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Bearing Leader-Follower Formation Control under Persistence of Excitation

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Abstract: This paper addresses the problem of formation control with a leader-follower structure in three dimensional space by exploring persistence of excitation (PE) of the desired bearing reference, which is provided by a possibly moving or time-varying desired formation. Using only bearing measurements, control laws are proposed for a group of agents with single-integrator dynamics. By defining a desired formation such that the corresponding inter-vehicle bearing measurements are persistently exciting, relaxed conditions on the interaction topology (which do not require bearing rigidity nor constraint consistence) can be used to derive distributed control laws that guarantee exponential stabilization of the desired formation in terms of relative position. A key outcome of this approach is that even if there is only one connection originated from each follower, exponential stability of the formation can be achieved as long as the excitation conditions are met on the desired formation. The approach generalizes stability results provided in prior work for leader-first follower (LFF) structure, based on bearing rigidity and constraint consistence that required at least two connections for each follower except for the first one. Simulations results are provided to illustrate the performance of the proposed control method.

Keywords: Multi-agent systems, Decentralized control, Nonlinear cooperative control, Distributed nonlinear control, Lyapunov methods, Stability of nonlinear systems

1. INTRODUCTION

The problem of formation control has been extensively studied over the last decades both by the robotics and the control communities. The main categories of solutions can be classified as Oh et al. (2015): 1) position-based formation control, Ren and Atkins (2007), 2) displacement-based formation control, Ren et al. (2005), 3) distance-based formation control, Anderson et al. (2007) and more recently 4) bearing-based formation control, Basiri et al. (2010). This latter control category has received growing attention due to its minimal requirements on the sensing ability of each agent. Early works on bearing-based formation control were mainly about controlling the subtended bearing angles which are measured in each agent's local coordinate frame and was limited to the planar formations only, Basiri et al. (2010); Bishop (2011). The main body of work however builds on concepts from bearing rigidity theory, which investigates the conditions for which a static geometric pattern of a framework is uniquely determined by the corresponding constant bearing measurements. Bearing rigidity theory in two-dimensional space (also termed parallel rigidity) is explored in Eren et al. (2003); Servatius and Whiteley (1999) and more recently it has been extended to arbitrary dimensions in Zhao and Zelazo (2016). More specifically, a formation control solution based on bearing measurements is proposed in Zhao and Zelazo (2016), under the assumption that the graph is undirected and infinitesimally rigid. The resulting bearing controller guarantees convergence to a target formation that is centroid invariant and scale invariant with respect to the initial conditions of the formation.

In the more challenging context of directed graphs, achieving stabilization of a formation requires not only bearing rigidity, as in the case of undirected graphs, but also the constraint consistence (also termed bearing persistence, in Zhao and Zelazo (2015)), which is the ability to maintain consistence between constraints induced by the desired bearing measurements. In Eren (2007), the conditions for directed bearing rigidity of a digraph in two-dimensional space are stated and a bearing control law for nonholomonic agents is proposed. The authors in Schiano et al. (2016) proposed a control strategy that relies on an extension of the rigidity theory to directed bearing frameworks defined in $\mathbb{R}^3 \times \mathcal{S}^1$ while at least one distance between two agents is required to correct the scale of the formation. In Trinh et al. (2019), bearing control laws were proposed to stabilize formations that satisfy a leader-first follower (LFF) interaction topology in an arbitrary dimensional space, obtained from a bearing Henneberg construction. LFF formations are

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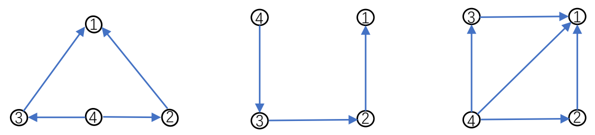

Fig. 1. Examples of frameworks with leader-follower structure. In (a), agent 4 is collinear with its two neighbors agent 2 and 3 and thus the framework is not bearing rigid. The framework in (b) forms a square with only one connection originating from each follower, and obviously it is not bearing rigid. The framework in (c) is bearing rigid but not constraint consistent. The stability of these three formations can not be guaranteed by the bearing controller relying only on bearing rigidity theory and constraint consistence Zhao and Zelazo (2015) but can be guaranteed by the proposed control law under the PE condition.

uniquely determined up to a translation (the leader's position) and a scaling factor, meaning that knowledge of the distance between the leader and the first follower is required to control the formation scale.

