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Distributed Formation Tracking for Multiple Quadrotor Helicopters ⋆

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Abstract: In this paper, we study the leader-following formation tracking problem for multiple quadrotor helicopters over static and connected communication networks via the distributed observer approach. With the virtual leader system being modeled by a linear exosystem, we develop a distributed control law that can accomplish the formation tracking for a large class of leader's trajectories.

Keywords: Formation tracking, quadrotor helicopters, state feedback, distributed control.

1. INTRODUCTION

Recently, the formation control of multiple quadrotor helicopters is drawing attention due to its potential in various applications such as persistent surveillance (Nigam et al. (2012)), drag reduction (Ning (2011)), and contour mapping (Han and Chen (2014)). In particular, the formation tracking problem aims to design a control law such that all the vehicles can track a desired reference trajectory asymptotically while maintaining a prescribed formation pattern. With the attitude being represented by unit quaternion, the formation tracking problem was studied over undirected and connected static networks in Abdessameud and Tayebi (2010, 2011) by a control law without relying on the linear velocity measurements of each quadrotor helicopter. Also with the attitude being represented by unit quaternion, the formation tracking problem was further studied over directed and every-time connected switching networks by state feedback control in Kabiri et al. (2018) in which the quadrotor helicopters are subject to constant unknown disturbances. It is noted that, though all vehicles can asymptotically track a desired reference velocity in Abdessameud and Tayebi (2010, 2011); Kabiri et al. (2018), the steady states of the position of the vehicles cannot be specified. Moreover, since the velocity and the acceleration of the desired formation trajectory are assumed to be available to all the vehicles and are used explicitly in the control laws in Abdessameud and Tayebi (2010, 2011); Kabiri et al. (2018), the control protocols are not fully distributed.

The formation tracking problem is also studied under the leader-following framework in Du et al. (2019) over directed and connected static networks, where the desired formation trajectory is generated by a virtual leader and all quadrotor helicopters are viewed as followers. However, the result of Du et al. (2019) has two limitations. First, the virtual leader in Du et al. (2019) can only produce a constant velocity. Second, since the Euler angles instead of unit quaternion are used to represent the attitude in Du et al. (2019), the issue of singularity remains.

In this paper, we study the leader-following formation tracking problem for multiple quadrotor helicopters over directed and connected static communication networks via the distributed observer approach (Su and Huang (2012)). Since our virtual leader is modeled by a linear exosystem, it can generate a large class of time signals including step functions of any magnitudes, quadratic functions of any coefficients, sinusoidal functions of any amplitudes, frequencies, and initial phases, and their combinations. Like in Abdessameud and Tayebi (2010, 2011); Kabiri et al. (2018), we also represent the attitude of the quadrotor helicopter by unit quaternion. But unlike in Abdessameud and Tayebi (2010, 2011); Kabiri et al. (2018), this paper offers at least two new features. First, since our communication network can be directed, the approaches in Abdessameud and Tayebi (2010, 2011) cannot handle our problem with directed graphs. Second, due to the employment of the distributed observer, our control law is distributed.

Throughout this paper, we adopt the following notation: $\|x\|$ denotes the Euclidean norm of vector x and $\|A\|$ denotes the induced norm of matrix A by the Euclidean norm. For a piecewise continuous bounded function $f : [0,\infty) \mapsto \mathbb{R}^n$, $\|f\|_\infty = \sup_{t\geq 0}\|f(t)\|$. $e_3 = [0,0,1]^T$. For $x_i \in \mathbb{R}^{n_i}$, $i = 1,\ldots,m$, $\mathrm{col}(x_1,\ldots,x_m) = [x_1^T,\ldots,x_m^T]^T$. $\mathrm{diag}(a_1,\ldots,a_n)$ denotes the diagonal matrix with $a_i \in \mathbb{R}$, $i = 1,\ldots,n$, on the diagonal. block diag $(A_1,\ldots,A_n)$ denotes the block diagonal matrix with A_i, $i = 1,\ldots,n$, on the diagonal. For $x = \mathrm{col}(x_1,x_2,x_3) \in \mathbb{R}^3$, $x^\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$. $\mathbb{Q}$ denotes the set of all quaternions, $\mathbb{Q} = \{q|q = \mathrm{col}(\hat{q},\bar{q}), \hat{q} \in \mathbb{R}^3, \bar{q} \in \mathbb{R}\}$. $\mathbb{Q}_u$ denotes the set of all unit quaternions, $\mathbb{Q}_u = \{q|q \in \mathbb{Q}, \|q\| = 1\}$. $SO(3)$ denotes the special orthogonal group of order three, $SO(3) = \{R \in \mathbb{R}^{3\times 3}|R^TR = RR^T = I_3, \mathrm{det}R = 1\}$. For q_i, $q_j \in \mathbb{Q}$, the

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quaternion product is defined by $q_i \odot q_j = \begin{bmatrix} \bar{q}_i\hat{q}_j + \bar{q}_j\hat{q}_i + \hat{q}_i^\times \hat{q}_j \\ \bar{q}_i\bar{q}_j - \hat{q}_i^T \hat{q}_j \end{bmatrix}$. For $q \in \mathbb{Q}_u$, the quaternion inverse is defined by $q^{-1} = \mathrm{col}(-\hat{q}, \bar{q})$. For $x \in \mathbb{R}^3$, $\mathbf{Q}(x) = \mathrm{col}(x, 0) \in \mathbb{R}^4$. For $q \in \mathbb{Q}_u$, $\mathcal{R}(q) = I_3 + 2\bar{q}\hat{q}^\times + 2(\hat{q}^\times)^2 \in SO(3)$. For two sets A and B, $B \setminus A = \{x \in B | x \notin A\}$.

2. PRELIMINARIES AND PROBLEM FORMULATION

A quadrotor helicopter is a nonlinear underactuated system. It has six degrees of freedom but only four control inputs, namely, T_i and $\tau_i = \mathrm{col}(\tau_x^i, \tau_y^i, \tau_z^i)$. The coordinate systems and the free body diagram of the ith quadrotor helicopter are shown in Fig. 1, where M_j^i and f_j^i denote the rotor torque and the rotor thrust generated by the jth rotor of the ith quadrotor helicopter, respectively. The inertial frame $\mathcal{I}$ is defined by $\mathcal{I} = \{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3\}$, where $\mathcal{I}_3$ is pointing downward vertically. The body-fixed frame $\mathcal{B}_i = \{\mathcal{B}_1^i, \mathcal{B}_2^i, \mathcal{B}_3^i\}$ is attached to the center of mass of the ith quadrotor helicopter, with $\mathcal{B}_1^i$ pointing towards the prescribed forward direction and $\mathcal{B}_3^i$ pointing downward. Rotors 1 to 4 are on the positive $\mathcal{B}_1^i$ axis, the negative $\mathcal{B}_2^i$ axis, the negative $\mathcal{B}_1^i$ axis, and the positive $\mathcal{B}_2^i$ axis, respectively.

