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Parameters adaptive identification of Bouc-Wen hysteresis

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Abstract: The adaptive identification method is designed for estimation of system parameters with the Bouc-Wen hysteresis. It is based on the use of adaptive observers and eliminates the problems inherent the identification procedures of such systems. Adaptive algorithms for identifying processes in an adaptive system are developed. The limitation of processes in the adaptive system is proved. The exponential dissipativity of the adaptive system is proved.

Keywords: Bouc-Wen hysteresis, adaptive observers, vector Lyapunov function, uncertainty.

1. INTRODUCTION

The Bouc-Wen (BWM) model is widely used to describe a hysteresis. System with BWM has the form (Bouc, 1967; Wen, 1976)

$$m\ddot{x} + c\dot{x} + F(x,z,t) = f(t)\,, \quad (1)$$

$$F(x,z,t) = \alpha kx(t) + (1-\alpha)kdz(t)\,, \quad (2)$$

$$\dot{z} = d^{-1}\left(a\dot{x} - \beta|\dot{x}||z|^{n}\, sign(z) - \gamma\dot{x}|z|^{n}\right), \quad (3)$$

where $m > 0$ is mass, $c > 0$ is damping, $F(x,z,t)$ is the recovering force, $d > 0$, $n > 0$, $k > 0$, $\alpha \in (0,1)$, $f(t)$ is exciting force, a, β, γ are some numbers.

Equation (3) is the BWM. Many modifications of BWM (Ismail et al., 2009) are proposed. Each proposed model considers features of the considered object. The BWM successful application depends on the identification of its parameters. The solution of the nonlinear equation (3) is the main problem of BWM identification. A three-level algorithm (Loh et al., 1993) is applied to Bouc-Wen model identification. This is based on regression analysis, least squares or Gauss-Newton methods, and the extended Kalman filter. The relevant approach is applied in (Baber et al., 1981; Kunnath et al., 1997). Adaptive algorithms are proposed in (Chassiakos et al., 1998; Smith et al., 1999) for the BWM parameters estimation with the data forgetting (Ioannou et al., 1996). Paper (Lin et al., 2001) presents an adaptive on-line identification methodology with a variable trace method to adjust the adaptation gain matrix.

Examples (Danilin et al., 2016) are known when BWM parameters estimations do not coincide with results obtained for other inputs. Such examples confirm the identification ambiguity which causes instability of the model relatively the input. Explain it with the fact that the Bouc-Wen the model should be stable and ensure the adequacy to a physical process (Ismail et al., 2009).

So, the analysis of publications shows that the set of algorithms and procedures for the Bouc-Wen model parameters identification is proposed. The proposed models consider the features of an examined object. The main difficulties of the BWM parameters estimation are (i) the ensuring model stability (ii) the input choice. As a rule, the variation range of BWM parameters specifies often. Some parameters, for example n, are set. It is also often supposed that all derivatives of the object are measured. Such situation does not always arise that bring in not performability of proposed algorithms.

Below the adaptive identification method based on adaptive observer application (Karabutov, 2019) is used for the problem solution of the model (3) stability. The system specified by the equations (1)-(3), is considered. It is supposed that the input $f(t)$ and the output $x(t)$ are measured. The study was financially supported by the Russian Foundation for Basic Research and the Lipetsk Region as part of the scientific project 19-48-480007 p_a.

2. PROBLEM STATEMENT

Consider the system S_{BW} (1)-(3). Let y be the output of the system. The set of experimental data has the form $\mathrm{I}_o = \{f(t), y(t), t \in J\}$, where $J \subset R$ is the given time slice.

Designate by the parameters vector of the system as $A = [m, c, a, k, \alpha, \beta, \gamma, n]^T$.

Problem: design the adaptive observer for vector estimation A of the system S_{BW} that satisfy the condition

$$\lim_{t\to\infty} |\hat{y}(t) - y(t)| \le \pi_y\,, \quad (4)$$

where $\hat{y} \in R$ is the output of the adaptive observer, $\pi_y \ge 0$.

