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ASPR based Output Regulation with Adaptive PFC and Feedforward Input via Kernel Method

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Abstract: An adaptive control system design problem based on the almost strictly positive real-ness (ASPR-ness) is dealt with. For ASPR systems, one can easily design a stable adaptive output feedback control system, however, in the case where the system is not ASPR, in order to guarantee the stability of the adaptive system, an parallel feedforward compensator (PFC), which makes the resulting augmented system ASPR, is introduced. In the proposed method, an adaptive PFC design scheme for making the resulting augmented system ASPR and an adaptive feedforward input design scheme for attaining the output tracking are proposed by applying the kernel method for uncertain non-ASPR linear systems. The effectiveness of the proposed method is confirmed through numerical simulations for a simple uncertain system.

Keywords: Adaptive control, ASPR, parallel feedforward compensator, kernel method

1. INTRODUCTION

The system is said to be 'almost strictly positive real' (or ASPR) if there exists a static output feedback such that the resulting closed loop system is strictly positive real (SPR) (Bar-Kana, 1991). It is well recognized that one can easily design a stable output feedback control for uncertain but ASPR systems and also can easily design a stable adaptive control system, and the ASPR based (adaptive) output feedback control has good robustness with respect to disturbances and systems uncertainties unlike the conventional adaptive strategies including general model reference adaptive controls (Kaufman et al., 1997; Mizumoto et al., 2007, 2010; Barkana et al., 2014; Mizumoto and Iwai, 1996; Fradkov and Hill, 1998). However, since the most practical systems do not have ASPR property such as (1) the system has relative degree of 1, (2) the system is minimum-phase, and (3) the high frequency of the system is positive. With this in mind, several alleviation methods against ASPR conditions have been proposed by Fradkov (1996); Bar-kana (1987); Mizumoto and Iwai (1996); Mizumoto et al. (2010); Mizumoto and Kawabe (2017). The introduction of a parallel feedforward compensator (PFC) is well recognized as one of the common and simple method to overcome the ASPR restriction. In this strategy, the PFC is added in parallel with the considered non-ASPR controlled system so as to render the resulting augmented system with the PFC ASPR, and the ASPR based control strategy can be applied to this ASPR augmented system.

Unfortunately, however, how to design such PFC has become a new issue. Moreover, although the stable adaptive output feedback control can be designed with an appropriate PFC which make the augmented system ASPR, the desired output tracking for the practical output had not obtained because of the affect from the PFC output even though the perfect output tracking was obtained for augmented system's output. As for the design problem of PFC, several kinds of PFC design schemes have proposed including model-based design schemes (Mizumoto et al., 2010), robust design schemes (Mizumoto and Iwai, 1996), ladder network design schemes Iwai and Mizumoto (1994) and adaptive design schemes (Takagi et al., 2015; Mizumoto and Kawabe, 2017). The adaptive type methods can design the PFC without information of nominal parameters. Unfortunately, however, the information about the order of the system (or the information of the order of the desired PFC) is required. As for the tracking problem for the practical output, internal model filter approaches (Mizumoto et al., 2010), augmented model approaches (Mizumoto and Iwai, 1996) and adaptive feedforward input approaches have been proposed. The adaptive RBF NN feedforward input approach (Mizumoto et al., 2009; Mizumoto and Kawabe, 2017) is one of the effective way to alleviate the affect from the PFC output for both linear and nonlinear systems. However, in order to obtain the desired tracking performance, the node of NN might be selected by higher order.

In this paper, we consider solving both problems on the PFC design and the actual tracking via Kernel method (Engel et al., 2004; Liu et al., 2008; Kurzrock and Ohmori, 2016) based on the concept of an adaptive PFC design and a RBF NN feedforward approach. An adaptive PFC design scheme and an adaptive feedforward input design scheme by the Kernel method to attain the actual output tracking for control system with the PFC is proposed for uncertain linear systems. The effectiveness of the proposed method is confirmed through numerical simulations.

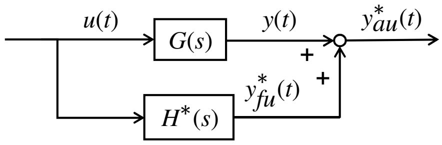


Fig. 1. Augmented ASPR system

2. PROBLEM STATEMENT

Consider the following n-th order LTI continuous-time SISO system:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + \boldsymbol{b}u(t)\\ y(t) &= \boldsymbol{c}^T\boldsymbol{x}(t)\end{aligned} \tag{1}$$

with the transfer function of $G(s)$, where $\boldsymbol{x}(t) \in \mathrm{R}^n$ is the state vector, $u(t) \in \mathrm{R}$ is the control input and $y(t) \in \mathrm{R}$ is the output of the system. $A \in \mathrm{R}^{n\times n}$, $\boldsymbol{b} \in \mathrm{R}^n$, $\boldsymbol{c} \in \mathrm{R}^n$ are uncertain system parameter matrix and vectors.

The objective of this paper is to design an adaptive output regulation control system so as to have the output $y(t)$ track the given reference signal $r(t)$ by constructing TDOF control system with adaptive output feedback and appropriate feedforward input.

To this end, we impose the following assumptions for the system:

Assumption 1. The system (1) is not ASPR, but, for a given ideal ASPR model $G^*_a(s)$, there exists an ideal PFC $H^*(s) = H(s, \boldsymbol{\rho}^*_f)$ with the ideal parameter vector $\boldsymbol{\rho}^*_f$ such that

$$G^*_a(s) = G(s) + H(s, \boldsymbol{\rho}^*_f) \tag{2}$$

is satisfied.

