constraints.

2.1 WEC dynamic modelling

A particular type of WEC called single point absorber is chosen as an example to show the efficacy of the proposed control method. Fig. 1 shows part of a possible hydraulic power take-off (PTO) design: a hydraulic cylinder is vertically installed below the float and is fixed to the bottom of the seabed; one possible realization of this design can be found in Weiss et al. (2012). z_w and z_v are the water level and the height of the mid-point of the float respectively. The PTO torque is proportional to the force f_u acting on the piston inside the cylinder. The extracted power is $P := -f_u v$, where the velocity on the piston is $v := \dot{z}_v$.

According to Newton's second law, the dynamic equation (Yu and Falnes (1995)) for the float of the point absorber is

$$m_s \ddot{z}_v = -f_s - f_r + f_e + f_u \tag{1}$$

where m_s is the float mass; the restoring force f_s is given by

$$f_s = k_s z_v \tag{2}$$

with the hydrostatic stiffness $k_s = \rho g s$, and ρ as water density, g as standard gravity, and s as the cross-sectional area of the float. f_r is the radiation force determined by

$$f_r = m_\infty \ddot{z}_v + \int_{-\infty}^{\infty} h_r(\tau)\dot{z}_v(t-\tau)d\tau \tag{3}$$

where m_∞ is the added mass; h_r is the kernel of the radiation force that can be computed via hydraulic software packages (e.g. WAMIT Lee (1995)). Following Yu and Falnes (1995), the convolutional term in (3) $f_R := \int_{-\infty}^{\infty} h_r(\tau)\dot{z}_v(t-\tau)d\tau$ can be approximated by a causal finite dimensional state-space model

$$\dot{x}_r = A_r x_r + B_r \dot{z}_v \tag{4}$$

$$f_r = C_r x_r \approx \int_{-\infty}^{t} h_r(\tau)\dot{z}_v(t-\tau)d\tau \tag{5}$$

where $(A_r, B_r\ C_r, 0)$ and x_r are the state-space realisation and the state respectively. Following Yu and Falnes (1995), the wave excitation force f_e can be determined by

$$f_e = \int_{-\infty}^{\infty} h_e(\tau) z_w(t-\tau)d\tau \tag{6}$$

where h_e is the kernel of the excitation force and the state-space approximation is given by

$$\dot{x}_e = A_e x_e + B_e z_w \tag{7}$$

$$f_e = C_e x_e \approx \int_{-\infty}^{t} h_e(\tau) z_w(t-\tau)d\tau \tag{8}$$

where $(A_e, B_e\ C_e, 0)$ and x_e are the state-space realization and the state respectively.

With the realizations of (4) and (7) and by approximations of the convolution terms of the radiation force and excitation force, i.e. $f_R = D_r \dot{z}_v$ and $f_e = D_e z_w$ with D_r and D_e being the radiation coefficient and the excitation coefficient respectively, a second-order model (Li and Belmont (2014)) can be obtained as follows

$$\begin{cases} \dot{x} = A_c x + B_{uc} u + B_{wc} w + \epsilon \\ y = C_c x \end{cases} \tag{9}$$

where

$$A_c = \begin{bmatrix} 0 & 1 \\ -\frac{k_s}{m} & -\frac{D_r}{m} \end{bmatrix}\ B_{wc} = \begin{bmatrix} 0 \\ \frac{D_e}{m} \end{bmatrix}\ B_{uc} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}\ C_c = [0\ 1]$$

where $m := m_s + m_\infty$, and $w := z_w$ is the wave elevation whose prediction is incorporated into the controller design, $y := \dot{z}_v$, $x := [z_v, \dot{z}_v]$, $u := f_u$. ϵ represents the matched modelling mismatch caused by approximations of radiation force (5) and excitation force (8). The continuous-time model (9) can be converted to a discrete time model

$$\begin{cases} x(k+1) = Ax(k) + B_u u(k) + B_w w(k) + \epsilon(k) \\ y(k) = Cx(k) \end{cases} \tag{10}$$

where the quadruple (A, B_u, B_w, C) is the discrete-time form of the quadruple $(A_c, B_{uc}, B_{wc}, C_c)$.