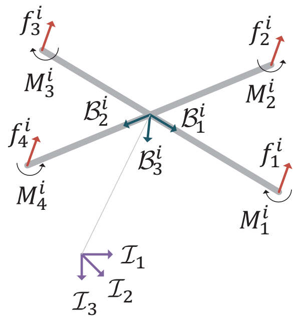

Fig. 1. Free body diagram.

Consider the quadrotor helicopters described by the equations of motion as follows:

$$m_i \ddot{p}_i = m_i g e_3 - T_i \mathcal{R}(q_i) e_3 \tag{1a}$$

$$\dot{q}_i = \frac{1}{2} q_i \odot \mathbf{Q}(\Omega_i) \tag{1b}$$

$$J_i \dot{\Omega}_i = -\Omega_i^\times J_i \Omega_i + \tau_i, \ i = 1, \ldots, N \tag{1c}$$

where, for the ith quadrotor helicopter, $i = 1, \ldots, N$, $m_i \in \mathbb{R}$ is the mass, $g \in \mathbb{R}$ is the gravitational acceleration, $p_i = \mathrm{col}(p_x^i, p_y^i, p_z^i) \in \mathbb{R}^3$ is the position vector of the center of mass with respect to $\mathcal{I}$, $q_i = \mathrm{col}(\hat{q}_i, \bar{q}_i) \in \mathbb{Q}_u$ is the unit quaternion representation of the attitude of $\mathcal{B}_i$ relative to $\mathcal{I}$, $J_i \in \mathbb{R}^{3\times 3}$ is the symmetric and positive definite inertia matrix expressed in $\mathcal{B}_i$, $\Omega_i \in \mathbb{R}^3$ is the angular velocity of $\mathcal{B}_i$ relative to $\mathcal{I}$ expressed in $\mathcal{B}_i$, $\tau_i = \mathrm{col}(\tau_x^i, \tau_y^i, \tau_z^i) \in \mathbb{R}^3$ is the control torque vector, and $T_i \in \mathbb{R}$ is the total thrust.

Given $N > 0$, a desired formation is specified by the trajectory $p_0(t) \in \mathbb{R}^3$ of the virtual leader, and N other vectors $h_i(t) \in \mathbb{R}^3$, $i = 1, \ldots, N$, representing the desired relative displacements between the ith quadrotor helicopter and the trajectory of the virtual leader. Thus, let

$$p_{ri} \triangleq h_i + p_0, \ i = 1, \ldots, N. \tag{2}$$

Then, p_{ri} is the desired trajectory of the ith quadrotor helicopter. The N quadrotor helicopters (1a)–(1c) are said to achieve asymptotic formation tracking, if the trajectory of each quadrotor helicopter exists for all $t \geq 0$, and is such that

$$\lim_{t\to\infty} (p_i(t) - p_{ri}(t)) = 0 \tag{3}$$

$$\lim_{t\to\infty} (\dot{p}_i(t) - \dot{p}_{ri}(t)) = 0, \ i = 1, \ldots, N. \tag{4}$$

Fig. 2 shows a triangular formation composed of three quadrotor helicopters.

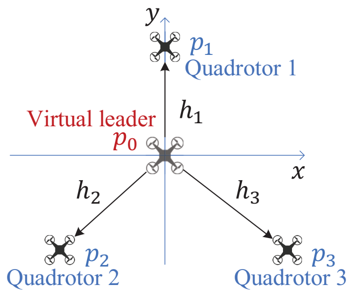

Fig. 2. A triangular formation.

The formation tracking problem described above has been studied in several papers. In particular, under the assumption that $p_{ri}(t)$, $i = 1, \ldots, N$, are sufficiently smooth with bounded second to fourth derivatives, using the approach in Abdessameud and Tayebi (2013), the above problem can be solved by a state feedback control law of the following form:

$$\begin{aligned} T_i &= \alpha_i(p_i, \dot{p}_i, p_{ri}, \dot{p}_{ri}, \ddot{p}_{ri}) \\ \tau_i &= \beta_i(p_i, \dot{p}_i, q_i, \Omega_i, p_{ri}, \dot{p}_{ri}, \ddot{p}_{ri}, p_{ri}^{(3)}, p_{ri}^{(4)}) \end{aligned} \tag{5}$$

where, for $i = 1, \ldots, N$, the specific forms of the functions α_i and β_i can be found in Abdessameud and Tayebi (2013).

A control law of the form (5) is called a purely decentralized full information control law since, for each i, $i = 1, \ldots, N$, α_i and β_i not only depend on the state of the ith quadrotor helicopter but also the reference trajectory p_{ri} as well as its first to fourth derivatives.

In practice, due to the communication constraints, the control of some subsystems of (1a)–(1c) may not access the information of the reference trajectory. To overcome this difficulty, it is desirable to design a so-called distributed control law to solve the above formation tracking problem. For this purpose, denote the three components of $p_0(t)$ and $h_i(t)$, $i = 1, \ldots, N$, by $p_0 \triangleq \mathrm{col}(p_x^0, p_y^0, p_z^0)$ and $h_i \triangleq \mathrm{col}(h_x^i, h_y^i, h_z^i)$, $i = 1, \ldots, N$. Then we need to limit the $(N+1)$ functions $p_0(t)$ and $h_i(t)$, $i = 1, \ldots, N$, such that they can be generated by the following exosystems:

$$\dot{v}_x = S_x v_x, \ \ p_x^0 = F_x^0 v_x, \ \ h_x^i = G_x^i v_x \tag{6}$$

$$\dot{v}_y = S_y v_y, \ \ p_y^0 = F_y^0 v_y, \ \ h_y^i = G_y^i v_y \tag{7}$$

$$\dot{v}_z = S_z v_z, \ \ p_z^0 = F_z^0 v_z, \ \ h_z^i = G_z^i v_z \tag{8}$$

where, for $k = x, y, z$, $v_k \in \mathbb{R}^n$, and $S_k \in \mathbb{R}^{n\times n}$, $F_k^0 \in \mathbb{R}^{1\times n}$, and $G_k^i \in \mathbb{R}^{1\times n}$ are constant matrices. Also denote the three components of p_{ri} by $p_{ri} \triangleq \mathrm{col}(p_x^{ri}, p_y^{ri}, p_z^{ri})$, and let $F_k^i \triangleq G_k^i + F_k^0$, $k = x, y, z$. Then, for $i = 1, \ldots, N$, $k = x, y, z$, $p_k^{ri} = F_k^i v_k$.

