

A New Data-Driven Approach of Reference Shaping

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Abstract: In this paper, we propose a data-driven method of reference shaping to improve the tracking performance of uncertain linear systems. The proposed method can be implemented on a system based on its input-output data; the finite-time L_2 -norm of the tracking error, estimated using the data, can be minimized. Moreover, the proposed algorithm can be extended for cases where the control inputs are constrained owing to actuation limits. The effectiveness of the proposed method is demonstrated through a numerical simulation and an experiment conducted using the cart system.

Keywords: Data-Driven Control, Reference Shaping, Input Constraints, Reference Governor

1. INTRODUCTION

Most of the real control systems, such as chemical plants and power grids, can undergo variations in their dynamic characteristics owing to failures or equipment modifications; see Gertler (1988); Mei et al. (2011). When the control objectives cannot be achieved due to such changes, one of the solutions is to appropriately redesign the controllers of the plant. However, this redesign requires stopping the operations of the plant, and hence, this approach is not very practical in terms of operating costs. Therefore, operators need another approach to avoid the stoppage of operations while improving the control performance.

One way to achieve this objective is to shape an external reference signal such that the control performance is improved. This approach is called *reference shaping* in literature such as Boettcher et al. (2011); Suzuki and Sugie (2008). In Boettcher et al. (2011), a method has been proposed for designing an optimal reference signal by solving a convex optimization problem. In Suzuki and Sugie (2008), an offline shaping algorithm was proposed based on model predictive control theory in literature, e.g., Allgöwer and Zheng (2012). Instead of offline designs of reference signals, for shaping a reference to a desired one in real-time, the design of a *reference governor*, which is a dynamical compensator added at the top of the control system, has been proposed in literature Angeli et al. (2001); Bemporad (1998); Gilbert et al. (1995); Gilbert and Kolmanovsky (2002). These designs require full knowledge of the system dynamics. Thus, owing to the massive complexity of real control systems, such model-based approaches are not very practical.

Recently, data-driven control in which the controllers are redesigned using only operational data, without utilizing any system models, is receiving attention. Many studies on reinforcement learning (RL)-based control are emerging, e.g., Vrabie et al. (2009); Jiang and Jiang (2014). However,

these RL-based approaches are limited to state-feedback control. In contrast, in literature Campi et al. (2002); Campi and Savaresi (2006); Kaneko (2013), methods for redesigning an output-feedback controller using a single input-output data have been proposed. These approaches enable one-shot learning of a feedback controller. However, to the best of our knowledge, data-driven approaches of reference shaping have not yet been proposed.

In this paper, we propose a new data-driven method of reference shaping for improving the tracking performance of unknown linear systems. The proposed method comprises two steps. First, by solving an optimization problem formulated by the single input-output data, we hypothetically redesign an output-feedback controller that can improve the tracking performance. Note that this redesign is hypothetical; the designed controller is not implemented in the plant. Next, we shape the reference signal offline, such that the control input when the reference is applied coincides with the ideal input generated by the hypothetical controller. This proposed method can be conducted without knowing the system model, and the finite-time L_2 -norm of the tracking error, estimated using the data, can be minimized. Moreover, the proposed algorithm can be extended for cases where the control inputs are constrained owing to actuation limits. The proposed methods are expected to be useful for real, complex control systems that facilitate modifications of system components. The effectiveness of the proposed methods is investigated through a numerical simulation and an experiment conducted using a cart system.

Notation: Given a transfer function $G(s)$ we omit the term (s) when no confusion occurs. Given an input signal $u(t)$ and a system whose transfer function is G , we denote the output in time-domain by G_u . The set of proper and stable transfer functions is denoted by \mathcal{RH}_∞ . Given $\tau > 0$, the finite-time L_2 -norm of a signal $x(t)$ is defined as

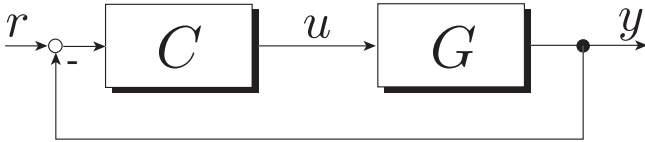


Fig. 1. The closed-loop system to be regulated

$$\|x(t)\|_{L_2, \tau} := \left(\int_0^\tau x^\top(t)x(t)dt \right)^{\frac{1}{2}}.$$

2. PROBLEM SETTING

We consider a closed-loop system as shown in Fig. 1, where G and C are assumed to satisfy the following assumptions.

