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Observer-based output feedback consensus protocol for double-integrator multi-agent systems under intermittent sampled position measurements

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Abstract: This paper deals with the leader-follower consensus problem for double-integrator multi-agent systems using sampled position data information. An observer-based output feedback controller is designed to study this problem while taking into account intermittent sensor failures. The non-uniformity and randomness of the sampling times due to intermittent information lead to a $\mu-$varying linear system on a discrete stochastic time domain for the closed-loop system dynamics (here μ is the graininess function). Some necessary and sufficient conditions for the observer and controller gains are derived, using positive perturbation and Lyapunov operators on the space of symmetric matrices, to guarantee mean-square exponential stability for the observation and tracking errors. Some simulation results illustrate the effectiveness of the proposed observer-based output feedback controller.

Keywords: Consensus, Stochastic time scale, Multi-agent systems, Intermittent information, Mean-square exponential stability, Sampled position data.

1. INTRODUCTION

In recent decades, the consensus problem for multi-agent systems (MASs) has been extensively studied due to its applications in many fields such as robots, spacecrafts and manufacturing systems to name a few (Dorri et al., 2018; Li et al., 2014; Defoort et al., 2016). This problem aims at designing a control law such that the state of each agent reaches an agreement based on available information exchanged between neighboring agents (Ren and Beard, 2005).

Many works have investigated the consensus problem into two different directions depending on whether the MAS is described via continuous-time or uniform discrete-time models (Shi and Shen, 2017; Zuo et al., 2019; Lin et al., 2016; Su et al., 2017; Cao et al., 2017). Nevertheless, one cannot consider, in most scenarios, continuous-time or uniform discrete-time domains (Ding et al., 2017; Taousser et al., 2019). Indeed, there could be some limitations on sensing ability due to obstacles for instance or intermittent sensor failures to name a few. In this paper, we consider the leader-follower consensus problem for MASs under intermittent measurements.

Recently, the consensus problem under intermittent information has been considered using different approaches. In (Gao and Wang, 2010; Wen et al., 2013), time delay based approaches have been considered. In (Huang et al., 2014; Taousser et al., 2016; Phillips and Sanfelice, 2019), Lyapunov stability tools for hybrid systems have been used to derive the controller. In (Ajwad et al., 2019), a continuous-discrete time observer has been designed for the continuous estimation of the state from discrete measurements. However, in most of these works, the maximum duration of the intermittent failures is known to derive the controller gains, leading to some conservatism.

The communication topology among the agents is described deterministically in the above mentioned works. However, it can be represented in a stochastic way due to intermittent information. Indeed, the time instant and duration of intermittent sensor failures are stochastic by nature. Using a stochastic framework, some works have studied the consensus problem for MASs (Zhao and Park, 2014; Rezaee and Abdollahi, 2017). These works based on Markov chains, need the knowledge of failure probabilities for each link and finitely many states for the sake of the transition matrix. To remove these drawbacks, in this paper, the consensus problem under intermittent sampled measurements is investigated using the time scale theory.

The theory of time scales has been investigated in (Bohner and Peterson, 2012) to study discrete-time and continuous-time theories in an unified framework. It also enables to study dynamical systems evolving on arbitrary time domains (hybrid, non uniform discrete-time domains, etc.). Some works have been derived to study the stability, stabilization, observability and reachability on arbitrary time scales (Bartosiewicz, 2019). Using this theory, the consensus problem has been recently analyzed for linear MASs (Shen and Cao, 2012; Babenko et al., 2018; Girejko and Malinowska, 2019). However, in these papers, the time scale is deterministic and known in advance, which is often not realistic in practice.

The stochastic time scale theory has been recently investigated in (Poulsen et al., 2019a) where necessary and sufficient conditions have been derived to guarantee mean square stability for linear systems evolving on a stochastic discrete-time domain. It has been used to study the consensus problem under intermittent information where both position and velocity for each agent are measured in (Poulsen et al., 2019b). However, in practice, agents have limited on-board measurement resources. Therefore, it is more relevant to only consider sampled position measurements. Hence, in this paper, we propose an observer-based output feedback consensus protocol for double-integrator MASs under intermittent sampled position measurements. It should be note that the design of an observer-based output controller is not an easy task since the separation principle does not hold (the time domain is stochastic) and the graininess function is unknown (μ is a random variable).

The rest of this paper is organized as follows. Section 2 recalls some preliminaries on time scale theory. Section 3 gives the problem formulation. In Section 4, the observer-based output feedback consensus protocol is derived and the mean square stability of the closed-loop system is analyzed. In Section 5, some numerical results show the effectiveness of the proposed scheme.

2. PRELIMINARIES ON TIME SCALE THEORY

Let us briefly recall the main basics on time scale theory. For more details, the reader is referred to (Bohner and Peterson, 2012).