In this paper, we consider the problem of controlling a multi-agent system with single-integrator dynamics to stabilize the actual formation to a reference one using bearing measurements only. We propose an approach that draws inspiration from the work in Trinh et al. (2019), which presents a first-order bearing formation control law, considering an LFF graph topology. A distinctive feature of the present work is the shift of focus from static formations to moving and time-varying formations. In this context, persistence of excitation (PE) can be explored to significantly relax the conditions on the graph topology that are needed to guarantee stabilization of the formations. As a result, we are able to define general PE criteria that incorporate and generalize convergence criteria used in prior work.

In particular, we will show that under the PE condition 1) the formation stabilization is achieved for a leader-follower structure that is not necessarily bearing rigid nor constraint consistent (i.e. a directed acyclic graph which has a spanning tree, as shown in Fig.1); 2) the desired formations may correspond to fixed geometric patterns that translate and rotate, as long as that the PE condition is provided by rotational motion; 3) scale ambiguity, which is a characteristic of bearing rigidity, can be removed and convergence of the desired formation in terms of scale can be guaranteed, without the need to measure the distance between any two agents. In general, a desired formation that is persistently exciting can be time-varying (in scale, translation, rotation, and even in shape), similar to those typically considered in position-based formation control Brinón-Arranz et al. (2014); Dong et al. (2015).

We also show that static formations under the LFF structure Trinh et al. (2019) can be treated as a special case, for which the proposed bearing control law guarantees formation stabilization up to a scale factor.

The body of the paper is organized as follows. Section II presents mathematical background on graph theory and introduces the leader-follower graph structure together with the bearing PE condition exploited in the paper. Section III presents the bearing formation control laws along with stability analysis. Section IV shows the performance of the proposed control strategy. The paper concludes with some final comments in Section V.

2. PERSISTENCE OF EXCITATION AND LEADER-FOLLOWER INTERACTION TOPOLOGY

2.1 *Persistence of excitation*

Let $\mathbb{S}^2 := \{y \in \mathbb{R}^3 : \|y\| = 1\}$ denote the 2-Sphere and $\|.\|$ the euclidean norm. The operator $[.]_\times$ yields the skew-symmetric matrix associated to its vector argument and $\lambda_{\max}(.)(\lambda_{\min}(.))$ represents the maximum (minimum) eigenvalue of its matrix argument.

For any $y \in \mathbb{S}^2$, we can define the projection operator π_y

$$\pi_y := I - yy^\top \geq 0, \tag{1}$$

which is such that, for any vector $x \in \mathbb{R}^3$, $\pi_y x$ provides the projection of x on the plane orthogonal to y.

Definition 1. A matrix $\Sigma(t) \in \mathbb{R}^{n\times n}$, is called *persistently exciting* (PE) if there exists $T > 0$ and $\mu > 0$ such that for all t

$$\int_t^{t+T} \Sigma(\tau)d\tau \geq \mu I. \tag{2}$$

Definition 2. A direction $y(t) \in \mathbb{S}^2$, is called *persistently exciting* (PE) if the matrix $\pi_{y(t)}$ satisfied the PE condition in Definition 1.

Lemma 1. Assume that $y(t) \in \mathbb{S}^2$ and $\dot{y}(t)$ is uniformly continuous, then relation (2) with $\Sigma(\tau) = \pi_{y(\tau)}$ is equivalent to:

There exists $(T, \epsilon) > 0$ and $\tau \in [t, t+T]$ such that $\|\dot{y}(\tau)\| \geq \epsilon$, $\forall \tau \in [t,\ t+T]$.

Proof. The proof of this lemma is given in (Le Bras et al., 2017, Appendix 6.1).

Lemma 2. Let $Q := \sum_{i=1}^{l} \pi_{y_i}$. Then the matrix is persistently exciting, if one of the following conditions is satisfied:

(1) there is at least one of the directions y_i that is persistently exciting,
(2) there are at least two direction y_i and y_j, $i, j \in \{1, ..., l\}$, $i \neq j$ such that they are uniformly non-collinear. That is, for all $t \geq 0$ there exists an $\epsilon_1 > 0$ such that $|y_i(t)^\top y_j(t)| \leq 1 - \epsilon_1$.

Proof. The proof of this lemma is given in (Le Bras et al., 2017, Lemma 3).

