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An interactive teaching/learning approach to the design of robust linear control systems using the closed-loop shaping methodology ⋆

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Abstract: Usually the controller is designed working on the open-loop transfer function. However, it is also possible to design the controller working on the closed-loop transfer functions. The closed-loop shaping methodology offers a straightforward framework which allows designers and students to focus on the required specification fulfillment and dealing with inherent linear systems limitations without complex computations or using difficult algorithms. This article summarizes the basic ideas of the manual closed-loop shaping methodology, and its application to the design of robust controllers for uncertainty linear systems. An interactive software tool for learning/teaching this methodology is also presented.

Keywords: Automatic Control Teaching, Robust Control, Interactive design.

1. INTRODUCTION

Automatic control is an interdisciplinary area which is taught to most engineering students and it is taking more and more relevance with time. In most cases, a basic introductory course is the only content most students have studied when finishing the career (Dorf, 1967; Astrom and Murray, 2012; Arevalo et al., 2020). Additionally, automatic control core contains different abstract concepts that are often difficult for many students to internalize.

Fortunately, most of these concepts have a nice an intuitive graphical representation: time series plot, poles-zeros map, root locus, and frequency domain plot (Bode plot, Nyquist plot, or Nichols plot).

The learning of the basic rules and interrelations between the different types of diagrams is an essential step to understand the different analysis and design techniques of control systems.

Our experience in teaching courses in control systems shows us that a good methodology to teach and learn these relations is using interactive software tools (Dormido, 2004). We have detected that a significant number of students have problems to grasp them and to recognize when a process is easy or difficult to control. The concept of interactive design presents two significant differences in relation to the conventional approach (non-interactive design):

1. The interactive procedure entails a characteristic feedback loop of iterative design to modify the controller parameters to fulfill the set of specifications required in the design process.
2. The student learns much more clearly and quickly which controller parameters must change and how to push the design in the direction of fulfilling the specifications involved and revealing at the same time the fundamental limitations of the control system (delays, non-minimum phase, . . .).

Loop shaping is one of the most popular control design techniques in linear systems. This methodology is based on shaping the frequency response of the different transfer functions. Usually this approach is based on shaping the open-loop frequency response (Díaz et al., 2017; Diaz et al., 2019), but recently the closed-loop shaping has also been proposed (Díaz et al., 2018). Differently from open-loop shaping, closed-loop shaping allows to take into account inherent linear systems design constraints (Seron, 2010).

To support this teaching an interactive tool, which can visualize the design procedure, has been designed and is currently used during the classes to illustrate the most relevant concepts of the closed-loop shaping methodology. Differently from our previous work (Diaz et al., 2019), this paper describes a methodology which takes into account plant uncertainty in the design procedure.

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The paper is organized as follows, section 2 contains the basic ideas of the manual closed-loop shaping methodology; section 3 describes the type of uncertain systems considered in this paper, and how to define the robust stability specification for them; section 4 describes the developed interactive tool; section 5 provides an illustrative example, and finally section6 contains some conclusions and future works.

2. BASIC IDEAS OF MANUAL CLOSED-LOOP SHAPING METHODOLOGY

2.1 System Definition

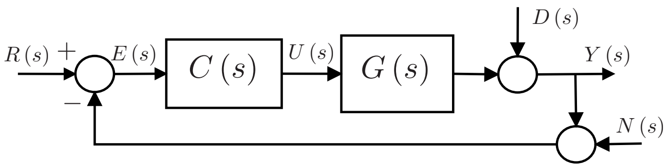

Fig. 1. Closed-loop control system block diagram.

Figure 1 shows the closed-loop control system under consideration. The closed-loop system is composed by the feedback connection of $G(s)$, which corresponds to the plant transfer function, and $C(s)$ that represents to the controller. The different signals involved in the closed-loop system are the output, $Y(s)$, the control action, $U(s)$ and the error, $E(s)$. Finally, the inputs are the reference, $R(s)$, the output disturbances $D(s)$ and the measurement noise, $N(s)$.