2.2 Physical Constraints

To ensure safe operations, multiple constraints are considered in this paper. The state constraints on the heave position and heave velocity of the float, which can be expressed by

$$|z_v| \leq z_{max} \tag{11}$$

and

$$|\dot{z}_v| \leq v_{max} \tag{12}$$

where $z_{max} > 0$ and $v_{max} > 0$ are maximal heave displacement and heave velocity respectively, which are constants.

Since the PTO has its limitation, the control input constraint is

$$|f_u| \leq u_{max} \tag{13}$$

where $u_{max} > 0$ denotes the maximal force produced by the PTO mechanism.

Hypothesis 1. The wave elevation and the prediction error at each step are bounded, i.e. $|w| \leq w_{max}$ and $|\tilde{w}| \leq \tilde{w}_{max}$ with $w_{max} > 0$ and $\tilde{w}_{max} > 0$ being constants. The model mismatch ϵ is norm bounded, i.e. $\|\epsilon\| \leq \xi_{max}$ with $\xi_{max} > 0$ being constant.

To formulate the MPC design, these constraints are represented as follows

$$x \in \mathbb{X},\ u \in \mathbb{U},\ \epsilon \in \mathbb{M},\ w \in \mathbb{W},\ \tilde{w} \in \tilde{\mathbb{W}} \tag{14}$$

with

$$\mathbb{X} := \{x \in \mathbb{R}^{n_x} : |x_1| \leq z_{max}, |x_2| \leq v_{max}\}$$
$$\mathbb{U} := \{u \in \mathbb{R} : |u| \leq u_{max}\},\ \mathbb{M} := \{\epsilon \in \mathbb{R}^{n_x} : \|\epsilon\| \leq \xi_{max}\}$$
$$\mathbb{W} := \{w \in \mathbb{R} : |w| \leq w_{max}\},\ \tilde{\mathbb{W}} := \{\tilde{w} \in \mathbb{R} : |\tilde{w}| \leq \tilde{w}_{max}\}$$

2.3 Energy Maximization Problem Formulation

The constrained optimal WEC control problem without considering prediction error and model mismatch is

$$\min_{\boldsymbol{u}} \lim_{N\to\infty} \frac{1}{N} \Sigma_{k=0}^{N-1} \left\{ y(k)u(k) + \frac{1}{2}x(k)^\top Q x(k) + \frac{1}{2}Ru^2(k) \right\} \tag{15}$$

subject to

$$x(k+1) = Ax(k) + B_u u(k) + B_w w(k) \tag{16}$$
$$x(k) \in \mathbb{X},\ u(k) \in \mathbb{U},\ w(k) \in \mathbb{W},\ \forall k \in \mathbb{I}_{\geq 0}$$

To minimize the first term $y(k)u(k)$ is to maximize the power output, and the last two terms $\frac{1}{2}x(k)^\top Q x(k) + \frac{1}{2}Ru^2(k)$ is to penalise the state and control input.

This constrained optimal control problem has been solved by a feedback non-causal model predictive control (Zhan et al. (2019, 2017)), which is briefly introduced in the next subsection.