2.2 *Preliminaries on graph theory*

Consider a system of n connected agents. The underlying interaction topology can be modelled as a digraph (directed graph) $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the set of vertices and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges. In this work, the graph is interpreted as sensing graph, meaning that if the ordered pair $(i, j) \in \mathcal{E}$ then agent i can access or sense information about agent j, which is called a neighbor of agent i. Note that the information flow is in the opposite direction. The set of neighbors of agent i is denoted by $\mathcal{N}_i := \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. Define $m_i = |\mathcal{N}_i|$, where $|.|$ denotes the cardinality of a set. A directed path is a finite sequence of distinct vertices $\nu_1, \nu_2, \ldots, \nu_{k-1}, \nu_k$, such that (ν_{i-1}, ν_i), $2 \leq i \leq k$ belongs to

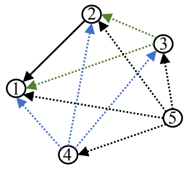

Fig. 2. Possible connections of a leader-follower structure when $n = 5$. The black solid line represents the only neighbor of the first follower (agent 2) which is the leader (agent 1). The green, blue and black dashed lines represent possible neighbors of agent 2, 3 and 4, respectively.

$\mathcal{E}$. A directed cycle is a directed path with the same start and end vertices, i.e. $\nu_1 = \nu_k$. A digraph $\mathcal{G}$ is called an acyclic digraph if it has no directed cycle. The digraph $\mathcal{G}$ is called a directed tree with a root vertex i, $i \in \mathcal{V}$, if for any vertex $j \neq i$, $j \in \mathcal{V}$, there exists only one directed path connecting j to i. Note that a directed tree is acyclic. We say that $\mathcal{G}$ has a directed spanning tree, if there exists a subgraph of $\mathcal{G}$ that is a directed tree and contains all the vertices of $\mathcal{G}$.

2.3 *Leader-follower structure*

In this section, we will define a leader-follower structure that is not necessarily bearing rigid nor constraint consistent.

Definition 3. A digraph $\mathcal{G}$ is called a leader-follower structure if it is acyclic and has a spanning tree.

Definition 4. A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is called a minimal leader-follower structure if it is a directed tree.

In this setting of a leader-follower structure, the leader is the root vertex which has no neighbors and each of the other followers has at least one neighbor. Without loss of generality, the agents are numbered (or can be renumbered) such that agent 1 is the leader, i.e. $\mathcal{N}_1 = \varnothing$, and for each agent $i \geq 2$ the set of neighbors satisfies $\mathcal{N}_i \subseteq \{1, 2, \ldots, i-1\}$. An example of a possible 5-agent leader-follower graph is shown in the Figure 2. The first follower (agent 2) has only one neighbor which is the leader (agent 1). The second follower (agent 3) has two possible neighbors: agent 1 and 2, and so forth.

Remark 1. Note that the leader-follower structure defined above is more general than the leader-first follower structure (LFF) considered in Trinh et al. (2019), for which each follower has two neighbors except the first follower which is only connected to the leader.

Given the digraph $\mathcal{G}$ as defined in Definition 3, let each agent $i \in \mathcal{V}$ be modeled as a point $p_i \in \mathbb{R}^3$ in a common inertial frame. Then, the stacked vector $p = [p_1^\top, ..., p_n^\top]^\top \in \mathbb{R}^{3n}$ is called a configuration of $\mathcal{G}$. The digraph $\mathcal{G}$ and the configuration p together define a formation $\mathcal{G}(p)$ in the 3-dimensional space. Defining the relative position vectors

$$p_{ij} := p_j - p_i,\ i,\ j \in \mathcal{V},\ i \neq j \tag{3}$$

then as long as $\|p_{ij}\| \neq 0$, the bearing of agent j relative to agent i is given by the unit vector

$$g_{ij} := p_{ij}/\|p_{ij}\| \in \mathbb{S}^2. \tag{4}$$

Let $v_i := \dot{p}_i$ denote the velocity of agent i. Let $p^*(t) = [p_1^{*\top}(t), ..., p_n^{*\top}(t)]^\top \in \mathbb{R}^{3n}$ be the desired configuration. Let $\{v_i^*(t)\}_{i\in\{1,\ldots,n\}}$ and $\{g_{ij}^*(t)\}_{i,j\in\mathcal{E}}$ be the set of all velocity vectors and the set of all bearing vectors, respectively, of the desired configuration $p^*(t)$.