Assumption 1. The system G is an unknown SISO linear system.

Assumption 2. The feedback controller C is known, and that stabilizes the closed-loop system in Fig. 1.

Assumption 3. The controller C is minimum-phase.

Assumption 1 implies that although we know that the system of our interest is a SISO LTI system we do not know its model. Assumption 2 is usually satisfied in many real control systems such as chemical plants and power grids. Assumption 3 simplifies the design though the requirement of the minimum-phase property can be removed. In addition, throughout this paper initial states of systems are assumed to be zero.

Given a reference signal r in Fig. 1 and a desired closed-loop transfer function $T_d \in \mathcal{RH}_\infty$, let the output y follow a desired trajectory $T_d r$, i.e., $y \approx T_d r$. Now, we suppose that the dynamics of G is drastically changed from the previous one owing to failures or equipment modifications. As a result, the output y no longer follows $T_d r$. One way to improve the tracking performance is to redesign C in Fig. 1 such that $y \approx T_d r$ is satisfied again. However, when control systems are in operation, this redesign requires stopping the operations of the plant, and hence, this approach is not very practical in terms of operating costs. Therefore, instead the redesign of C , we aim to shape the reference signal r in Fig. 1 so that y follows $T_d r$ as much as possible. This problem is formulated as follows.

Problem 1. Consider a closed-loop system in Fig. 1. Let Assumptions 1-3 be satisfied. Given $T_d \in \mathcal{RH}_\infty$ and r , find r^* such that

$$y^* := \frac{GC}{1+GC} r^* \quad (1)$$

is close to $T_d r$ as much as possible.

If the model of G was known, one can construct r^* by taking $r^* = T_d(GC/(1+GC))^{-1}r$. However, since G is completely unknown, this model-based approach no longer applies. Instead a data-driven approach needs to be developed, which is shown in the next section.

3. PROPOSED METHOD

We show a method to solve Problem 1 by using one pair of input-output data of r and y , denoted by $\{r_0, y_0\}$. Throughout this paper, we assume that r_0, y_0 and the desired output trajectory, denoted by y_d , are finite-time

and have an identical time length denoted by τ . Then, r_0, y_0 , and y_d satisfy

$$y_0 = \frac{GC}{1+GC} r_0, \quad y_d = T_d r_0 \quad (2)$$

where T_d is a desired closed-loop transfer function. The proposed method consists of the following two steps:

- i) Design a new feedback controller C^* *virtually* so that the closed-loop dynamics is close to T_d .
- ii) Shape the reference signal such that the actual control input coincides with the one generated by the virtual controller.

Note that the controller C^* is not implemented to the actual plant. We will use this as an intermediate component for the reference shaping. Before stating the detail of the step i), in the next subsection we show how the step ii) can be carried out.

3.1 Reference Shaping based on Virtual Controller

For our derivation, we hypothetically assume that the virtual controller design is done perfectly, i.e., we have C^* satisfying

$$T_d = \frac{GC^*}{1+GC^*}. \quad (3)$$

The control input generated by this C^* and the original controller C are defined as

$$u^* := \frac{C^*}{1+GC^*} r_0, \quad u_0 := \frac{C}{1+GC} r_0, \quad (4)$$

respectively. Hence, we have

$$u^* - u_0 = \frac{C^* - C}{(1+GC^*)(1+GC)} r_0. \quad (5)$$

On the other hand, it follows from (1) that

$$u^* = \frac{C}{1+GC} r^*. \quad (6)$$

The equation (6) implies that u^* , which is the control input generated by the virtual controller, coincides with the input when r^* is applied to the closed-loop system in Fig. 1. By substituting (6) and the second equation in (4) into (5), the relation (5) can be equivalently written as

$$r^* - r_0 = \frac{C^* - C}{(1+GC^*)(1+GC)} \frac{1+GC}{C} r_0 = \frac{C^* - C}{(1+GC^*)C} r_0. \quad (7)$$

By substituting (3) into this equation, we have

$$r^* = r_0 + (1 - T_d) \frac{C^* - C}{C} r_0. \quad (8)$$

In (8), the second term in the right-hand side represents the compensation signal determined by the virtual controller C^* . The following theorem straightforwardly follows the above discussion.

