Distributed Consensus Control for General Linear Multi-agent Systems via a Dynamic Event-triggered Strategy

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Abstract: This paper puts forward a distributed dynamic event-triggered strategy to solve the distributed event-triggered consensus problem of linear multi-agent systems under directed graphs. Based on dynamic triggering function, each agent can reach consensus asymptotically. Different from existing static triggering schemes, the proposed dynamic triggering scheme, where an internal dynamic variable is involved, results in larger inter-event times and also leads to less communication overheads among agents, which is conducive to guaranteeing that Zeno behavior is excluded for each agent. In addition, under the proposed strategy, neither controller updates nor triggering threshold detections require continuous communication. Finally, the effectiveness of the theoretical analysis is demonstrated by numerical simulations.

Keywords: Multi-agent systems, dynamic event-triggered strategy, distributed control.

1. INTRODUCTION

In the past decades, the problem of coordinated control of multi-agent systems (MASs) has received increasing attention (Olfati et al. [2004], Ren et al. [2008]). Modeling of MASs originates from social animals such as insects, fish, and birds, which benefits the accomplishment of the tasks that are difficult for individuals. MASs also exist in many engineering fields, such as distributed optimization (Li et al. [2017]), wireless sensor networks (Ding et al. [2017]), mobile robot collaboration (Meng et al. [2013]), drone/satellite formation flight (Anderson et al. [2007], Marshall et al. [2006]) and so on. Thus, MASs have enabled many scholars around the world to do a lot of research work in this field in highly interconnected present world (see Huang et al. [2019] - Tan et al. [2018] and the references therein).

In general, designing a suitable control strategy to make all agents' states reach a common quantity related to certain control performance is a pivotal issue on consensus of MASs. It is worth mentioning that most of the aforementioned works on consensus problems are obtained under the assumption that continuous communication among agents is available. However, it is difficult to maintain such an environment with continuous communication and unnecessary communication also leads to a waste of energy. To this end, distributed controllers with intermittent communication have been studied recently. In Wen et al. [2013], a periodic sampling control protocol was studied, where the sampling is triggered after a fixed time interval. However, the controller still update periodically even after the control target has been achieved. In many cases, due to the limitations of energy supply or communication bandwidth, it is desired that information exchanges among agents only occur at some discrete and non-periodic sampling points, so event-triggered control schemes have been introduced. By viewing a triggered event as a moment when a certain measurement error exceeds a pre-designed threshold, the communication among agents is required under the event-triggered control strategy only when an event is triggered. The event-triggered consensus control problems for MASs with single- or double-integrator dynamics were investigated in Dimarogonas et al. [2012] - Nowzari et al. [2016]. Subsequently, plenty of scholars conducted research on event-triggered consensus control of general linear MASs. Event-triggered consensus control protocols were designed for general linear MASs over undirected and directed graphs in Zhang et al. [2014] and Zhu et al. [2014], respectively. However, in the above-mentioned works, event-triggered functions still need to continuously access to neighbors' state information. To solve this problem, considering general linear MASs under directed graphs, Yang et al. [2016] designed a triggering threshold based on on exponential function and Liu et al. [2018] proposed a triggering threshold related to the sum of relative state estimations from itself and its neighbors for each agent. A model-based event-triggered controller and a dynamic threshold approaching zero in finite time are put forward in Du et al. [2019] to achieve finite-time consensus without the requirement on continuous state transmission.

It should be pointed out that all aforementioned results are obtained under the framework of static event-triggered control mechanisms. However, a new class of dynamic event-triggered control mechanisms, where internal dynamic variables are involved, have several merits with respect to the commonly studied static one including the significant larger inter-event time, which is beneficial to prevent Zeno behavior in practical application.