Definition 5. We say that a desired formation $\mathcal{G}(p^*(t))$ satisfies the PE condition if for all agent $i \geq 2$, the matrices $\sum_{j\in\mathcal{N}_i} \pi_{g_{ij}^*(t)}$ satisfy the PE condition as defined in Lemma 2.

The following lemma will show the uniqueness of the desired formation of a leader-follower structure under PE condition.

Theorem 1. Consider a leader-follower formation. Assume that the leader's position $p_1^*(t)$, the velocity vectors $\{v_i^*(t)\}_{i\in\{1,\ldots,n\}}$ and the bearing vectors $\{g_{ij}^*(t)\}_{(i,j)\in\mathcal{E}}$ are well-defined, known, and bounded. Let $\hat{p}_1^* \triangleq p_1^*$ and $\hat{p}_i^*$ denote the estimate of p_i^*, for $i = 2, \ldots, n$ with the following dynamics:

$$\dot{\hat{p}}_i^* = v_i^* - K \sum_{j\in\mathcal{N}_i} \pi_{g_{ij}^*}(\hat{p}_i^* - \hat{p}_j^*),\ \forall i \geq 2, \tag{5}$$

with arbitrary initial conditions and K a positive definite matrix. Assume the desired formation satisfies the PE condition. Then $\hat{p}_i^*$ converges uniformly globally exponentially (UGE) to the unique p_i^*.

Proof. Consider the error variables $\tilde{p}_i^* := \hat{p}_i^* - p_i^*$ defined for $i = 2, \ldots, n$ and the corresponding dynamics obtained from (5). For $i = 2$, we have $\mathcal{N}_2 = \{1\}$ and it is straightforward to verify that the dynamics of $\tilde{p}_2^*$ is given by

$$\dot{\tilde{p}}_2^* = -K\pi_{g_{21}^*}\tilde{p}_2^* \tag{6}$$

and that $\tilde{p}_2^* = 0$ is UGE stable under the PE condition (by direct application of (Le Bras et al., 2017, Lemma 4)). For $i = 3$ and $\mathcal{N}_3 = \{1\}$, the proof is exactly the same as for agent 2. For $\mathcal{N}_3 = \{2\}$ or $\{1, 2\}$, the dynamics of $\tilde{p}_3^*$ can be written as

$$\dot{\tilde{p}}_3^* = -K \sum_{j\in\mathcal{N}_3} \pi_{g_{3j}^*}\tilde{p}_3^* + K\pi_{g_{32}^*}\tilde{p}_2^* \tag{7}$$

which together with (6) forms a cascaded system with $\tilde{p}_2^*$ as input to (7). Using the fact that $\tilde{p}_2^* = 0$ is UGE stable and system (7) is continuously differentiable and globally Lipschitz in $(\tilde{p}_3^*, \tilde{p}_2^*)$, it follows (by direct application of (Le Bras et al., 2017, Proposition 1)) that $\tilde{p}_3^* = 0$ is also UGE stable. In the general case, we can write

$$\dot{\tilde{p}}_i^* = -K \sum_{j\in\mathcal{N}_i} \pi_{g_{ij}^*}\tilde{p}_i^* + K \sum_{j\in\mathcal{N}_i\setminus\{1\}} \pi_{g_{ij}^*}\tilde{p}_j^*, \tag{8}$$

for $i = 2, \ldots, n$ and the proof of that $\tilde{p}_i^* = 0$ is UGE stable can be obtained in a similar way.

Remark 2. For the static case where $v_i^* - v_j^* = 0,\ \forall (i,j) \in \mathcal{E}$, we obviously conclude that g_{21}^* is not persistently exciting. However, if each agent i ($i \geq 3$) has two neighbors $1 \leq j \neq k < i$ with $g_{ij}^* \neq \pm g_{ik}^*$, the desired formation becomes exactly the same as the desired LFF formation described in Trinh et al. (2019) and uniqueness of the target formation can still be guaranteed if the distance $d_{21}^* = \|p_1^* - p_2^*\|$ is provided. Under the proposed controller, which will be defined in the next section, the formation will converge to the desired shape up to a scaling factor as discussed in Trinh et al. (2019).

Note that under the condition of Theorem 1, the shape and the size of the desired formation may be time-varying. There are some situations in which a rigid motion of the desired formation ensures the PE condition. More specifically, the rigid motion should include a rotational motion as stated in the following Corollary.