Remark 1. The identification effectiveness of the system S_{BW} depends on features of the input $f(t)$. Requirements to $f(t)$ in identification problems are known. The force $f(t)$ satisfies the condition of constant excitation (CE). This condition is necessary, but not sufficient (Karabutov, 2018). The input having the CE property can not ensure the identifiability of the hysteresis structure. The structural identifiability of the hysteresis is a guarantee with the input $f(t)$ having the S-stabilisation property of the system S_{BW} (Karabutov, 2018). Conditions of this property verification give in (Karabutov, 2018).

3. SYSTEM IDENTIFICATION

Consider the case $d=1$, $a=1$. Substitute $F(x,z,t)$ in (1), and divide it by $s+\mu$, where $\mu>0$ and does not coincide with roots of the polynomial $s^2+a_1s+a_2$, $s=d/dt$. Then transform (1) to the form

$$\dot{x}=a_1x+a_2p_x+a_3p_z+bp_f, \tag{5}$$

$$\begin{aligned}&\dot{p}_x=-\mu p_x+x, \dot{p}_f=-\mu p_f+f,\\ &\dot{p}_z=-\mu p_z+z,\end{aligned} \tag{6}$$

where $a_1=-(c-\mu m)/m$, $a_2=-(\alpha k-\mu(c-\mu m)/m$, $a_3=-(1-\alpha)k/m$.

Equations (5), (6) contain only measurable variables, except z. It complicates the identification process of the system S_{BW} parameters. Apply the model

$$\dot{\hat{x}}=-k_x(\hat{x}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{a}_3p_z+\hat{b}p_f \tag{7}$$

to the parameters estimation of (5), where $k_x>0$ is the specified number; $\hat{a}_i(t)$, $i=1,2,3$, and $\hat{b}(t)$ are adjusted parameters.

Designate $e=\hat{x}-x$. Obtain the equation for the identification error from (5), (7)

$$\dot{e}=-k_xe+\Delta a_1x+\Delta a_2p_x+\Delta a_3p_z+\Delta bp_f, \tag{8}$$

where $\Delta b=\hat{b}(t)-b$, $\Delta a_1=\hat{a}_1(t)-a_1$, $\Delta a_2=\hat{a}_2(t)-a_2$, $\Delta a_3=\hat{a}_3(t)-a_3$.

The (8) is not solvable as the variable z is unknown in (6). Obtain the current estimation for $z(t)$. Consider model

$$\dot{\hat{x}}_{\bar{z}}=-k_x(\hat{x}_{\bar{z}}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{b}p_f. \tag{9}$$

Determine by the misalignment $\varepsilon_z=x-\hat{x}_{\bar{z}}$ and use it for the variable z estimation. Consider ε_z as the current estimation z. Apply the model to the estimation z

$$\dot{\hat{z}}=-k_z\left(\hat{z}-\varepsilon_z\right)+\tilde{\dot{x}}-\hat{\beta}\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign(\hat{z})-\hat{\gamma}\tilde{\dot{x}}\left|\hat{z}\right|^n, \tag{10}$$

where $\tilde{\dot{x}}=\left(x(t+\tau)-x(t)\right)/\tau$, $k_z>0$ is the given number; $\hat{\beta}$, $\hat{\gamma}$ are the hysteresis (3) parameters estimations;, τ is the integration step.

Introduce the misalignment $\varepsilon=\hat{z}-\varepsilon_z$ and obtain the equation for ε

$$\begin{aligned}\dot{\varepsilon}=-k_z\varepsilon+\Delta\dot{x}+\Delta\beta\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right)+\beta\eta_\beta+\\ \Delta\gamma\tilde{\dot{x}}\left|\hat{z}\right|^n+\gamma\eta_\gamma\end{aligned}, \tag{12}$$

$$\eta_\beta=\left|\dot{x}\right|\left|z\right|^n sign\left(z\right)-\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right),\ \eta_\gamma=\dot{x}\left|z\right|^n-\tilde{\dot{x}}\left|\hat{z}\right|^n,$$

where $\Delta\dot{x}=\tilde{\dot{x}}-\dot{x}$, $\Delta\beta=\beta-\hat{\beta}$, $\Delta\gamma=\gamma-\hat{\gamma}$.