Therefore, it has been widely investigated that the dynamic event-triggered control method has been used to solve consensus problems of MASs in recent years. Girard et al. [2015] occupied internal dynamic variables in event-triggered control for nonlinear systems. Yi et al. [2017] improved the form of the dynamic event-triggered mechanism in a distributed manner and extended it to a single-integrator MAS. Based on this work, Yi et al. [2019] used internal dynamic variables in self-triggered control to overcome the drawback of continuous sensing and listening of the triggering. In Ge et al. [2017], a dynamic event-triggered communication mechanism was used to address a distributed resource-efficient formation control problem of a networked MAS with general linear system dynamics. In George et al [2018], the dynamic average consensus problem was solved by the proposed dynamic event-triggered algorithm, which was robust to network disruptions. However, all interaction topologies among agents in the above-mentioned works are assumed as undirected graphs. In fact, an undirected graph can be seen as a special case of a directed graph. Therefore, it's very meaningful and practical to investigate the consensus problem over directed graphs. Motived by the previous works, we discuss the dynamic event-triggered consensus problem for general linear MASs on directed graphs.

In a word, we will discuss the dynamic event-triggered consensus problem of general linear MASs under directed graphs and exhibit Zeno behavior in this paper. The principal contributions are summarized as follows:

(1) Compared to the static event-triggered mechanism, such as Zhu et al. [2014]-Liu et al. [2018], the dynamic event-triggered function with an internal dynamic variable proposed in this paper yields the larger triggering intervals, which benefits the exclusion of Zeno behavior in practice. Moreover, the communication instants are reduced significantly which also saves the communication energy greatly.
(2) Different from most of the existing works on dynamic triggering mechanisms, which mainly focus on the integrator-type dynamics and undirected graphs (Girard et al. [2015]-George et al [2018]), this paper investigates the consensus problem with dynamic event-triggered strategy for general linear MASs on directed graphs, which in turn poses more challenges in the consensus stability analysis and Zeno behavior exclusion due to the more general agents' models and more complex communication topologies.
(3) The issue that continuous access to neighbors' states is still required in agent's own triggering detection is ignored in many existing works on both static and dynamic triggering mechanisms (see Dimarogonas et al. [2012], Zhang et al. [2014], Zhu et al. [2014], Girard et al. [2015]), which brings about a paradox to the original purpose of saving communication energy by introducing the event-triggered strategy. In this endeavor, this paper aims to avoid the continuous communication in not only the controller update but also the triggering detection, which poses more challenges in the triggering function design under the framework of the dynamic event-triggered strategy.

The rest of this paper is organized as follows. Some preliminaries including useful knowledge and the dynamics are introduced in Section 2. Section 3 presents the main result and Section 4 illustrates the result through simulation examples. Section 5 concludes the paper.

Notation: Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}^{m\times n}$ be the set of $m \times n$ real matrices, respectively. $\mathcal{I}$ is a set of positive integers and $\mathcal{I}_N = \{1, 2, \dots, N\}$. $\mathbf{0}_N$ and $\mathbf{1}_N$ mean the $N \times 1$ column vector of all zeros and ones, respectively. For a vector $x \in \mathbb{R}^n$, $x > 0$ means that every entry $x_i > 0$ with $i = 1, 2, \dots, n$. For a symmetric matrix P, $P > 0$ means that P is positive definite and $\lambda_{\max}(P)$ ($\lambda_{\min}(P)$) means the maximum (minimum) eigenvalues of P. The superscript T and the symbol $\otimes$ represent the transpose for matrices and the Kronecker product, respectively. Denote $\|\cdot\|$ as the Euclidean norm for vectors and the induced 2-norm for matrices.

2. PRELIMINARIES

In this section, we introduce some definitions in algebraic graph theory and the considered MAS briefly.

2.1 Graph Theory

Consider a group of MASs with N agents. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of a nonempty finite node set $\mathcal{V} = \{v_1, \dots, v_N\}$, an edge set $\mathcal{E} \in (\mathcal{V} \times \mathcal{V})$ and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. The edge $(v_i, v_j) \in \mathcal{E}$ indicates that the node v_j can receive information from the node v_i or the node v_i can broadcast information to the node v_j. The neighbor set of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. The adjacency matrix $\mathcal{A}$ of a directed graph is given by $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The Laplacian matrix of $\mathcal{G}$ is defined as $L = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ with $i \neq j$. If between any pair of distinct nodes v_i and v_j in a directed graph $\mathcal{G}$, there exists a directed path from v_i to v_j, $i, j = 1, 2, \dots, N$, $\mathcal{G}$ is strongly connected. For the purpose of drawing forth our main result, we need the following assumptions and lemmas.