2.4 Feedback Non-causal MPC With Perfect Prediction and Accurate Model

As proposed in (Zhan et al. (2019)), the solution of the optimal problem (15) without considering prediction error and model mismatch is

$$u(k) = \begin{cases} K_x x(k) + K_d E^k w_{k,n_p} + v(k),\ k \in \mathbb{I}_{[0,n_p-1]} \\ K_x x(k),\ k \in \mathbb{I}_{\geq n_p} \end{cases} \tag{17}$$

where K_x and K_d are constant vectors determined by the method proposed in (Zhan and Li (2018)), and $v(k)$ is to cope with constraints and is solved by the following optimization problem

$$\boldsymbol{v}^*_{[0,n_p-1]} = \arg \min_{v_{[0,n_p-1]}} \Sigma_{k=0}^{n_p-1} v^2(k) \tag{18}$$

subject to

$$\bar{x}(k+1) = A\bar{x}(k) + B_u \bar{u}(k)$$
$$\bar{u}(k) = K_x \bar{x}(k) + K_d E^k w_{k,n_p} + v(k)$$
$$\bar{x}(0) = x(0)$$
$$\bar{x}(k) \in \mathbb{X}_k,\ \bar{u}(k) \in \mathbb{U}_k, \forall k \in \mathbb{I}_{0,n_p-1}, \bar{x}(n_p) \in \mathbb{X}_T$$

with $\bar{x}(k)$ and $\bar{u}(k)$ as state and input of the auxiliary system $\bar{x}(k+1) = A\bar{x}(k) + B_u\bar{u}(k)$, E is the translation matrix defined by

$$E := \begin{bmatrix} 0_{(n_p-1)\times 1} & I_{n_p-1} \\ 0 & 0_{1\times(n_p-1)} \end{bmatrix}$$

and the tightened constraint sets are

$$\mathbb{E}_k := \Sigma_{i=0}^{k-1} A_K^i B_w \mathbb{W},\ \mathbb{X}_k := \mathbb{X} \sim \mathbb{E}_k \tag{19}$$
$$\mathbb{U}_k := \mathbb{U} \sim K_x \mathbb{E}_k,\ \mathbb{X}_T := \Sigma \sim \mathbb{E}_{n_p} \tag{20}$$

where Σ is the maximal output admissible set (MOAS) (Kolmanovsky and Gilbert (1995)) of the system (16) with terminal controller $u = K_x x$ is

$$\Sigma := \left\{ x(0) \in \mathbb{X} : \begin{array}{c} x(k+1) = A_K x(k) + B_w w(k), \\ x(k) \in \mathbb{X}, K_x x(k) \in \mathbb{U}, \\ w(k) \in \mathbb{W}, \forall k \in \mathbb{I}_{\geq 0} \end{array} \right\} \tag{21}$$

where $A_K := A + B_u K_x$.

Based on this result, the prediction error and the model mismatch caused by wave approximations are considered and explicitly handled in this paper, which lead to a new constrained optimal WEC control problem as follows

$$\min_{\boldsymbol{u}} \lim_{N\to\infty} \frac{1}{N} \Sigma_{k=0}^{N-1} \left\{ y(k)u(k) + \frac{1}{2}x(k)^\top Q x(k) + \frac{1}{2}Ru^2(k) \right\} \tag{22}$$

$$x(k+1) = Ax(k) + B_u u(k) + B_w \hat{w}(k) + B_w \tilde{w}(k) + \epsilon(k)$$
$$x(k) \in \mathbb{X},\ u(k) \in \mathbb{U},\ \epsilon(k) \in \mathbb{M},\ \forall k \in \mathbb{I}_{\geq 0}$$
$$w(k) \in \mathbb{W},\ \tilde{w} \in \tilde{\mathbb{W}},\ \forall k \in \mathbb{I}_{\geq 0}$$

3. PREDICTIVE CONTROL WITH PREDICTION ERROR TOLERANCE

In this section, a novel compensator based model predictive control scheme is proposed to tackle the problems of prediction error and model mismatch.