We assume the exosystems (6)–(8) satisfy the following:

Assumption 2.1. None of the eigenvalues of the matrices S_x and S_y have positive real parts. All the non-zero imaginary eigenvalues of the matrices S_x and S_y are semi-simple, and all the eigenvalues of the matrix S_z are at the origin. Moreover, the dimensions of the Jordan blocks corresponding to the eigenvalue at the origin of the matrices S_x and S_y are not greater than three, and the dimensions of the Jordan blocks corresponding to the eigenvalue at the origin of the matrix S_z are not greater than two.

Remark 2.1. Under Assumption 2.1, $p_z^{ri}(t)$ can be step and ramp functions of any coefficients while $p_x^{ri}(t)$ and $p_y^{ri}(t)$ can be quadratic functions of any coefficients, sinusoidal functions of any magnitudes, frequencies, and initial phases, and their combinations. Thus, for $i = 1, \ldots, N$, the reference accelerations $\ddot{p}_{ri} = \mathrm{col}(\ddot{p}_x^{ri}, \ddot{p}_y^{ri}, \ddot{p}_z^{ri})$ are bounded and $\ddot{p}_z^{ri}(t) = 0$ for all $t \geq 0$.

Let $v \triangleq \mathrm{col}(v_x, v_y, v_z)$, $S \triangleq$ block diag(S_x, S_y, S_z), $F_i \triangleq$ block diag(F_x^i, F_y^i, F_z^i), $i = 0, 1, \ldots, N$. Then, the exosystems (6)–(8) as well as the reference trajectories can be compactly put as follows:

$$\dot{v} = Sv, \quad p_0 = F_0 v, \quad p_{ri} = F_i v, \ i = 1, \ldots, N. \qquad (9)$$

We can view the system composed of (1a)–(1c) and (9) as a multi-agent system of $(N+1)$ agents with (9) as the virtual leader and the N subsystems of (1a)–(1c) as followers. Given systems (1a)–(1c) and (9), we can define a digraph [1] $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \{0, 1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Here, node 0 is associated with the virtual leader (9) and node $i, i = 1, \ldots, N$, is associated with the ith subsystem of (1a)–(1c), and, for $i = 1, \ldots, N$, $j = 0, 1, \ldots, N$, $i \neq j$, $(j, i) \in \mathcal{E}$ if and only if agent i can use the information of agent j for control.

We now describe our control law as follows:

$$\begin{aligned} T_i &= f_i(\vartheta_i, \xi_i) \\ \tau_i &= g_i(\vartheta_i, \xi_i, \xi_j, j \in \mathcal{N}_i) \\ \dot{\xi}_i &= k_i(\xi_i, \xi_j, j \in \mathcal{N}_i), \ i = 1, \ldots, N \end{aligned} \qquad (10)$$

where, for $i = 1, \ldots, N$, $f_i(\cdot)$, $g_i(\cdot)$, $k_i(\cdot)$ are some nonlinear functions, $\vartheta_i = \mathrm{col}(p_i, \dot{p}_i, q_i, \Omega_i)$ is the state of the ith quadrotor helicopter, ξ_i is the state of the distributed dynamic compensator, $\xi_0 = v$, and $\mathcal{N}_i$ denotes the neighbor set of the ith quadrotor helicopter.

Since the control law (10) satisfies the communication constraints described by the digraph $\mathcal{G}$, we call such a control law a distributed control law.

We now describe our problem as follows:

Problem 1. Given the plant (1a)–(1c), the virtual leader (9), and a digraph $\mathcal{G}$, find a control law of the form (10) such that, for any initial condition of the closed-loop system satisfying $\|q_i(0)\| = 1$, $i = 1, \ldots, N$, the solution of the closed-loop system exists for all $t \geq 0$ and satisfies (3) and (4). Moreover, $T_i(t)$ and $\tau_i(t)$, $i = 1, \ldots, N$, are bounded over $t \in [0, \infty)$.

For the solvability of Problem 1, we make one more assumption as follows:

[1] See Su and Huang (2012) for a summary of digraph.

Assumption 2.2. The digraph $\mathcal{G}$ contains a directed spanning tree with node 0 as the root, that is, every node i, $i = 1, \ldots, N$, is reachable from node 0 in the digraph $\mathcal{G}$.

Remark 2.2. Under Assumption 2.2, the digraph $\mathcal{G}$ is said to be connected.

3. MAIN RESULTS

3.1 Existing Results

Our approach is a composition of the purely decentralized full information control law developed in Abdessameud and Tayebi (2013) and a so-called distributed observer. Let us summarized some results in Abdessameud and Tayebi (2013) in this subsection.

For $x = \mathrm{col}(x_1, x_2, x_3) \in \mathbb{R}^3$, let

$$\chi(x) \triangleq \mathrm{col}(\tanh(x_1), \tanh(x_2), \tanh(x_3))$$
$$h(x) \triangleq \mathrm{diag}\left(\frac{d\tanh(x_1)}{dx_1}, \frac{d\tanh(x_2)}{dx_2}, \frac{d\tanh(x_3)}{dx_3}\right)$$

and let $\dot{h}(\cdot)$ be the time derivative of $h(\cdot)$.

Remark 3.1. It can be verified that $\tanh : \mathbb{R} \mapsto (-1, 1)$ is a strictly increasing and continuously differentiable function with the following properties:

(1) $\tanh(0) = 0$ and $x \tanh(x) > 0$ for all $x \neq 0$.
(2) $|\tanh(x)| < 1$ for all $x \in \mathbb{R}$.
(3) $\frac{d\tanh(x)}{dx} = 1 - \tanh^2(x)$ is bounded for all $x \in \mathbb{R}$.