Present (7) as

$$\dot{\hat{x}}=-k_x(\hat{x}-x)+\hat{a}_1x+\hat{a}_2p_x+\hat{a}_3p_{\hat{z}}+\hat{b}p_f,$$

where

$$\dot{p}_{\hat{z}}=-\mu p_{\hat{z}}+\hat{z}. \tag{13}$$

Then (8) rewrite as

$$\dot{e}=-k_xe+\Delta a_1x+\Delta a_2p_x+\Delta a_3p_{\hat{z}}+\Delta bp_f, \tag{14}$$

and adaptive algorithms describe as

$$\Delta\dot{a}_1=-\gamma_1ex, \Delta\dot{a}_2=-\gamma_2e_x, \Delta\dot{a}_3=-\gamma_3ep_{\hat{z}}, \Delta b=-\gamma_bep_f, \tag{15}$$

where $\gamma_i>0, i=1,2,3$; $\gamma_b>0$.

Tuning algorithms for $\Delta\beta$ and $\Delta\gamma$ in (10) have the form

$$\begin{aligned}&\Delta\dot{\beta}=-\chi_\beta\varepsilon\left|\tilde{\dot{x}}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right),\\ &\Delta\dot{\gamma}=-\chi_\gamma\varepsilon\tilde{\dot{x}}\left|\hat{z}\right|^n,\end{aligned} \tag{16}$$

where $\chi_\beta>0, \chi_\gamma>0$ are parameters ensuring a convergence of algorithms.

Several algorithms are applicable for the indicator n estimation in (10). Their effectiveness of their work depends on several factors. The simple algorithm has the form

$$\dot{\hat{n}}=\begin{cases}-\gamma_n\varepsilon\hat{\beta}\left|\hat{z}\right|^{\hat{n}-1}\hat{z}\tilde{\dot{x}}, if\left|\dfrac{\varepsilon}{\varepsilon_z}\right|\in\left[\upsilon_0,\upsilon_1\right],\\ 0, \quad if\left|\dfrac{\varepsilon}{\varepsilon_z}\right|\notin\left[\upsilon_0,\upsilon_1\right],\end{cases} \tag{17}$$

where υ_0, υ_1 are given positive numbers, $\gamma_n>0$.

Remark 2. Stability of the identification procedure is the main problem the solution of the system with BWM. We proposed the method based on adaptive observers application. Another solution to the stability problem is to change the structure of the equation (3). We proposed the equation[1]

$$\dot{z} = d^{-1}\left(-\rho z|\dot{x}|^{\omega} + a\dot{x} - \beta|\dot{x}||z|^{n} sign(z) - \gamma\dot{x}|z|^{n}\right) \quad (3a)$$

for describe hysteresis, where $\rho > 0$, $\omega > 0$.

4. PROPERTIES OF ADAPTIVE SYSTEM

Consider the subsystem AS_X described by (14), (15). Let $\Delta K(t) \triangleq \left[\Delta a_1(t), \Delta a_2(t), \Delta a_3(t), \Delta b(t)\right]^T$,

$$V_K(t) \triangleq 0.5\Delta K^T(t)\Gamma^{-1}\Delta K(t) , \quad (18)$$

$$V(t) = V_e(t) + V_K(t) , \quad (19)$$

where $\Gamma = diag\left(\gamma_1, \gamma_2, \gamma_3, \gamma_b\right)$.

Assumption 1. The input $f(t)$ is constantly exciting and limited.

Theorem 1. Let 1) functions (18), $V_K(t)$ are positive definite and satisfy the condition $\inf_{|e|\to\infty} V_e(e) \to \infty$, $\inf_{\|\Delta K\|\to\infty} V_K(\Delta K) \to \infty$; 2) assumption 1 for the system (1)-(3) is satisfied. Then all trajectories of the system AS_X are limited belong area $\mathrm{G}_t = \left\{(e, \Delta K) : V(t) \le V(t_0)\right\}$ and the estimation

$$\int_{t_0}^{t} 2k_x V_e(\tau)\, d\tau \le V(t_0) - V(t)$$

is fair.