3.1 Overall strategy

Define the prediction error of the wave elevation as

$$\tilde{w} = w - \hat{w} \tag{23}$$

where $\hat{w}$ is the predicted wave elevation. The error of n_p-step-ahead prediction is $\tilde{w}_{k,n_p} := [\tilde{w}_k, \tilde{w}_{k+1}, ..., \tilde{w}_{k+n_p-1}]$ with n_p being a positive integer. The continuous-time state-space model (9) can be rewritten as

$$\dot{x} = A_c x + B_{uc} u + B_{wc} \hat{w} + B_{wc} \tilde{w} + \epsilon \tag{24}$$

where the term of $B_{wc}\tilde{w} + \epsilon$ is unavailable. The nominal model which only involves available information is as follows

$$\dot{z} = A_c z + B_{uc} u_n + B_{wc} \hat{w} \tag{25}$$

where z is the nominal state and u_n is the nominal input.

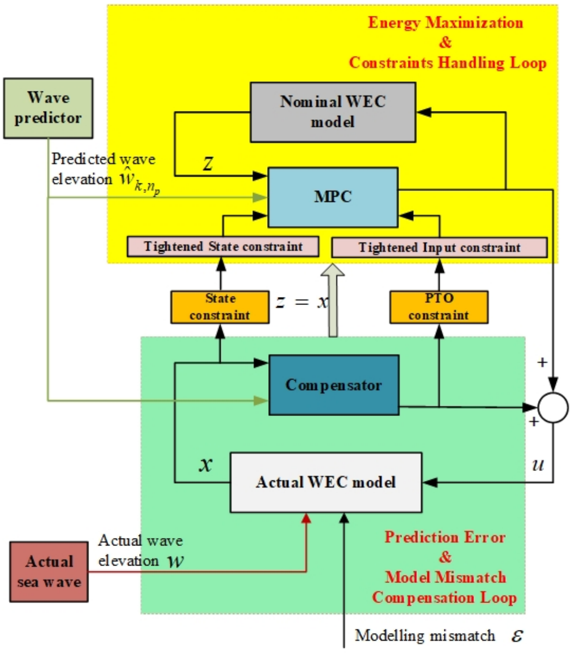

Fig. 2. Diagram of the proposed control strategy

The discrete-time form of (25) is

$$z(k+1) = A_z z(k) + B_u u_n(k) + B_w \hat{w}(k) \tag{26}$$

As shown in Fig. 2, the basic idea of the proposed control strategy is to design a compensator that fully eliminates the unavailable term of $B_{wc}\tilde{w} + \epsilon$ so that a MPC can be designed based on the nominal model (26) by only using the available information.

The control input is proposed as

$$u = u_{COM} + u_{MPC} \tag{27}$$

The first term of the controller u_{COM} is to compensate for the prediction error and the model mismatch, and the second term of the controller u_{MPC} is to maximize the energy output subject to multiple constraints.

3.2 Design of Compensator

To compensate for the prediction error and the model mismatch, a compensator is designed as follows

$$u_{COM} = -\rho \text{sign } (\sigma) \tag{28}$$

where ρ calculated by

$$\rho = \frac{\|GB_{wc}\|}{\|GB_{uc}\|}\tilde{w}_{max} + \frac{\|G\|}{\|GB_{uc}\|}\xi_{max} + 2n_p\|K_d\|\tilde{w}_{max} + \alpha \tag{29}$$

with α as a positive constant and σ is a sliding variable designed as

$$\begin{aligned}\sigma =& G[x(t) - x(t_0) - \int_{t_0}^{t} (A_c x(\tau) + B_{uc} u_{MPC} \\ &+ B_{wc}\hat{w}(\tau) - n_p B_{uc}\|K_d\|\tilde{w}_{max}\text{sign}(\sigma))d\tau]\end{aligned} \tag{30}$$

with G as a matrix such that GB_{uc} is invertible and t_0 represents the initial time instant, which is a non-negative constant.

Theorem 2. The prediction error and model mismatch can be compensated for by the proposed compensator (28) so that the closed-loop dynamics of (24) approximates the closed-loop dynamics of (25).

The proof is omitted due to page limits.

3.3 Design of MPC

Since the addition control u_{COM} is introduced in the controller, two issues need to be fully considered for the MPC design:

- the input constraint for MPC subsystem can be further tightened in order to ensure the constraint of the total input to be satisfied;
- the state constraint for MPC subsystem can be further tightened to rule out the possibilities of constraint violations caused by prediction error and model mismatch.

