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Impulsive observer design for switched linear systems with time varying sampling and (a)synchronous switching rules

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Abstract: Observer synthesis for switched linear systems with time-varying sampled-data measurements is addressed in this paper. Between sampling times, the behaviour of the observer under consideration has the particularity to be that of the estimated active mode while, at the sampling times a correction (or an update) is made using the estimated active mode and the sampled output signal. Two cases are discussed: *i*) the situation where no delay occurs in the estimation of the active mode and, *ii*) the more practical case where the estimated switching signal that defines the proposed observer is a delayed replica of the plant's switching signal. In these two situations, appropriate time-varying piecewise quadratic Lyapunov functions are used to establish convergence conditions of the proposed observer in the linear matrix inequalities (LMIs) framework.

Keywords: Continuous-discrete observers, linear switched systems, sampled-data systems, LMIs

1. INTRODUCTION

Switched systems has been the subject of a growing amount of interests in the past decade and are by now a well studied topic (Daafouz et al., 2002; Liberzon and Morse, 1999; Liberzon, 2012). Switched systems are a subclass of hybrid systems where different dynamics describe the different operating modes. The transition from one mode to another is given by a switching rule. The motivation for considering such a class of systems is vast and encompasses robust stabilization (modeling and control) of systems subject to sampling uncertainties and singularly perturbed systems. An important problem arising when considering switched systems is the stability problem. Indeed given a switching rule, stability (resp. instability) of all modes does not guaranty the stability (resp. instability) of the overall switched system (Liberzon, 2012). Several results exist on stability of switched systems in different settings (for a survey on the topic, see for instance Briat., 2013; Allerhand and Shaked, 2011; Zhang and Gao, 2010; Xiang, 2016; Lin and Antsaklis, 2009).

While full state feedback is often assumed to be measured, in many practical applications due to technical or economic constraints, such an hypothesis is not justified. Therefore observation for linear and nonlinear systems constitutes a large area of study. For this reason observer synthesis for switched systems has received some attention in the literature (Alessandri and Coletta, 2003; Pettersson, 2006; Barbot et al., 2007; Bejarano and Pisano, 2010). However when the output is transmitted over a network the assumption of continuous transmission is no longer warranted and close attention has to be paid to the aperiodic and sampled nature of the output. This consideration has, in the last decade, led to numerous works in the field of observation with aperiodic sampling for both continuous time linear and nonlinear systems. For such sampled data systems, the so called impulsive observer (sometimes referred as continuous-discrete observer) offer interesting properties in terms of both performance and simplicity of analysis (see for instance Raff and Allgower, 2007; Dinh et al., 2014; Etienne et al., 2017). However, to the best of the author's knowledge, observation of continuous time switched systems subject to time varying sampling has not received attention. While some results have recently been proposed for discrete time systems (Han et al., 2019), such missing analysis motivates the present study.

In this work we will address the problem of observer synthesis for switched linear systems with aperiodic sampled measurements. Between the sampling times, the observer is just a copy of the dynamics given by the estimated active mode. At sampling times, an instantaneous correction (or an update) is made using the sampled output signal as well as the estimated active mode. For the design of this observer, two cases are considered:

- The first case assumes that the switching sequence is known in real time.
- For the second case, since in practice the identification of the active mode may takes some times thus introducing a delay (Tian et al., 2011; Yang et al., 2015), we will also consider the asynchronous situation where the switching times are not instan-

taneously estimated. In this context, we will assume that there can be a bounded delay between the real active mode and its estimation provided by the observer. Indeed if one considers sampled data outputs, then it is natural to see the switching signal (which can be seen as an output) as not continuously measured and available in real-time .

In these two situations, using appropriate time-varying piecewise quadratic Lyapunov functions, we propose convergence conditions for the impulsive observers in the linear matrix inequalities framework.

The rest of the paper is organized as follows. The considered class of switched linear systems and the impulsive observers under consideration are described in Section 2. LMIs-based conditions proposed for the observer synthesis, obtained from piecewise quadratic Lyapunov functions, are presented in Section 3. To show the relevance of the proposed observer synthesis methods, numerical examples are also provided in Section 4. Concluding remarks are finally given in Section 5.

Notation: For a vector or a matrix v, $v^\top$ denotes its transpose. We define for a matrix A, $He(A) := A + A^\top$. $\mathbb{R}_{\geq 0}$ corresponds to the set of non-negative real numbers. $\mathbb{S}_n$ denotes the set of $n \times n$ symmetric matrices while $\mathbb{S}_n^+$ denotes the set of $n \times n$ positive definite symmetric matrices. For a matrix $M \in \mathbb{S}_n$, $\lambda_{min}(M)$ and $\lambda_{max}(M)$ denote its smallest and largest eigenvalues, respectively. For two positive definite (resp. positive semidefinite) matrices P and Q, we write $P \succ Q$ (resp. $P \succeq Q$) if $P - Q$ is positive definite (resp. positive semidefinite).