Assumption 2. The reference signal $r(t)$ for which the output $y(t)$ is required to track is generated by the following reference model.

$$\begin{aligned}\dot{\boldsymbol{\omega}}(t) &= \boldsymbol{q}(\boldsymbol{\omega})\\ r(t) &= p(\boldsymbol{\omega})\\ \boldsymbol{q}(\boldsymbol{0}) &= \boldsymbol{0},\, p(\boldsymbol{0}) = 0\end{aligned} \tag{3}$$

Where $\boldsymbol{\omega}(t) \in \mathrm{R}^q$ is the state of the reference model, and the reference model is set as neutrally stable, i.e. the linear approximation $A_Q = [\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\omega}}]_{\boldsymbol{\omega}=0}$ of the vector field $\boldsymbol{q}(\boldsymbol{\omega})$ at $\boldsymbol{\omega} = 0$ has all its eigenvalues on the imaginary axis (Isidori, 1995).

Assumption 3. The system (1) is stabilizable and has no zeros on the imaginary axis.

3. IDEAL CONTROL SYSTEM DESIGN

Before showing the proposed controller design scheme via Kernel method, we recall an ideal TDOF control with ideal PFC and ideal RBF NN feedforward input provided by Mizumoto and Kawabe (2017).

3.1 Ideal PFC Representation

Define the ideal output of the ideal augmented ASPR model $G^*_a(s)$ with the input $u(t)$ by (See Fig. 1)

$$y^*_{au}(t) := G^*_a(s)[u(t)] \tag{4}$$

The notation: $y(t) := W(s)[u(t)]$ denotes the output $y(t)$ of a system with the transfer function of $W(s)$ and the input $u(t)$.

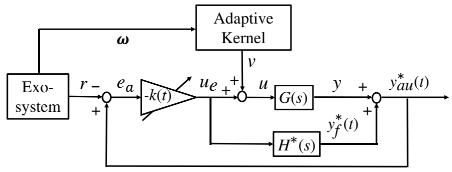


Fig. 2. Block Diagram of the ideal control system

Defining the ideal PFC output by

$$y^*_{fu}(t) := H^*(s)[u(t)] \tag{5}$$

it follows from $G^*_a(s) = G(s) + H^*(s)$ that

$$y^*_{fu}(t) = y^*_{au}(t) - y(t) \tag{6}$$

Supposing that the ideal PFC model is given by

$$\begin{aligned}H^*(s) &= \frac{N^*_H(s)}{D^*_H(s)}\\ &= \frac{b^*_1 s^{n_h-1} + b^*_2 s^{n_h-2} + \cdots + b^*_{n_h}}{s^{n_h} + a^*_1 s^{n_h-1} + \cdots + a^*_{n_h}}\end{aligned} \tag{7}$$

the ideal PFC output is obtained as follows

$$y^*_{fu}(t) = \frac{Z(s)}{F(s)}[y^*_{fu}(t)] + \frac{N^*_H(s)}{F(s)}[u(t)] = \boldsymbol{\rho}^{*T}_f \boldsymbol{z}^*_u(t) \tag{8}$$

$$\boldsymbol{\rho}^*_f = [z^*_1\ z^*_2\ \cdots\ z^*_{n_h}\ b^*_1\ b^*_2\ \cdots\ b^*_{n_h}]^T\ ,\ \ (z^*_i = f_i - a^*_i)$$

$$\boldsymbol{z}^*_u(t) = \boldsymbol{z}(y^*_{fu}, u) = \left[\frac{s^{n_h-1}}{F(s)}[y^*_{fu}], \cdots, \frac{1}{F(s)}[y^*_{fu}]\right. \tag{9}$$
$$\left.\frac{s^{n_h-1}}{F(s)}[u], \cdots, \frac{1}{F(s)}[u]\right]^T$$

by introducing a n_h-th order stable filter

$$\frac{1}{F(s)} = \frac{1}{s^{n_h} + f_1 s^{n_h-1} + \cdots + f_{n_h}} \tag{10}$$

(8) gives a parametric representation of the ideal PFC (Mizumoto and Kawabe, 2017).

3.2 Approximated Ideal Feedforward Input

Under Assumptions 2 and 3, there exists ideal feedforward input $v^*(t)$ which attains the perfect output tracking $y(t) \equiv r(t), \forall t \geq 0$ and $v^*(t)$ can be obtained by a function of $\boldsymbol{\omega}$ as $v^*(t) = c(\boldsymbol{\omega})$. Here, we consider approximating the ideal input by a RBF NN as

$$\bar{v}^*(t) = \boldsymbol{\rho}^{*T}_m \boldsymbol{\phi}_\omega(\boldsymbol{\omega}) \tag{11}$$

where $\boldsymbol{\phi}_\omega(\boldsymbol{\omega})$ is a radial basis function vector and $\boldsymbol{\rho}^*_m$ is the ideal weight vector which satisfies

$$\boldsymbol{\rho}^*_m \triangleq \arg\min_{\boldsymbol{\rho}_m \in R^l}\{\sup_{\boldsymbol{\omega}\in\Omega_\omega} |v^* - \boldsymbol{\rho}^T_m \boldsymbol{\phi}_\omega(\boldsymbol{\omega})|\} \tag{12}$$

It is noted that there exists the ideal weight vector $\boldsymbol{\rho}^*_m$ given in (12) for a sufficient large node of l and a compact set Ω_ω such that the ideal input $v^*(t)$ can be approximated by (Ge et al., 2002)

$$v^*(t) = \boldsymbol{\rho}^{*T}_m \boldsymbol{\phi}_\omega(\boldsymbol{\omega}) + \epsilon(\boldsymbol{\omega}),\ |\epsilon(\boldsymbol{\omega})| \leq \epsilon^* \tag{13}$$

with bounded approximation error $\epsilon(\boldsymbol{\omega})$.

3.3 Ideal Control System

Again define a PFC output with any input $q(t)$, $y_{fq}(t) := H(s)[q(t)]$, by

$$y_{fq}(t) = \boldsymbol{\rho}^T_f \boldsymbol{z}_q(t), \quad \boldsymbol{z}_q(t) = \boldsymbol{z}(y_{fq}, q) \tag{14}$$

with a stable filter given in (10) as a parametric representation of the PFC.