Theorem 1. Consider Problem 1 and let r^* be given by (8). Then, y^* in (1) satisfies $y^*(t) \equiv y_d(t)$ for $t \in [0, \tau]$.

Proof: The claim follows from the above discussion. ■

Theorem 1 implies that the shaped reference signal r^* in (8) is an ideal solution of Problem 1. It should be noted that the unknown system dynamics G does not appear in (8). Thus, we can compute r^* without knowing the system model.

3.2 Data-Driven Design of Virtual Controller

We show how to design the virtual controller C^* in (8) by using the measured data $\{r_0, y_0\}$. In the previous subsection, we have assumed that C^* satisfies (3). In reality, however, it may be very difficult to find C^* which satisfies this relation exactly. Thus, for designing C^* , we consider an evaluation function

$$J := \left\| y_d - \frac{GC}{1+GC} r^* \right\|_{L_2, \tau}. \quad (9)$$

From simple calculation by using (2), J can be written as

$$\begin{aligned} J &= \left\| y_d - \frac{GC}{1+GC} \left(1 + (1-T_d) \frac{C^* - C}{C} \right) r_0 \right\|_{L_2, \tau} \\ &= \left\| y_d - \left(T_d y_0 + (1-T_d) \frac{C^*}{C} y_0 \right) \right\|_{L_2, \tau}. \end{aligned} \quad (10)$$

It should be emphasized that all of y_d, T_d, C and y_0 in (10) are known quantities. Thus, we can carry out the minimization of J over C^* . To solve this minimization problem in a tractable manner, a parametrization of C^* described below would be effective. Let $\rho \in \mathbb{R}^n$ and C^* be parametrized as

$$C^*(s; \rho) = \sum_{k=1}^n \rho_k c_k(s) \quad (11)$$

where ρ_k is the k -th element of ρ and $c_k(s)$ is a given stable transfer function. Note that $c_k(s)$ can be non-proper because the signal shaping (8) can be performed in offline¹. Using the notations

$$q_0 := y_d - T_d y_0, \quad q_k := -(1-T_d) \frac{c_k}{C} y_0, \quad (12)$$

the equation (10) can be rewritten as

$$J = \left\| q_0 + \sum_{k=1}^n \rho_k q_k \right\|_{L_2, \tau}. \quad (13)$$

Clearly, the minimization problem $\min_{\rho} J$ can be easily solved by a standard least square method.

The proposed algorithm is summarized as follows.

Algorithm 1: Reference shaping

1. Collect input-output data $\{r_0, y_0\}$.
 2. Find C^* minimizing J in (10).
 3. Compute r^* in (8).
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Remark 1. From a different viewpoint, the evaluation function J in (10) is derived in the third author's recent paper; Ikezaki and Kaneko (2019).

3.3 Reference Governor Design

Algorithm 1 can be computationally heavy when r^* is very long (i.e. τ is long). An alternative way instead of this step is to implement a dynamical compensator so-called *reference governor* at the top of the control system,

¹ When $c_k(s)$ is non-proper, the computation of r^* in (8) is carried out for $t \in [0, \tau - \epsilon]$ where $\epsilon > 0$ is a given small value.



Fig. 2. The closed-loop system with a reference governor as shown in Fig. 2. From (8), one can expect that the reference governor designed as

$$R = (1-T_d) \frac{C^*}{C} + T_d \quad (14)$$

improves tracking performance. This can be shown as follows.