The theorem 1 shows the limitation of adaptive system trajectories. The asymptotical stability ensuring the system demands to impose additional conditions.

Let $P(t) \triangleq \left[x(t)\ p_x(t)\ p_{\dot{z}}(t) p_f(t)\right]^T$.

Definition 1. The vector P is constantly excited with a level ν or have property $\mathcal{PE}_\nu$ if

$$\mathcal{PE}_\nu :\ P(t)P^T(t) \ge \nu I_4$$

fairly for $\exists\, \nu > 0$ and $\forall t \ge t_0$ on some interval $T > 0$, where $I_4 \in R^4$ is the unity matrix.

If the vector $P(t)$ has property $\mathcal{PE}_\nu$ then we will write $P(t) \in \mathcal{PE}_\nu$.

The system S_{BW} is stable, and the input $f(t)$ is restricted. Therefore, present the property $\mathcal{PE}_\nu$ for the matrix $B_P(t) = P(t)P^T(t)$ as

$$\mathcal{PE}_{\nu,\bar{\nu}} :\ \nu I_4 \le B_P(t) \le \bar{\nu} I_4 \quad \forall t \ge t_0 , \quad (20)$$

where $\bar{\nu} > 0$ is some number.

Let the estimation to $V_K(t)$ be fair

$$0.5\, \beta_l^{-1}(\Gamma)\|\Delta K(t)\|^2 \le V_K(t) \le 0.5\, \beta_1^{-1}(\Gamma)\|\Delta K(t)\|^2 , \quad (21)$$

where $\beta_1(\Gamma)$, $\beta_l(\Gamma)$ are minimum and maximum eigenvalues of the matrix Γ .

Apply inequalities (20), (21) and obtain estimations for $\dot{V}_e, \dot{V}_K$

$$\dot{V}_e \le -k_x V_e + \frac{\bar{\nu}\beta_l(\Gamma)}{k_x} V_K , \quad (22)$$

$$\dot{V}_K \le -\frac{3}{4}\vartheta\nu\beta_1(\Gamma)V_K + \frac{8}{3}\vartheta V_e , \quad (23)$$

Obtain estimates (22), (23) applying the approach from (Karabutov, 2018).

Theorem 2. Let conditions be satisfied 1) positive definite Lyapunov functions $V_e(t)$ and (18) allow the indefinitely small highest limit at $|e(t)| \to 0$, $\|\Delta K(t)\| \to 0$; 2) $P(t) \in \mathcal{PE}_{\nu,\bar{\nu}}$; 3) equality

$$e\Delta K^T P = \vartheta\left(\Delta K^T B \Delta K + e^2\right)$$

is fair in the area $O_\nu(O)$ with $0 < \vartheta$, where $O = \{0,\ 0^{3m}\} \subset R \times R^{3m} \times J_{0,\infty}$, O_ν is some neighbourhood of the point O ; 4) the function $V_K(t)$ satisfies (21); 5) $\dot{V}_\varepsilon, \dot{V}_K$ satisfy the system of inequalities

$$\begin{bmatrix} \dot{V}_e \\ \dot{V}_K \end{bmatrix} \le \underbrace{\begin{bmatrix} -k_x & \dfrac{\bar{\nu}\,\beta_l(\Gamma)}{k_x} \\ \dfrac{8}{3}\vartheta & -\dfrac{3\nu\vartheta\beta_1(\Gamma)}{4} \end{bmatrix}}_{A_V} \begin{bmatrix} V_e \\ V_K \end{bmatrix};$$