Corollary 1. Consider a minimal leader-follower formation subjected to a continuous rigid motion such that:

$$g_{ij}^*(t) = R(t)^\top g_{ij}^*(0), \forall (i,j) \in \mathcal{E}$$

where $R(t) \in SO(3)$ is the orientation matrix of a virtual frame attached to the formation (at some point) with respect to the common inertial frame. Let $\Omega(t)$ be the orientation velocity of the formation expressed in the virtual frame, such that $\dot{R}(t) = R(t)[\Omega(t)]_\times$. Assume that $\Omega(t)$ is a uniformly continuous vector. If there exists $(T, \epsilon) > 0$ and $\tau \in [t, t+T]$, $\forall t$ such that $\Omega(t)$ and $g^*_{ij}(t)$ are uniformly non-collinear: $\|\Omega(\tau) \times g^*_{ij}(\tau)\| \geq \epsilon$, $\forall (i,j) \in \mathcal{E}$, then the desired formation satisfies the PE condition.

Proof. Recalling that in the case of a minimal leader-follower structure, $|\mathcal{N}_i| = 1$ ($\forall i \geq 2$), it then follows that the desired formation satisfies the PE condition if and only if $g^*_{ij}(t),\ \forall (i,j) \in \mathcal{E}$ satisfies the PE condition of Definition 2. Using the fact that $\|\dot{g}^*_{ij}(t)\| = \|[\Omega(t)] \times g^*_{ij}(t)\|$ along with Lemma 1, it is straightforward to verify that the PE condition is satisfied as long as $\Omega(t)$ is uniformly non-collinear to $g^*_{ij}(t), \forall (i,j) \in \mathcal{E}$.

3. BEARING FORMATION CONTROL

3.1 Problem formulation

Consider the framework $\mathcal{G}(p)$, where each agent $i \in \mathcal{V}$ is modelled as a single integrator with the following dynamics:

$$\dot{p}_i = v_i \tag{9}$$

where $p_i \in \mathbb{R}^3$ is the position and $v_i \in \mathbb{R}^3$ the velocity input, all expressed in a common inertial frame.

We assume that the n-agent system satisfies the following assumptions.

Assumption 1. The desired velocity $v^*_i(t)$ are bounded for all t. The desired positions $p^*_i(t)$ are such that the resulting desired bearings $g^*_{ij}(t)$ are well-defined for all t and the resulting desired formation satisfies the PE condition.

Assumption 2. The sensing topology of the group is described by a digraph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ that satisfies the leader-follower structure defined in Definition 3. Each agent $i \geq 2$ can measure the relative bearing vectors g_{ij} to its neighbors $j \in \mathcal{N}_i$.

Assumption 3. As the formation evolves in time, no inter-agent collisions and occlusions occur. In particular, we assume that the bearing information $g_{ij}(t),\ (i,j) \in \mathcal{E}$ is all the time well-defined.

With all these ingredients, we can define the bearing formation control problem as follows.

Problem 1. Consider the system (9) and the underlying framework $\mathcal{G}(p)$. Under Assumptions 1-3, design stabilizing distributed control laws based on only bearing measurements that guarantee convergence to the desired formation.

3.2 Exponential stabilization of the formations

For distinct agents $i,\ j \in \mathcal{V}$, define the desired relative position vectors p^*_{ij} according to (3) and the relative position error $\tilde{p}_{ij} := p_{ij} - p^*_{ij}$. In addition, for $i \in \mathcal{V}$ and $j \in \mathcal{N}_i$, define the dynamics of the error:

$$\dot{\tilde{p}}_{ij} = v_j - v^*_j - (v_i - v^*_i). \tag{10}$$

The relative error $\tilde{p}_{ij}$ is defined for all $i,\ j \in \mathcal{V}$ (with $i \neq j$), which may include pairs for which $(i,j) \notin \mathcal{E}$. However, the dynamics of the error is only defined for $\tilde{p}_{ij}$ $(i,j) \in \mathcal{E}$.

The following control law is proposed for each agent $i \in \mathcal{V}$

$$v_i = -\sum_{j \in \mathcal{N}_i} k_i \pi_{g_{ij}} p^*_{ij} + v^*_i, \tag{11}$$

where k_i is positive gains.