Let the transfer function from r to y in Fig. 2 be denoted as

$$T^* := R \frac{GC}{1+GC}. \quad (15)$$

Note that if C^* is stable, then T^* is stable under Assumptions 1-3. The stability of C^* can be easily guaranteed by parametrizing C^* with stable functions, as described in Section 3.2. In view of this, we assume that C^* is stable. Let C_d be a desired feedback controller satisfying

$$T_d = \frac{GC_d}{1+GC_d}. \quad (16)$$

Note that this ideal controller is, in general, different from C^* obtained by minimizing J in (10). Let

$$\Delta := C^* - C_d. \quad (17)$$

Using these notations, T^* in (15) can be written as

$$\begin{aligned} T^* &:= \frac{GC}{1+GC} \left(1 + (1-T_d) \frac{C_d + \Delta - C}{C} \right) \\ &= \frac{GC}{1+GC} T_d \left(1 + \frac{1}{GC_d} \frac{C_d + \Delta}{C} \right) \\ &= \frac{GC}{1+GC} T_d \left(1 + \frac{1}{GC} + \frac{1}{GCC_d} \Delta \right) \\ &= T_d + (1-T_d) S G^{-1} \Delta \end{aligned} \quad (18)$$

where $S := 1/(1+GC)$. From this error analysis, we can see that, as C^* is closer to C_d , the tracking performance of the entire control system with R in (14) is theoretically shown to improve. Later, we will demonstrate this through a numerical simulation.

4. EXTENSION TO INPUT-CONSTRAINED SYSTEMS

In this section, we propose a reference shaping for input-constrained systems. We consider a closed-loop system as shown in Fig. 1. Let Assumptions 1-3 be satisfied. Moreover, we assume that u in Fig. 1 must satisfy

$$\underline{u} \leq u(t) \leq \bar{u}, \quad \forall t \geq 0 \quad (19)$$

where \underline{u} and \bar{u} are given and known constant values. In these settings, we design r^* so that y^* in (1) is close to y_d in (2) as much as possible while satisfying the input constraint (19).

The control input when the updated reference signal r^* in (8) is applied can be written as

$$u^* = \left(T_d + (1-T_d) \frac{C^*}{C} \right) u_0. \quad (20)$$

Hence, the constraint (19) on u^* over a finite-time horizon $t \in [0, \tau]$, where τ is defined as the time length of the measured data, can be equivalently written as

$$\underline{u} \leq \left(T_d + (1 - T_d) \frac{C^*}{C} \right) u_0 \leq \bar{u}. \quad (21)$$

As long as $y^*(\tau)$ is close to $y_d(\tau)$, one can expect that $u^*(t)$ for $t > \tau$ satisfies the original input constraint (19). It should be emphasized that the constraint (21) is convex with respect to C^* . When the controller C^* is parametrized as (11), the constraint (21) can be equivalently written as a linear constraint

$$A\rho \leq b \quad (22)$$

where $A := [-1, 1]^T \otimes [a_1, \dots, a_n]$, $b := [\underline{b}^T, \bar{b}^T]^T$, \otimes is the Kronecker product, and

$$a_k := (1 - T_d) \frac{C^k}{C} u_0, \quad \underline{b} := -\underline{u} + T_d u_0, \quad \bar{b} := \bar{u} - T_d u_0.$$

The proposed reference shaping under input constraints can be summarized as follows.

Algorithm 2: Reference shaping under input constraints

1. Collect input-output data $\{r_0, y_0\}$.
 2. Find C^* minimizing J in (10) while satisfying (21).
 3. Compute r^* in (8).
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We end this section by shortly describing the reference governor design under the input constraint (19). Even though the control input is constrained, the reference governor R in Fig. 2 can be constructed as (14) as long as C^* is designed.

Remark 2. While the proposed method above can handle a more generic input constraint such as $\mathcal{T}u(t) \leq \nu(t)$ where \mathcal{T} and $\nu(t)$ are a known linear dynamical system and signal respectively, for simplicity, we have considered (19) as an input constraint to be imposed. Similarly, we can also consider an output constraint.