Let $\mathbb{T}$ be a time scale, i.e. a closed subset of $\mathbb{R}$. In this paper, it is considered unbounded above time scales, i.e. $\sup\{t \in \mathbb{T}\} = \infty$. Let us define the following useful operators:

- The forward jump operator is $\sigma : \mathbb{T} \to \mathbb{T}$ defined by $\sigma(t) = \inf\{s > t \mid s \in \mathbb{T}\}$
- The graininess operator is $\mu : \mathbb{T} \to \mathbb{R}$ defined by $\mu(t) = \sigma(t) - t$

On $\mathbb{T}$, the $\Delta-$derivative of $f : \mathbb{T} \to \mathbb{R}^n$ is defined by

$$f^{\Delta}(t) = \frac{f(\sigma(t)) - f(t)}{\mu(t)},$$

evaluated as $\mu(t) \to 0$ if $\mu(t) = 0$. In particular, for continuous-time systems, the $\Delta-$derivative is the classical derivative, and for discrete-time systems (i.e. $\mathbb{T} = \mathbb{Z}$), the $\Delta-$derivative is the forward difference operator. A function $f : \mathbb{T} \to \mathbb{R}^n$ is differentiable on $\mathbb{T}$ if its $\Delta-$derivative exists at every point $t \in \mathbb{T}$.

$f : \mathbb{T} \to \mathbb{R}^n$ is rd-continuous on $\mathbb{T}$ if at points $t \in \mathbb{T}$ with $\sup\{s \in \mathbb{T} : s < t\} = t$, f has finite left-sided limits and at points $t \in \mathbb{T}$ with $\sigma(t) = t$, f is continuous. An $m \times n$ matrix-valued function g on $\mathbb{T}$ is rd-continuous if each of its entries is rd-continuous. Furthermore, when $m = n$, g is regressive if $I + \mu(t)g(t)$ is invertible for all $t \in \mathbb{T}$, where I is the identity matrix. One can notice that every function g on $\mathbb{T}$ is rd-continuous if $\mathbb{T}$ is purely discrete.

If g is regressive and rd-continuous, then system

$$X^{\Delta}(t) = g(t)X(t), \quad X(t_0) = I, \tag{1}$$

has a unique solution, i.e. $e_g(\cdot, t_0)$, called the generalized exponential function on $\mathbb{T}$. For $x(t) \in \mathbb{R}$, the unique solution of system

$$x^{\Delta}(t) = g(t)x(t), \quad x(t_0) = x_0, \tag{2}$$

is given by $\exp\left(\int_{t_0}^{t} \xi_s(g(\tau))\Delta\tau\right) x_0$ where $\xi_h(z) = \lim_{s\to h^+} \frac{Log(1+zs)}{s}$ is the cylinder transformation and Log is the principal logarithm.

Let us briefly recall some basics on stochastic time scale theory. For more details, the reader is referred to (Poulsen et al., 2019a). Let us consider the initial time $t_0 \in \mathbb{R}$ and a sequence of random variables $\{\mu_i\}_{i=0}^{\infty}$ with range $(0, \infty)$. The corresponding discrete stochastic time scale is

$$\tilde{\mathbb{T}} := \{t_0\} \cup \left\{ t_0 + \sum_{i=0}^{n} \mu_i \mid n \in \mathbb{N}_0 \right\} := \{t_n\}_{n=0}^{\infty}.$$

Let μ be a random variable. If the set of random variables $\{\mu\} \cup \{\mu_i\}_{i=0}^{\infty}$ consists of independent, identically distributed (i.i.d.) random variables, the corresponding stochastic time scale is called an i.i.d. discrete stochastic time scale generated by μ.

Let us consider $\tilde{\mathbb{T}}$ a discrete stochastic time scale generated by the random variables $\{\mu_i\}_{i=0}^{\infty}$. In this case, the analysis of system (2) on $\tilde{\mathbb{T}}$ is reduced to the analysis of the random sequence defined by $x(t_0) = x_0$ and

$$x(t_{n+1}) = (I + \mu_n g(t_n))x(t_n).$$

For almost all realizations of $\tilde{\mathbb{T}}$, $\{x(t_n)\}_{n=0}^{\infty}$ is the unique solution of (2) on the realization of $\tilde{\mathbb{T}}$ if g is regressive almost surely. To define the mean-square stability of system (2) on $\tilde{\mathbb{T}}$, the following norm of the solution is defined as the L^2 norm in the underlying sample space Ω, i.e.

$$\|x(t_m)\|_{\Omega} := \mathrm{E}\left[x^T(t_m)x(t_m)\right],$$

with E the expectation. Using this notation, one can introduce the following definition