Remark 3. Assume for now the results that will be shown in the following sections hold, i.e., assume that the equilibrium point $\tilde{p}_{ij} = 0,\ (i,j) \in \mathcal{E}$ of the system (10) is Exponentially Stable (ES). To express this result in terms of the formation and the positions of the agents p_i, note that under the given assumption it also holds that $\tilde{p}_{i1} = 0,\ 2 \leq i \leq n$ is ES. Then, we can conclude that $\forall 2 \leq i \leq n,\ p_i$ exponentially converges to p^*_i up to a translation given by $p_1 - p^*_1$. The latter can be driven to zero provided that agent 1 can sense its position and velocity.

Stability and convergence of the first follower

Lemma 3. For $i = 2$ consider the dynamics of the error (10) along with the control law (11). If the Assumptions 1-3 are satisfied, then the equilibrium point $\tilde{p}_{21} = 0$ is exponentially stable (ES).

Proof. The control law (11) for agent 1 is $v_1 = v^*_1$ and for agent 2 is $v_2 = -\pi_{g_{21}} \tilde{p}_{21} + v^*_2$. Recalling (10), the closed-loop system for the state $\tilde{p}_{21}$ is expressed as

$$\dot{\tilde{p}}_{21} = -k_2 \pi_{g_{21}} \tilde{p}_{21}. \tag{12}$$

Consider the following Lyapunov function candidate:

$$\mathcal{L}_{21} = \frac{1}{2}\|\tilde{p}_{21}\|^2 \tag{13}$$

Taking its time-derivative, it yields

$$\dot{\mathcal{L}_{21}} = -k_2 \tilde{p}^\top_{21} \pi_{g_{21}} \tilde{p}_{21}, \tag{14}$$

which is negative-semidefinite, one concludes that the state $\tilde{p}_{21}$ is bounded. Since $g^{*\top}_{21} \pi_{g_{21}} g^*_{21} = g^\top_{21} \pi_{g^*_{21}} g_{21}$, it is straightforward to verify that

$$\begin{aligned} \dot{\mathcal{L}}_{21} &= -k_2 p^{*\top}_{21} \pi_{g_{21}} p^*_{21} \\ &= -k_2 \frac{\|p^*_{21}\|^2}{\|p_{21}\|^2} p^\top_{21} \pi_{g^*_{21}} p_{21} \leq -k_2 \gamma_2 \tilde{p}^\top_{21} \pi_{g^*_{21}} \tilde{p}_{21} \end{aligned} \tag{15}$$

with $\gamma_2 = \frac{\min \|p^*_{21}(t)\|^2}{(\|\tilde{p}_{21}(0)\| + \max \|p^*_{21}(t)\|)^2}$. Using the PE condition of g^*_{21} along with a direct application of (Loria and Panteley, 2002, Lemma 5) one can conclude that $\tilde{p}_{21} = 0$ is ES.

Remark 4. Note that in the above lemma, assumption 3 relies on the evolution of state variables. This assumption serves here to show that if there is no collision or occlusion, the bearings are well-defined and the proposed control design yields the desired convergence properties (Lemma 3 and even in the following results: Lemma 4 and Theorem 2). Trying to more specifically characterize the set of initial conditions for which the system's solutions avoid collision and occlusion is out of the scope of the paper.

Stability and convergence of the second follower

Lemma 4. For $i = 3$ consider the dynamics of the error (10) along with the control law (11). If the Assumptions 1-3 are satisfied and Lemma 3 is valid, then the equilibrium point $\tilde{p}_{3j} = 0,\ \forall j \in \mathcal{N}_3$ is ES.

Proof. According to the leader-follower structure described in subsection 2.2, the second follower (agent 3) can have three possible sets of neighbors: $\mathcal{N}_3 = \{1\}$, $\mathcal{N}_3 = \{2\}$ and $\mathcal{N}_3 = \{1, 2\}$.

Case i): $\mathcal{N}_3 = \{1\}$, the proof is identical to the proof of Lemma 3.

Case ii): $\mathcal{N}_3 = \{2\}$ or $\mathcal{N}_3 = \{1, 2\}$. The closed-loop system for the state $\tilde{p}_{3j},\ j \in \mathcal{N}_3$ is expressed as

$$\dot{\tilde{p}}_{3j} = -\sum_{l\in\mathcal{N}_3} k_3 \pi_{g_{3l}} \tilde{p}_{3l} + v_j - v_j^*. \tag{16}$$

Since $v_1 = v_1^*$ and v_2 is a function of variables $\tilde{p}_{21}, \tilde{p}_{31} = \tilde{p}_{32} + \tilde{p}_{21}$, we can interpret (16) as a cascaded system that has $\tilde{p}_{21}$ as input to the unforced system

$$\dot{\tilde{p}}_{3j} = -k_3 \sum_{l\in\mathcal{N}_3} \pi_{g_{3l}} \tilde{p}_{3j}. \tag{17}$$