5. INVESTIGATION OF PROPOSED ALGORITHMS

5.1 Numerical Simulation

We show the effectiveness of the proposed methods through a numerical simulation. Consider a control system in Fig. 1 with

$$G = \frac{12s + 8}{20s^4 + 113s^3 + 147s^2 + 62s + 8}, \quad C = 3 + \frac{1}{2s}.$$

Let

$$T_d = \frac{27}{s^3 + 9s^2 + 27s + 27}$$

and the original reference signal r_0 be a step signal. In Fig. 3, the red dotted and blue solid lines show the actual output y_0 and the desired trajectory y_d in (2), respectively. Clearly, y_0 does not follow y_d .

To improve the tracking performance, we apply Algorithm 1 to the control system. At Step 2 in the algorithm, we parametrize C^* as a standard PID controller described as

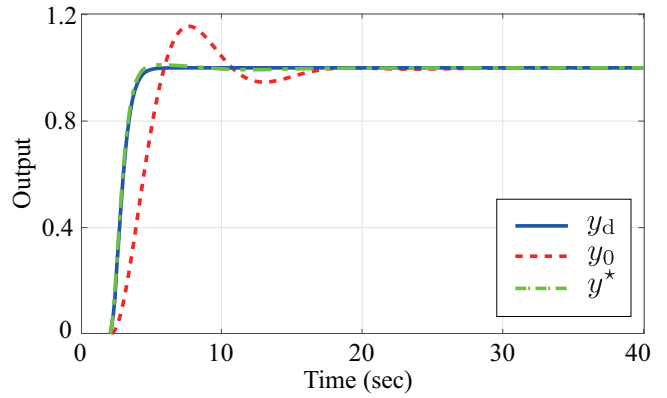


Fig. 3. Trajectory of desired output y_d , actual output y_0 in (2) when a step signal r_0 is applied, and y^* in (1) when a shaped reference signal r^* is applied, respectively.

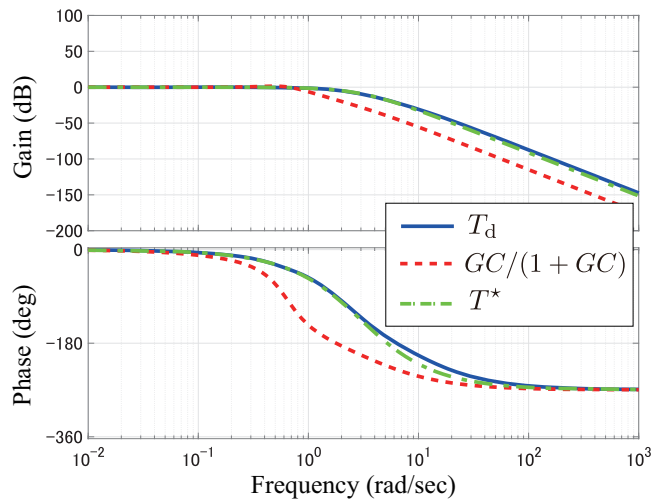


Fig. 4. Bode diagrams of T_d , $GC/(1+GC)$, and T^* in (15), respectively.

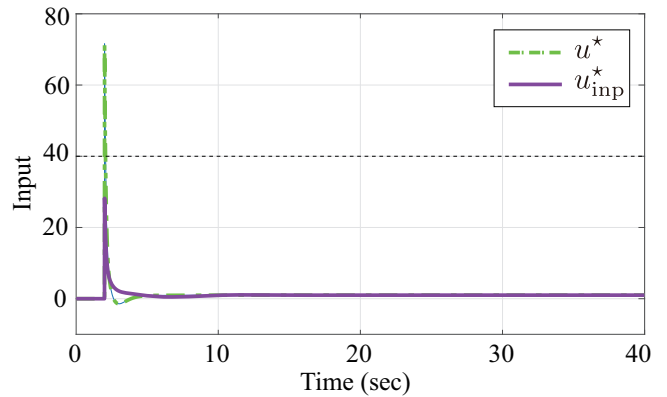


Fig. 5. The control input signals where u^* and u_{inp}^* are the cases when r^* and r_{inp}^* is applied to the system, respectively.