Definition 1. If for all $\varepsilon > 0$, there exists a $\delta > 0$ such that for each $x_0 \in \mathbb{R}^n$ with $\|x_0\| < \delta$ and all $m \in \mathbb{N}$, one gets $\|x(t_m)\|_{\Omega} < \varepsilon$, then the zero solution of (2) on $\tilde{\mathbb{T}}$ is mean-square stable. If there is no dependence on δ, the zero solution of (2) on $\tilde{\mathbb{T}}$ is globally mean-square stable. If in addition, $\|x(t_m)\|_{\Omega} \to 0$ as $m \to \infty$ for sufficiently small $\|x_0\|$, then the zero solution of (2) on $\tilde{\mathbb{T}}$ is mean-square asymptotically stable. If the convergence is exponential, the zero solution of (2) on $\tilde{\mathbb{T}}$ is mean-square exponentially stable.

A special case worth considering is the case that the system is time-variant only due to the disruption in timing. In this case, (2) can be written as

$$x^{\Delta}(t) = g(\mu(t))x(t), \quad x(t_0) = x_0. \tag{3}$$

The mean square exponential stability of (3) on discrete, i.i.d. stochastic time scales is equivalent to a spectral condition on Lyapunov-like operators, as stated in the following theorem.

Theorem 1. (Poulsen et al. (2019a)). Let $\tilde{\mathbb{T}}$ be a stochastic time scale and let $\mathcal{H}(n)$ be the space of all $n \times n$ symmetric matrices. Define $\bar{\mathfrak{g}} := \mathrm{E}[\mu g(\mu)]$, and $\Pi(X) = \mathrm{E}[\mu g(\mu) -$

$\overline{\mathfrak{g}})^T X(\mu g(\mu) - \overline{\mathfrak{g}})]$. Define the operator $\mathcal{L}_{\overline{\mathfrak{g}}} : \mathcal{H}(n) \to \mathcal{H}(n)$ by

$$\mathcal{L}_{\overline{\mathfrak{g}}}(P) = (I + \overline{\mathfrak{g}})^T P (I + \overline{\mathfrak{g}}).$$

Let $\mathcal{I}$ be the identity operator.

Then (3) is globally exponentially mean-square stable if and only if all eigenvalues of $\overline{\mathfrak{g}} + I$ are in the open unit disk and

$$r\left(\left(\mathcal{I} - \mathcal{L}_{\overline{\mathfrak{g}}}\right)^{-1} \Pi\right) < 1, \tag{4}$$

where r denotes spectral radius.

3. PROBLEM STATEMENT

In this paper, we consider a group of $N+1$ cooperative agents, i.e. one leader and N followers. The dynamics of the agents are described by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t), \qquad i \in \{1, \cdots, N\}, \\ y_i(t) &= Cx_i(t) \\ \\ \dot{x}_0(t) &= Ax_0(t), \\ y_0(t) &= Cx_0(t) \end{aligned} \tag{5}$$

where $x_0 \in \mathbb{R}^2$ (resp. $x_i \in \mathbb{R}^2$) is the leader state (resp. state of agent i) and $u_i \in \mathbb{R}$ is the control input of agent i. $y_0 \in \mathbb{R}$ (resp. $y_i \in \mathbb{R}$) is the output measurement for the leader (resp. for agent i). $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $C = (1\ 0)$ are known constant real matrices.

The communication topology between the N followers is described by the graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of all followers, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the communication links among the followers. The adjacency matrix $[a_{ij}] \in \mathbb{R}^{N \times N}$ is defined by $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$, otherwise $a_{ij} = 0$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ with $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. The sampled information of the leader is only transmitted to some followers (not all of them). The communication topology among all the agents (i.e. followers and leader) is described by the graph $\bar{\mathcal{G}}$. This graph is defined by the matrix $P = L + M \in \mathbb{R}^{N \times N}$, where $M = diag(m_1, \ldots, m_N)$ with $m_i = 1$ if the sampled information of the leader is accessible by the agent i and with $m_i = 0$ otherwise.

Assumption 1. It is assumed that the communication topology among all the agents $\bar{\mathcal{G}}$ is fixed and has a directed spanning tree (the communincation topology contains a directed spanning tree if there is a node, called root node, such that there is a directed path from this node to every other node).

Assumption 2. It is considered that sampled information is exchanged between neighboring agents regularly at each sampling time except in the presence of an interruption of measurement. This may occur due to the intermittent sensor failures, for instance. Furthermore, it is assumed that the time instant and duration of intermittent sensor failure are stochastic and have the memoryless property.