MPC is designed based on the nominal model (26), which can be rewritten as

$$z(k+1) = A_z z(k) + B_u u_{MPC}(k) + B_w \hat{w}(k) \tag{31}$$

by applying the nominal control input as MPC, i.e. $u_n = u_{MPC}$, where available but inaccurate information are used. The dual-mode control policy is applied as follows.

$$u_{MPC}(k) = \begin{cases} K_x z(k) + K_d E^k \hat{w}_{k,n_p} + v_n(k), & k \in \mathbb{I}_{[0,n_p-1]} \\ K_x z(k), & k \in \mathbb{I}_{\geq n_p} \end{cases} \tag{32}$$

where $v_n(k)$ is introduced to cope with the constraint.

The system (31) with the dual-mode control (32) is

$$z(k+1) = \begin{cases} A_K z(k) + B_u K_d E^k \hat{w}_{k,n_p} + B_u v_n(k) + B_w \hat{w}(k), & k \in \mathbb{I}_{[0,n_p-1]} \\ A_K z(k) + B_w \hat{w}(k), k \in \mathbb{I}_{\geq n_p} \end{cases} \tag{33}$$

Following (Chisci et al. (2001)) and (Zhan et al. (2019)), the feasibility is ensured by introducing an auxiliary prediction system and the tightened constraint as follows.

A. Auxiliary prediction model Define an auxiliary prediction model for $k \in \mathbb{I}_{[0,n_p-1]}$ as

$$\begin{aligned}\bar{z}(k+1) =& A\bar{z}(k) + B_u \bar{u}_{MPC}(k) \\ \bar{u}_{MPC}(k) =& K_x \bar{z}(k) + K_d E^k \hat{w}_{k,n_p} + v_n(k) \\ \bar{z}(0) =& z(0) = x(0)\end{aligned} \tag{34}$$

where $\bar{z}$ and $\bar{u}_{MPC}$ are auxiliary state and input.

B. Tightened constraints The MOAS Σ_n of the system (31) with terminal controller $u_{MPC} = K_x z$ is

$$\Sigma_n := \left\{ z(0) \in \mathbb{X} : \begin{array}{c} z(k+1) = A_K z(k) + B_w \hat{w}(k), \\ z(k) \in \mathbb{X}, K_x z(k) \in \mathbb{U}_{MPC}, \\ \hat{w}(k) \in \hat{\mathbb{W}}, \ \forall k \in \mathbb{I}_{\geq 0} \end{array} \right\} \tag{35}$$

From (27) and (28), the input constraint for MPC subsystem is

$$\mathbb{U}_{MPC} := \{u_{MPC} \in \mathbb{R} : |u_{MPC}| \leq u_{max} - \rho\} \tag{36}$$

From (42) and Hypothesis 1, the set $\hat{\mathbb{W}}$ can be obtained as

$$\hat{\mathbb{W}} := \{\hat{w} \in \mathbb{R} : |\hat{w}| \leq w_{max} + \tilde{w}_{max}\} \tag{37}$$

The tightened constraints for MPC subsystem are

$$\bar{z}(k) \in \mathbb{Z}_k, \bar{u}_{MPC}(k) \in \mathbb{U}_{kMPC} \tag{38}$$

Table 1. WEC parameters of accurate model and physical constraints

Description	Notation	values
Stiffness	k_s	6.39×10^5 N/m
Float mass	m_s	7×10^3 kg
Added mass	m_∞	1×10^3 kg
Total mass	m	8×10^3 kg
Radiation coefficient	D_r	2×10^5kg/s
Excitation coefficient	D_e	$4 \times 10^3 \mathrm{kg}/s^2$
Input force limit	$u_{\max}$	21 kN
Float heave limit	$z_{\max}$	1 m
Heave velocity limit	$v_{\max}$	3 m/s

