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Simple Control Scheme for Robust Tracking and Model Following of Uncertain Systems with Unknown Dead-Zone Inputs and Nonlinear Perturbations

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Abstract: The robust tracking and model following problem is investigated for a class of uncertain systems with completely unknown dead-zone inputs and any nonlinear perturbations. It is supposed that the uncertain nonlinearities are any continuous and bounded nonlinear functions which are unknown. Based on Wu inequality, a new design method is developed so that (i) the resulting robust tracking control schemes are direct, instead of the complicated ones consisting of two parts, to overcome the constraint of control input; (ii) it is unnecessary to understand any information on the nonlinear upper bound functions of uncertain nonlinearities, which results in the structural simplicity of robust tracking control schemes. In addition, it is also proved that the tracking error between the output of an actual nonlinear system with input constraints and the dynamical signals of the given reference model is uniformly exponentially bounded.

Keywords: Uncertain systems, uncertain nonlinearity, robust tracking, dead-zone input, model following, adaptive control.

1. INTRODUCTION

In many engineering control problems, the robust tracking and model following have been widely investigated for uncertain linear or/and nonlinear systems. In fact, the research of robust tracking and model following have been one of the main topics in control literature. Some design approaches for a engineering system to tracking some dynamical signals of the reference model have been also developed (see, e.g. Hopp and Schmitendorf (1990); Sugie and Osuka (1993); Wu (2004); Shigemaru and Wu (2007); Wu (2008); Shyu and Chen (1995), and the references therein).

On the other hand, any engineering system often consists of many electrical and mechanical components, such as hydraulic and pneumatic valves, electronic circuits, electric servomotors, and other devices. In general, such electrical and mechanical devices should have some dead-zone nonlinearity. It is obvious fact that the dead-zone nonlinearity included in systems should degrade the performances of systems, and may even lead to the instability of systems. Therefore, it is rather important to study the uncertain dynamical systems with dead-zone input nonlinearities. Moreover, some methods have been also developed to design a robust stabilizing or tracking controllers for such uncertain dynamical systems with dead-zone inputs (see, e.g. Tao and Kokotovic (1994); Selmic and Lewis (2000); Wang et al. (2004); Zhou et al. (2006); Ibrir et al. (2007); Wu (2017a,b), and the references therein).

In the current design methods, reported in contol literature, to deal with the model following control problems, an actual plant is generally described by some linear differential equations. That is, uncertain linear systems without any constraint control input are considered, and the uncertainties are assumed to be linear or to be linear norm-bounded in the state. Under such assumptions, the standard robust tracking control schemes are always composed of two parts, i.e. the linear feedback due to the state of reference model and an additional control function called auxiliary control. Then, by introducing an auxiliary state vector, one can transform uncertain linear systems and reference model into an uncertain linear auxiliary system with the auxiliary control, and further synthesize some types of auxiliary control schemes such that the stability of uncertain linear auxiliary dynamical systems can be guaranteed. However, in the conventional design methods, there are two main shortcomings which should limit the applications of conventional design methods to practical model following control problems. The first one is that the standard robust tracking control schemes with two parts cannot be used to obtain an uncertain linear auxiliary dynamical system, since the control input has some constraints, e.g. the dead-zone input nonlinearities. The second one, more critical shortcoming of the conventional design methods, is that when the uncertainties of the considered systems are completely nonlinear, i.e. not linear norm-bounded in the states, it is quite difficult to synthesize some types of robust tracking control schemes with a simple structure.

In this paper, we investigate the robust tracking and model following problem for a class of uncertain nonlinear systems with completely unknown dead-zone inputs and any nonlinear perturbations. We assume that the uncertain nonlinear functions are continuous and bounded in their arguments, where their nonliner upper bounds are not required to be known. For such a class of robust tracking and model following problem, on the basis of a new integral inequality (Wu (2018a)), which has been highlighted and called Wu inequality in AIE [1], we present a novel design method so that

(i) the resulting adaptive robust tracking control schemes are direct, instead of the complicated ones consisting of two parts, to overcome the constraint of control input;
(ii) it is unnecessary to understand any information on the nonlinear upper bound functions of uncertain nonlinearities, which results in the structive simplicity of robust tracking control schemes.