Assumption 2.1. The directed graph $\mathcal{G}$ is strongly connected.

Lemma 1. (Hardy et al. [1952]) The general algebraic connectivity of a strongly connected graph $\mathcal{G}$ associated with the Laplacian matrix $\mathcal{L}$ is defined by

$$a(\mathcal{L}) = \min_{\mathbf{r}^T x=0, x\neq 0} \frac{x^T \widetilde{\mathcal{L}} x}{x^T R x},$$

where $\widetilde{\mathcal{L}} = \frac{1}{2}(R\mathcal{L} + \mathcal{L}R^T)$, $R = \mathrm{diag}(r_1, \dots, r_N)$ with $\mathbf{r} = (r_1, \dots, r_N)^T$ satisfying $\mathbf{r}^T\mathcal{L} = \mathbf{0}_N$ and $\sum_{i=1}^{N} r_i = 1$.

Lemma 2. (Hardy et al. [1952]) Given any $x, y \in \mathbb{R}^N$, Young's inequality states that for any $\phi > 0$, $x^T y \leq \frac{x^T x}{2\phi} + \frac{\phi y^T y}{2}$.

Lemma 3. (Yu et al. [2010]) For a strongly connected directed graph $\mathcal{G}$, zero is a simple eigenvalue of $\mathcal{L}$ with the corresponding right eigenvector $\mathbf{1}_N$, that is $\mathcal{L}\mathbf{1}_N = 0$, and there exists a positive vector $\mathbf{r} = (r_1, \dots, r_N)^T$ satisfy $\mathbf{r}^T\mathbf{1}_N = 1$ such that $\mathbf{r}^T\mathcal{L} = \mathbf{0}_N$.

2.2 Multi-Agent System Model

Consider a linear MAS consisting of N identical agents, indexed by $1, \dots, N$. The dynamics of the ith agent is described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \mathcal{I}_N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the agent state, $u_i(t) \in \mathbb{R}^p$ is the control input. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times p}$.

Assumption 2.2. The matrix pair (A, B) in (1) is stabilizable.

The objective of this paper is to design a distributed dynamic event-triggered consensus strategy for each agent such that the states of all agents achieve consensus while avoiding Zeno behavior.

3. MAIN RESULT

In this section, a dynamic event-triggered control strategy will be proposed to deal with consensus problems for the linear MASs under strongly connected directed graphs without Zeno behavior.

A dynamic event-triggered consensus control protocol is proposed for each agent as follows:

$$u_i(t) = cKz_i(t), \tag{2}$$

where $c > 0$, and the feedback gain matrix $K \in \mathbb{R}^{p\times n}$ is chosen by $K = -B^TP$ with a positive matrix P to be decided. Moreover,

$$z_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(e^{A(t-t^i_{k_i})}x_i(t^i_{k_i}) - e^{A(t-t^j_{k_j})}x_j(t^j_{k_j})), \tag{3}$$

where $k_i \in \mathcal{I}_N$, a_{ij} is the ijth entry of the adjacency matrix $\mathcal{A}$, $t^i_{k_i}$ and $x_i(t^i_{k_i})$ are the latest event-triggered time and the latest broadcast state of agent i, respectively. For the sake of simplicity, we denote $e^{A(t-t^i_{k_i})}x_i(t^i_{k_i})$ as $\bar{x}_i(t)$, so the measurement error is defined as

$$e_i(t) = e^{A(t-t^i_{k_i})}x_i(t^i_{k_i}) - x_i(t) = \bar{x}_i(t) - x_i(t). \tag{4}$$

In addition, closed-loop form of the MAS can be expressed as

$$\dot{x}(t) = (I_N \otimes A + c\mathcal{L} \otimes BK)x(t) + (c\mathcal{L} \otimes BK)e(t), \tag{5}$$

where $x(t) = [x_1^T(t), \dots, x_N^T(t)]^T$, $e(t) = [e_1^T(t), \dots, e_N^T(t)]^T$.