Furthermore, define

$$\bar{y}_{fq}(t) := G_a^{*-1}(s)[y_{fq}(t)] := \boldsymbol{\rho}_f^T \bar{\boldsymbol{z}}_q(t), \quad \bar{\boldsymbol{z}}_q(t) := G_a^{*-1}(s)[\boldsymbol{z}_q(t)] \tag{15}$$

If the ideal order of the PFC: $H^*(s)$ is known, then the ideal PFC output is obtained by

$$\begin{aligned} y_f^*(t) = y_{fu_e}^*(t) &= \boldsymbol{\rho}_f^{*T} \boldsymbol{z}_{u_e}^*(t) \\ &= G_a^*(s)[\bar{y}_{fu_e}^*(t)] \\ &= G_a^*(s)[\boldsymbol{\rho}_f^{*T} \bar{\boldsymbol{z}}_{u_e}^*(t)] \end{aligned} \tag{16}$$

Considering a TDOF control system as shown in Fig. 2, the ideal controller is designed by

$$u^*(t) = u_e^*(t) + v^*(t) \tag{17}$$

$$u_e(t) = -k^* e_a^*(t) \tag{18}$$

$$e_a^*(t) = y_a^*(t) - r(t) \ , \quad y_a^*(t) = y(t) + y_f^*(t)$$

Unfortunately, since the system is uncertain, we are not able to obtain ideal PFC parameter vector $\boldsymbol{\rho}_f^*$ and suitable order of the PFC. Furthermore, ideal feedforward input v^* and the ideal feedback gain k^* which makes the resulting closed-loop system SPR are also unknown. In the next section, a new adaptive strategy through the Kernel method is proposed.

4. ADAPTIVE CONTROL SYSTEM DESIGN VIA KERNEL METHOD

4.1 Adaptive Control System Design

Now, consider a case where the ideal order of the PFC is uncertain, but appropriate (approximated or nominal) order n_f is known, and suppose that the following assumption is satisfied.

Assumption 4. There exists a lowest order $\underline{n}_f$ such that the ideal PFC output $\bar{y}_{fq}^*(t)$ with any input $q(t)$ can be approximated with a order of $n_f \geq \underline{n}_f$ by

$$\bar{y}_{fq}^* = \boldsymbol{\rho}_\phi^{*T} \boldsymbol{\phi}(\bar{\boldsymbol{z}}_q^*(t)) + \epsilon(\bar{\boldsymbol{z}}_q^*(t)), \quad |\epsilon(\bar{\boldsymbol{z}}_q^*(t))| \leq \epsilon_q^* \tag{19}$$

with a basis function $\boldsymbol{\phi}(\bullet)$ of a sufficiently large node l_ϕ and an ideal weight vector $\boldsymbol{\rho}_\phi^*$ such as

$$\boldsymbol{\rho}_\phi^* \triangleq \arg \min_{\boldsymbol{\rho}_\phi^* \in R^l} \{ \sup_{\bar{\boldsymbol{z}}_q^*(t) \in \Omega_{zq^*}} |\bar{y}_{fq}^* - \boldsymbol{\rho}_\phi^{*T} \boldsymbol{\phi}(\bar{\boldsymbol{z}}_q^*(t))| \}$$

in a compact set Ω_{zq^*}.

Under Assumption 4, we define the following new signals.

$$y_{fu}(t) = G_a^*(s)[\bar{y}_{fu}(t)] \tag{20}$$

$$\bar{y}_{fu}(t) = \boldsymbol{\rho}_\phi^T(t) \boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(t)), \quad \bar{\boldsymbol{z}}_u^*(t) = G_a^{*-1}(s)[\boldsymbol{z}_u^*(t)] \tag{21}$$

with $\boldsymbol{z}_u^*(t)$ defined in (9) and $y_{fu}^*(t)$ obtained in (6), and

$$y_{fv}(t) = G_a^*(s)[\bar{y}_{fv}(t)] \tag{22}$$

$$\bar{y}_{fv}(t) = \boldsymbol{\rho}_\phi^T(t) \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)), \quad \bar{\boldsymbol{z}}_v(t) = G_a^{*-1}(s)[\boldsymbol{z}_v(t)] \tag{23}$$

with $\boldsymbol{z}_v(t) = \boldsymbol{z}(y_{fv}, v)$. Further, we define

$$y_f(t) = y_{fu}(t) - y_{fv}(t) \tag{24}$$

as the PFC output. Where $\boldsymbol{\rho}_\phi(t)$ is the estimated parameter vector of $\boldsymbol{\rho}_\phi^*$ and $G_a^*(s)$ is the given ASPR model.

Using the PFC output $y_f(t)$ obtained by (24) the adaptive controller is designed as follows:

$$u(t) = u_e(t) + v(t) \tag{25}$$

$$y_a(t) = y(t) + y_f(t) \ , \quad e_a(t) = y_a(t) - r(t)$$

$$u_e(t) = -k(t) e_a(t) - \rho_z \boldsymbol{\phi}^T(\bar{\boldsymbol{z}}_v(t)) \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)) e_a(t) \tag{26}$$

$$v(t) = \boldsymbol{\rho}_m^T(t) \boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t)) \tag{27}$$

Where $k(t)$ is the adaptively adjusted output feedback gain of k^* and $\boldsymbol{\rho}_m(t)$ is the adaptively adjusted weight vector of $\boldsymbol{\rho}_m^*$ in (13). The second term in (26) is an additional feedback term for maintaining the robustness of the obtained adaptive control system. $k(t)$, $\boldsymbol{\rho}_\phi(t)$ and $\boldsymbol{\rho}_m(t)$ are adjusted by the following parameter adjusting laws:

$$\dot{k}(t) = \gamma_k e_a(t)^2 - \sigma_k k(t) \tag{28}$$

$$\dot{\boldsymbol{\rho}}_\phi(t) = -\gamma_\phi \big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)) \big) e_a(t) - \sigma_\phi \boldsymbol{\rho}_\phi(t) \tag{29}$$

$$\dot{\boldsymbol{\rho}}_m(t) = -\gamma_m \boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t)) e_a(t) - \sigma_m \boldsymbol{\rho}_m(t) \tag{30}$$

We have the following theorem concerning the stability of the obtained adaptive control system.