We also show that the output of the uncertain nonlinear systems with dead-zone input can track the output of the given reference model in the sense of uniform exponential boundedness.

2. PROBLEM FORMULATION

We consider a class of uncertain dynamical systems with any dead-zone input constraint and uncertain nonlinearities, described by

$$\frac{dx(t)}{dt} = Ax(t) + B\mathcal{D}(u(t)) + \Delta\mathcal{F}(x(t), t) \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, and $y(t) \in R^l$ is the system output , and A, B, and C are the known constant matrices. Here, the sufficiently smooth function $\Delta\mathcal{F}(\cdot) : R^n \times R^+ \to R^n$ represents the system uncertain nonlinearities.

Moreover, the nonlinear vector function $\mathcal{D}(u(t)) : R^m \to R^m$ represents any dead-zone input in the form of

$$\mathcal{D}(u) := \Big[\, \mathcal{D}_1(u_1) \;\; \mathcal{D}_2(u_2) \;\; \cdots \;\; \mathcal{D}_m(u_m) \,\Big]^\top \tag{2a}$$

where for any $i \in \{1, 2, \ldots, m\}$,

$$\mathcal{D}_i(u_i) = \begin{cases} \tilde{m}_{ir}\Big(u_i - \tilde{b}_{ir}\Big), & \text{if } u_i \geq \tilde{b}_{ir} \\ 0, & \text{if } -\tilde{b}_{il} \leq u_i \leq \tilde{b}_{ir} \\ \tilde{m}_{il}\Big(u_i + \tilde{b}_{il}\Big), & \text{if } u_i \leq -\tilde{b}_{il} \end{cases} \tag{2b}$$

where for any $i \in \{1, 2, \ldots, m\}$, the parameters $\tilde{m}_{ir}$ and $\tilde{m}_{il}$ represent the right and the left slope of the dead-zone characteristic, and $\tilde{b}_{ir}$ and $\tilde{b}_{il}$ are the breakpoints of the input nonlinearity. Such parameters $\tilde{m}_{ir}$, $\tilde{m}_{il}$, $\tilde{b}_{ir}$, and $\tilde{b}_{il}$ are generally assumed to be any positive constants.

For this robust tracking and model following problem, the reference signal $y_r(t)$ is assumed to be the output of a reference model given by

$$\frac{dx_r(t)}{dt} = A_r x_r(t) + B_r r(t) \tag{3a}$$

$$y_r(t) = C_r x_r(t) \tag{3b}$$

where $x_r(t) \in R^{n_r}$ and $y_r(t) \in R^{l_r}$ are respectively the state vector and the output vector of the reference model, $r(t) \in R^{m_r}$ is the input vector of the reference model, and A_r, B_r, C_r are known constant matrices. Here, $y_r(t)$ has the same dimension as $y(t)$, i.e. $l_r = l$. Since for any practical robust tracking and model following problems, the model state should be required to be bounded, we assume that for the reference model, there exists a finite positive constant $\mathcal{M}$ such that for all $t \geq t_0$, $\|x_r(t)\| \leq \mathcal{M}$. Moreover, without loss of generality, the input vector of reference model is also assumed to be bounded, i.e. $\|r(t)\| \leq \bar{r}$, where $\bar{r}$ is any positive constant.

Similar to Hopp and Schmitendorf (1990); Shigemaru and Wu (2007); Wu (2008), for the reference model described by (3), there exist some the matrices $\mathcal{G}_r \in R^{n\times n_r}$, $\mathcal{H}_r \in R^{m\times n_r}$, $\mathcal{F}_r \in R^{m\times m_r}$, such that the following matrix algebraic equation holds.

$$\begin{bmatrix} A & B & 0 \\ C & 0 & 0 \\ 0 & 0 & B \end{bmatrix} \begin{bmatrix} \mathcal{G}_r \\ \mathcal{H}_r \\ \mathcal{F}_r \end{bmatrix} = \begin{bmatrix} \mathcal{G}_r A_r \\ \mathcal{C}_r \\ \mathcal{G}_r B_r \end{bmatrix} \tag{4}$$

It is worth noticing that some approaches to finding the solution of this algebraic matrix equation are also discussed in detail (see, e.g. Hopp and Schmitendorf (1990); Shigemaru and Wu (2007); Shyu and Chen (1995) and the references therein).