2. DEFINITIONS AND PROBLEM STATEMENT

Let's consider the following class of switched linear systems:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \quad t \neq s_k, t \neq t_k, \\ x(s_k^+) = x(s_k), \\ y(t_k) \ = C_{\sigma(t_k)}x(t_k), \end{cases} \tag{1}$$

where $\forall t \in [0\,;\infty)$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^\ell$ and $y(t) \in \mathbb{R}^m$ are the state vector, the control input and the output vector. The signal $\sigma\colon [0\,;\infty) \longrightarrow \mathscr{S} \subset \mathbb{N}$ is a piecewise constant function, continuous from the right, representing the switching signal of the system. $\{s_k\}_{k\in\mathbb{N}}$ is the strictly increasing sequence of the switching instants and $\{t_k\}_{k\in\mathbb{N}}$ is the sequence of sampling times. By convention, we set $s_0 = t_0 = 0$ and for technical simplicity we assume that $\forall k \neq 0$, $\{t_k\}_{k\in\mathbb{N}} \cap \{s_k\}_{k\in\mathbb{N}} = \emptyset$. For every $i \in \mathscr{S}$, A_i, B_i and C_i are known matrices with appropriate dimensions. Solution of the dynamical system are defined iteratively.

The switching signal is assumed to admit an average dwell-time (ADT), i.e.:

Assumption 1. There exists $\underline{s} \in \mathbb{R}_+^\star$ such that for every $s, t \in [0\,;\infty)$ with $t \geq s$, the number $\mathrm{N}_s(s,t)$ of switching of σ in the interval $[s\,;t[$ verifies $\mathrm{N}_s(s,t) \leq N_0 + \frac{t-s}{\underline{s}}$ for some $N_0 \in \mathbb{N}$; $\underline{s}$ is the ADT and N_0 is called the chatter bound.

Furthermore, sampling sequences with ranged dwell are considered, that is:

Assumption 2. There exists constants $\overline{\tau} \geq \underline{\tau} > 0$ such that $\forall k \in \mathbb{N}$, $t_{k+1} - t_k \in [\underline{\tau}\,;\overline{\tau}]$.

Remark 3. Note that for sampling sequences $\{t_k\}_{k\in\mathbb{N}}$ satisfying the ranged dwell hypothesis in Assumption 2, for every $s, t \in [0\,;\infty)$ with $t \geq s$, the number $\mathrm{N}_c(s,t)$ of sampling t_k in $[s\,;t)$ verifies $\mathrm{N}_c(s,t) \geq \frac{t-s}{\overline{\tau}} - 1$.

For the class of switched linear systems of the form (4), the concept of global uniform exponential stability (GUES) needed in our subsequent analysis is recalled.

Definition 4. (GUES). System (1) is said to be globally uniformly exponentially stable with convergence rate β and overshoot M if, $\forall t \in [0\,;\infty)$ and $\forall x_0 = x(0) \in \mathbb{R}^n$, $\|x(t)\| \leq M \exp(-\beta\, t)\, \|x_0\|$.

The impulsive observer under consideration is described by:

$$\begin{cases} \dot{\hat{x}}(t) = A_{\hat{\sigma}(t)}\hat{x}(t) + B_{\hat{\sigma}(t)}u(t), \quad t \neq s_k, t \neq t_k, \\ \hat{x}(s_k^+) = \hat{x}(s_k), \\ \hat{x}(t_k^+) = \hat{x}(t_k) + G_{\hat{\sigma}(t_k)}\big(C_{\sigma(t_k)}x(t_k) - C_{\hat{\sigma}(t_k)}\hat{x}(t_k)\big), \end{cases} \tag{2}$$

where $\forall t \in [0\,;\infty)$, $\hat{x}(t) \in \mathbb{R}^n$ is the observer state vector (i.e. the estimated state vector) and $\hat{\sigma}\colon [0\,;\infty) \longrightarrow \mathscr{S}$ is an estimate of the real switching signal σ. The switching times of $\hat{\sigma}$ are denoted by the strictly increasing sequence $\{\hat{s}_k\}_{k\in\mathbb{N}}$. It is assumed next that the estimated switching times are obtained with a maximum time delay $\delta \geq 0$, that is:

Assumption 5. $\forall k \in \mathbb{N}$, $\hat{s}_k - s_k \leq \delta$.

Let us stress that the identification method of the real active mode is outside the scope of this paper, but our work can be used for any method that can reconstruct the real state in an time upper bounded by δ.

Note that, according to (2), between sampling, the observer is a copy of the system that run the estimated active mode $\hat{\sigma}$. Moreover, when a sampling occurs, an instantaneous correction is made using the sampled output and the estimated active mode $\hat{\sigma}(t)$.

A simpler specific case occurs when no delay in the active mode detection is present (i.e. $\delta = 0$). In this case, the observer (2) yields to:

$$\begin{cases} \dot{\hat{x}}(t) = A_{\sigma(t)}\hat{x}(t) + B_{\sigma(t)}u(t), \quad t \neq s_k, t \neq t_k \\ \hat{x}(s_k^+) = \hat{x}(s_k), \\ \hat{x}(t_k^+) = \hat{x}(t_k) + G_{\sigma(t_k)}C_{\sigma(t_k)}\big(x(t_k) - \hat{x}(t_k)\big). \end{cases} \tag{3}$$

In this delay-free situation, we are interested by the following observer synthesis problem (OSP):

Problem 6. Solving the observer synthesis problem (OSP) is to find a set of impulsive observation gains G_i, $i \in \mathscr{S}$ for observer (3) such that the estimated error $e = x - \hat{x}$ is GUES.