Theorem 1. Under Assumptions 1 to 4, all the signals in the resulting control system with control input given in (25) to (27) with parameter adjusting laws (28) to (30) are bounded.

Proof. See Appendix

4.2 Adaptive Control via Kernel Method

In practical cases, it may be difficult to find the basis function vectors $\boldsymbol{\phi}(\bullet)$, $\boldsymbol{\phi}_\omega(\bullet)$ which satisfy assumptions. We consider solving this problem by applying the kernel trick (Bishop, 2006).

If the basis function $\boldsymbol{\phi}(\bullet)$ is known, then unknown parameter vector $\boldsymbol{\rho}_\phi(t)$ can be obtained from (29) as

$$\dot{\boldsymbol{\rho}}_\phi(t) = -\gamma_\phi \big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)) \big) e_a(t) - \sigma_\phi \boldsymbol{\rho}_\phi(t)$$

Supposing that the initial condition is $\boldsymbol{\rho}_\phi(0) = \mathbf{0}$, then $\boldsymbol{\rho}_\phi(t)$ is obtained by

$$\boldsymbol{\rho}_\phi(t) = -\gamma_\phi \int_0^t e^{-\sigma_\phi(t-\tau)} \big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(\tau)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(\tau)) \big) e_a(\tau)\, d\tau \tag{31}$$

Thus, defining

$$\bar{y}_f(t) = \bar{y}_{fu}(t) - \bar{y}_{fv}(t), \tag{32}$$

we have from (21), (23) and (31) that

$$\begin{aligned} \bar{y}_f(t) = -\gamma_\phi \int_0^t & e^{-\sigma_\phi(t-\tau)} e_a(\tau) \big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(\tau))^T - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(\tau))^T \big) \\ & \times \big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)) \big)\, d\tau \end{aligned} \tag{33}$$

Applying the kernel trick such as $\boldsymbol{\phi}(\bar{\boldsymbol{z}}_{u1}(\tau))^T \boldsymbol{\phi}(\bar{\boldsymbol{z}}_{u2}(t)) = k(\bar{\boldsymbol{z}}_{u1}(\tau), \bar{\boldsymbol{z}}_{u2}(t))$ with an appropriate kernel function $k(\bullet, \bullet)$, $\bar{y}_f(t)$ given in (33) can be obtained with the kernel function instead of unknown basis functions as follows:

$$\begin{aligned} \bar{y}_f(t) = -\gamma_\phi \int_0^t & e^{-\sigma_\phi(t-\tau)} e_a(\tau) \\ & \times \big\{ k(\bar{\boldsymbol{z}}_u^*(\tau), \bar{\boldsymbol{z}}_u^*(t)) - k(\bar{\boldsymbol{z}}_v(\tau), \bar{\boldsymbol{z}}_u^*(t)) \\ & \quad - k(\bar{\boldsymbol{z}}_u^*(\tau), \bar{\boldsymbol{z}}_v(t)) + k(\bar{\boldsymbol{z}}_v(\tau), \bar{\boldsymbol{z}}_v(t)) \big\}\, d\tau \end{aligned} \tag{34}$$

We implement $\bar{y}_f(t)$ obtained in (34) by approximating via Riemann Sum at a time instant $t = t_k$as follows:

$$\begin{aligned}\bar{y}_f(t) &\simeq \bar{y}_f(t_k)\\ &= -\gamma_\phi \sum_{i=0}^{k} e^{-\sigma_\phi(t_k-t_i)} e_a(t_i)\\ &\quad \times \left\{k(\bar{\boldsymbol{z}}_u^*(t_i), \bar{\boldsymbol{z}}_u^*(t_k)) - k(\bar{\boldsymbol{z}}_v(t_i), \bar{\boldsymbol{z}}_u^*(t_k))\right.\\ &\quad \left. - k(\bar{\boldsymbol{z}}_u^*(t_i), \bar{\boldsymbol{z}}_v(t_k)) + k(\bar{\boldsymbol{z}}_v(t_i), \bar{\boldsymbol{z}}_v(t_k))\right\} \times \Delta t \end{aligned} \tag{35}$$

with $\Delta t = t_k - t_{k-1}$.

Now, define

$$\begin{aligned}&\bar{y}_{f,k}(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &:= -\mu_\phi \sum_{i=0}^{k} e^{-\sigma_\phi(t_k-t_i)} e_a(t_i) k(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_k)) \end{aligned} \tag{36}$$

with $\mu_\phi = \gamma_\phi \Delta t$. $\bar{y}_{f,k}(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_k))$ can be expanded as follows:

$$\begin{aligned}&\bar{y}_{f,k}(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &= -\mu_\phi \sum_{i=0}^{k-1} e^{-\sigma_\phi(t_k-t_i)} e_a(t_i) k(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &\quad - \mu_\phi e_a(t_k) k(\bar{\boldsymbol{z}}_{u1}(t_k), \bar{\boldsymbol{z}}_{u2}(t_k)) \end{aligned} \tag{37}$$

Moreover, since it follows from the property of kernel that

$$\begin{aligned}&k(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &= k(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_{k-1})) k(\bar{\boldsymbol{z}}_{u2}(t_{k-1}), \bar{\boldsymbol{z}}_{u2}(t_k))\end{aligned}$$

we have

$$\begin{aligned}&\bar{y}_{f,k}(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &= -\mu_\phi \sum_{i=0}^{k-1} e^{-\sigma_\phi(t_k-t_i)} e_a(t_i)\\ &\quad \times k(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_{k-1})) k(\bar{\boldsymbol{z}}_{u2}(t_{k-1}), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &\quad - \mu_\phi e_a(t_k) k(\bar{\boldsymbol{z}}_{u1}(t_k), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &= -\mu_\phi e^{-\sigma_\phi(t_k-t_{k-1})} \sum_{i=0}^{k-1} e^{-\sigma_\phi(t_{k-1}-t_i)} e_a(t_i)\\ &\quad \times k(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_{k-1})) k(\bar{\boldsymbol{z}}_{u2}(t_{k-1}), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &\quad - \mu_\phi e_a(t_k) k(\bar{\boldsymbol{z}}_{u1}(t_k), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &= e^{-\sigma_{\phi t}} \bar{y}_{f,k-1}(\bar{\boldsymbol{z}}_{u1}(t_i), \bar{\boldsymbol{z}}_{u2}(t_{k-1})) k(\bar{\boldsymbol{z}}_{u2}(t_{k-1}), \bar{\boldsymbol{z}}_{u2}(t_k))\\ &\quad - \mu_\phi e_a(t_k) k(\bar{\boldsymbol{z}}_{u1}(t_k), \bar{\boldsymbol{z}}_{u2}(t_k)) \end{aligned} \tag{38}$$

with $\sigma_{\phi t} = \sigma_\phi \Delta t$.