$$C^*(s; \rho) = \rho_1 + \frac{\rho_2}{s} + \rho_3 s. \quad (23)$$

Clearly, this controller can be written as (11) with $n = 3$. Next, we compute q_k in (12), and subsequently, we minimize J in (13) over $\rho \in \mathbb{R}^3$ by a standard least square method. Then $\rho^* = [6.1288, 0.9878, 6.5459]$. Using the virtual controller C^* with this optimized parameter,

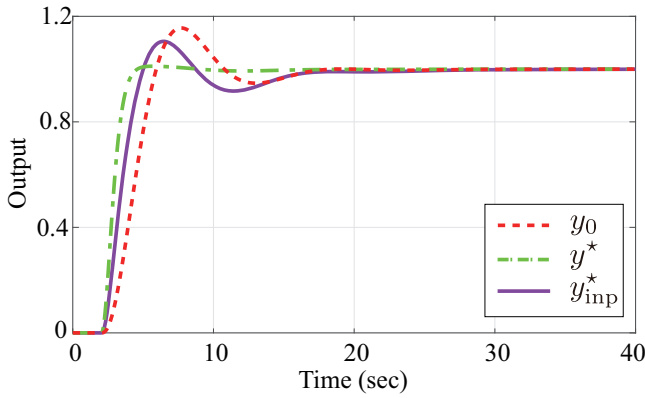


Fig. 6. The output trajectories where y^* and y_{inp}^* are the cases when r^* and r_{inp}^* is applied to the system, respectively.

we design r^* in (8). In Fig. 3, the green chained line shows the output when r^* is applied to the system. By comparing the lines in this figure, we can see that the tracking performance is improved by shaping the reference signal.

To compare the tracking performance in the frequency domain, we design a reference governor R in (14), where C^* is chosen as (23) with the aforementioned optimized parameter ρ^* . In Fig. 4, blue solid, red dotted, and green chained lines show the Bode diagrams of T_d , $GC/(1+GC)$, and T^* in (15), respectively. We can see that the addition of the reference governor R makes the whole control system dynamics close to the desired one.

In Fig. 5 the green chained line shows u^* in (6), which is the the control input when r^* is applied to the system in Fig. 1. The peak value of u^* is 71.6. One may need to suppress this peak to protect the actuator. To this end, we consider an input constraint (19) with $\underline{u} = -25$ and $\bar{u} = 25$. Since the virtual controller is parametrized as (23), this input constraint can be described as a linear constraint (21). Then the problem of minimizing J in (10) while satisfying (21) can be solved by the quadratic programming. We denote the obtained virtual controller and reference signal by C_{inp}^* and r_{inp}^* . Let u_{inp}^* and y_{inp}^* denote the control input and output when r_{inp}^* is applied to the system, respectively. The control input u_{inp}^* is depicted by the purple solid line in Fig. 5. Moreover, in Fig. 6 we show y_0 in (2), y^* , and y_{inp}^* by the red dotted, green chained, and purple solid lines, respectively. From these figures we can see that r_{inp}^* achieves a better tracking performance than r_0 while satisfying the input constraint.

5.2 Experimental Investigation

Next we investigate the effectiveness of Algorithm 2 through experiment. We use a cart control system as shown in Fig. 7. The cart on the belt can be moved by applying a voltage to the motor. The position of the cart can be measured by the potentiometer. Let u (V) and y (m) denote the applied voltage and the measured position, respectively. Let the sampling interval for both of the control and measurement be 0.01 (sec). A discrete-time controller, which is implemented to the motor driver, controls the motor by feeding back the measured position