6) the upper solution for $V_{e,K}(t) = \left[V_e(t)\ V_K(t)\right]^T$ satisfies to the comparison equation $\dot{S} = A_V S$ if

$$V_\rho(t) \le s_\rho(t) \quad \forall\left(t \ge t_0\right) \& \left(V_\rho\left(t_0\right) \le s_\rho\left(t_0\right)\right),$$

where $\rho = e, K$, $S = \left[s_e\ s_K\right]^T$, $A_V \in R^{2\times 2}$ is M -matrix. Then the system AS_X is exponentially stable with the estimation

$$V_{e,K}(t) \le e^{A_V(t-t_0)} S(t_0) ,$$

if

$$k_x \ge \frac{4}{3}\sqrt{\frac{2\bar{\nu}\beta_l(\Gamma)}{\nu\beta_1(\Gamma)}} . \quad (24)$$

Theorem 2 shows that the adaptive system AS_X gives the true

[1] Karabutov N., About identification of system with Bouc-Wen hysteresis. Modeling of non-linear processes and systems, Fourth international conference, Moscow, 2019 (in print).

estimates for parameters of the system (1). This is fair at the fulfilment of conditions (24). We supposed that the variable $p_{\tilde{z}}$ restricted.

The boundedness of the variable $\hat{x}_{\tilde{z}}$ follows from the boundedness of the system AS_X trajectories.

Consider subsystem AS_Z described by equations (12), (16). Introduce Lyapunov functions

$$\begin{gathered} V_{\varepsilon\beta\gamma}(t) = V_\varepsilon(t) + V_{\beta,\gamma}(t), \\ V_{\beta,\gamma}(t) = 0.5\chi_\beta^{-1}\left(\Delta\beta(t)\right)^2 + 0.5\chi_\gamma^{-1}\left(\Delta\gamma(t)\right)^2 . \end{gathered} \tag{25}$$

Theorem 3. Let 1) functions $V_\varepsilon(t)$, $V_{\beta,\gamma}(t)$ are positive definite and satisfy conditions

$$\inf_{|\varepsilon|\to\infty} V_\varepsilon(\varepsilon) \to \infty \,, \quad \inf_{\|[\Delta\beta,\Delta\gamma]\|\to\infty} V_{\beta,\gamma}\left(\Delta\beta,\Delta\gamma\right) \to \infty \,;$$

2) the function $V_{\varepsilon\beta\gamma}(t)$ has the form (25); 3) the function

$$\tilde{g}_1(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{n+1}(t)}{V_\varepsilon(t,\varepsilon)} \,, \quad g_1 = \sup_{\varepsilon\in\Omega} \tilde{g}_1(t) \,, \tag{26}$$

exists, where Ω is the definition range of the subsystem AS_Z; 4) $|\Delta\dot{x}| \le \delta_\Delta$, $\delta_\Delta \ge 0$; 5) $|\dot{x}| \le \upsilon$, $\upsilon > 0$; 6) the assumption 1 holds for the system (1)-(3). Then all trajectories of the system AS_Z are bounded, belong in the area $\mathrm{G}_\varepsilon = \left\{\left(\varepsilon,\Delta\beta,\Delta\gamma\right) : V_{\varepsilon\beta\gamma}(t) \le V_{\varepsilon\beta\gamma}\left(t_0\right)\right\}$, and the estimation

$$\begin{gathered} \int_{t_0}^{t}\left(k_z - \upsilon(\beta+\gamma)g_1\right)V_\varepsilon(\tau)d\tau + \\ \frac{1}{2\left(k_z - \upsilon(\beta+\gamma)g_1\right)\left(t-t_0\right)}\left(\delta_\Delta\right)^2 \le \\ V_{\varepsilon\beta\gamma}(t_0) - V_{\varepsilon\beta\gamma}(t) \end{gathered} \tag{27}$$

is fair if

$$k_z > \upsilon(\beta+\gamma)g_1 \,. \tag{28}$$

So, the boundedness of trajectories in the adaptive system is proved. The analysis showed that the subsystem AS_X is asymptotically stable. The prove of trajectories boundedness for the subsystem AS_Z is more complex problem in parametrical and output spaces. This problem is solvable if the condition (28) is satisfied. The estimation (27) shows that the quality of processes in the AS_Z-system depends on the output derivative of the S_{BW}-system. The following result given more exact estimations for processes in the AS_Z-system.