Thus, for any $x \in \mathbb{R}^3$, $\chi(x)$ and $h(x)$ are bounded.

Let us first rephrase Lemma 2.9 of Abdessameud and Tayebi (2013) as follows.

Lemma 3.1. Consider the following second-order nonlinear system:

$$\ddot{\theta} = -k_p \chi(\theta) - k_d \chi(\dot{\theta}) + \zeta(t) \qquad (11)$$

where $\theta \in \mathbb{R}^3$, k_p and k_d are positive constants. If $\zeta(t) \in \mathbb{R}^3$ is piecewise continuous and bounded over $t \in [0, \infty)$, and $\lim_{t\to\infty} \zeta(t) = 0$, then, for any initial condition $\theta(0)$, $\dot{\theta}(0)$, the solution θ, $\dot{\theta}$ of system (11) are bounded and satisfy $\lim_{t\to\infty} \theta(t) = 0$, $\lim_{t\to\infty} \dot{\theta}(t) = 0$. □

For $i = 1, \ldots, N$, let $q_{ri}(t) \in \mathbb{Q}_u$ be any sufficiently smooth time functions defined over $t \in [0, \infty)$ called the reference attitudes. Then, we can put the translational dynamics (1a) to the form

$$\ddot{p}_i = \frac{T_i}{m_i}\left(\mathcal{R}(q_{ri}) - \mathcal{R}(q_i)\right) e_3 - \frac{T_i}{m_i} \mathcal{R}(q_{ri}) e_3 + g e_3. \qquad (12)$$

Motivated by Cao and Lynch (2016), performing on (12) the following transformation:

$$u_i = -\frac{T_i}{m_i} \mathcal{R}(q_{ri}) e_3 + g e_3, \ i = 1, \ldots, N \qquad (13)$$

gives

$$\ddot{p}_i = u_i + \frac{T_i}{m_i} (\mathcal{R}(q_{ri}) - \mathcal{R}(q_i)) e_3, \ i = 1, \ldots, N. \qquad (14)$$

For $i = 1, \ldots, N$, let $\bar{p}_i \triangleq p_i - p_{ri}$ and

$$u_i = \ddot{p}_{ri} - k_p \chi(\bar{p}_i) - k_d \chi(\dot{\bar{p}}_i) \qquad (15)$$

where k_p and k_d are any positive constants. Then, (14) becomes the following

$$\ddot{\bar{p}}_i = -k_p\chi(\bar{p}_i) - k_d\chi(\dot{\bar{p}}_i) + \frac{T_i}{m_i}(\mathcal{R}(q_{ri}) - \mathcal{R}(q_i))e_3. \quad (16)$$

Thus, by Lemma 3.1, if, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} \frac{T_i(t)}{m_i}(\mathcal{R}(q_{ri}(t)) - \mathcal{R}(q_i(t)))e_3 = 0, \quad (17)$$

then, for $i = 1, \ldots, N$, we have $\lim_{t\to\infty} \bar{p}_i(t) = 0$ and $\lim_{t\to\infty} \dot{\bar{p}}_i(t) = 0$.

Nevertheless, the true control is T_i not u_i. In order to obtain T_i from u_i through (13), as noted in Abdessameud and Tayebi (2011), for any $u_i \in \mathbb{R}^3$, $i = 1, \ldots, N$, (13) is satisfied if T_i and q_{ri}, $i = 1, \ldots, N$, are such that

$$T_i = m_i\|ge_3 - u_i\| \quad (18)$$

and

$$T_i\mathcal{R}(q_{ri})e_3 = m_ige_3 - m_iu_i. \quad (19)$$

Moreover, as a special case of Lemma 4 of Abdessameud and Tayebi (2011), the following lemma presents a method to compute a $q_{ri}(t)$ that satisfies the constraint (19).

Lemma 3.2. Let $u_i \triangleq \mathrm{col}(u_x^i, u_y^i, u_z^i)$, $i = 1, \ldots, N$. Suppose T_i, $i = 1, \ldots, N$, is given by (18). If $\|u_z^i\|_\infty < g$, then a $q_{ri} = \mathrm{col}(\hat{q}_{ri}, \bar{q}_{ri})$ satisfying (19) is given by

$$\bar{q}_{ri} = \sqrt{\frac{1}{2} + \frac{m_i(g - u_z^i)}{2T_i}}, \quad \hat{q}_{ri} = \frac{m_i}{2T_i\bar{q}_{ri}}\begin{bmatrix} u_y^i \\ -u_x^i \\ 0 \end{bmatrix}. \quad (20)$$

$\square$

Remark 3.2. Suppose u_i is differentiable and q_{ri} is as described in Lemma 3.2. Then, q_{ri} satisfies the following equation (Abdessameud and Tayebi (2011)):

$$\dot{q}_{ri} = \frac{1}{2}q_{ri} \odot \mathbf{Q}(\Omega_{ri}) \quad (21)$$

where

$$\Omega_{ri} = \Xi(u_i)\dot{u}_i \quad (22)$$

and

$$\Xi(u_i) = \frac{1}{\gamma_{1i}^2\gamma_{2i}}\begin{bmatrix} -u_x^iu_y^i & -{u_y^i}^2 + \gamma_{1i}\gamma_{2i} & u_y^i\gamma_{2i} \\ {u_x^i}^2 - \gamma_{1i}\gamma_{2i} & u_x^iu_y^i & -u_x^i\gamma_{2i} \\ u_y^i\gamma_{1i} & -u_x^i\gamma_{1i} & 0 \end{bmatrix}$$

with $\gamma_{1i} = \frac{T_i}{m_i}$, $\gamma_{2i} = \gamma_{1i} + g - u_z^i$. Differentiating Ω_{ri} gives

$$\dot{\Omega}_{ri} = \bar{\Xi}(u_i, \dot{u}_i)\dot{u}_i + \Xi(u_i)\ddot{u}_i \quad (23)$$

where $\bar{\Xi}(u_i, \dot{u}_i)$ is the time derivative of $\Xi(u_i)$. In the sequel, for $i = 1, \ldots, N$, we call Ω_{ri} and $\dot{\Omega}_{ri}$ the reference angular velocities and the reference angular accelerations, respectively.