Remark 1. Assumption 1 is classical to solve the consensus problem for multi-agent systems. This assumption implies that matrix P is a nonsingular M-matrix, that is, the eigenvalues of P have nonnegative real parts. Assumption 2 is less restrictive than many existing works dealing with intermittent information. Indeed, in (Taousser et al., 2016), it is assumed that the maximum duration of intermittent sensor failures is known to derive the controller gains. In (Poulsen et al., 2019b), it is required information on both position and velocity states of the neighbors. However, due to limited on-board measurement resources, it is often difficult to measure position and velocity of the agents.

To deal with intermittent sampled position measurements, a discrete stochastic time scale is used to represent the time domain. Indeed, the number of time steps when the position is measured in the case of intermittent sensor failures is a geometric random variable due to Assumption 2. More formally, we denote p as the probability that no sensor failure happens at a given time instant in the time scale and h is the normal (uninterrupted) sampling time of the system. Based on Assumption 2, the time domain can be modeled as the i.i.d. discrete stochastic time scale $\tilde{\mathbb{T}}$. It is obtained using variable μ with the probability mass function

$$f(\mu) = \begin{cases} p & \mu = h \\ (1-p)\gamma(\mu) & \mu > h \end{cases}. \tag{6}$$

In (6), $\gamma(\mu)$ is a probability distribution function and its support is a subset of (h, ∞).

Remark 2. In this paper, the distribution could belong to the class of mixed probability distributions since γ could be a continuous probability distribution. Contrary to existing works as discussed in Remark 1, in this paper we do not require a known maximum duration of intermittent sensor failures.

Discretizing system (5) onto the i.i.d. discrete stochastic time scale $\tilde{\mathbb{T}}$ yields

$$\begin{aligned} x_i^{\Delta}(t) &= A_d x_i(t) + B_d(\mu(t)) u_i(t), \qquad i \in \{1, \cdots, N\}, \\ y_i(t) &= Cx_i(t) \\ \\ x_0^{\Delta}(t) &= A_d x_0(t,) \\ y_0(t) &= Cx_0(t) \end{aligned} \tag{7}$$

$t \in \tilde{\mathbb{T}}$, with $A_d = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B_d(\mu(t)) = \begin{pmatrix} \frac{\mu(t)}{2} \\ 1 \end{pmatrix}$. One should highlight that system (7) is the exact discretization of system (5) in a sampled data setting.

The problem under investigation is to design an observer-based output feedback controller to solve the leader-follower consensus problem for system (7) evolving on the i.i.d. discrete stochastic time scale $\tilde{\mathbb{T}}$ due to intermittent sampled position measurements.

4. OBSERVER-BASED OUTPUT FEEDBACK CONSENSUS PROTOCOL

Let $\tilde{\mathbb{T}} = \{t_n\}_{n=0}^{\infty}$ be the sequence of sampling times obtained using the random variable μ (i.e. distance between sampling times which is non-uniform due to intermittent information) which satisfies (6). Using available sampled position data, the following discrete-time observer-based output feedback consensus protocol is proposed, $\forall i \in \{1, \cdots, N\}$, $\forall t \in \tilde{\mathbb{T}}$

$$u_i(t) = K \left(\sum_{j=1}^{N} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + m_i(\hat{x}_0(t) - \hat{x}_i(t)) \right), \tag{8}$$

where $\hat{x}_j$ is the estimated state of agent j ($j \in \{0, \cdots, N\}$) and $K \in \mathbb{R}^{1\times 2}$ is the control gain matrix.

An agent j estimates its own state using its measurement y_j and transmits this estimation to its neighbors. For each agent, the discrete-time observer dynamics is given by:

$$\begin{aligned} \hat{x}_j(t_{k+1}) &= \hat{x}_j(t_k) + \hat{A}\hat{x}_j(t_k) + \hat{B}u_j(t_k) \\ &\quad + H(y_j(t_k) - \hat{y}_j(t_k)), \\ \hat{y}_j(t_k) &= C\hat{x}_j(t_k) \end{aligned} \tag{9}$$

where $H \in \mathbb{R}^2$ is the observer gain matrix, for the leader $u_0 = 0$, and $\hat{A}$ and $\hat{B}$ are design choices.

Remark 3. It should be noted that since μ is a random variable, one cannot directly use μA_d and $\mu B_d(\mu)$ in Eq. (9) (instead, we use $\hat{A}$ and $\hat{B}$). Indeed, the next sampling time at time t_k is unknown. From Eq. (8), the control input for each agent is updated regularly at each sampling time except in the case of an intermittent sensor failure.