Table 2. WEC parameters of inaccurate model

Description	Notation	values
Stiffness	k_s	5.5×10^5 N/m
Float mass	m_s	6.5×10^3 kg
Added mass	m_∞	0.5×10^3 kg
Total mass	m	7×10^3 kg
Radiation coefficient	D_r	1.8×10^5kg/s
Excitation coefficient	D_e	$4.6 \times 10^3 \mathrm{kg}/s^2$

with

$$\hat{\mathbb{E}}_k := \Sigma_{i=0}^{k-1} A_K^i B_w \hat{\mathbb{W}},\ \mathbb{Z}_k := \mathbb{X} \sim \hat{\mathbb{E}}_k \tag{39}$$

$$\mathbb{U}_{kMPC} := \mathbb{U}_{MPC} \sim K_x \hat{\mathbb{E}}_k,\ \mathbb{Z}_T := \Sigma_n \sim \hat{\mathbb{E}}_{n_p} \tag{40}$$

The constraint-handling term $v_n(k)$ is solved by the following optimization problem

$$\boldsymbol{v}^*_{n_{[0,n_p-1]}} = \arg \min_{v_{n_{[0,n_p-1]}}} \Sigma_{k=0}^{n_p-1} v_n^2(k) \tag{41}$$

subject to (34) and

$$\bar{z}(k) \in \mathbb{Z}_k,\ \bar{u}_{MPC}(k) \in \mathbb{U}_{kMPC}, \forall k \in \mathbb{I}_{0,n_p-1}, \bar{z}(n_p) \in \mathbb{Z}_T$$

4. SIMULATION RESULTS

The parameters of the WEC model and the hydrodynamic coefficients are adopted from those used in (Zhan et al. (2019, 2017)) for comparison purpose. The simulations based on a reduced-order model is provided for geometric visualisation of the satisfaction of recursive feasibility. The simulation based on a higher-order model of WEC is omitted due to page limits. The coefficients and physical constraints are listed in Table 1. The model of the prediction error is

$$\tilde{w}(k+1) = \lambda \tilde{w}(k) + \xi_k, k = 1, ..., N \tag{42}$$

where $N > 0$ is the prediction step, $\lambda = 1.01$ is taken, making the filter unstable, to match with realistic prediction errors that grows with the prediction time. Both $\xi_k \sim \mathcal{N}(0, 0.1)$ and $\tilde{w}(1) \sim \mathcal{N}(0, 0.8)$ are Gaussian white noises. The parameters of the uncertain model that considered the modelling mismatch is listed in Table 2. The control horizon of MPC is set to be 5 steps.

It can be found in Fig. 3 that the prediction error and model mismatch degrade the control performance of conventional feedback MPC by 14.2% of energy loss, while with the compensation, the energy output is barely affected by only 0.3% of energy loss. Therefore, the proposed controller can effectively cope with both the prediction error and the model mismatch. The tightened constraints on both states and input are shown in Fig. 5. From Fig. 4, it can be seen that state constraints on heave position and velocity are satisfied, which ensures safe operations in large