2.2 Specifications

From Figure 1, it is possible to deduce the relationship between the reference, the disturbance and the noise:

$$Y(s) = T(s)R(s) + S(s)D(s) - S(s)N(s) \quad (1)$$

where $T(s) = \frac{L(s)}{1+L(s)}$ and $S(s) = \frac{1}{1+L(s)}$ with $L(s) = C(s)G(s)$. $L(s)$ is the open-loop transfer functions, $S(s)$ is named the sensitivity function and $T(s)$ is named the complementary sensitivity function (note that $T(s) + S(s) = 1$).

The usual specifications are that the output should track the reference while rejecting the disturbances and being insensitive to noise. To achieve this $T(s) \approx 1$, which implies $S(s) \approx 0$, although this would be appropriate for the reference and the disturbance it would not be from the noise point of view.

In order to handle this trade-off, usually it is assumed that references and disturbances have only relevant frequency components in the low-frequency range while noise have them in the high-frequency range. As shown in Figure 2, these assumptions induce a decomposition of the frequency range and suggests a shape for the frequency response of $S(s)$ and $T(s)$.

Desired specifications can be graphically visualized in different diagrams and different transfer functions. Figure 3 show the specifications for the complementary sensitiviy function in the Bode diagram. These representations need

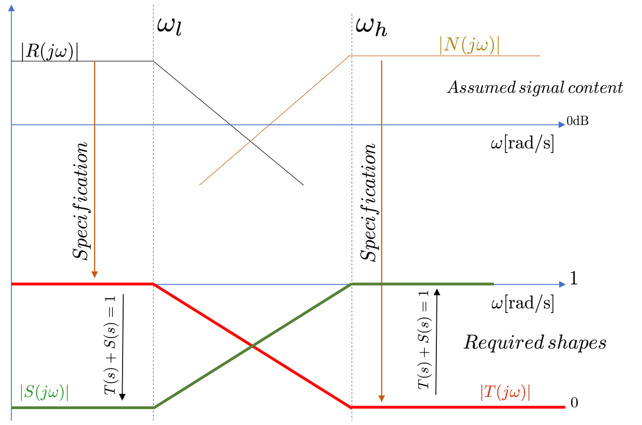

Fig. 2. Signal frequency decomposition and desired frequency responses.

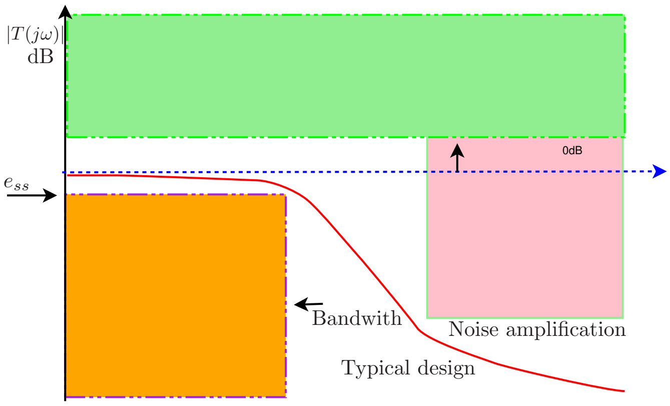

Fig. 3. Complementary Sensitivity function specifications over the Bode diagram.

to be adapted for each closed-loop function and diagram (Bode plot, Polar plot or Nichols plot), and they are different from the ones obtained for the open-loop transfer function.

2.3 Parametrization

To achieve the desired shapes defined in Figure 2, one should define an appropriate controller, $C(s)$. Unfortunately, the relationship between the controller and the closed-loop functions is non linear. This is the main reason why the loop-shaping is usually performed in open-loop (Sánchez-Peña and Sznaier, 1998).