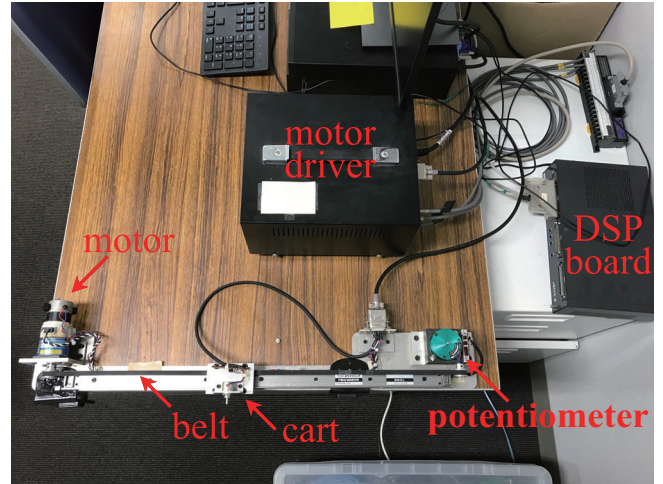


Fig. 7. The cart control system

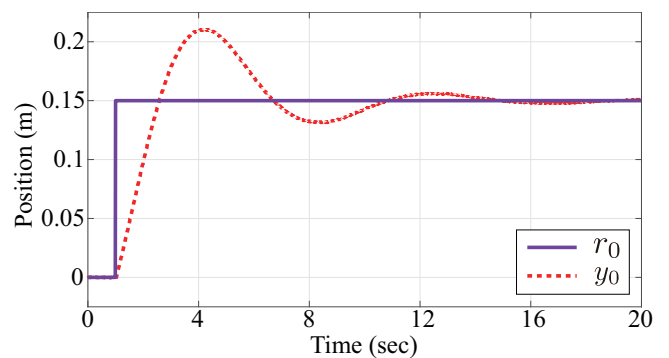


Fig. 8. Trajectories of the original reference signal r_0 and the output y_0 in (2), respectively.

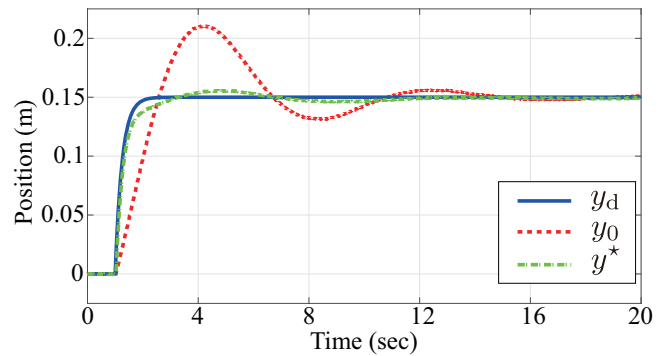


Fig. 9. Trajectories of the regulated output y^* in (1) and the desired output y_d in (2), respectively.

through the DSP board, so that the trajectory of the cart position follows a desired one. Let the controller be given as

$$C_{dis}(z; \rho) = \rho_1 + \rho_2 \frac{0.01}{z-1} + \rho_3 \frac{z-1}{0.01 + 10(z-1)}. \quad (24)$$

We regard the triple of this $C_{dis}(z; \rho)$, a sampler, and a zero-order holder as the continuous-time controller $C(s; \rho)$ in Fig. 1.

Let r_0 be a step signal so that the cart moves ahead by 0.15(m), and $\rho = [0.1, 0.1, 0.08]$ in (24). In Fig. 8, the purple solid and red dotted lines show the reference signal

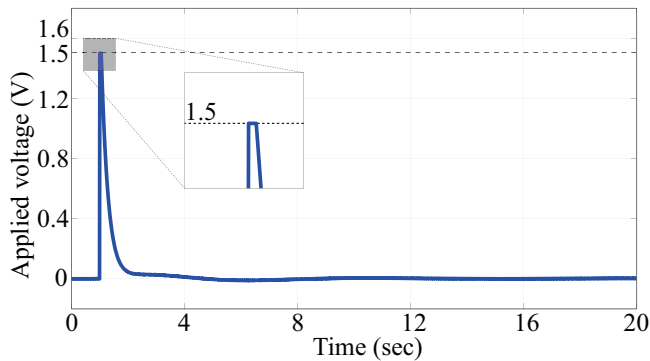


Fig. 10. Input signal generated by C^* .

r_0 and the measured output y_0 , respectively. This figure shows a poor tracking performance despite the position controller C is actuated.