Then, we define $\delta_i(t) = x_i(t) - \sum_{j=1}^N r_ix_j(t)$ as a disagreement vector for each agent i, where r_j is the jth row of $\mathbf{r}$ defined in Lemma 1. By putting δ_i in a stack, namely, $\delta(t) = [\delta_1^T(t), \delta_2^T(t), \dots, \delta_N^T(t)]$, one has

$$\delta(t) = x(t) - (\mathbf{1}_N\mathbf{r}^T \otimes I_n)x(t) = (M \otimes I_n)x(t)$$

with $M = (I_N - \mathbf{1}_N\mathbf{r}^T)$. Meanwhile, for agent i, we also have $\bar{\delta}(t) = (M \otimes I_n)\bar{x}(t)$.

Therefore, based on the closed-loop system (5), we can see the disagreement vector $\delta(t)$ satisfies

$$\dot{\delta}(t) = (I_N \otimes A + c\mathcal{L} \otimes BK)\delta(t) + (c\mathcal{L} \otimes BK)e(t), \tag{6}$$

where we have used the fact that $M\mathcal{L} = \mathcal{L}M = \mathcal{L}$.

Now, we introduce an internal dynamic variable satisfying

$$\dot{\eta}_i(t) = -\sigma_i\eta_i(t) + \xi_i(\frac{\epsilon}{\alpha^*}\|z_i(t)\|^2 - \|e_i(t)\|^2), \tag{7}$$

where $i \in \mathcal{I}_N$, $\eta(0) > 0$ and the parameters satisfy $\sigma_i > 0$, $\epsilon > 0$, $\alpha^* > \gamma\|\mathcal{L}\|^2$ and $\xi_i \in [0, \gamma]$ with $\gamma = (1 - \frac{1}{\alpha})\|M\|^2 + \frac{c}{\beta}\lambda_{max}(\mathcal{L}^TR^2\mathcal{L} \otimes PBB^TP) + 2c\left\|M^TR\mathcal{L} \otimes PBB^TP\right\|$, where R defined in Lemma 1. Moreover, α, β are the Young inequality parameters satisfying $0 < \alpha < 1$ and $\beta > 0$.

Moreover, inspired by Yi et al. [2019], we assume that the first triggering time $t_1^i = 0$, so the triggering times $\{t_k^i\}_{k=2}^\infty$ is determined by

$$t^i_{k_i+1} = \max_{r\geq t^i_{k_i}}\{(\|e_i\|^2 \leq \frac{\epsilon}{\alpha^*}\|z_i(t)\|^2 + \frac{\eta_i(t)}{\theta_i}, \forall t \in [t^i_{k_i}, r]\}, \tag{8}$$

where $\theta_i > \frac{\gamma-\xi_i}{\sigma_i}$.

Remark 1. The proposed mechanism (8) is called the dynamic event-triggered mechanism since it involves an internal variable $\eta_i(t)$. If setting $\eta_i(t) \equiv 0$, we can get the static event-triggered mechanism (9). In addition, it can also be seen a limit case of the dynamic triggering mechanism (8) when the parameter θ_i goes larger enough.

$$t^i_{k_i+1} = \max_{r\geq t^i_{k_i}}\{\|e_i(t)\| \leq \sqrt{\frac{\epsilon}{\alpha^*}}\|z_i\|, \forall t \in [t^i_{k_i}, r]\}. \tag{9}$$

Compared with (8), (9) does not involve any extra dynamic variables except $x_i(t), \bar{x}_i(t)$ and $\bar{x}_j(t), j \in \mathcal{N}_i$.

Nest, we present the following theorem to cope with the dynamic consensus problem.