In this paper, the question is how to synthesize a state feedback control scheme such that the output $y(t)$ of the uncertain system described by (1) and (2) can follow the output $y_r(t)$ of the reference model described by (5).

Assumption 2.1. The nominal dynamical system of (1) is assumed to be stabilizable. That is, the pair $\big(A,\, B\big)$ is completely controllable.

Assumption 2.2. For all $(x,\, t) \in R^n \times R^+$, there exists an uncertain nonlinear function $\mathcal{E}(\cdot)$ of appropriate dimensions such that $\Delta\mathcal{F}(\cdot) = B\mathcal{E}(\cdot)$. Moreover, the uncertain $\mathcal{E}(\cdot)$ is assumed to be bounded with respect

[1] https://advanceseng.com/wu-inequality-applications-control-theory/

to their arguments, in *Euclidean* norm. More specifically, there exist a nonlinear function $\xi_0(\cdot) : R^n \times R^+ \to R^h$ and a constant vector $\theta_0^* \in R^h$ such that for all $x \in R^n$ and all $t \geq t_0$,

$$\Big\|\mathcal{E}(x(t),t)\Big\| \leq \Big(\theta_0^*\Big)^\top \xi_0(x(t),t) \tag{5}$$

where the nonlinear nonnegative function $\xi_0(x,t)$ is assumed to be sufficiently smooth.

Remark 2.1. As given in most all control literature, Assumption 2.1 is very standard (see, e.g. Zhang et al. (1996)). In particular, it should be pointed out that the nonlinear function vector $\xi_0(\cdot)$ and constant vector θ_0^* do not need to be known for designing robust tracking control schemes by using the design method proposed later.

Remark 2.2. There are some methods to deal with dead-zone functions in the control literature. In this paper, we will utilize a direct method proposed in Wu (2017a,b) to deal with such a nonsymmetric dead-zone input nonlinearity where its information is completely unknown.

It follows from Assumption 2.1 that for any given matrix $Q \in R^{n\times n}$ where $Q = Q^\top > 0$, there exists an unique positive definite matrix $P \in R^{n\times n}$ as the solution of the Riccati equation in the form of

$$A^\top P + PA - \hat{\mu}PBB^\top P \;=\; -Q \tag{6}$$

where $\hat{\mu}$ is any given positive constant.

In this paper, we will make use of a new integral inequality, called *Wu inequality* (see, Wu (2018a)), to implement our stability analysis in the next section. Here, such an integral inequality is given in the following lemma.

Lemma 2.1. (Wu (2018a)) Let $\mathcal{Y}(t)$ and $\mathcal{Z}(t)$ be any continuous functions with $\mathcal{Y}(t_0) \neq 0$. Moreover, θ, γ, β_1, and β_2 are any given positive constants, and $\breve{\rho}_j(\mathcal{Y},\mathcal{Z})$ is any nonnegative continuous function. Then, there are always some positive constants ϵ_j, $j = 1, 2, \ldots, l$, such that the inequality

$$\begin{aligned}\Big|\mathcal{Y}(t)\Big| \;\leq\; &\gamma \mathrm{e}^{-\theta(t-t_0)} \\ &+\beta_1 \sum_{j=1}^{l} \epsilon_j^{-1} \int_{t_0}^{t} \mathrm{e}^{-\theta(t-s)} \breve{\rho}_j\Big(\mathcal{Y}(s), \mathcal{Z}(s)\Big) ds + \beta_2\end{aligned} \tag{7}$$

implies that

$$\Big|\mathcal{Y}(t)\Big| \leq \frac{\gamma}{1-\eta^*}\mathrm{e}^{-\theta_0(t-t_0)} + \frac{\beta_2}{1-\eta^*} \tag{8}$$

where θ_0 and η^* are some positive constants which satisfy $\theta_0 < \theta$ and $\eta^* < 1$, respectively.

In the rest of this paper, with respect to the concept of the conventional uniform ultimate boundedness, similar to Wu (2017b, 2018b), we also introduce a concept of uniform exponential boundedness as follows.