Remark 3.3. Similar to Luo et al. (2005); Chen and Huang (2009), given the reference attitudes $q_{ri} \in \mathbb{Q}_u$ and the reference angular velocities $\Omega_{ri} \in \mathbb{R}^3$, $i = 1, \ldots, N$, let

$$\epsilon_i \triangleq q_{ri}^{-1} \odot q_i \quad (24)$$

$$\tilde{\Omega}_i \triangleq \Omega_i - \mathcal{R}^T(\epsilon_i)\Omega_{ri}, \; i = 1, \ldots, N. \quad (25)$$

Then, for $i = 1, \ldots, N$, $\epsilon_i = \mathrm{col}(\hat{\epsilon}_i, \bar{\epsilon}_i) \in \mathbb{Q}_u$ are called the attitude tracking errors, and $\tilde{\Omega}_i \in \mathbb{R}^3$ are the relative angular velocities between Ω_i and Ω_{ri} expressed in $\mathcal{B}_i$. Since, for $i = 1, \ldots, N$, $\mathcal{R}(q_i) \in SO(3)$ is bounded and $\mathcal{R}(q_{ri}) - \mathcal{R}(q_i) = \mathcal{R}(q_i)(\mathcal{R}^T(\epsilon_i) - I_3)$, if $\lim_{t\to\infty} \hat{\epsilon}_i(t) = 0$, then $\lim_{t\to\infty} \mathcal{R}(\epsilon_i(t)) = I_3$ and $\lim_{t\to\infty}(\mathcal{R}(q_{ri}(t)) - \mathcal{R}(q_i(t))) = 0$.

3.2 Distributed Observer

We recall the distributed observer for the virtual leader (9) from Su and Huang (2012) as follows:

$$\dot{\eta}_i = S\eta_i + \mu\sum_{j=0}^{N} a_{ij}(\eta_j - \eta_i), \; i = 1, \ldots, N \quad (26)$$

where, for $i = 1, \ldots, N$, $\eta_i \in \mathbb{R}^{3n}$, $\eta_0 = v$, a_{ij} is the element of the weighted adjacency matrix of $\mathcal{G}$, and μ is any positive constant.

The following lemma is extracted from Remark 4 of Su and Huang (2012).

Lemma 3.3. Under Assumptions 2.1 and 2.2, for any $\mu > 0$ and any initial condition $\eta_i(0)$, $i = 1, \ldots, N$, and $v(0)$, we have

$$\lim_{t\to\infty}(\eta_i(t) - v(t)) = 0, \; i = 1, \ldots, N \quad (27)$$

exponentially. $\square$

Remark 3.4. For $i = 1, \ldots, N$, let $\eta_{di} \triangleq \mu\sum_{j=0}^{N} a_{ij}(\eta_j - \eta_i)$. From (27), we have $\lim_{t\to\infty} \eta_{di}(t) = 0$ exponentially, which together with (26) imply that $\lim_{t\to\infty}(\dot{\eta}_i(t) - \dot{v}(t)) = \lim_{t\to\infty}(S\eta_i(t) + \eta_{di}(t) - Sv(t)) = 0$ exponentially.

The control law (15) relies on p_{ri} as well as its first and second derivatives, and thus relies on v as well as its first and second derivatives. In order to obtain a distributed control law, for each i, $i = 1, \ldots, N$, in (15), we can replace v by η_i and $\dot{v}$ by $\dot{\eta}_i$. Nevertheless, from (26), $\ddot{\eta}_i$ depends on $\dot{\eta}_j$, $j \in \mathcal{N}_i \setminus \{0\}$, which is available to the ith quadrotor helicopter only if the neighbor sets satisfy $\mathcal{N}_j \subseteq \mathcal{N}_i$ for all $j \in \mathcal{N}_i \setminus \{0\}$. Such a condition does not hold in general. Therefore, we cannot replace $\ddot{v}$ by $\ddot{\eta}_i$. To circumvent this difficulty, we introduce another distributed dynamic compensator in the following lemma.

Lemma 3.4. Under Assumptions 2.1 and 2.2, consider a distributed dynamic compensator as follows:

$$\dot{\hat{\eta}}_i = S\hat{\eta}_i + L(\hat{\eta}_i - \eta_i), \; i = 1, \ldots, N \quad (28)$$

where, for $i = 1, \ldots, N$, $\hat{\eta}_i \in \mathbb{R}^{3n}$ and $L \in \mathbb{R}^{3n\times 3n}$ is such that $(S + L)$ is Hurwitz. Then, for any initial condition $\hat{\eta}_i(0)$, $\eta_i(0)$, $i = 1, \ldots, N$, and $v(0)$, we have

$$\lim_{t\to\infty}(\hat{\eta}_i(t) - v(t)) = 0 \quad (29)$$

$$\lim_{t\to\infty}(\dot{\hat{\eta}}_i(t) - \dot{v}(t)) = 0 \quad (30)$$

$$\lim_{t\to\infty}(\ddot{\hat{\eta}}_i(t) - \ddot{v}(t)) = 0, \; i = 1, \ldots, N \quad (31)$$

all exponentially.

Proof: For $i = 1, \ldots, N$, let $\tilde{\eta}_i \triangleq \hat{\eta}_i - \eta_i$. Then, from (26) and (28), we have

$$\dot{\tilde{\eta}}_i = (S + L)\tilde{\eta}_i - \eta_{di}, \; i = 1, \ldots, N. \quad (32)$$

Under Assumptions 2.1 and 2.2, by Remark 3.4, for $i = 1, \ldots, N$, we have $\lim_{t\to\infty} \eta_{di}(t) = 0$ exponentially. Since $(S+L)$ is Hurwitz, we have $\lim_{t\to\infty} \tilde{\eta}_i(t) = 0$ exponentially and $\lim_{t\to\infty} \dot{\tilde{\eta}}_i(t) = 0$ exponentially.

Under Assumptions 2.1 and 2.2, for $i = 1, \ldots, N$, it follows from Lemma 3.3 that $\lim_{t\to\infty}(\hat{\eta}_i(t) - v(t)) = \lim_{t\to\infty}(\tilde{\eta}_i(t) + \eta_i(t) - v(t)) = 0$ exponentially. By Remark 3.4, we have $\lim_{t\to\infty}(\dot{\hat{\eta}}_i(t) - \dot{v}(t)) = \lim_{t\to\infty}(\dot{\tilde{\eta}}_i(t) + \dot{\eta}_i(t) - \dot{v}(t)) = 0$ exponentially.