Theorem 4. Consider the linear MAS (1), and suppose that Assumptions 2.1 and 2.2 hold. Under the proposed distributed dynamic event-triggered consensus protocol composed of controller (2)-(3) and the dynamic triggering mechanism (8), the event-triggered consensus problem can be solved for any initial states if the parameters c, ϵ, α, β are selected such that $\frac{c\beta}{1-\alpha}\lambda_{max}(PBB^TP) + \frac{\epsilon}{1-\alpha} < -\lambda_{max}(R\otimes Q)$, where $Q = PA + A^TP - 2ca(\mathcal{L})PBB^TP < 0$. In addition, Zeno behavior can be excluded.

Proof.

According to (7) and (8), one has $\dot{\eta}_i(t) \geq -(\sigma_i + \frac{\xi_i}{\theta_i})\eta_i(t)$. So it is easy to get

$$\eta_i(t) > \eta_i(0)e^{-(\sigma_i+\frac{\xi_i}{\theta_i})t} > 0. \tag{10}$$

Therefore, considering the dynamic triggering function, we choose the following Lyapunov function:

$$W = \delta^T(R \otimes P)\delta + \sum_{i=1}^N \eta_i(t). \tag{11}$$

The time derivative of W along the closed-loop system (6) is given by

$$\begin{aligned}\dot{W} =& 2\delta^T(R \otimes P)\dot{\delta} + \sum_{i=1}^N \dot{\eta}_i(t)\\ =& \delta^T(R \otimes (PA + A^TP)\\ & - c(R\mathcal{L} + \mathcal{L}^TR) \otimes PBB^TP)\delta\\ & - 2\delta^T(cR\mathcal{L} \otimes PBB^TP)e + \sum_{i=1}^N \dot{\eta}_i(t).\end{aligned} \tag{12}$$

It follows from Lemma 1 that

$$-\delta^Tc((R\mathcal{L} + \mathcal{L}^TR) \otimes PBB^TP)\delta \leq -2ca(\mathcal{L})\delta^T(R \otimes PBB^TP)\delta.$$

Thus, we can get that

$$\begin{aligned}\dot{W} \leq &\delta^T(R\otimes Q)\delta - 2\delta^T(cR\mathcal{L}\otimes PBB^TP)e + \sum_{i=1}^{N}\dot{\eta}_i(t)\\ \leq &\lambda_{max}(R\otimes Q)[\bar{\delta}-(M\otimes I_n)e]^T[\bar{\delta}-(M\otimes I_n)e]\\ &-2[\bar{\delta}-(M\otimes I_n)e]^T(cR\mathcal{L}\otimes PBB^TP)e + \sum_{i=1}^{N}\dot{\eta}_i(t),\end{aligned} \tag{13}$$

where we have used the fact that $\bar{\delta} = \delta + (M\otimes I_n)e$.

By the Lemma 2, one has

$$\begin{aligned}&\lambda_{max}(R\otimes Q)[\bar{\delta}-(M\otimes I_n)e]^T[\bar{\delta}-(M\otimes I_n)e]\\ &\leq \lambda_{max}(R\otimes Q)[(1-\alpha)\bar{\delta}^T\bar{\delta} + (1-\frac{1}{\alpha})e^T(M^TM\otimes I_n)e],\end{aligned} \tag{14}$$

where $0 < \alpha < 1$.

Given $\beta > 0$, the second term in (13) can be handled according to the Lemma 2 as well

$$\begin{aligned}&-2[\bar{\delta}-(M\otimes I_n)e]^T(cR\mathcal{L}\otimes PBB^TP)e\\ \leq &c\beta\lambda_{max}(PBB^TP)\bar{\delta}^T\bar{\delta} + [\frac{c}{\beta}\lambda_{max}(\mathcal{L}^TR^2\mathcal{L}\otimes PBB^TP)\\ &+2c\left\|M^TR\mathcal{L}\otimes PBB^TP\right\|]\left\|e\right\|^2.\end{aligned} \tag{15}$$