Consequently, $\bar{y}_f(t_k)$ can be obtained as follows recursively.

$$\begin{aligned}&\bar{y}_f(t_k)\\ &= e^{-\sigma_{\phi t}} \Big(\bar{y}_{f,k-1}(\bar{\boldsymbol{z}}_u^*(t_i), \bar{\boldsymbol{z}}_u^*(t_{k-1}))\\ &\qquad - \bar{y}_{f,k-1}(\bar{\boldsymbol{z}}_v(t_i), \bar{\boldsymbol{z}}_u^*(t_{k-1})) \Big) \times k(\bar{\boldsymbol{z}}_u^*(t_{k-1}), \bar{\boldsymbol{z}}_u^*(t_k))\\ &\quad - \mu_\phi e_a(t_k) \big(k(\bar{\boldsymbol{z}}_u^*(t_k), \bar{\boldsymbol{z}}_u^*(t_k)) - k(\bar{\boldsymbol{z}}_v(t_k), \bar{\boldsymbol{z}}_u^*(t_k)) \big)\\ &\quad + e^{-\sigma_{\phi t}} \Big(\bar{y}_{f,k-1}(\bar{\boldsymbol{z}}_v(t_i), \bar{\boldsymbol{z}}_v(t_{k-1}))\\ &\qquad - \bar{y}_{f,k-1}(\bar{\boldsymbol{z}}_u^*(t_i), \bar{\boldsymbol{z}}_v(t_{k-1})) \Big) \times k(\bar{\boldsymbol{z}}_v(t_{k-1}), \bar{\boldsymbol{z}}_v(t_k))\\ &\quad - \mu_\phi e_a(t_k) \big(k(\bar{\boldsymbol{z}}_v(t_k), \bar{\boldsymbol{z}}_v(t_k)) - k(\bar{\boldsymbol{z}}_u^*(t_k), \bar{\boldsymbol{z}}_v(t_k)) \big) \end{aligned} \tag{39}$$

As for the feedforward input $v(t)$, taking into consideration from (27), (30) that

$$\begin{aligned} v(t) &= \boldsymbol{\rho}_m^T(t) \boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t))\\ \dot{\boldsymbol{\rho}}_m(t) &= -\gamma_m \boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t)) e_a(t) - \sigma_m \boldsymbol{\rho}_m(t)\end{aligned}$$

we have at time instant $t = t_k$

$$\begin{aligned} v(t) &\simeq v_k(t_k)\\ &= -\mu_m \sum_{i=0}^{k} e^{-\sigma_m(t_k-t_i)} e_a(t_i) k(\boldsymbol{\omega}(t_i), \boldsymbol{\omega}(t_k))\\ &= e^{-\sigma_{mt}} v_{k-1}(\boldsymbol{\omega}(t_i), \boldsymbol{\omega}(t_{k-1})) k(\boldsymbol{\omega}(t_{k-1}), \boldsymbol{\omega}(t_k))\\ &\quad - \mu_m e_a(t_k) k(\boldsymbol{\omega}(t_k), \boldsymbol{\omega}(t_k)) \end{aligned} \tag{40}$$

with $\mu_m = \gamma_m \Delta t$, $\sigma_{mt} = \sigma_m \Delta t$.

Furthermore, the feedback control $u_e(t)$ is given by

$$u_e(t) = -k(t) e_a(t) - \rho_z k(\bar{\boldsymbol{z}}_v(t), \bar{\boldsymbol{z}}_v(t)) e_a(t) \tag{41}$$

5. VALIDATION THROUGH NUMERICAL SIMULATIONS

The effectiveness of the proposed method is confirmed through the following numerical simulations.

In this simulation, we supposed that the considered system is suddenly changed from second order system to third order system as follows:

For $0 \le t < 100$, $250 \le t < 400$

$$G(s) = \frac{1}{s^2 + 20s + 50}$$

and for $100 \le t < 250$, $400 \le t \le 600$

$$G(s) = \frac{1}{s^3 + 50s^2 + 10s + 50}$$

We assume that the controlled system is unknown but we know the information that a first order PFC may make the ASPR augmented system for the initial system. Thus we set the ideal ASPR model and the stable filter for PFC estimation by

$$G_a^*(s) = \frac{1}{s + 10}, \quad \frac{1}{F(s)} = \frac{1}{s + 90}$$

Furthermore, we adopt the Gaussian kernel:

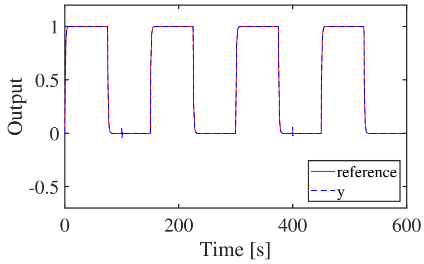
$$k(u, u') = e^{-\frac{1}{2\sigma^2}\|u-u'\|^2}$$

for PFC estimation and ideal feedforward input estimation.