To show (31), differentiating on both sides of (28) gives

$$\begin{aligned}\ddot{\hat{\eta}}_i &= S\dot{\hat{\eta}}_i + L(\dot{\hat{\eta}}_i - \dot{\eta}_i) \\ &= S^2\hat{\eta}_i + SL\tilde{\eta}_i + L\dot{\tilde{\eta}}_i,\ i = 1,\dots,N. \end{aligned} \tag{33}$$

Thus, for $i = 1,\dots,N$, we have $\lim_{t\to\infty}(\ddot{\hat{\eta}}_i(t) - \ddot{v}(t)) = \lim_{t\to\infty} S^2(\hat{\eta}_i(t) - v(t)) = 0$ exponentially. □

Remark 3.5. Differentiating on both sides of (33) gives $\hat{\eta}_i^{(3)} = S^2\dot{\hat{\eta}}_i + SL\dot{\tilde{\eta}}_i + L\ddot{\tilde{\eta}}_i$. Thus Lemma 3.4 implies that, for $i = 1,\dots,N$, $\lim_{t\to\infty}(\hat{\eta}_i^{(3)}(t) - v^{(3)}(t)) = \lim_{t\to\infty} S^2(\dot{\hat{\eta}}_i(t) - \dot{v}(t)) = 0$ exponentially.

3.3 Distributed Position Control

To define our distributed position control law, let $\hat{\eta}_i = \mathrm{col}(\hat{\eta}_x^i, \hat{\eta}_y^i, \hat{\eta}_z^i)$ be a partition of $\hat{\eta}_i$ with $\hat{\eta}_k^i \in \mathbb{R}^n$, $k = x, y, z$. For $i = 1,\dots,N$, let

$$\hat{p}_{ri} \triangleq \mathrm{col}(\hat{p}_x^{ri}, \hat{p}_y^{ri}, \hat{p}_z^{ri}),\quad \varphi_i \triangleq \mathrm{col}(\varphi_x^i, \varphi_y^i, \bar{\varphi}_z^i) \tag{34}$$

where, for $i = 1,\dots,N$, $k = x, y, z$,

$$\hat{p}_k^{ri} \triangleq F_k^i \hat{\eta}_k^i,\quad \varphi_k^i \triangleq F_k^i S_k^2 \hat{\eta}_k^i,\quad \bar{\varphi}_z^i \triangleq \frac{\varphi_z^i}{1 + {\varphi_z^i}^2} \tag{35}$$

and let

$$\tilde{p}_i \triangleq p_i - \hat{p}_{ri}. \tag{36}$$

Then our distributed intermediary control law is proposed as follows:

$$u_i = \varphi_i - k_p\chi(\tilde{p}_i) - k_d\chi(\dot{\tilde{p}}_i) \tag{37}$$

$$\dot{\eta}_i = S\eta_i + \mu\sum_{j=0}^{N} a_{ij}(\eta_j - \eta_i) \tag{38}$$

$$\dot{\hat{\eta}}_i = S\hat{\eta}_i + L(\hat{\eta}_i - \eta_i),\ i = 1,\dots,N \tag{39}$$

where k_p and k_d are positive constants to be specified, μ is any positive constant, and L is a constant matrix such that $(S + L)$ is Hurwitz.

Lemma 3.5. Under Assumptions 2.1 and 2.2, for $i = 1,\dots,N$, we have

$$\lim_{t\to\infty}(\varphi_i(t) - \ddot{p}_{ri}(t)) = 0 \tag{40}$$

$$\lim_{t\to\infty}(\dot{\varphi}_i(t) - p_{ri}^{(3)}(t)) = 0 \tag{41}$$

$$\lim_{t\to\infty}(\ddot{\varphi}_i(t) - p_{ri}^{(4)}(t)) = 0, \tag{42}$$

all exponentially.

Proof: Under Assumptions 2.1 and 2.2, by Lemma 3.4, for $i = 1,\dots,N$, $k = x, y, z$, we have $\lim_{t\to\infty}(\varphi_k^i(t) - \ddot{p}_k^{ri}(t)) = 0$, $\lim_{t\to\infty}(\dot{\varphi}_k^i(t) - {p_k^{ri}}^{(3)}(t)) = 0$, and $\lim_{t\to\infty}(\ddot{\varphi}_k^i(t) - {p_k^{ri}}^{(4)}(t)) = 0$, all exponentially. By Remark 2.1, $\ddot{p}_z^{ri}(t) = 0$ for all $t \geq 0$, thus $\lim_{t\to\infty}\varphi_z^i(t) = 0$, $\lim_{t\to\infty}\dot{\varphi}_z^i(t) = 0$, and $\lim_{t\to\infty}\ddot{\varphi}_z^i(t) = 0$, all exponentially. Since $\bar{\varphi}_z^i = \frac{\varphi_z^i}{1+{\varphi_z^i}^2}$, $\dot{\bar{\varphi}}_z^i = \frac{\dot{\varphi}_z^i(1-{\varphi_z^i}^2)}{(1+{\varphi_z^i}^2)^2}$, and $\ddot{\bar{\varphi}}_z^i = \frac{(2{\varphi_z^i}^3 - 6\varphi_z^i){\dot{\varphi}_z^i}^2 + (1-{\varphi_z^i}^4)\ddot{\varphi}_z^i}{(1+{\varphi_z^i}^2)^3}$, we further have $\lim_{t\to\infty}\bar{\varphi}_z^i(t) = 0$, $\lim_{t\to\infty}\dot{\bar{\varphi}}_z^i(t) = 0$, and $\lim_{t\to\infty}\ddot{\bar{\varphi}}_z^i(t) = 0$, all exponentially. The proof is completed upon noting (34). □

Remark 3.6. Since, under Assumption 2.1, $\ddot{p}_{ri}$, $p_{ri}^{(3)}$, $p_{ri}^{(4)}$, $i = 1,\dots,N$, are bounded, Lemmas 3.4 and 3.5 imply that $\ddot{\hat{p}}_{ri}$, φ_i, $\dot{\varphi}_i$, $\ddot{\varphi}_i$, $i = 1,\dots,N$, are bounded. Thus, there exists a positive constant δ_d such that $\|\varphi_i\|_\infty \leq \delta_d$, $i = 1,\dots,N$. By Remark 3.1, the distributed intermediary control law (37) is bounded by $\|u_i\|_\infty \leq \delta_d + \sqrt{3}(k_p + k_d)$. Let $\delta_m \triangleq \max\{m_i | i = 1,\dots,N\}$ and $\bar{T} \triangleq \delta_m(g + \delta_d + \sqrt{3}(k_p + k_d))$. Then, for all $t \geq 0$ and $i = 1,\dots,N$, we have $T_i(t) \leq \bar{T}$.