Thus, substituting (7), (14) and (15) into (13) yields that

$$\begin{aligned}\dot{W} \leq &[\lambda_{max}(R\otimes Q)(1-\alpha) + c\beta\lambda_{max}(PBB^TP)]\left\|\bar{\delta}\right\|^2\\ &+\sum_{i=1}^{N}(\gamma-\xi_i)\left\|e_i(t)\right\|^2 + \sum_{i=1}^{N}\xi_i\frac{\epsilon}{\alpha^*}\left\|z_i(t)\right\|^2 - \sum_{i=1}^{N}\sigma_i\eta_i(t).\end{aligned} \tag{16}$$

Since the fact that $z = (\mathcal{L}\otimes I_n)\bar{x} = (\mathcal{L}\otimes I_n)\bar{\delta}$, we have $\left\|z\right\|^2 \leq \left\|\mathcal{L}\right\|^2\left\|\bar{\delta}\right\|^2$. Then, according to the triggering function (8), (16) can be rewritten as

$$\begin{aligned}\dot{W} \leq &[\lambda_{max}(R\otimes Q)(1-\alpha) + c\beta\lambda_{max}(PBB^TP)\\ &+\epsilon]\left\|\bar{\delta}\right\|^2 - \sum_{i=1}^{N}(\sigma_i - \frac{\gamma-\xi_i}{\theta_i})\eta_i(t) < 0.\end{aligned} \tag{17}$$

Therefore, we can conclude that the disagreement vector $\delta \to \mathbf{0}$ as $t \to \infty$, which means the MAS (1) can achieve the consensus asymptotically.

Now, we prove that Zeno behavior is strictly ruled out for each agent. Firstly, suppose that Zeno behavior is existed, which implies that there exists an agent i, such that $\lim_{k_i\to+\infty} t_{k_i}^i = T_0$, where T_0 is a positive constant.

Let $\varepsilon_0 = \frac{1}{2\|A\|}\ln(\frac{1}{\varpi}\sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\sigma_i+\frac{\xi_i}{\theta_i})T_0} + 1) > 0$, where $\varpi = \frac{\|A\|}{c\rho\|BK\|}$. Then according to the property of limits, there exists a positive integer $N(\varepsilon_0)$ such that

$$t_{k_i}^i \in [T_0-\varepsilon_0, T_0], \forall k_i \geq N(\varepsilon_0). \tag{18}$$

Noting that (10) holds, we can conclude that one sufficient condition to guarantee that the inequality in (8) holds is

$$\left\|e_i(t)\right\| \leq \sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\sigma_i+\frac{\xi_i}{\theta_i})t} \tag{19}$$

It follows from the fact that the interval between two consecutive triggering events is bounded, so it is apparent that $e^{A(t-t_{k_i}^i)}$ is bounded for $\forall t \in [t_{k_i}^i, t_{k_i+1}^i)$. In light of (5), it is not challenging to verify that $(\mathbf{r}^T\otimes e^{-At})x$ is an invariant quantity. Therefore, deriving from $x_i = \delta_i + \sum_{j=1}^{N} r_ix_j$, we obtain that $x(t)$ is finite for any finite t. Thus, we can get that for $\forall t \in [t_{k_i}^i, t_{k_i+1}^i)$, the triggering error $e(t) = \bar{x}_i(t) - x_i(t) = e^{A(t-t_{k_i}^i)}x_i(t_{k_i}^i) - x_i(t)$ is also bounded. According to the fact $z = (\mathcal{L}\otimes I_n)(x+e) = (\mathcal{L}\otimes I_n)(\delta+e)$, we know z is bounded. Thus, we use ρ to denote the upper bound of $\|z_i(t)\|$. Based on (1), it can be obtained that

$$\dot{e}_i(t) = Ae_i(t) - cBKz_i(t). \tag{20}$$

Next, based on the fact that the measurement error is reset to zero once an event is triggered, the solution of (20) follows that

$$\begin{aligned}\left\|e_i(t)\right\| \leq &c\left\|BK\right\|\int_{t_{k_i}^i}^{t}\left\|e^{A(t-s)}z_i(s)\right\|ds\\ \leq &\frac{c\left\|BK\right\|}{\|A\|}\rho(e^{\|A\|(t-t_{k_i}^i)}-1).\end{aligned}$$