Definition 2.1. (*Uniform exponential boundedness*) The dynamical systems described by a differential equation are said to be uniformly exponentially bounded, if there exist some positive constants ε, α, and $\kappa(\delta) > 0$ such that for any $\delta > 0$ and for any $t > t_0$,

$$\Big\|x(t)\Big\| \;\leq\; \kappa(\delta)\mathrm{e}^{-\alpha(t-t_0)} + \varepsilon$$

where $x(t)$ is the solutions of differential equation with initial condition $x(t_0)$, and $\Big\|x(t_0)\Big\| < \delta$.

3. ROBUST TRACKING CONTROL SCHEMES

In this section, we synthesize a class of tracking control schemes. Firstly, we define the tracking error as follows.

$$e(t) \;=\; y(t) - y_r(t) \tag{9}$$

Secondly, we also define a new state vector $z(t)$, called the auxiliary state, as follows:

$$z(t) \;=\; x(t) - \mathcal{G}_r x_r(t) \tag{10}$$

where $\mathcal{G}_r \in R^{n\times n_r}$ satisfies the matrix algebraic equation given in (4).

It is obvious from (4) and (10) that the relationship between the tracking error $e(t)$ and the auxiliary state vector $z(t)$ can be obtained in the form of

$$e(t) \;=\; Cz(t) \tag{11}$$

Now, from (1), (4), and (10), we can obtain an auxiliary dynamical system in the form of

$$\begin{aligned}\frac{dz(t)}{dt} &= Az(t) + B\mathcal{D}(u(t)) \\ &\quad + \Delta\tilde{\mathcal{F}}(z(t),t) + Bg(x_r(t), r(t))\end{aligned} \tag{12}$$

where

$$g(x_r(t), r(t)) := -\mathcal{H}_r x_r(t) - \mathcal{F}_r r(t) \tag{13a}$$

$$\Delta\tilde{\mathcal{F}}(z(t),t) := \Delta\mathcal{F}\Big(z(t) + \mathcal{G}_r x_r(t),\; t\Big) \tag{13b}$$

Here, because of the boundedness of the given reference model, we can define for (13a) a positive constant β_0^* as follows.

$$\beta_0^* \;:=\; \max\Big\{ \Big\|\mathcal{H}_r x_r(t) + \mathcal{F}_r r(t)\Big\| \;:\; t \in R^+ \Big\}$$

Thus, we propose a class of robust tracking control schemes with a relatively simple structure of form

$$\begin{aligned}u(t) &= \mathcal{K}(z(t), \hat{\varrho}(t), t) \\ &= -\frac{1}{2}\hat{\mu}\hat{\varrho}(t)B^\top Pz(t)\end{aligned} \tag{14}$$

where $\hat{\mu}$ is any given positive constant, P is a solution of algebraic Riccati equation (6), and $\hat{\varrho}(t)$ is a self-tuning control gain updated by

$$\frac{d\hat{\varrho}(t)}{dt} \;=\; -\gamma\sigma\hat{\varrho}(t) + \frac{1}{2}\gamma\hat{\mu}\left\|B^\top Pz(t)\right\|^2 \tag{15}$$

where γ and σ are some given positive constants, and the initial condition of the self-tuning control gain $\hat{\varrho}(t)$ is given as $\hat{\varrho}(t_0) \geq 0$.

Remark 3.1. As stated in the introduction section, in order to transform (1) and (3) into an auxiliary dynamical system, in the conventional design methods, the control schemes should be composed of two parts, i.e.

$$u(t) \;=\; \mathcal{K}_1(x_r(t), r(t)) + \mathcal{K}_2(z(t), t)$$

where

$$\mathcal{K}_1(x_r(t), r(t)) \;=\; \mathcal{H}_r x_r(t) + \mathcal{F}_r r(t)$$

and $\mathcal{K}_2(z(t), t)$ will be redesigned. However, if the control input has some constraints, e.g. the dead-zone input nonlinearities, by using such a control scheme with two parts, it is not possible to obtain an auxiliary system.