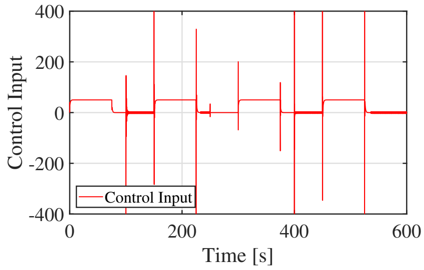
The design parameters in the adaptive controller were set as

$$\begin{aligned}&\gamma_k = 1.0 \times 10^9,\ \sigma_k = 1.0 \times 10^{-6},\ \rho_z = 10\\ &\mu_\phi = 50,\ \ \sigma_\phi = 1.0 \times 10^2\\ &\mu_m = 50,\ \ \sigma_m = 1.0 \times 10^{-6},\ \ \sigma = 0.5\end{aligned}$$

Fig. 3 shows the results with the proposed adaptive controller with kernel method and Fig. 4 shows the results with the previous adaptive control method presented by Mizumoto and Kawabe (2017). Although the result with previous adaptive method was degraded after the change of the controlled system and the output diverged finally, the result with the proposed method maintain the good control performance for whole operation. Thus the adaptive algorithm via kernel method make it robust with respect to systems uncertainties on the order.

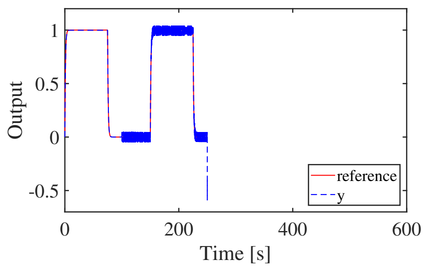


(a) Output and reference signal

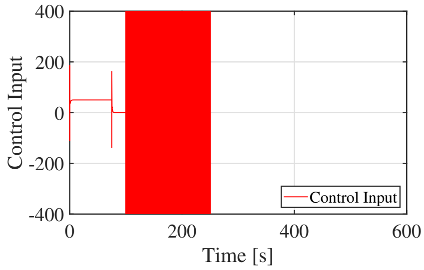


(b) Control input

Fig. 3. Results with the proposed adaptive kernel method



(a) Output and reference signal



(b) Control input

Fig. 4. Results with the previous adaptive PFC method: output and reference signal

6. CONCLUSION

In this paper, an adaptive control system design method based on the almost strictly positive real-ness (ASPR-ness) is proposed via kernel method. In order to guarantee the stability of the adaptive system, an adaptive parallel feed-forward compensator (PFC) based on the kernel method, which makes the resulting augmented system ASPR, was derived. Moreover, an adaptive feedforward input design scheme for attaining the output tracking was proposed by applying the kernel method in stead of the RBF NN for uncertain non-ASPR linear systems. The effectiveness of the proposed method was confirmed through numerical simulations.

Appendix A. SKETCH OF THE PROOF OF THEOREM 1

The augmented system's output with the PFC can be represented by

$$\begin{aligned} y_a(t) &= y(t) + y_f(t) \\ &= y^*_{au}(t) - y^*_{fu}(t) + y_f(t) \end{aligned} \tag{A.1}$$

Moreover, $y_f(t)$ is expressed from the definition given in (24) that

$$\begin{aligned} y_f(t) &= y_{fu}(t) - y_{fv}(t) \\ &= \big(y_{fu}(t) - y^*_{fu}(t)\big) - y_{fv}(t) \\ &\quad + y^*_{fu}(t) \end{aligned} \tag{A.2}$$

It follows form

$$\begin{aligned} y^*_{au}(t) &= G^*_a(s)[u(t)] \\ y_{fu}(t) &= G^*_a(s)[\boldsymbol{\rho}^T_\phi(t)\boldsymbol{\phi}(\bar{\boldsymbol{z}}^*_u(t))] \\ y^*_{fu}(t) &= G^*_a(s)[\boldsymbol{\rho}^{*T}_\phi\boldsymbol{\phi}(\bar{\boldsymbol{z}}^*_u(t)) + \epsilon(\bar{\boldsymbol{z}}^*_u(t))] \\ y_{fv}(t) &= G^*_a(s)[\boldsymbol{\rho}^T_\phi(t)\boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t))] \end{aligned}$$

that

$$\begin{aligned} y_a(t) = {} & G^*_a(s)[u_e(t) + \Delta\boldsymbol{\rho}^T_m(t)\boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t)) - \epsilon(\boldsymbol{\omega})] \\ & + G^*_a(s)[v^*(t)] \\ & + G^*_a(s)[\Delta\boldsymbol{\rho}^T_\phi(t)\big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}^*_u(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t))\big) - \epsilon(\bar{\boldsymbol{z}}^*_u)] \\ & - G^*_a(s)[\boldsymbol{\rho}^{*T}_\phi\boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t))] \end{aligned} \tag{A.3}$$

Taking the fact that $r(t) = G(s)[v^*(t)]$ into consideration, the error system can be obtained as

$$\begin{aligned} e_a(t) = G^*_a(s)\big[& u_e(t) + \Delta\boldsymbol{\rho}^T_m(t)\boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t)) \\ & + \Delta\boldsymbol{\rho}^T_\phi(t)\big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}^*_u(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t))\big) \\ & + \boldsymbol{\rho}^{*T}_\phi\boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)) - \epsilon(\boldsymbol{\omega}) - \epsilon(\bar{\boldsymbol{z}}^*_u) + \epsilon_v(t)\big] \end{aligned} \tag{A.4}$$

$$\Delta\boldsymbol{\rho}_m(t) = \boldsymbol{\rho}_m(t) - \boldsymbol{\rho}^*_m,\ \Delta\boldsymbol{\rho}_\phi(t) = \boldsymbol{\rho}_\phi(t) - \boldsymbol{\rho}^*_\phi$$