Thus, it can be concluded that one sufficient condition to guarantee that the above inequality holds is

$$\frac{c\left\|BK\right\|}{\|A\|}\rho(e^{\|A\|(t-t_{k_i}^i)}-1) \leq \sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\sigma_i+\frac{\xi_i}{\theta_i})t}. \tag{21}$$

Now suppose that $t_{N(\varepsilon_0+1)}^i$ denote the next triggering time determined by (8). Then one gets

$$\begin{aligned}t_{N(\varepsilon_0+1)}^i - t_{N(\varepsilon_0)}^i \geq &\frac{1}{\|A\|}\ln(\frac{1}{\varpi}\sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\sigma_i+\frac{\xi_i}{\theta_i})T_0}+1)\\ = &2\varepsilon_0,\end{aligned}$$

which contradicts to (18). Therefore, Zeno behavior is excluded for each agent.

Thus, the proof is accomplished.

Remark 2. For arbitrary (A, B) satisfying Assumption 2.2, one can always find a positive definite matrix P such that $Q < 0$ holds. Moreover, the existence of parameters in Theorem 4 can be guaranteed by selecting parameters more conservatively off-line as long as they satisfy their bounds. Therefore, the proposed dynamic triggering scheme is implementable.

4. SIMULATION EXAMPLE

In this section, we demonstrate the theoretical result by the following numerical example and make the comparison between the dynamic event-triggered control strategy and traditional static one. Consider a group of 6 agents with general linear dynamics (1) with $A = [0\ 7; -1\ 1]$, $B = [2;\ 1]$. The directed graph is shown in Fig. 1.

The initial states are given by $x_1(0) = [-30;\ 20], x_2(0) = [15;\ 60], x_3(0) = [10;\ -26], x_4(t) = [3;\ 43], x_5(t) = [13;\ 65], x_6(t) = [28;\ -30]$. Choose the feedback gain matrix $K = [-0.9850\ -0.7367]$. And the parameters are selected as $c = 3.7788$, $\alpha^* = 100$, $\epsilon = 1$. Moreover, according to the dynamic triggering scheme (8), we choose

$\eta_1(0) = \eta_3(0) = \eta_4(0) = \eta_6(0) = 8$, $\eta_2(0) = 3$, $\eta_5(0) = 5$, $\sigma_i = \xi_i = 60$, and $\theta_i = 2$, where $i = 1, 2, \ldots 6$.

The simulation results are shown in Figs.2-5. Fig. 2 depicts state evolutions under the dynamic triggering scheme and the static one, respectively. The trajectories of dynamic variable $\eta_i(t)$ is present in Fig. 3. Fig. 4 shows the evolution of each agent's triggering error with respect to the threshold under the proposed dynamic triggering scheme. These two figures imply that the dynamic variable, tracking errors and corresponding thresholds all converge to zero asymptotically. The corresponding triggering instants under dynamic and static triggering schemes are shown in Fig. 5. For a clearer comparison, we record the triggering numbers for each agent with the dynamic and static triggering schemes in Table 1, which can be obtained that the triggering numbers are greatly reduced under the proposed dynamic triggering scheme.

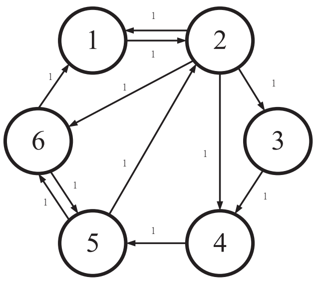
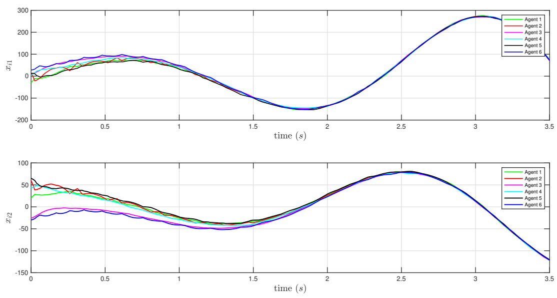
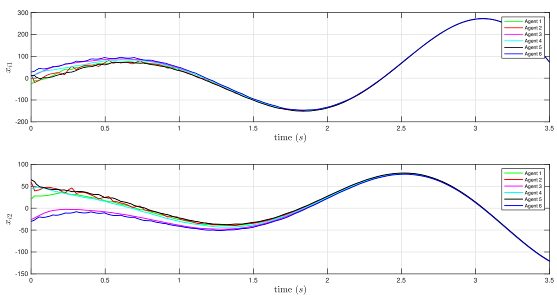