Now the proof becomes analogue to the proof of Lemma 3. Consider the following Lyapunov function candidate:

$$\mathcal{L}_{3j} = \frac{1}{2}\|\tilde{p}_{3j}\|^2, \tag{18}$$

and its time-derivative is given by

$$\dot{\mathcal{L}}_{3j} = -k_3 \tilde{p}_{3j}^\top \sum_{l\in\mathcal{N}_3} \pi_{g_{3l}} \tilde{p}_{3j} \tag{19}$$

which is negative-semidefinite. Thus state $\tilde{p}_{3j}$ is bounded. Due to the fact that $\tilde{p}_{3j} = \tilde{p}_{3k} + \tilde{p}_{kj},\ k \neq j,\ k, j \in \{1, 2\}$ and $\tilde{p}_{21} = 0$ in the unforced system (17), one has $\tilde{p}_{3j} = \tilde{p}_{3k}$. It is straightforward to verify that

$$\begin{aligned}\dot{\mathcal{L}}_{3j} &= -k_3 \sum_{l\in\mathcal{N}_3} \frac{\|p_{3l}^*\|^2}{\|p_{3l}\|^2} \tilde{p}_{3l}^\top \pi_{g_{3l}^*} \tilde{p}_{3l} \\ &\leq -k_3\gamma_3 \tilde{p}_{3j}^\top \sum_{l\in\mathcal{N}_3} \pi_{g_{3l}^*} \tilde{p}_{3j} \leq 0,\end{aligned} \tag{20}$$

with $\gamma_3 = \frac{\min_{l\in\mathcal{N}_j} \|p_{3l}^*\|^2}{(\|\tilde{p}_{3j}(0) + \max_{l\in\mathcal{N}_j} \|p_{3l}^*\|)^2}$. Using the PE condition along with direct application of (Loria and Panteley, 2002, Lemma 5), we can conclude that the equilibrium point $\tilde{p}_{3j} = 0$, $j \in \mathcal{N}_3$ of the unforced system (17) is ES. This in turn implies that the equilibrium point $\tilde{p}_{3j} = 0,\ j \in \mathcal{N}_3$ is ES for the system (16).

The n-agents system

Theorem 2. For all $i \in \mathcal{V}\backslash\{1\}$ and $\forall j \in \mathcal{N}_i$, consider the system (10) in closed-loop with the proposed control law (11). If the Assumptions 1-3 are satisfied, then the equilibrium point $\tilde{p}_{ij} = 0,\ i = 2, \ldots, n,\ \forall j \in \mathcal{N}_i$ is ES.

Proof. We will prove the convergence of $\tilde{p}_{ij} = 0$ by mathematical induction. Firstly, for $k = 2$ we have $\tilde{p}_{21} = 0$ is ES based on Lemma 3. Thus Theorem 2 is true for $k = 2$. It is also true for $k = 3$ from the conclusion of Lemma 4.

Secondly, we suppose Theorem 2 is true for $4 \leq k \leq i-1$, that is $\tilde{p}_{kj} = 0,\ \forall j \in \mathcal{N}_k$ is ES for all $4 \leq k \leq i-1$ and we will prove that it is also true for $k = i$. Recall (10), the closed-loop system for the states $\tilde{p}_{ij},\ j \in \mathcal{N}_i$ is represented as

$$\dot{\tilde{p}}_{ij} = -k_i \pi_{g_{ij}} \tilde{p}_{ij} - \sum_{q\in\mathcal{N}_i\backslash\{j\}} k_i \pi_{g_{iq}} \tilde{p}_{iq} + v_j - v_j^*, \tag{21}$$

where v_j is a function of variables $\tilde{p}_{jm},\ m \in \mathcal{N}_j,\ \tilde{p}_{iq} = \tilde{p}_{ij} + \tilde{p}_{jq}$. Note that since the graph is connected, $\tilde{p}_{jq}$ can be represented by the error variables $\tilde{p}_{km},\ 2 \leq k \leq i-1,\ m \in \mathcal{N}_k$. System (21) can then be considered as a cascaded system with $\tilde{p}_{km},\ 2 \leq k \leq i-1,\ m \in \mathcal{N}_k$, being inputs of the unforced system analogously to system (17). Using a similar argument as shown in Lemma 4, one can conclude that the equilibrium point $\tilde{p}_{ij} = 0,\ \forall j \in \mathcal{N}_i$ of the unforced system is ES. Because Theorem 2 is true for $2 < k \leq i-1$, we can conclude that the equilibrium point $\tilde{p}_{ij} = 0,\ \forall j \in \mathcal{N}_i$ for system (21) is also ES. This in turn implies that Theorem 2 is true for $k = i$.