Using the $\Delta-$derivative, system (9) becomes $\forall j \in \{0, \cdots, N\}$, $\forall t \in \tilde{\mathbb{T}}$

$$\begin{aligned} \hat{x}_j^{\Delta} &= \frac{1}{\mu}\hat{A}\hat{x}_j + \frac{1}{\mu}\hat{B}u_j + \frac{1}{\mu}H(y_j - \hat{y}_j), \\ \hat{y}_j &= C\hat{x}_j \end{aligned} \tag{10}$$

Let us define the tracking error $e = (e_1^T, \ldots, e_N^T)^T$ where

$$e_i = x_i - x_0,$$

is the difference between the state of follower i and the leader. The estimated tracking error is denoted $\hat{e} = (\hat{e}_1^T, \ldots, \hat{e}_N^T)^T$ where

$$\hat{e}_i = \hat{x}_i - \hat{x}_0,$$

and its dynamics, obtained using (10), can be written as:

$$\hat{e}^{\Delta} = \frac{1}{\mu}(I_N \otimes \hat{A})\hat{e} - \frac{1}{\mu}(P \otimes \hat{B}K)\hat{e} + \frac{1}{\mu}(I_N \otimes HC)(e - \hat{e}), \tag{11}$$

where $I_N \in \mathbb{R}^{N\times N}$ is the identity matrix.

The tracking error dynamics can be expressed as follows:

$$e^{\Delta} = (I_N \otimes A_d)e - (P \otimes B_d(\mu)K)\hat{e}. \tag{12}$$

Let us denote the error

$$\epsilon = e - \hat{e}.$$

Its $\Delta-$derivative is given by

$$\begin{aligned} \epsilon^{\Delta} = &\left[(I_N \otimes (A_d - \frac{1}{\mu}\hat{A})) - (P \otimes (B_d(\mu) - \frac{1}{\mu}\hat{B})K) \right] e \\ &+ \left[(P \otimes (B_d(\mu) - \frac{1}{\mu}\hat{B})K) + (I_N \otimes \frac{1}{\mu}\hat{A}) - \frac{1}{\mu}(I_N \otimes HC) \right] \epsilon \end{aligned} \tag{13}$$

In a compact form, the tracking error dynamics can be expressed as:

$$\begin{pmatrix} e \\ \epsilon \end{pmatrix}^{\Delta} = \Lambda(\mu, K, H) \begin{pmatrix} e \\ \epsilon \end{pmatrix}, \tag{14}$$

with $\Lambda(\mu, K, H)$ obtained from (12)-(13).

To design the observer-based output feedback consensus protocol for double-integrator MASs under intermittent sampled position measurements, the following corollary to Theorem 1, which guarantees the exponential mean-square stability of the zero solution of (14), is introduced.

Corollary 1. Let $K \in \mathbb{R}^{1\times 2}$ and $H \in \mathbb{R}^2$ be fixed and let $\Lambda(\mu, K, H)$ be as in (14). Let us introduce the operators: $\mathcal{L}_{\overline{\Lambda}(K,H)} : \mathcal{H}(2N) \to \mathcal{H}(2N)$ where

$$\mathcal{L}_{\overline{\Lambda}(K,H)}(X) = (I + \overline{\Lambda}(K,H))^T X (I + \overline{\Lambda}(K,H)),$$

with $\overline{\Lambda}(K,H) = \mathrm{E}[\mu\Lambda(\mu,K,H)]$ and $\Pi_{(K,H)}(X) = \mathrm{E}[(\mu\Lambda(\mu,K,H) - \overline{\Lambda}(K,H))^T X (\mu\Lambda(\mu,K,H) - \overline{\Lambda}(K,H))]$. The zero solution of (14) is globally exponentially mean-square stable if and only if all eigenvalues of $\overline{\Lambda}(K,H) + I$ are in the open unit disk and

$$r\left(\left(\mathcal{I} - \mathcal{L}_{\overline{\Lambda}(K,H)} \right)^{-1} \Pi_{(K,H)} \right) < 1. \tag{15}$$

where $\mathcal{I} : H(2N) \to H(2N)$ is the identity operator and r is the spectral radius.

Proof: The proof follows from the fact that the error dynamics, for a fixed pair of gain matrices (K, H) are of the form (3). Therefore Theorem 1, is applicable. □

Remark 4. Since $\tilde{\mathbb{T}}$ is a discrete stochastic time scale, the design of an observer-based output feedback controller is not an easy task. Furthermore, it is clear that matrix Λ is not block triangular, making the decoupling between the observer and controller parts not possible. Nevertheless, by choosing $\hat{A} = E[\mu A_d]$ and $\hat{B} = E[\mu B_d(\mu)]$, we see $\Lambda(\mu, K, H)$ is a block triangular matrix on average, i.e.

$$\overline{\Lambda}(K,H) = \begin{pmatrix} (I_N \otimes \hat{A}) - (P \otimes \hat{B}K) & (P \otimes \hat{B}K) \\ 0 & I_N \otimes (\hat{A} - HC) \end{pmatrix}.$$