Fig. 1. The communication graph among the agents



(a) State evolutions of MAS under dynamic triggering scheme (8)



(b) State evolutions of MAS under static triggering scheme (9)

Fig. 2. State evolutions under two event-triggered schemes

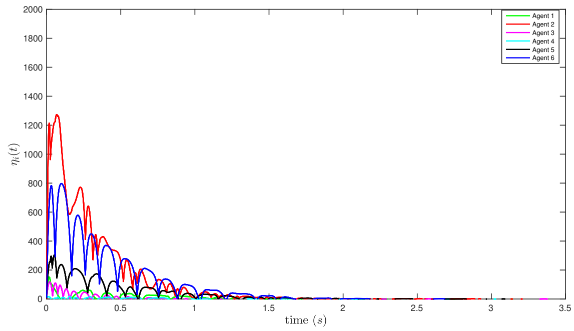


Fig. 3. The dynamic variable $\eta_i(t)$ given in (7) for dynamic trigging scheme (8)

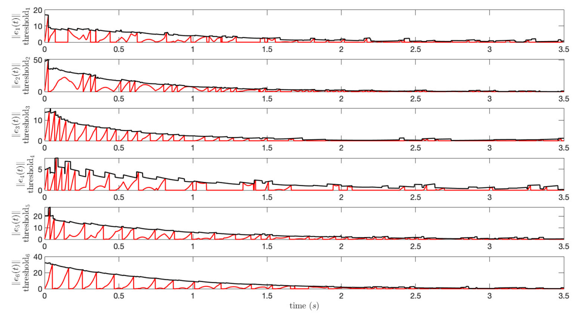
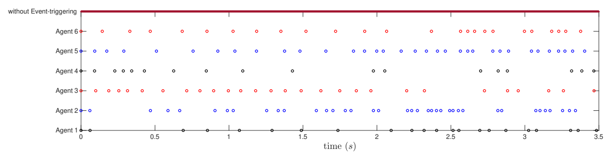
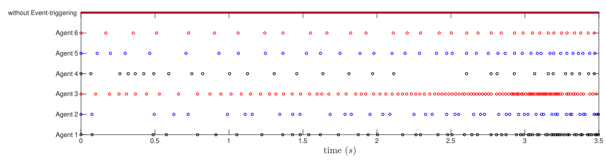


Fig. 4. Triggering errors and thresholds for each agent under dynamic triggering scheme (8)



(a) Triggering times under dynamic triggering scheme (8)



(b) Triggering times under static triggering scheme (9)

Fig. 5. Triggering times for each agent under two schemes and comparison with the case without event-triggered strategy

5. CONCLUSION

The consensus problem for linear MASs under directed graphs has been investigated. A distributed dynamic event-triggered control strategy along with a dynamic triggering function has been addressed. Under the proposed dynamic event-triggered control strategy, there is no need

Table 1. Triggering numbers on each agent under different triggering schemes

Agent index i	1	2	3	4	5	6
Dynamic triggering scheme	22	38	28	18	34	25
Static triggering scheme	59	47	109	30	40	34

for continuous communication in either controller update or triggering condition monitoring. In addition, each agent does not exhibit Zeno behavior by proving the event time interval is strictly positive. Our future research will be devoted to extending the event-triggered consensus problem for heterogenous MASs under directed graphs. At the same time, it is also necessary to further consider the case when MASs contain some uncertainties or external disturbances.

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