Using the stabilizing controller parametrization (Sánchez-Peña and Sznaier, 1998; Diaz et al., 2019) this problem can be avoided. For stable plants, all stabilizing controller can be written as (Kwok and Davison, 2007)

$$C(s) = \frac{Q(s)}{1 - C(s)G(s)} \quad (2)$$

where $Q(s)$ is a stable system to be designed. With this selection, the sensitivity and the complementary sensitivity function becomes:

$$T(s) = Q(s)G(s) \quad (3)$$
$$S(s) = 1 - Q(s)G(s). \quad (4)$$

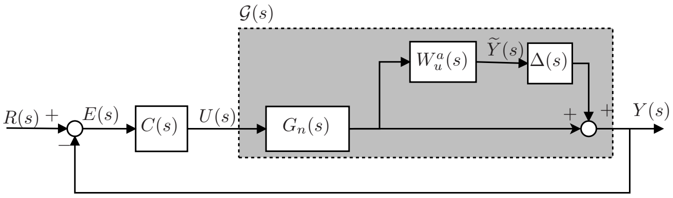

Fig. 4. Closed-loop scheme with a plant with multiplicative uncertainty.

As it can be seen, $Q(s)$ has an affine relation with the closed-loop transfer function and makes the interactive design easier (Díaz et al., 2018). It can be proven that any stabilizing controller can be written in the form of (2), consequently the selection of this form does not introduce limitations on the design.

3. ROBUST STABILITY

3.1 Uncertainty description

The plant will be described as a nominal plant, $G_n(s)$, an uncertain block, $\Delta(s)$, which fulfills $\|\Delta(s)\|_\infty < 1$ and a transfer function, $W_u^m(s)$, which defines the per unit uncertainty size at each frequency. These elements are connected as shown in Figure 6, i.e. $G(s) = G_n(s)\,(1 + W_u^m(s)\Delta(s))$. Multiplicative uncertainty, $W_m^a(s)$, can be transformed in an additive, or absolute, uncertainty as:

$$W_u^a(s) = W_u^m(s)G_n(s).$$

Consequently, $G(s) = G_n(s) + W_u^a(s)\Delta(s)$.

3.2 Robust stability conditions

The robust stability condition determines the required conditions so that the closed-loop system, including the uncertainty, can be guaranteed stable (Doyle et al., 1992):

$$\|T_n(s)W_u^a(s)\|_\infty < 1 \tag{5}$$

where $T_n(s)$ represents the complementary sensitivity function obtained with the nominal plant. When using the controller introduced in section 2.3, the stability condition becomes :

$$\|Q(s)G_n(s)W_u^m(s)\|_\infty < 1. \tag{6}$$

This condition can be seen as a relationship between curves as follows:

$$|Q(j\omega)| < \frac{1}{|G_n(j\omega)W_u^m(j\omega)|}\ \forall\,\omega \tag{7}$$

which can be graphically represented. As $G_n(s)$ and $W_u^m(s)$ are obtained from the modeling procedure, the condition can be forced by selecting $Q(s)$ appropriately. Interactive applications can be a good methodology to address this issue.

4. INTERACTIVE TOOL

Figure 5 shows the main view of RCLSD (Robust Closed-Loop Shaping Design). In the upper left part of the applications the assumed control scheme is shown, by clicking in each element of the block scheme it is possible to make it active (it can be defined). In the lower left part, a pole zero map is shown, it can be used to defined the poles and zeros of the different elements.

In the right part four different figures are shown, the upper ones correspond to the frequency response while the lower ones are the closed-loop system time response. In the frequency response figures, the poles and zeros of the designed elements can be interactively modified.

RCLSD can also handle plants without uncertainty, plants with additive uncertainty or a plants with multiplicative uncertainty (Figure 7). The nominal plant shape can be selected from a set of predefined ones (Figure 8) or it can be completely defined by its poles, zeros and gain.

As previously introduced, most specifications can be drawn in the different frequency response diagrams and for the different closed-loop functions. RCLSD includes all these possibilities, and it can be configured according to the required needs (Figure 9).

RCLSD offers a very nice way to introduce students in most relevant concepts in robust control, visualizing the trade-off between performance and robustness(Boulet and Duan, 2007) and analyzing the effect of non-minimum phase zeros (Hoagg and Bernstein, 2007).