Remark 3.2. It can be observed from (14) that our robust tracking control schemes are structurally linear in the state, and have a time-varying control gain function. Moreover, it is obvious from (14) and (15) that the nonlinear upper bound function $\xi_0(\cdot)$ of the system uncertainties are not used to construct such a class of robust tracking control schemes. That is, the proposed robust control schemes are completely independent of uncertain nonlinearity $\Delta\mathcal{F}(x(t), t)$, which results in the simplicity of our design method.

We also define that $\tilde{\varrho}(t) := \hat{\varrho}(t) - \varrho^*$, and ϱ^* is an unknown positive constant which will be defined latter. Thus, we can rewrite (15) as the adaptation error systems in the form of

$$\frac{d\tilde{\varrho}(t)}{dt} \;=\; -\gamma\sigma\tilde{\varrho}(t) + \frac{1}{2}\gamma\hat{\mu}\left\|B^\top P z(t)\right\|^2 - \gamma\sigma\varrho^* \quad (16)$$

Moreover, we will define $(z, \tilde{\varrho})(t)$ as a solution of the closed-loop auxiliary systems and the adaptation error systems. Thus, we can have the following results.

Theorem 3.1. Consider the adaptive closed-loop auxiliary systems with dead-zone input, described by (12) and (14) with (15), which satisfy *Assumptions 2.1* and *Assumptions 2.2*. Then, the solutions $(z, \tilde{\varrho})$ $(t;\, t_0, z(t_0), \tilde{\varrho}(t_0))$ of the closed-loop auxiliary dynamical systems and the adaptation error systems uniformly exponentially bounded. That is, the auxiliary state $z(t)$ converges uniformly exponentially towards a ball.

Proof: Here, similar to some known method (see, e.g. Wu (2017a,b)), for any $i \in \{1, 2, \ldots, m\}$, we will rewrite the dead-zone function $\mathcal{D}_i(u_i)$ described in (2) as follows.

$$\mathcal{D}_i(u_i(t)) \;=\; \beta_i(t)u_i(t) + d_i(t) \quad (17)$$

where $\beta_i(t)$ and $d_i(t)$ are respectively defined as follows.

$$\beta_i(t) := \begin{cases} \tilde{m}_{il}, & \text{if } u_i(t) \le 0 \\ \tilde{m}_{ir}, & \text{if } u_i(t) > 0 \end{cases} \quad (18)$$

and

$$d_i(t) \;:=\; \begin{cases} -\tilde{m}_{ir}\tilde{b}_{ir}, & \text{if } u_i(t) \ge \tilde{b}_{ir} \\ -\beta_i(t)u_i(t), & \text{if } -\tilde{b}_{il} < u_i(t) < \tilde{b}_{ir} \\ \tilde{m}_{il}\tilde{b}_{il}, & \text{if } u_i(t) \le -\tilde{b}_{il} \end{cases} \quad (19)$$

Moreover, we can also define that $\bar{m}_i := \max\Big\{\tilde{m}_{il},\ \tilde{m}_{ir}\Big\}$ and $\bar{b}_i := \max\Big\{\tilde{b}_{il},\ \tilde{b}_{ir}\Big\}$. Thus, it is obvious from these definitions that

$$\Big|d_i(t)\Big| \;\le\; \bar{d}_i \;:=\; \bar{m}_i \cdot \bar{b}_i \quad (20)$$

Thus, the nonlinear dead-zone input vector given in (2) can be redescribed as follows.

$$\mathcal{D}(u(t)) \;=\; M(t)u(t) + d(t) \quad (21a)$$

where

$$M(t) := \mathrm{diag}\Big\{\ \beta_1(t),\ \beta_2(t),\ \ldots,\ \beta_m(t)\ \Big\} \quad (21b)$$

$$d(t) := \Big[\ d_1(t)\ \ d_2(t)\ \ \ldots\ \ d_m(t)\ \Big]^\top \quad (21c)$$

From (18)–(21), it can be known that for any $t \ge t_0$, the matrix $M(t)$ is positive definite, and the vector $d(t)$ is norm-bounded. Therefore, we can introduce the following definitions.

$$\eta^* := \min\Big\{\ \lambda_{\min}\left(M(t)\right),\quad t \in R^+\ \Big\} \quad (22a)$$

$$\rho^* := \max\Big\{\ \lambda_{\max}\left(d(t)d^\top(t)\right),\quad t \in R^+\ \Big\} \quad (22b)$$

where for any $i \in \{1, 2, \ldots, N\}$, η^* and ρ^* are two unknown positive constants.