Theorem 4. Let 1) positive definite Lyapunov functions $V_{\beta,\gamma}(t)$ and $V_\varepsilon(t)$ allow the indefinitely small higher limit $\|[\Delta\beta(t),\Delta\gamma(t)]\| \to 0$ to $|\varepsilon(t)| \to 0$; 2) $P(t) \in \mathcal{PE}_{\nu,\bar{\nu}}$; 3) such $c_1 > 0, c_2 > 0$ exist that conditions

$$\varepsilon\Delta\gamma\tilde{x}|\hat{z}|^n = c_2\left[\left(\Delta\gamma\right)^2\left(\tilde{x}|\hat{z}|^n\right)^2 + \varepsilon^2\right],$$

$$\varepsilon\Delta\beta\left|\tilde{x}\right|\left|\hat{z}\right|^n sign\left(\hat{z}\right) = c_1\left[\left(\Delta\beta\right)^2\left(\left|\tilde{x}\right|\left|\hat{z}\right|^n\right)^2 + \varepsilon^2\right]$$

are satisfied in the area $O_\nu(O)$, where $O = \{0,\ 0^2\} \subset R \times R^2 \times J_{0,\infty}$, O_ν is some neighbourhood of the point O; 4) inequality $\left(\varepsilon - \varepsilon_z\right)^{2n} \ge c_z$ holds for almost all t where $c_z \ge 0$; 5) such $\pi_x \ge 0$ and $\omega > 0$ exist that $\left(\tilde{\dot{x}}\right)^2 \ge \pi_x$ и $|\varepsilon - \varepsilon_z| \le \omega|\varepsilon|$; 6) the function

$$g_2(t) = \sup_{\varepsilon\in\Omega} \frac{|\varepsilon|^{2(n+1)}(t)}{V_\varepsilon(t,\varepsilon)} \,, \quad g_2 = \sup_{\varepsilon\in\Omega} \tilde{g}_2(t)$$

exists, where Ω the subsystem AS_Z definition domain; 7) $\dot{V}_\varepsilon$, $\dot{V}_{\beta,\gamma}$ satisfy the system of inequalities

$$\begin{bmatrix} \dot{V}_\varepsilon \\ \dot{V}_{\beta,\gamma} \end{bmatrix} \le \underbrace{\begin{bmatrix} -\left(k_z - 2\tilde{\upsilon}g_1 - \omega\upsilon g_2\right) & \lambda\chi\omega\upsilon \\ c & -\dfrac{d_s}{2} \end{bmatrix}}_{A_\varepsilon} \begin{bmatrix} V_\varepsilon \\ V_{\beta,\gamma} \end{bmatrix} + \underbrace{\begin{bmatrix} \dfrac{1}{2k_z} \\ 0 \end{bmatrix}}_{B_\varepsilon} \left(\delta_\Delta\right)^2 ;$$

8) the upper solution for $V_{\varepsilon,\beta,\gamma} = \left[V_\varepsilon(t)\, V_{\beta,\gamma}(t)\right]^T$ satisfies to the equation

$$\dot{\tilde{S}} = A_\varepsilon\tilde{S} + B_\varepsilon\left(\delta_\Delta\right)^2$$

If

$$V_{\tilde{\rho}}(t) \le \tilde{s}_{\tilde{\rho}}(t) \quad \forall\left(t \ge t_0\right) \& \left(V_{\tilde{\rho}}\left(t_0\right) \le \tilde{s}_{\tilde{\rho}}\left(t_0\right)\right),$$

where $\tilde{\rho} = \varepsilon,\ \beta,\gamma$, $\tilde{S} = \left[\tilde{s}_\varepsilon\ \tilde{s}_{\beta,\gamma}\right]^T$, $A_\varepsilon \in R^{2\times 2}$ is M-matrix. Then the system AS_Z is exponentially dissipative with the estimation

$$V_{\varepsilon,\beta,\gamma}(t) \le e^{A_\varepsilon(t-t_0)}\tilde{S}(t_0) + \left(\delta_\Delta\right)^2 \int_{t_0}^{T} e^{A_\varepsilon(t-\tau)}B_\varepsilon d\tau \,,$$