Remark 3.7. From (37), for $i = 1,\dots,N$, direct calculation gives

$$\dot{u}_i = \dot{\varphi}_i - k_p h(\tilde{p}_i)\dot{\tilde{p}}_i - k_d h(\dot{\tilde{p}}_i)\ddot{\tilde{p}}_i \tag{43}$$

$$\begin{aligned}\ddot{u}_i = \ddot{\varphi}_i &- k_p \dot{h}(\tilde{p}_i)\dot{\tilde{p}}_i - k_p h(\tilde{p}_i)\ddot{\tilde{p}}_i \\ &- k_d \dot{h}(\dot{\tilde{p}}_i)\ddot{\tilde{p}}_i - k_d h(\dot{\tilde{p}}_i)\tilde{p}_i^{(3)}. \end{aligned} \tag{44}$$

3.4 Distributed Attitude Control

In what follows, we propose a distributed attitude control law. For this purpose, let

$$\bar{\Omega}_i \triangleq \tilde{\Omega}_i + k_1\hat{\epsilon}_i,\ i = 1,\dots,N \tag{45}$$

where k_1 is any positive constant. Then, from (1b)–(1c), (24), and (25), a direct extension of the result in Luo et al. (2005) gives

$$\dot{\hat{\epsilon}}_i = \frac{1}{2}(\bar{\epsilon}_i I_3 + \hat{\epsilon}_i^\times)(\bar{\Omega}_i - k_1\hat{\epsilon}_i) \tag{46}$$

$$\dot{\bar{\epsilon}}_i = -\frac{1}{2}\hat{\epsilon}_i^T(\bar{\Omega}_i - k_1\hat{\epsilon}_i) \tag{47}$$

$$\begin{aligned} J_i\dot{\bar{\Omega}}_i = &-\Omega_i^\times J_i\Omega_i + J_i(\tilde{\Omega}_i^\times \mathcal{R}^T(\epsilon_i)\Omega_{ri} - \mathcal{R}^T(\epsilon_i)\dot{\Omega}_{ri}) \\ &+ \frac{k_1}{2}J_i(\bar{\epsilon}_i I_3 + \hat{\epsilon}_i^\times)\tilde{\Omega}_i + \tau_i,\ i = 1,\dots,N. \end{aligned} \tag{48}$$

Let

$$\hat{\dot{\Omega}}_{ri} \triangleq \bar{\Xi}(u_i, \dot{u}_i)\dot{u}_i + \Xi(u_i)\hat{\ddot{u}}_i,\ i = 1,\dots,N \tag{49}$$

where, for $i = 1,\dots,N$,

$$\begin{aligned}\hat{\ddot{u}}_i \triangleq \ddot{\varphi}_i &- k_p\dot{h}(\tilde{p}_i)\dot{\tilde{p}}_i - k_p h(\tilde{p}_i)\ddot{\tilde{p}}_i \\ &- k_d\dot{h}(\dot{\tilde{p}}_i)\ddot{\tilde{p}}_i - k_d h(\dot{\tilde{p}}_i)(p_i^{(3)} - \dot{\varphi}_i). \end{aligned} \tag{50}$$

Then our distributed attitude control law for the ith quadrotor helicopter, $i = 1,\dots,N$, is proposed as follows:

$$\begin{aligned}\tau_i = \Omega_i^\times J_i\Omega_i &- J_i(\tilde{\Omega}_i^\times\mathcal{R}^T(\epsilon_i)\Omega_{ri} - \mathcal{R}^T(\epsilon_i)\hat{\dot{\Omega}}_{ri}) \\ &- \frac{k_1}{2}J_i(\bar{\epsilon}_i I_3 + \hat{\epsilon}_i^\times)\tilde{\Omega}_i - k_2(\tilde{\Omega}_i + k_1\hat{\epsilon}_i) \end{aligned} \tag{51}$$

where k_1 and k_2 are any positive constants.

3.5 Solvability of Problem 1

The solvability of Problem 1 can be summarized as follows:

Theorem 3.1. Under Assumptions 2.1 and 2.2, let k_p and k_d be such that

$$k_p + k_d < g - \frac{1}{2}. \tag{52}$$

Then Problem 1 is solvable by combining the distributed position control law (18) with u_i obtained from (37)–(39) and the distributed attitude control law (51).

Proof: By definition of $\bar{\varphi}_z^i$ in (35), for all $t \geq 0$, $|\bar{\varphi}_z^i(t)| \leq |\varphi_z^i(t)|/(1 + |\varphi_z^i(t)|^2) \leq 1/2$. If k_p and k_d are such that (52) holds, then, from (37), we have $\|u_z^i\|_\infty \leq \|\bar{\varphi}_z^i\|_\infty + k_p + k_d \leq 1/2 + k_p + k_d < g$, $i = 1,\dots,N$. By Lemma 3.2, the reference attitude defined by (20) satisfies (19).

Since, for $i = 1,\dots,N$, T_i is given by (18) and q_{ri} is such that (19) holds, via the transformation (13), the translational dynamics (1a) can be transformed to (14). Then, substituting (37) into (14) gives

$$\ddot{\tilde{p}}_i = -k_p\chi(\tilde{p}_i) - k_d\chi(\dot{\tilde{p}}_i) + \zeta_i,\ i = 1,\ldots,N \quad (53)$$

where $\zeta_i = \varphi_i - \ddot{\hat{p}}_{ri} + \frac{T_i}{m_i}(\mathcal{R}(q_{ri}) - \mathcal{R}(q_i))e_3$.