With these choices of $\hat{A}$ and $\hat{B}$, due to the block triangular structure of $\overline{\Lambda}(K,H)$, the condition that all eigenvalues of $\overline{\Lambda}(K,H) + I$ are in the open unit disk is equivalent to all the eigenvalues of $I_{2N} + (I_N \otimes \hat{A}) - (P \otimes \hat{B}K)$ and $I_{2N} + I_N \otimes (\hat{A} - HC)$ are in the open unit disk. Thus, a practitioner could design the controller gain and observer gain separately, provided controllability and observability criteria are satisfied, then check if (15) also held for the choice of (K, H). If the condition in (15) failed, the practitioner could design a new pair of gain matrices.

5. EXAMPLE

In this example, let us consider a multi-agent system which consists in two followers, denoted as 1 and 2, and one leader denoted as 0. The communication topology $\bar{\mathcal{G}}$ among all the agents, defined by:

$$P = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

is fixed and has a directed spanning tree. It is considered that sampled information is exchanged between neighboring agents regularly at each sampling time (i.e. the time step h is $0.1s$) except in the presence of an interruption of measurement. Hence, the time domain is modeled as an i.i.d. discrete stochastic time scale obtained using the probability mass function (6) with $h = 0.1$, $p = 5/6$ and $\gamma(\mu)$ the discrete uniform distribution over the values $\{0.2, 0.3, \ldots, 1.1\}$.

The problem under investigation is to design an observer-based output feedback controller to solve the leader-follower consensus problem for system (7) evolving on the i.i.d. discrete stochastic time scale $\tilde{\mathbb{T}}$ due to intermittent sampled position measurements.

We find that for the choices $K = (1.6, 1.6), H = (1.6, 0.6)^T$, $\hat{A} = E[\mu A_d]$, and $\hat{B} = E[\mu B_d(\mu)]$, the following inequalities are satisfied:

$$r(I_{2N} + I_N \otimes (\hat{A} - HC)) < 1$$
$$r(I_{2N} + (I_N \otimes \hat{A}) - (P \otimes \hat{B}K)) < 1$$
$$r\left(\left(\mathcal{I} - \mathcal{L}_{\overline{\Lambda}(H,K)}\right)^{-1} \Pi_{(K,H)}\right) < 1.$$

Therefore, by Corollary 1 and Remark 4, we conclude that the error dynamics of system (14) are mean-square exponentially stable, and therefore mean-square exponential consensus is achieved. We illustrate this conclusion with simulation results.

In the following, the initial configuration of the agents is $x_0(0) = (0, 1)^T$, $x_1(0) = (5, 2)^T$ and $x_2(0) = (-2, -2)^T$. The initial state estimations are given by $\hat{x}_0(0) = (1, 2)$, $\hat{x}_1(0) = (4, 1)$ and $\hat{x}_2(0) = (-1, -0.5)$. In all of the figures that follow, real time is on the x-axis. One can see the interruption in timing via the horizontal nonuniformity between plot points. Figure 1 depicts the trajectories of the leader $x_0 = (x_{0,1}, x_{0,2})^T$ and of the two followers $x_i = (x_{i,1}, x_{i,2})^T$, $i = 1, 2$. Figure 2 shows the estimated state and actual state of follower 2 versus time. A similar figure could be made for follower 1.

It is worth noting that the estimated state is equal to the actual state on average, but the variance around the mean will not converge to zero, but instead to some finite value. When estimating the velocity, the observer uses the position of the agent (since C has a zero in the second entry corresponding to velocity), but the observer has no sense of how much time has passed since the last communication instance. The variance of the position residuals is smaller due to the measured position being available, but the same argument about the time step not being available leads to some variance. Therefore, the estimated state forms a zero mean, (in general) non-Gaussian noise about the true state due to the timing variability. We show a histogram of the difference between the true state and the estimated state over 2000 seconds in Figure 3.

In spite of intermittent sampled position measurements, it is worth noting that the consensus is achieved using the proposed observer-based feedback control since the tracking errors converge to zero.