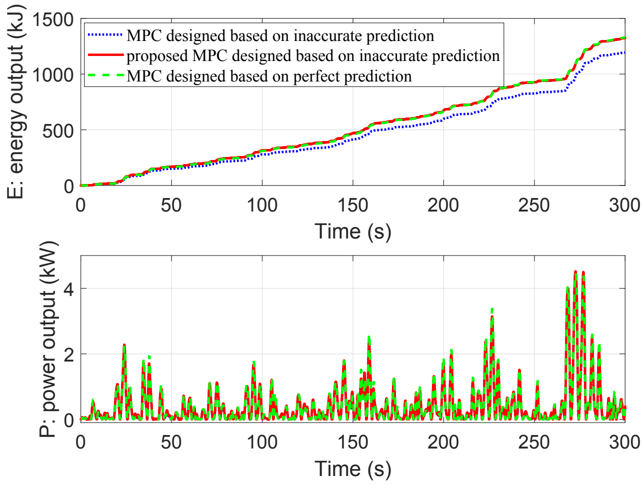

Fig. 3. Generated power and energy output vs time

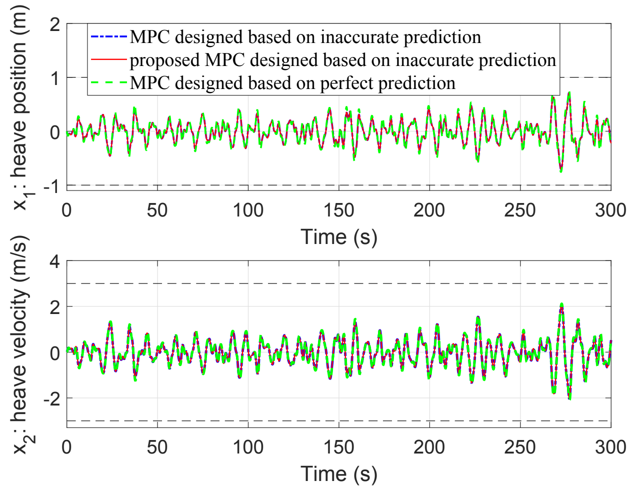

Fig. 4. Heave position and velocity vs time

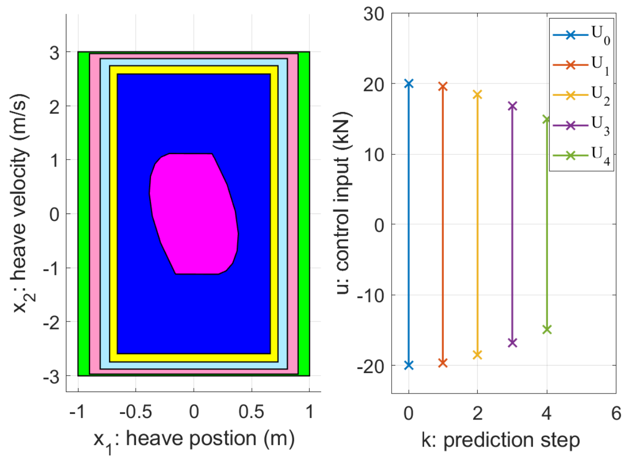

Fig. 5. Tightened constraints

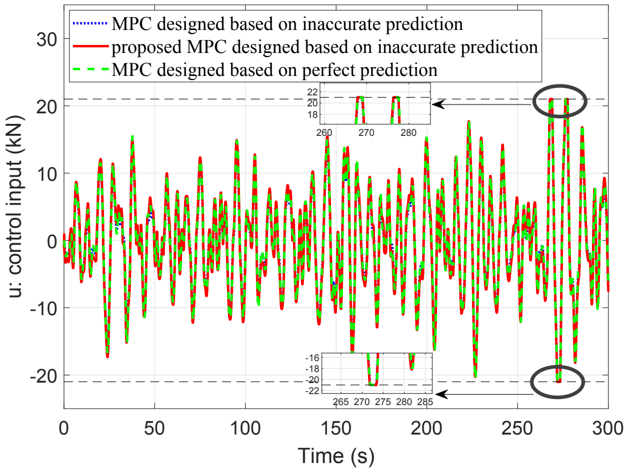

Fig. 6. PTO force (i.e. control input) vs time

wave conditions. Fig. 6 shows that the input constraint is active when the proposed compensator based feedback MPC is applied. Therefore, the ability of handling constraints is verified.

5. CONCLUSION

This paper aims to cope with the prediction error and model mismatch in non-causal WEC control problems. The proposed compensator based feedback MPC scheme maintains the energy output and simultaneously handle multiple constraints to ensure safe operations. Simulation results show that the control performance degradation is significantly reduced by the proposed controller.

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