To improve the performance we consider shaping the reference signal by using the data $\{r_0, y_0\}$. Let

$$T_d = \frac{4}{s + 4}. \quad (25)$$

To protect the motor, we need to make the magnitude of the applied voltage be less than 1.5(V), i.e., (19) with $\underline{u} = -1.5(\text{V})$ and $\bar{u} = 1.5(\text{V})$. We parametrize C^* as the form of (24). In these settings, by applying Algorithm 2 to the cart control system, we obtain an optimal parameter $\rho^* = [1.47, -0.0957, -9.57]$. Based on this, we design r^* in (8). In Fig. 9, the blue solid, red dotted, and green chained lines show the desired trajectory y_d , the initial output y_0 , and the shaped output y^* in (1), respectively. Furthermore, Fig. 10 shows the control input u when r^* is applied to the cart control system. These results imply that the proposed algorithm can be effective for real physical systems under input constraints.

6. CONCLUSION

In this paper, we have proposed a data-driven approach of reference shaping to improve tracking performance of uncertain linear systems. The proposed method can be implemented on a system based on its input-output data; the finite-time L_2 -norm of the tracking error, estimated by using the data, can be minimized. Moreover, the proposed algorithm can be extended for cases where the control inputs are constrained owing to actuation limits. The effectiveness of the proposed methods has been shown through a numerical simulation and an experiment of a cart system. The proposed methods are expected to be useful for real, complex control systems that facilitate modifications of system components. The experimental result implies that the proposed approaches are to what extent robust against noises. Extension to noisy cases is one of future works.

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REFERENCES

Allgöwer, F. and Zheng, A. (2012). *Nonlinear model predictive control*, volume 26. Birkhäuser.

- Angeli, D., Casavola, A., and Mosca, E. (2001). On feasible set-membership state estimators in constrained command governor control. *Automatica*, 37(1), 151–156.
- Bemporad, A. (1998). Reference governor for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 43(3), 415–419.
- Boettcher, U., Fetzner, D., Li, H., de Callafon, R.A., and Talke, F.E. (2011). Reference signal shaping for closed-loop systems with application to seeking in hard disk drives. *IEEE Transactions on Control Systems Technology*, 20(2), 335–345.
- Campi, M.C., Lecchini, A., and Savaresi, S.M. (2002). Virtual reference feedback tuning: a direct method for the design of feedback controllers. *Automatica*, 38(8), 1337–1346.
- Campi, M.C. and Savaresi, S.M. (2006). Direct nonlinear control design: the virtual reference feedback tuning (vrft) approach. *IEEE Transactions on Automatic Control*, 51(1), 14–27.
- Gertler, J.J. (1988). Survey of model-based failure detection and isolation in complex plants. *IEEE Control systems magazine*, 8(6), 3–11.
- Gilbert, E. and Kolmanovsky, I. (2002). Nonlinear tracking control in the presence of state and control constraints: a generalized reference governor. *Automatica*, 38(12), 2063–2073.
- Gilbert, E.G., Kolmanovsky, I., and Tan, K.T. (1995). Discrete-time reference governors and the nonlinear control of systems with state and control constraints. *International Journal of robust and nonlinear control*, 5(5), 487–504.
- Ikezaki, T. and Kaneko, O. (2019). A new approach of data-driven controller tuning method by using virtual internal model structure -virtual internal model tuning-. In *Proc. of IFAC Workshop on Adaptive and Learning Control Systems*. To appear.
- Jiang, Y. and Jiang, Z.P. (2014). Robust adaptive dynamic programming and feedback stabilization of nonlinear systems. *IEEE Transactions on Neural Networks and Learning Systems*, 25(5), 882–893.
- Kaneko, O. (2013). Data-driven controller tuning: Frit approach. *IFAC Proceedings Volumes*, 46(11), 326–336.
- Mei, S., Zhang, X., and Cao, M. (2011). *Power grid complexity*. Springer Science & Business Media.
- Suzuki, H. and Sugie, T. (2008). Off-line reference shaping of periodic trajectories for constrained systems with uncertainties. *IEEE Transactions on Automatic Control*, 53(6), 1531–1535.
- Vrabie, D., Pastravanu, O., Abu-Khalaf, M., and Lewis, F.L. (2009). Adaptive optimal control for continuous-time linear systems based on policy iteration. *Automatica*, 45(2), 477–484.