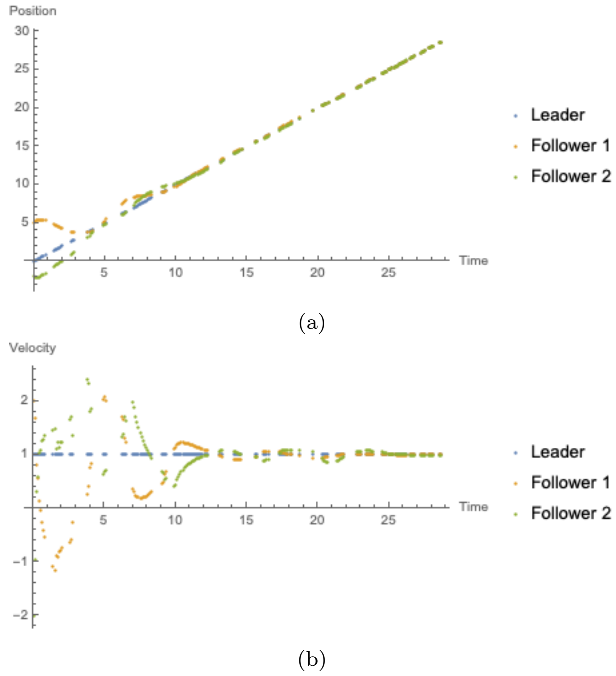


Fig. 1. Trajectories of the leader $x_0 = (x_{0,1}, x_{0,2})^T$ and of the two followers $x_i = (x_{i,1}, x_{i,2})^T$, $i = 1, 2$. (a) Evolution of the position for each agent. (b) Evolution of the velocity for each agent.

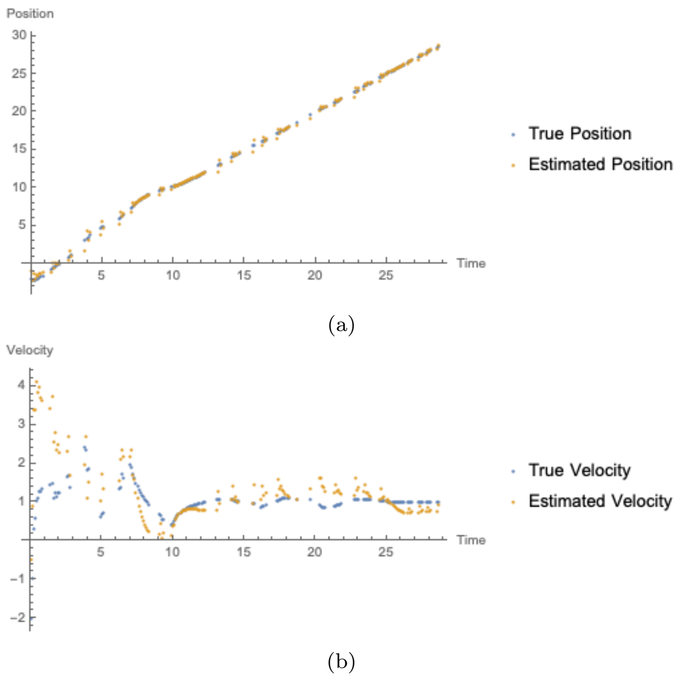


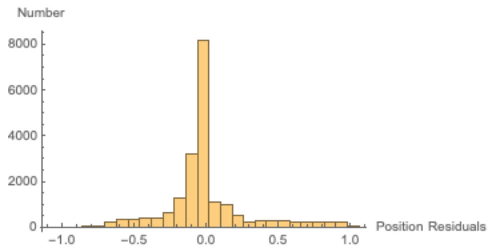
Fig. 2. The estimated states and the actual states of follower 2 versus time. (a) The estimated position and actual position of follower 2. (b) The estimated velocity and actual velocity of follower 2.

6. CONCLUSION

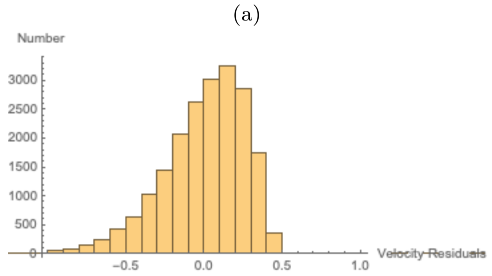
This paper has introduced an observer-based output feedback controller to investigate the leader-follower consensus problem for double-integrator MASs using sampled intermittent position data information. Using the stochastic time scale theory, some necessary and sufficient conditions to design the observer and controller gains have been derived to guarantee mean-square exponential consensus. Some simulation results have illustrated the effectiveness of the proposed scheme.

REFERENCES

Ajwad, S.A., Ménard, T., Moulay, E., Defoort, M., and Coirault, P. (2019). Observer based leader-following consensus of second-order multi-agent systems with nonuniform sampled position data. *Journal of the Franklin Institute*, 356(16), 10031–10057.



(a)



(b)

Fig. 3. Histograms for (a) the position residuals, and (b) the velocity residuals, over 2000 seconds of measurement. We see the distribution has mean approximately equal to zero, but the distribution is skewed.

Babenko, S., Defoort, M., Djemai, M., and Nicaise, S. (2018). On the consensus tracking investigation for multi-agent systems on time scale via matrix-valued lyapunov functions. *Automatica*, 97, 316–326.