if $\left(k_z - 2\tilde{\upsilon}g_1 - \omega\upsilon g_2\right)d_s > 2c\lambda\chi\omega\upsilon$, $k_z > 2\tilde{\upsilon}g_1 - \omega\upsilon g_2$, $d_s > 0$,

$$\begin{gathered} \overline{\chi} = \min\left(\chi_\beta, \chi_\gamma\right), \overline{c} = \min\left(c_1, c_2\right), \\ \chi = \max\left(\chi_\beta, \chi_\gamma\right), d_s = \chi\pi_x\overline{c}c_z. \end{gathered}$$

So, we have shown that the system AS_Z is exponentially dissipative. The area of the dissipativity depends on the informational set I_o the S_{BW}-system. The obtained results justify the application possibility of adaptive observers for the S_{BW}-system identification.

5. SIMULATION RESULTS

Consider the system (1)-(3) with parameters $n = 1.5$, $c = 2$, $m = 1$, $\beta = 0.5$, $\alpha = 0.7$, $k = 0.6$. Let $d = a = 1$. The exciting force $f(t) = 2 - 2\sin(0.15\pi t)$. The system S_{BW} modelled with initial conditions $x(0) = 1$, $\dot{x}(0) = 0$, $z(0) = 1$. Form the set I_o. The system phase portrait and output of the hysteresis shown in Fig. 1.

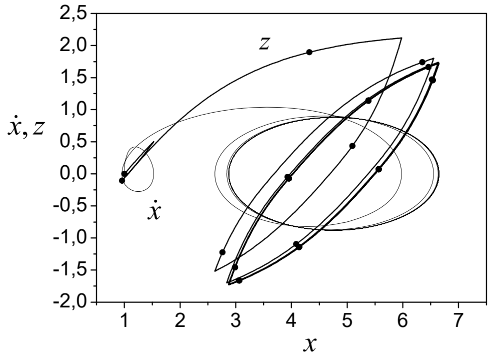

Fig. 1. System phase portrait and hysteresis change.

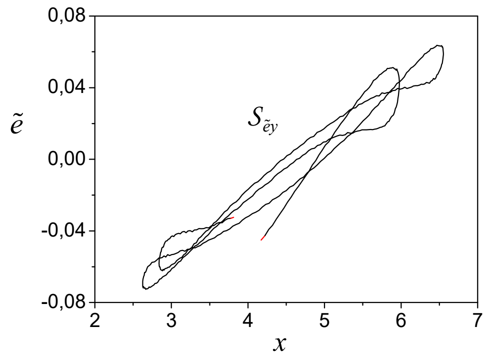

Fig. 2. System phase portrait and hysteresis change.

Construct the framework $S_{\tilde{e}y}$ (Fig. 2), using the method (Karabutov, 2018), and evaluate the structural identifiability of the system S_{BW}. The variable $\tilde{e} \in R$ is equal $\tilde{e} = \dot{x} - \hat{\dot{x}}_h$. $\hat{\dot{x}}_h$ is the estimation of the steady state (process) in the S_{BW}-system for $\forall t \geq 9.85$ s, and $\tilde{e}$ is the hysteresis output estimation. Fig. 1, 2 shows that definition ranges z and $\tilde{e}$ coincides. The analysis $S_{\tilde{e}y}$ shows that the system S_{BW} is structurally identifiable, and the input $f(t)$ is S-stabilizing.

Consider the identification of the system S_{BW} parameters. Determine by the parameter μ of the system Eq. (13) using the transient process analysis for $\tilde{e}$ and $t < 9.85$ s. Calculate Lyapunov exponents (LE) (Karabutov, 2015). The estimation for the maximum LE is –0.9. Therefore, we set $\mu = 0.8$. Initial conditions in (6) are equal to zero.

Adaptive system work results presented in Fig. 3-6. Parameters k_x, k_z are equal to 2.5 and 0.75. The tuning process of AS_X-systems (the model (7)) parameters shown in Fig. 3, and in Fig. 4 is the tuning of the model (10) parameters.