5. EXAMPLE

Let's assume a generalized plant composed by a nominal plant defined by:

$$G_n(s) = \frac{31.233(s+2)}{(s+3.23)(s^2+2.26s+19.34)}$$

and an additive uncertainty weighting function defined by:

$$W_u^a(s) = \frac{1.654(s+143.4)}{(s+1.85)}.$$

Both elements can be interactively introduced in RCLSD. Figure 10 shows $G_n(s)$ and $(W_u^a(s))^{-1}$ magnitude Bode diagrams as seen in RCLSD. As it is the case in most common scenarios the uncertainty weighting function takes relevant values in the high frequency range.

Firstly, a set of specifications must be defined. In this case, a null steady-state error for steps will be required, an error below 5% for frequencies up to 2 rad/s and a noise attenuation bigger than 0.1 for frequencies bigger to 20 rad/s. This can be graphically represented in RCLSD as shown in Figure 11. The design goal will be keeping the complementary sensitivity function magnitude frequency response out of the regions in yellow.

Before beginning the design, performance specifications will be complemented with the robust stability one (section 3.2), which represented in RCLSD takes the form shown in Figure 12. Again the goal is to find a design filter, $Q(s)$, which keeps the magnitude frequency response curve out from the yellow areas.

In order to guarantee that the closed-loop system has null steady-state for the step references, a $Q(s)$ which guarantees that $Q(0) = \frac{1}{G_n(0}$) is selected (this is automatically handled by RCLSD). It is tested by trial and error that specifications can not be achieved with a first order $Q(s)$ so a second order one is selected. One of the poles of $Q(s)$ is placed over the plant zero, and the second one is placed around -10. These selections generate the following filter:

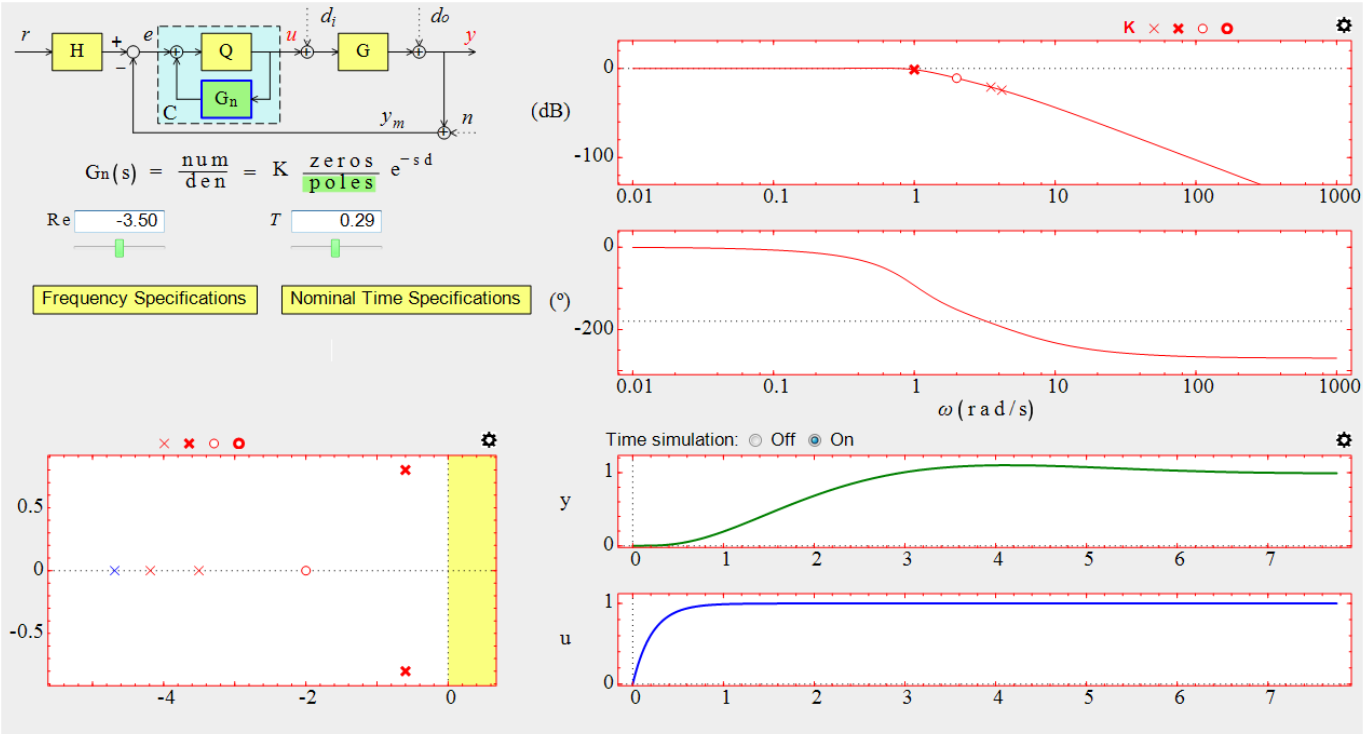

Fig. 5. RCLSD main view

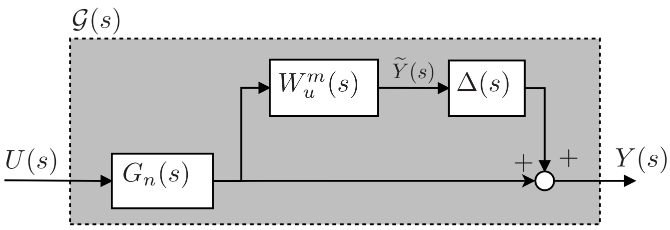

Fig. 6. Plant with multiplicative uncertainty.

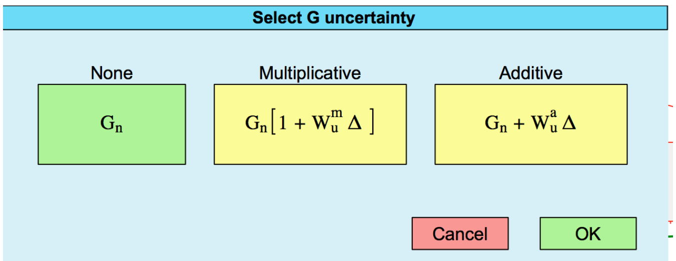

Fig. 7. RCLSD Plant types

$$Q(s) = \frac{40}{(s+2)(s+20)}. \tag{8}$$

From this selection the following complementary sensitivity function, $T(s)$, and controller, $C(s)$, are obtained:

$$T(s) = \frac{1249.3}{(s+20)(s+3.23)(s^2+2.26s+19.34)} \tag{9}$$

$$C(s) = \frac{40(s+3.23)(s^2+2.26s+19.34)}{s(s+20.19)(s+2)(s^2+5.297s+29.48)}. \tag{10}$$