Then, we construct a Lyapunov-like function for the adaptive closed-loop auxiliary dynamical systems, which is described by

$$\mathcal{V}(z, \tilde{\varrho}) = z^\top(t)Pz(t) + \eta^*\gamma^{-1}\tilde{\varrho}^2(t) \quad (23)$$

By taking the derivative of $\mathcal{V}(\cdot)$ along the trajectories of the adaptive closed-loop auxiliary dynamical systems, we have that for any $t \ge t_0$,

$$\begin{aligned}\frac{d\mathcal{V}(z, \tilde{\varrho})}{dt} &= z^\top(t)\Big[A^\top P + PA\Big]z(t) \\ &\quad + 2z^\top(t)PB\mathcal{D}(u(t)) + 2z^\top(t)P\Delta\tilde{\mathcal{F}}(z(t), t) \\ &\quad + 2z^\top(t)PBg(x_r(t), r(t)) + 2\eta^*\gamma^{-1}\tilde{\varrho}(t)\frac{d\tilde{\varrho}(t)}{dt} \end{aligned} \quad (24)$$

Moreover, according to Assumption 2.2, (6), and (21) we can obtain that for any $t \ge t_0$,

$$\begin{aligned}\frac{d\mathcal{V}(z, \tilde{\varrho})}{dt} &\le z^\top(t)\Big[-Q + \hat{\mu}PBB^\top P\Big]z(t) \\ &\quad + 2z^\top(t)PBM(t)u(t) + 2z^\top(t)PBd(t) \\ &\quad + 2\left\|B^\top Pz(t)\right\|\Big(\theta_0^*\Big)^\top\hat{\xi}_0(z(t), t) \\ &\quad + 2\beta_0^*\left\|B^\top Pz(t)\right\| + 2\eta^*\gamma^{-1}\tilde{\varrho}(t)\frac{d\tilde{\varrho}(t)}{dt} \end{aligned} \quad (25)$$

where

$$\hat{\xi}_0(z(t), t) \;:=\; \xi_0\Big(z(t) + \mathcal{G}_r x_r(t),\ t\Big) \quad (26)$$

Now, in the light of the fact that for any positive constant $\varepsilon > 0$, $2x^\top y < \varepsilon\|x\|^2 + \varepsilon^{-1}\|y\|^2$, it follows from (25) that for any $t \geq t_0$,

$$\begin{aligned}\frac{d\mathcal{V}(z,\tilde{\varrho})}{dt} \leq & -z^\top(t)Qz(t) + 2z^\top(t)PBM(t)u(t) \\ & +\epsilon_1^{-1} + \epsilon_3^{-1} + \hat{\mu}\eta^*\varrho^* \left\|B^\top Pz(t)\right\|^2 \\ & +\epsilon_2^{-1}\left\|\hat{\xi}_0(z(t),t)\right\|^2 + 2\eta^*\gamma^{-1}\tilde{\varrho}(t)\frac{d\tilde{\varrho}(t)}{dt} \end{aligned} \quad (27)$$

where for any $j \in \{1,2,3\}$, ε_j is any positive constant, and the unknown positive constant ϱ^* is defined by

$$\varrho^* := \frac{1}{\hat{\mu}\eta^*}\left(\hat{\mu} + \epsilon_1\rho^* + \epsilon_2\left\|\theta_0^*\right\|^2 + \epsilon_3\left(\beta_0^*\right)^2\right) \quad (28)$$