Bartosiewicz, Z. (2019). Exponential stability of nonlinear positive systems on time scales. *Nonlinear Analysis: Hybrid Systems*, 33, 143–150.

Bohner, M. and Peterson, A. (2012). Dynamic equations on time scales: An introduction with applications.

Cao, Y., Sun, Y., and Xie, X.J. (2017). Explicit condition for consensus of third-order discrete-time multi-agent systems without accelerated velocity measurements. *Journal of The Franklin Institute*, 354(12), 5056–5066.

Defoort, M., Demesure, G., Zuo, Z., Polyakov, A., and Djemai, M. (2016). Fixed-time stabilisation and consensus of non-holonomic systems. *IET Control Theory & Applications*, 10(18), 2497–2505.

Ding, L., Han, Q.L., Ge, X., and Zhang, X.M. (2017). An overview of recent advances in event-triggered consensus of multiagent systems. *IEEE transactions on cybernetics*, 48(4), 1110–1123.

Dorri, A., Kanhere, S.S., and Jurdak, R. (2018). Multi-agent systems: A survey. *IEEE Access*, 6, 28573–28593.

Gao, Y. and Wang, L. (2010). Asynchronous consensus of continuous-time multi-agent systems with intermittent measurements. *International Journal of Control*, 83(3), 552–562.

Girejko, E. and Malinowska, A.B. (2019). Leader-following consensus for networks with single-and double-integrator dynamics. *Nonlinear Analysis: Hybrid Systems*, 31, 302–316.

Huang, N., Duan, Z., and Zhao, Y. (2014). Leader-following consensus of second-order non-linear multi-agent systems with directed intermittent communication. *IET Control Theory & Applications*, 8(10), 782–795.

Li, S., Du, H., and Shi, P. (2014). Distributed attitude control for multiple spacecraft with communication delays. *IEEE Transactions on Aerospace and Electronic Systems*, 50(3), 1765–1773.

Lin, P., Ren, W., and Gao, H. (2016). Distributed velocity-constrained consensus of discrete-time multi-agent systems with nonconvex constraints, switching topologies, and delays. *IEEE Transactions on Automatic Control*, 62(11), 5788–5794.

Phillips, S. and Sanfelice, R.G. (2019). Robust distributed synchronization of networked linear systems with intermittent information. *Automatica*, 105, 323–333.

Poulsen, D., Davis, J., and Gravagne, I. (2019a). Mean square stability for systems on stochastically generated discrete time scales. *Nonlinear Analysis: Hybrid Systems*, 31, 41–55.

Poulsen, D., Defoort, M., and Djemai, M. (2019b). Mean square consensus of double-integrator multi-agent systems under intermittent control: A stochastic time scale approach. *Journal of the Franklin Institute*, 356(16), 9076–9094.

Ren, W. and Beard, R.W. (2005). Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control*, 50(5), 655–661.

Rezaee, H. and Abdollahi, F. (2017). Discrete-time consensus strategy for a class of high-order linear multiagent systems under stochastic communication topologies. *Journal of the Franklin Institute*, 354(9), 3690–3705.

Shen, J. and Cao, J. (2012). Consensus of multi-agent systems on time scales. *IMA Journal of Mathematical Control and Information*, 29(4), 507–517.

Shi, P. and Shen, Q. (2017). Observer-based leader-following consensus of uncertain nonlinear multi-agent systems. *International Journal of Robust and Nonlinear Control*, 27(17), 3794–3811.

Su, H., Qiu, Y., and Wang, L. (2017). Semi-global output consensus of discrete-time multi-agent systems with input saturation and external disturbances. *ISA transactions*, 67, 131–139.

Taousser, F., Defoort, M., and Djemai, M. (2016). Consensus for linear multi-agent system with intermittent information transmissions using the time-scale theory. *International journal of Control*, 89(1), 210–220.

Taousser, F.Z., Defoort, M., Djemai, M., Djouadi, S.M., and Tomsovic, K. (2019). Stability analysis of a class of switched nonlinear systems using the time scale theory. *Nonlinear Analysis: Hybrid Systems*, 33, 195–210.

Wen, G., Duan, Z., Yu, W., and Chen, G. (2013). Consensus of multi-agent systems with nonlinear dynamics and sampled-data information: a delayed-input approach. *International Journal of Robust and Nonlinear Control*, 23(6), 602–619.

Zhao, H. and Park, J.H. (2014). Group consensus of discrete-time multi-agent systems with fixed and stochastic switching topologies. *Nonlinear Dynamics*, 77(4), 1297–1307.

Zuo, Z., Defoort, M., Tian, B., and Ding, Z. (2019). Distributed consensus observer for multi-agent systems with high-order integrator dynamics. *IEEE Transactions on Automatic Control*.