Then, by mathematical induction, if follows that the claim is true for all $k \in \mathcal{V}\backslash\{1\}$.

4. SIMULATION RESULTS

In this section, we consider a four-agent system defined in $\mathbb{R}^3$, $\mathcal{V} = \{1, 2, 3, 4\}$, with digraphs that satisfy a minimal leader-follower graph formed by a single directed path, that is, each follower has only one neighbor such that $\mathcal{N}_i = \{i-1\},\ i \in \mathcal{V}\backslash\{1\}$. According to Assumption 1, the desired formation is chosen such that the four agents form a squared shape in $\mathbb{R}^2$ that rotates about agent 1. Note that the desired configuration is not bearing rigid. For the sake of simplicity, the leader (agent 1) is static at position $p_1 = [0\ 0\ 0]^\top$. The desired trajectories for the followers are $p_2^* = 1.5[\sin(\frac{t}{4})\ 0\ \cos(\frac{t}{4})]^\top$, $p_3^* = 1.5[\sin(\frac{t}{4}) - \frac{\pi}{4}\ 0\ \cos(\frac{t}{4}) - \frac{\pi}{4}]^\top$, $p_4^* = 1.5[\sin(\frac{t}{4}) - \frac{\pi}{2}\ 0\ \cos(\frac{t}{4}) - \frac{\pi}{2}]^\top$. The initial conditions are $p_2(0) = [-1\ 3\ 1]^\top$, $p_3(0) = [-2\ 4\ -1]^\top$, $p_4(0) = [-1.5\ 3\ 0]^\top$. The gains used are $k_i = 3$. Fig. 3 depicts the trajectories of the four agents during the time evolution of the formation and we can see that the four agents converge to the desired trajectories. Fig. 4 shows the time evolution of the error state $\|\tilde{p}\|$. We can conclude that the proposed control law stabilizes the formation efficiently without requiring of bearing rigidity.

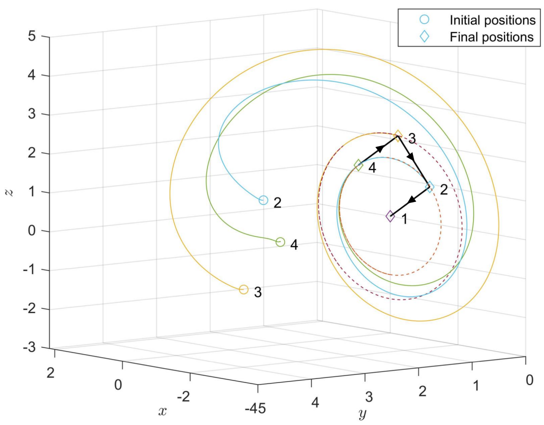

Fig. 3. 3-D Trajectory described by a formation under a single directed path topology. Colored solid lines represent the agents' trajectories and dashed lines represent the desired trajectories. The black solid lines represent the connections between agents.

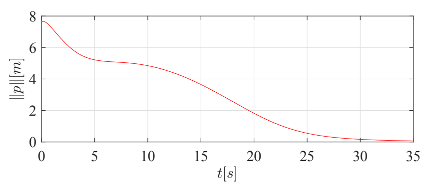

Fig. 4. Time evolution of the total position error $\|\tilde{p}\|$.

5. CONCLUSION

This paper studies bearing formation control using persistence of excitation and an underlying interaction graph with a leader-follower structure. The desired formation is determined by the desired relative positions between agents, which are chosen such that the corresponding desired bearing vectors satisfy the persistence of excitation condition. Under the proposed control laws, the desired formation is achieved with exponential rate of convergence, that is, the relative positions of the group of agents converge exponentially to the desired values, without relying on bearing rigidity nor on estimation of range. Simulation results are provided to validate the control laws. Future work will be dedicated to the incorporation of collision avoidance in the bearing control laws to bypass assumption 3 in order to ensure at least semi-global exponential stability.

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