Where $\epsilon_v(t) := G^{*-1}_a(s)[H^*(s)[v^*(t)]]$. Since $G^*_a(s)$ is ASPR, i.e. it has relative degree of 1 and is minimum-phase, a realization of $G(s)$ can be represented by the following canonical form:

$$\begin{aligned} \dot{e}_a(t) = {} & a_e e_a(t) + b_e\Big(u_e(t) + \Delta\boldsymbol{\rho}^T_m(t)\boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t)) \\ & + \Delta\boldsymbol{\rho}^T_\phi(t)\big(\boldsymbol{\phi}(\bar{\boldsymbol{z}}^*_u(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t))\big) \\ & + \boldsymbol{\rho}^{*T}_\phi\boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)) - \epsilon(\boldsymbol{\omega}) - \epsilon(\bar{\boldsymbol{z}}^*_u) + \epsilon_v(t)\Big) \\ & + \boldsymbol{c}_\eta\boldsymbol{\eta}_a(t) \end{aligned} \tag{A.5}$$

$$\dot{\boldsymbol{\eta}}_a(t) = A_\eta\boldsymbol{\eta}_a(t) + \boldsymbol{b}_\eta e_a(t) \tag{A.6}$$

with appropriate parameters a_e, b_e, $\boldsymbol{b}_\eta$, $\boldsymbol{c}_\eta$ and A_η. Since the system is minimum-phase, A_η should be a stable matrix.

Now, consider the following positive definite function $V(t)$:

$$\begin{aligned} V(t) = {} & e^2_a(t) + \boldsymbol{\eta}^T_a(t)P_\eta\boldsymbol{\eta}_a(t) + \frac{b_e}{\gamma_k}\Delta k(t)^2 \\ & + \frac{b_e}{\gamma_\phi}\Delta\boldsymbol{\rho}^T_\phi(t)\Delta\boldsymbol{\rho}_\phi(t) + \frac{b_e}{\gamma_m}\Delta\boldsymbol{\rho}^T_m(t)\Delta\boldsymbol{\rho}_m(t) \end{aligned} \tag{A.7}$$

where $\Delta k(t) = k(t) - k^*$ and k^* is the ideal feedback gain to be determined later. P_η is a symmetric positive definite matrix such that the following Lyapunov equation is satisfied for any symmetric positive definite matrix Q_η.

$$A^T_\eta P_\eta + P_\eta A_\eta = -Q_\eta < 0$$

Taking the time derivative of $V(t)$, it follows that

$$\begin{aligned}\dot{V}(t) &= -2(b_e k^* - a_e)e_a^2(t) - 2b_e\Delta k(t)e_a^2(t)\\ &+ 2b_e\Delta\boldsymbol{\rho}_m^T(t)\boldsymbol{\phi}(\boldsymbol{\omega}(t))e_a(t)\\ &+ 2b_e\Delta\boldsymbol{\rho}_\phi(t)(\boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)))e_a(t)\\ &- 2b_e\rho_z\boldsymbol{\phi}^T(\bar{\boldsymbol{z}}_v(t))\boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t))e_a^2(t)\\ &- 2b_e\boldsymbol{\rho}_\phi^{*T}\boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t))e_a(t)\\ &- 2b_e\epsilon(\bar{\boldsymbol{z}}_u^*)e_a(t) - 2b_e\epsilon(\boldsymbol{\omega})e_a(t)\\ &+ 2b_e\epsilon_v(t)e_a(t) + 2\boldsymbol{c}_\eta^T\boldsymbol{\eta}_a(t)e_a(t)\\ &- \boldsymbol{\eta}_a^T(t)Q_\eta\boldsymbol{\eta}_a(t) + 2\boldsymbol{b}_\eta^T P_\eta\boldsymbol{\eta}_a(t)e_a(t)\\ &+ 2b_e\Delta k(t)e_a^2(t) - \frac{2b_e\sigma_k}{\gamma_k}\Delta k^2(t) - \frac{2b_e\sigma_k}{\gamma_k}\Delta k(t)k^*\\ &- 2b_e\Delta\boldsymbol{\rho}_\phi^T(t)(\boldsymbol{\phi}(\bar{\boldsymbol{z}}_u^*(t)) - \boldsymbol{\phi}(\bar{\boldsymbol{z}}_v(t)))e_a(t)\\ &- \frac{2b_e\sigma_\phi}{\gamma_\phi}\|\Delta\boldsymbol{\rho}_\phi(t)\|^2 - \frac{2b_e\sigma_\phi}{\gamma_\phi}\Delta\boldsymbol{\rho}_\phi^T(t)\boldsymbol{\rho}_\phi^*\\ &- 2b_e\Delta\boldsymbol{\rho}_m^T(t)\boldsymbol{\phi}_\omega(\boldsymbol{\omega}(t))e_a(t)\\ &- \frac{2b_e\sigma_m}{\gamma_m}\|\Delta\boldsymbol{\rho}_m(t)\|^2 - \frac{2b_e\sigma_m}{\gamma_m}\Delta\boldsymbol{\rho}_m^T(t)\boldsymbol{\rho}_m^* \qquad \text{(A.8)}\end{aligned}$$

and thus the time derivative of $V(t)$ can be evaluated by

$$\begin{aligned}\dot{V}(t) &\le -2\bigg(b_e k^* - a_e - \frac{b_e}{2\delta_1} - \frac{b_e}{2\delta_2} - \frac{b_e}{2\delta_3} - \frac{1}{2\delta_4}\\ &\qquad - \frac{1}{2\delta_5}\|P_\eta\boldsymbol{b}_\eta\|^2\bigg)e_a(t)^2\\ &- (\lambda_{\min}[Q_\eta] - \delta_4\|\boldsymbol{c}_\eta^T\|^2 - \delta_5)\|\boldsymbol{\eta}_a(t)\|^2\\ &- \frac{b_e\sigma_k}{\gamma_k}\Delta k^2(t) - \frac{b_e\sigma_\phi}{\gamma_\phi}\|\Delta\boldsymbol{\rho}_\phi(t)\|^2\\ &- \frac{b_e\sigma_m}{\gamma_m}\|\Delta\boldsymbol{\rho}_m(t)\|^2 + b_e\delta_1\epsilon_\omega^{*2} + b_e\delta_2\epsilon_u^{*2} + b_e\delta_3\epsilon_v^{*2}\\ &+ \frac{b_e\sigma_k}{\gamma_k}k^{*2} + \big(\frac{b_e}{2\rho_z} + \frac{b_e\sigma_\phi}{\gamma_\phi}\big)\|\boldsymbol{\rho}_\phi^*\|^2 + \frac{b_e\sigma_m}{\gamma_m}\|\boldsymbol{\rho}_m^*\|^2 \qquad \text{(A.9)}\end{aligned}$$