Substituting the robust tracking control schemes with adaptation laws into (27) yields

$$\begin{aligned}\frac{d\mathcal{V}(z,\tilde{\varrho})}{dt} \leq & -z^\top(t)Qz(t) - \hat{\mu}\eta^*\hat{\varrho}(t)\left\|B^\top Pz(t)\right\|^2 \\ & +\epsilon_1^{-1} + \epsilon_3^{-1} + \hat{\mu}\eta^*\varrho^*\left\|B^\top Pz(t)\right\|^2 \\ & +\epsilon_2^{-1}\left\|\hat{\xi}_0(z(t),t)\right\|^2 - 2\sigma\eta^*\tilde{\varrho}^2(t) \\ & +\hat{\mu}\eta^*\tilde{\varrho}(t)\left\|B^\top Pz(t)\right\|^2 - 2\sigma\eta^*\tilde{\varrho}(t)\varrho^* \\ \leq & -z^\top(t)Qz(t) - \sigma\eta^*\tilde{\varrho}^2(t) \\ & +\epsilon_2^{-1}\left\|\hat{\xi}_0(z(t),t)\right\|^2 + \epsilon^* \end{aligned} \quad (29)$$

where

$$\epsilon^* := \epsilon_1^{-1} + \epsilon_3^{-1} + \sigma\eta^*\left|\varrho^*\right|^2 \quad (30)$$

Since P and Q are positive definite, it can be obtained from (23) and (29) that for any $t \geq t_0$,

$$\frac{d\mathcal{V}(z,\tilde{\varrho})}{dt} \leq -\theta_{\min}\mathcal{V}(z,\tilde{\varrho}) + \epsilon_2^{-1}\left\|\hat{\xi}_0(z(t),t)\right\|^2 + \epsilon^* \quad (31)$$

where

$$\theta_{\min} := \min\left\{\lambda_{\min}(Q)\lambda_{\max}^{-1}(P), \quad \sigma\gamma\right\}$$

Then, if defining $\mathcal{V}(t) := \mathcal{V}(z(t),\tilde{\varrho}(t))$, from (23) and (31), we can obtain that for any $t \in R^+$,

$$\begin{aligned}\left\|z(t)\right\|^2 \leq & \lambda_{\min}^{-1}(P)\exp\{-\theta_{\min}(t-t_0)\}\mathcal{V}(t_0) \\ & +\lambda_{\min}^{-1}(P)\epsilon_2^{-1}\int_{t_0}^{t}\exp\{-\theta_{\min}(t-s)\}\left\|\hat{\xi}_0(z(s),s)\right\|^2 ds \\ & +\lambda_{\min}^{-1}(P)\theta_{\min}^{-1}\epsilon^* \end{aligned} \quad (32)$$

Finally, noting that ϵ_2 is any positive constant, and in terms of Wu inequality given in *Lemma 2.1*, from (32) it is not difficult that we can obtain a bound on the auxiliary state as follows.

$$\left\|z(t)\right\|^2 \leq \frac{\lambda_{\min}^{-1}(P)\mathcal{V}(t_0)}{1-\hat{\kappa}}\mathrm{e}^{-\theta_0(t-t_0)} + \frac{\lambda_{\min}^{-1}(P)\theta_{\min}^{-1}\epsilon^*}{1-\hat{\kappa}} \quad (33)$$

where θ_0 and $\hat{\kappa}$ are some positive constants which satisfy $\theta_0 < \theta_{\min}$ and $\hat{\kappa} < 1$, respectively.

Thus, it is obvious from (33) that the auxiliary state $z(t)$ converges uniformly exponentially to a ball $\mathcal{B}(\hat{\rho}_0)$ described by

$$\mathcal{B}(\hat{\rho}_0) := \left\{ z \;\Big|\; \|z\| \leq \hat{\rho}_0 := \sqrt{\frac{\lambda_{\min}^{-1}(P)\theta_{\min}^{-1}\epsilon^*}{1-\hat{\kappa}}} \right\} \quad (34)$$

Thus, we complete the proof of Theorem 3.1. $\nabla\nabla\nabla$

Moreover, from Theorem 3.1 we can easily obtain the following theorem.

Theorem 3.2. Consider the model following problem of the uncertain nonlinear systems with dead-zone inputs, described by (1) and (2). Suppose that *Assumption 2.1* and *Assumption 2.2* is satisfied. Then, by using the state feedback control schemes $u(t)$ given in (14) with (15), one can guarantee that the tracking error $e(t)$ between the uncertain systems and the reference model is uniformly exponentially bounded. That is, the output $y(t)$ of the uncertain nonlinear systems described by (1) and (2) can track the output $y_r(t)$ of the reference model described by (5) in the sense of uniform exponential boundedness.