To apply Lemma 3.1 to (53), we need to show that $\lim_{t\to\infty}\zeta_i(t) = 0$. Under Assumptions 2.1 and 2.2, by Lemmas 3.4 and 3.5, we have $\lim_{t\to\infty}(\varphi_i(t) - \ddot{\hat{p}}_{ri}(t)) = 0$ exponentially. Since the boundedness of T_i follows from Remark 3.6, we only need to show $\lim_{t\to\infty}(\mathcal{R}(q_{ri}(t)) - \mathcal{R}(q_i(t))) = 0$. By Remark 3.3, it suffices to show

$$\lim_{t\to\infty}\hat{\epsilon}_i(t) = 0,\ i = 1,\ldots,N. \quad (54)$$

By Lemma 3.1 of Chen and Huang (2009), if

$$\lim_{t\to\infty}\bar{\Omega}_i(t) = 0,\ i = 1,\ldots,N, \quad (55)$$

then, for any initial condition $\epsilon_i(0)$ satisfying $\|\epsilon_i(0)\| = 1$, $i = 1,\ldots,N$, the solution of (46)–(47) is bounded and satisfies $\lim_{t\to\infty}\hat{\epsilon}_i(t) = 0$, $i = 1,\ldots,N$. Thus, in what follows, we show (55). For this purpose, substituting (51) into (48) gives

$$J_i\dot{\bar{\Omega}}_i + k_2\bar{\Omega}_i = J_i\mathcal{R}^T(\epsilon_i)(\dot{\hat{\Omega}}_{ri} - \dot{\Omega}_{ri}),\ i = 1,\ldots,N. \quad (56)$$

From (23), (49), (44), and (50), we have

$$\begin{aligned}\dot{\hat{\Omega}}_{ri} - \dot{\Omega}_{ri} &= \Xi(u_i)(\hat{\ddot{u}}_i - \ddot{u}_i)\\ &= k_d\Xi(u_i)h(\dot{\tilde{p}}_i)(\dot{\varphi}_i - \hat{p}_{ri}^{(3)}),\ i = 1,\ldots,N. \quad (57)\end{aligned}$$

First note that, by Remarks 3.1, 3.6, and 3.2, $h(\dot{\tilde{p}}_i)$, u_i, and $\Xi(u_i)$, $i = 1,\ldots,N$, are bounded, and system (56) can be viewed as a stable first order linear system subject to the nonlinear input $J_i\mathcal{R}^T(\epsilon_i)(\dot{\hat{\Omega}}_{ri} - \dot{\Omega}_{ri})$. Since Remark 3.5 and Lemma 3.5 imply that $\lim_{t\to\infty}(\dot{\varphi}_i(t) - \hat{p}_{ri}^{(3)}(t)) = 0$ exponentially, from (57), we have

$$\lim_{t\to\infty}(\dot{\hat{\Omega}}_{ri}(t) - \dot{\Omega}_{ri}(t)) = 0,\ i = 1,\ldots,N \quad (58)$$

exponentially. For $i = 1,\ldots,N$, since J_i is symmetric and positive definite and $\mathcal{R}^T(\epsilon_i) \in SO(3)$ is bounded, equations (56) and (58) imply that $\lim_{t\to\infty}\bar{\Omega}_i(t) = 0$ exponentially.

By Lemma 3.1, for any initial condition $\tilde{p}_i(0)$, $\dot{\tilde{p}}_i(0)$, $i = 1,\ldots,N$, the solution $\tilde{p}_i$, $\dot{\tilde{p}}_i$ of system (53) are bounded and satisfy $\lim_{t\to\infty}\tilde{p}_i(t) = 0$, $\lim_{t\to\infty}\dot{\tilde{p}}_i(t) = 0$, $i = 1,\ldots,N$.

Since Lemma 3.4 implies that $\lim_{t\to\infty}(\hat{p}_{ri}(t) - p_{ri}(t)) = 0$ exponentially and $\lim_{t\to\infty}(\dot{\hat{p}}_{ri}(t) - \dot{p}_{ri}(t)) = 0$ exponentially, we have

$$\begin{aligned}\lim_{t\to\infty}\bar{p}_i(t) &= \lim_{t\to\infty}(\tilde{p}_i(t) + \hat{p}_{ri}(t) - p_{ri}(t))\\ &= 0,\ i = 1,\ldots,N\end{aligned}$$

and

$$\begin{aligned}\lim_{t\to\infty}\dot{\bar{p}}_i(t) &= \lim_{t\to\infty}(\dot{\tilde{p}}_i(t) + \dot{\hat{p}}_{ri}(t) - \dot{p}_{ri}(t))\\ &= 0,\ i = 1,\ldots,N.\end{aligned}$$

Since, by Remark 3.6, T_i is bounded, it remains to show that τ_i is bounded. By Remark 3.6, $\dot{\varphi}_i$ and $\ddot{\varphi}_i$ are bounded. From (53), $\ddot{\tilde{p}}_i$ is bounded. Then, the boundedness of $\dot{u}_i$ and Ω_{ri} follows from (43) and (22), respectively. Since $\tilde{\Omega}_i$ is bounded, in view of (25), Ω_i and thus $\Omega_i^\times$ are bounded. Since $\frac{d}{dt}\|f(t)\| = \frac{f^T(t)f(t)}{\|f(t)\|}$ for $f(t) \neq 0$, from (18), we have $\dot{T}_i = \frac{-m_i\dot{u}_i^T(ge_3-u_i)}{\|ge_3-u_i\|}$. Since u_i and $\dot{u}_i$ are bounded, $\dot{T}_i$ is bounded. Noting that $\dot{\mathcal{R}}(q_{ri}) = \mathcal{R}(q_{ri})\Omega_{ri}^\times$ and $\dot{\mathcal{R}}(q_i) = \mathcal{R}(q_i)\Omega_i^\times$, $\frac{d}{dt}(\frac{T_i}{m_i}(\mathcal{R}(q_{ri}) - \mathcal{R}(q_i))e_3)$ is bounded. Since $(\dot{\varphi}_i - \hat{p}_{ri}^{(3)})$ is bounded, so is $\dot{\zeta}_i$. Then, the boundedness of $\tilde{p}_i^{(3)}$ and $\ddot{u}_i$ follows from (53) and (44), respectively. Thus, from (23), $\dot{\Omega}_{ri}$ is bounded, which together with (58) imply that $\dot{\hat{\Omega}}_{ri}$ is bounded. The boundedness of τ_i then follows from (51). □

Remark 3.8. Since the control law composed of (18), (51), and (37)–(39) is in the form of (10) with $\xi_i = \mathrm{col}(\eta_i, \hat{\eta}_i)$, it is a distributed control law.

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