with any positive constants $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5$, and $|\epsilon(\boldsymbol{\omega}(t))| \le \epsilon_\omega^*$, $|\epsilon(\bar{\boldsymbol{z}}_u^*(t))| \le \epsilon_u^*$, $|\epsilon_v(t))| \le \epsilon_v^*$. Finally, considering sufficiently large k^* and small δ's such that

$$\begin{aligned}&b_e k^* - a_e - \frac{b_e}{2\delta_1} - \frac{b_e}{2\delta_2} - \frac{b_e}{2\delta_3}\\ &\qquad\qquad - \frac{1}{2\delta_4} - \frac{1}{2\delta_5}\|P_\eta\boldsymbol{b}_\eta\|^2 = \alpha_1 > 0\\ &\lambda_{min}[Q_\eta] - \delta_4\|\boldsymbol{c}_\eta^T\|^2 - \delta_5 = \alpha_2 > 0\end{aligned}$$

we have

$$\begin{aligned}\dot{V}(t) &\le -\alpha_1 e_a^2(t) - \alpha_2\|\boldsymbol{\eta}_a(t)\|^2 - \alpha_3\Delta k^2(t)\\ &\quad - \alpha_4\|\Delta\boldsymbol{\rho}_\phi(t)\|^2 - \alpha_5\|\Delta\boldsymbol{\rho}_m(t)\|^2 + R \qquad \text{(A.10)}\end{aligned}$$

with

$$\begin{aligned}R &\ge b_e\delta_1\epsilon_\omega^{*2} + b_e\delta_2\epsilon_u^{*2} + b_e\delta_3\epsilon_v^{*2}\\ &+ \frac{b_e\sigma_k}{\gamma_k}k^{*2} + (\frac{b_e}{2\rho_z} + \frac{b_e\sigma_\phi}{\gamma_\phi})\|\boldsymbol{\rho}_\phi^*\|^2 + \frac{b_e\sigma_m}{\gamma_m}\|\boldsymbol{\rho}_m^*\|^2\end{aligned}$$

Consequently, we can conclude that $V(t)$ is bounded and then all the signals in the control system are bounded.

REFERENCES

Bar-kana, I. (1987). Parallel feedforward and simplified adaptive control. *International Journal of Adaptive Control and Signal Processing*, 1(2), 95–109.

Bar-Kana, I. (1991). Positive realness in multivariable continuous-time systems. *Journal of the Franklin Institute*, 328(4), 403–418.

Barkana, I., Rusnak, I., and Weiss, H. (2014). Almost passivity and simple adaptive control in discrete-time systems. *Asian Journal of Control*, 16(4), 1–12.

Bishop, C.M. (2006). *Pattern Recognition and Machine Learning*. Springer-Verlag New York, LLC.

Engel, Y., Mannor, S., and Meir, R. (2004). The kernel recursive least-squares algorithm. *IEEE Trans. on Signal Process*, 52(8), 2275–2285.

Fradkov, A.L. (1996). Shunt output feedback adaptive controller for nonlinear plants. *Proc. of 13th IFAC World Congress, San-Francisco, July*, K, 367–372.

Fradkov, A.L. and Hill, D.J. (1998). Exponentially feedback passivity and stabilizability of nonlinear systems. *Automatica*, 34(6), 697–703.

Ge, S.S., Wang, C.C., Lee, T.H., and Zhang, T. (2002). *Stable Adaptive Neural Network Control*. Kluwer Academic Pub.

Isidori, A. (1995). *Nonlinear Control Systems*. Springer, 3rd edition.

Iwai, Z. and Mizumoto, I. (1994). Realization of simple adaptive control by using parallel feedforward compensator. *Int. J. of Control*, 59(6), 1543–1565.

Kaufman, H., Barkana, I., and Sobel, K. (1997). *Direct Adaptive Control Algorithms*. Springer, 2nd edition.

Kurzrock, J. and Ohmori, H. (2016). Diesel engine injection timing control using feedback error learning in reproducing kernel hilbert space. *Symposium for Combustion Control 2016, Novotel Aachen City, Germany, June 15-16*, 191–199.

Liu, W., Pokharel, P.P., and Principe, J.C. (2008). The kernel least-mean-square algorithm. *IEEE Trans. on Signal Process.*, 56(2), 543–554.

Mizumoto, I., Chen, T., Ohdaira, S., Kumon, M., and Iwai, Z. (2007). Adaptive output feedback control of general mimo systems using multirate sampling and its application to a cart-crane system. *Automatica*, 43(12), 2077–2085.

Mizumoto, I., Ikeda, D., Hirahata, T., and Iwai, Z. (2010). Design of discrete time adaptive pid control systems with parallel feedforward compensator. *Control Engineering Practice*, 18(2), 168–176.

Mizumoto, I. and Iwai, Z. (1996). Simplified adaptive model output following control for plants with unmodelled dynamics. *Int. J. of Control*, 64(1), 61–80.

Mizumoto, I. and Kawabe, N. (2017). Adaptive output feedback based output tracking control with adaptive parallel feedforward compensator. *Proceedings of 20th IFAC World Congress, Toulouse, France, July 9-14*, 486–5492.

Mizumoto, I., Okamatsu, Y., Tanaka, H., and Iwai, Z. (2009). Adaptive output regulation of nonlinear systems based on output feedback and nn feedforward control. *Proc. of the European Control Conference 2009, Budapest, August*, 981–986.

Takagi, T., Mizumoto, I., and Tsunematsu, J. (2015). Performance-driven adaptive output feedback control with direct design of pfc. *Journal of Robotics and Mechatronics*, 27(5), 461–468.