Remark 3.3. In the proof of Theorem 3.1, some positive parameters θ_0, ϵ^*, $\hat{\kappa}$, ϵ_j, $j = 1,2,3$, have been utilized. However, these parameters are completely independent of the proposed robust tracking control schemes described by (14) with (15). That means that these parameters are employed only for the theoretical proof of our results. Therefore, we do not need to know or select them in this paper.

4. ILLUSTRATIVE EXAMPLE

In this section, we provide a numerical example to describe the design procedure of the presented method, and to demonstrate the efficiency of the results through its simulations.

$$\frac{dx(t)}{dt} = \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathcal{D}(u(t)) + \Delta\mathcal{F}(x(t),t) \quad (35a)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \quad (35b)$$

where $\mathcal{D}(u)$ is a dead-zone function. For simulation, the uncertain nonlinear function $\Delta\mathcal{F}\left(x(t),t\right)$ is given by

$$\Delta\mathcal{F}\left(x(t),t\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\left(\theta_1\frac{1-\mathrm{e}^{-x_1(t)}}{1+\mathrm{e}^{-x_1(t)}} + \theta_2\sin(x_2(t))\right)$$

and where θ_1 and θ_2 are some unknown parameters.

The reference model is described by the differential equation in the form of

$$\frac{dx_r(t)}{dt} = \begin{bmatrix} 2 & 4 \\ -2 & -2 \end{bmatrix} x_r(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t) \quad (36a)$$

$$y_r(t) = \begin{bmatrix} 2 & -1 \end{bmatrix} x_r(t) \quad (36b)$$

Now, for this numerical example, in the light of (4), (37), and (38), it is not difficult to obtain the following matrices.

$$\mathcal{G}_r = \begin{bmatrix} 2 & 4 \\ 2 & -1 \end{bmatrix}, \quad \mathcal{H}_r = [\,0 \quad 13\,], \quad \mathcal{F}_r = 2$$

For the given constant $\hat{\mu} = 5$ and matrix $Q = \text{diag}\{1,\ 1\}$, from (8) we can have that

$$P = \begin{bmatrix} 1.433 & -0.167 \\ -0.167 & 0.430 \end{bmatrix}$$

and for the adaptation law, the following parameters are selected.

$$\gamma = 0.5, \qquad \sigma = 0.1$$

Moreover, for simulation, we also give the characteristic parameters of the dead-zone function $\mathcal{D}(u(t))$ as follows.

$$\begin{aligned} \tilde{m}_l &= 1.0, \qquad \tilde{m}_r = 0.7 \\ \tilde{b}_l &= 3.0, \qquad \tilde{b}_r = 1.0 \end{aligned}$$

Finally, we select the reference input $r(t)$, the unknown parameters θ_1, θ_2, and all the initial conditions as follows.

$$\begin{aligned} r(t) &= 0.2, \qquad \theta_1 = \theta_2 = 0.5 \\ x(0) &= [\,-3.0 \quad 3.0\,]^\top \\ x_r(0) &= [\,-8.0 \quad 8.0\,]^\top \\ \hat{\varrho}(0) &= 5.0 \end{aligned}$$

Thus, the results of simulation can be depicted in *Fig.1* and *Fig.2*. It can be observed from *Fig.1* and *Fig.2* that the output $y(t)$ of uncertain system (37) can indeed track the output $y_r(t)$ of reference model (38) in the sense of uniform exponential boundedness.

(The details of the illustrative numerical example and the figures of the simulation will be displayed in the oral presentation.)

5. CONCLUDING REMARKS

In the paper, the problem of robust tracking and model following has been investigated for a class of uncertain nonlinear systems with completely unknown dead-zone inputs and uncertain nonlinearity. The system uncertainties have been assumed to be any continuous and bounded nonlinear functions. Based on *Wu inequality*, a new design method has been proposed by which some adaptive robust tracking control schemes with a relatively simple structure can be obtained. It has been also shown that the output of the uncertain nonlinear systems can be guaranteed to track the output of the given reference model in the sense of uniform exponential boundedness.

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