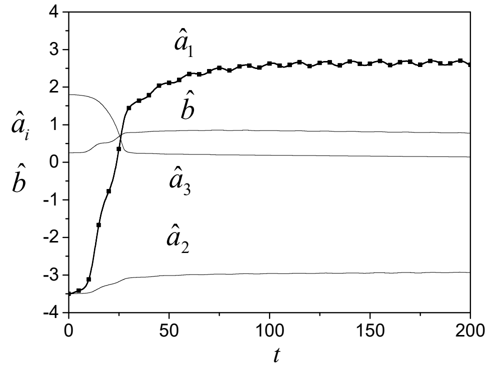

Fig. 3. Tuning of model (7) parameters.

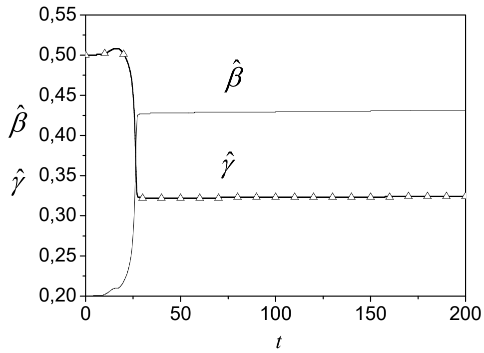

Fig. 4. Tuning of model (10) parameters.

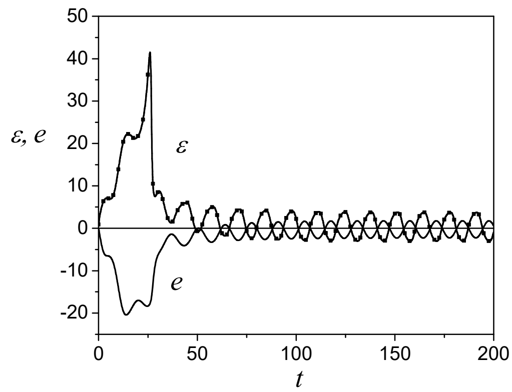

Fig. 5. Outputs modification of systems AS_X, AS_Z.

Show the modification of identification errors e, ε in Fig. 5. We see that the accuracy of obtained estimations depends on numbers of tuned parameters and the $\dot{x}$ level, and properties

$f(t)$. Obtained results confirm statements of theorems 3, 4. The AS_Z-system work results influence on the tuning processes in the AS_X-system. Gain coefficients in (15), (16) and (17) are $\chi_\beta = 0.0000002$, $\chi_\gamma = 0.0000002$, $\gamma_4 = 0.00005$ $\gamma_1 = 0.0002$, $\gamma_2 = 0.00001$, $\gamma_3 = 0.00002$.

The hysteresis output estimation process shown in Fig. 6.

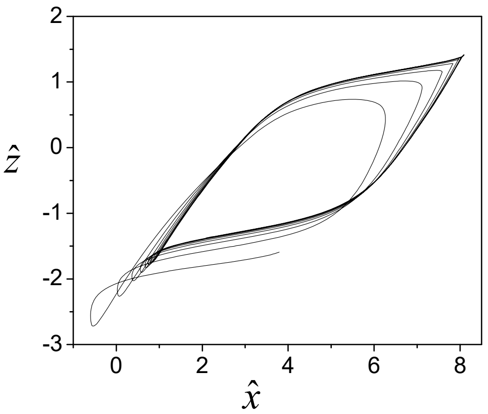

Fig. 6 Hysteresis estimation at adaptation of AS_{BW}-system.

So, simulation results confirm the exponential dissipativity of the designed system.

6. CONCLUSION

The adaptive parameter identification method designed for the system with Bouc-Wen hysteresis. The method is based on the application of adaptive observers and removes the identification system stability problem. Adaptive algorithms of model parameter tuning designed, and the boundedness of trajectories shown in the adaptive system. The current estimation of uncertainty which used for the tuning of hysteresis model parameters is obtained. Quality estimations of the work adaptive identification system obtained. We show that the boundary of the system exponential dissipativity area determined by the system output derivative level.

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