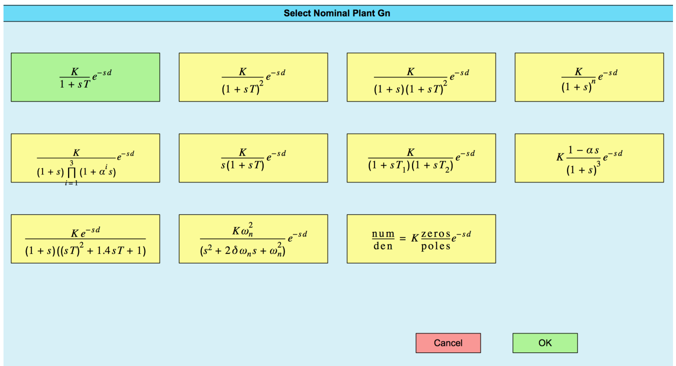

Fig. 8. RCLSD Plant predefined shapes

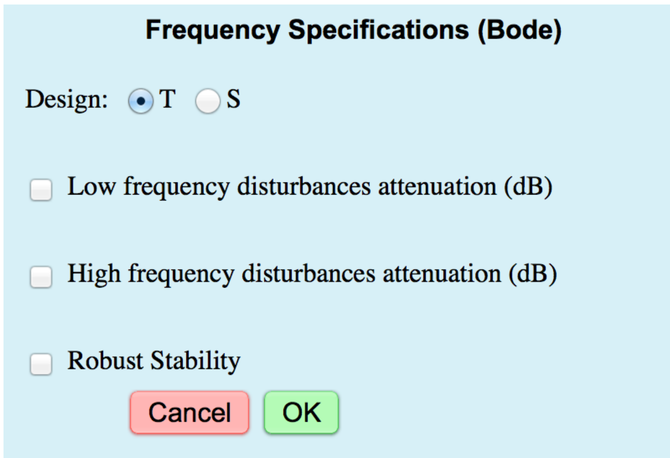

Fig. 9. RCLSD : Frequency domain specification

This controller fulfills the desired specifications as can be seen in Figure 13. Additionally, Figure 14 shows achieved time response, both for the output and the control action.

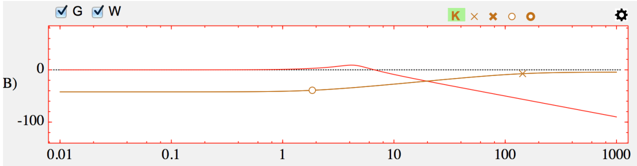

Fig. 10. Example: $G_n(s)$ and $(W_u^a)^{-1}$ magnitude Bode diagrams as seen in RCLSD ($G_n(s)$ in red, $(W_u^a)^{-1}$ in brown).

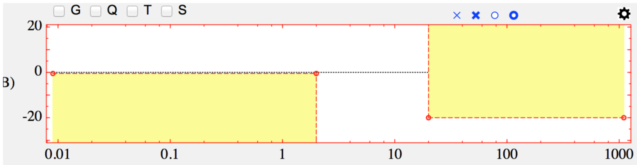

Fig. 11. Example: Performance specification as seen in RCLSD.

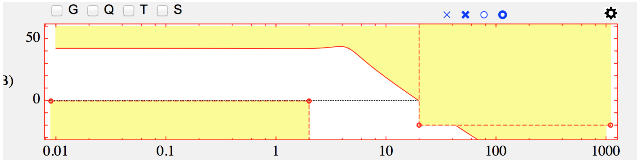

Fig. 12. Example: Performance and robust stability specifications as seen in RCLSD.

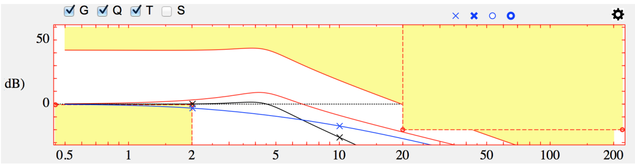

Fig. 13. Example: Proposed design ($T(s)$ in black, $G_n(s)$ in red, $Q(s)$ in blue).

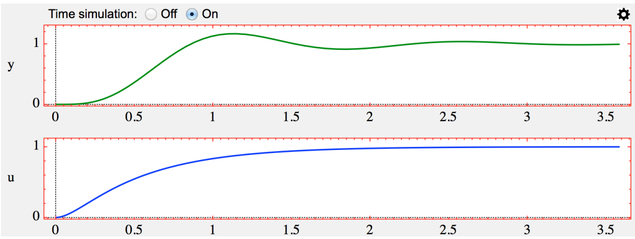

Fig. 14. Example: Proposed design time response.

6. CONCLUSIONS AND FUTURE WORKS

6.1 Conclusions

In this work it has been shown how an interactive tool can be used to shape closed-loop transfer functions. This methodology allows to simultaneously take into account performance and robustness in graphical and interactive manner. The procedure has been illustrated with a complete example.

Although presented tool is currently used for teaching purposes, the concepts behind them can be used to design controllers in framework which is much simple than the one based on H_∞ optimization. Additionally, obtained control would be low-order than the ones obtained by H_∞ optimization.

6.2 Future Works

Proposed methodology using RCLSD allows to achieve designs that guarantee nominal performance and robust stability. The authors are working to develop a tool which allows to achieve robust performance design using interactive and graphical methods.

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