When the estimated switching signal is the delayed replica of the system's switching signal, the following observer based stability analysis problem (OBSAP) will be examined:

Problem 7. Solving the observer based stability analysis problem (OBSAP) is to find conditions such that $(x^\top, x^\top - \hat{x}^\top)^\top$ is GUES when both matrices K_i and G_i, $i \in \mathscr{S}$ are given.

3. MAIN RESULT

3.1 Case of synchronous switching signals

In this part, we consider the case where $\delta = 0$ (i.e. σ and $\hat{\sigma}$ are synchronous). Considering x given by (1) and $\hat{x}$ defined by (3), the observation error has the following dynamics:

$$\begin{cases} \dot{e}(t) \ = A_{\sigma(t)}e(t), \quad t \neq s_k, & (4a)\\ e(s_k^+) = e(s_k), & (4b)\\ e(t_k^+) = (I - G_{\sigma(t_k)}C_{\sigma(t_k)})e(t_k^-). & (4c)\end{cases}$$

LMIs-based conditions are proposed in Theorem 8 to solve the OSP problem:

Theorem 8. Consider the switched linear system (4) with switching times $\{s_k\}_{k\in\mathbb{N}}$ and sampling sequences $\{t_k\}_{k\in\mathbb{N}}$ satisfying Assumptions 1 and 2. If there exist symmetric matrices $P_m^i \in \mathbb{S}_n$, $m \in \{0,1\}$, $i \in \mathscr{S}$ and the scalars $\underline{\alpha} \in \mathbb{R}$, $\mu_s \geq 1$ and $\underline{\mu}_c \in \,]0\,;1]$ such that the following statements hold:

$$\forall i \in \mathscr{S},\ \forall \tilde{\tau} \in \{0, \overline{\tau}\}, \quad P^i(\tilde{\tau}) \succ 0, \tag{5}$$

$$\forall i \in \mathscr{S},\ \forall \tilde{\tau} \in \{0, \overline{\tau}\}, \quad He(P^i(\tilde{\tau})\, A_i) + P_1^i + \underline{\alpha} P^i(\tilde{\tau}) \prec 0, \tag{6}$$

$$\forall i \in \mathscr{S},\ \forall \tilde{\tau} \in \{\underline{\tau}, \overline{\tau}\}, \quad \begin{bmatrix} \underline{\mu}_c\, P^i(\tilde{\tau}) & P_0^i - L_i\, C_j \\ * & P_0^i \end{bmatrix} \succ 0, \tag{7}$$

$$\forall i,j \in \mathscr{S},\ \forall \tilde{\tau} \in \{0, \overline{\tau}\},\ P^i(\tilde{\tau}) \prec \mu_s\, P^j(\tilde{\tau}), \tag{8}$$

$$\underline{\alpha} > \frac{\ln \mu_s}{\underline{s}} + \frac{\ln \underline{\mu}_c}{\overline{\tau}} \tag{9}$$

where

$$\forall i \in \mathscr{S},\ \forall \tilde{\tau} \in \mathbb{R}, \quad P^i(\tilde{\tau}) = P_0^i + \tilde{\tau} P_1^i, \tag{10}$$

then the system (4) with impulsive observation gains $G_i = (P_0^i)^{-1}L_i$, $i \in \mathscr{S}$ is GUES (i.e. the OSP is solved) with the convergence rate:

$$\beta = \frac{1}{2}\left(\underline{\alpha} - \frac{\ln \mu_s}{\underline{s}} - \frac{\ln \underline{\mu}_c}{\overline{\tau}}\right). \tag{11}$$

Proof 1. Let's consider the piecewise continuous Lyapunov function candidate $V(t) = e^\top(t)\, P^{\sigma(t)}(t - \tau(t))\, e(t)$, $t \in [0\,;\infty)$ where $\tau(t) = \max\{t_k \colon t \geq t_k\}$ is a piece wise constant function that denotes at time t the last sampling that occurred. $P^{\sigma(t)}(t-\tau(t))$ is introduced in (10) i.e.:

$$P^{\sigma(t)}(t - \tau(t)) = P_0^{\sigma(t)} + (t - \tau(t))\, P_1^{\sigma(t)}.$$

For any $t \in [0\,;\infty)$, by construction of $\tau(t)$, one has $t - \tau(t) \in [0\,;\overline{\tau}]$ and thus by convexity argument, one deduce from condition (5) that V is a positive definite function.

Denote by $\{\bar{\mathbf{t}}_k\}_{k\in\mathbb{N}}$ the strictly increasing sequence associated to $\{t_k\}_{k\in\mathbb{N}} \cup \{s_k\}_{k\in\mathbb{N}}$. Let $t \in \,]\bar{\mathbf{t}}_k\,;\bar{\mathbf{t}}_{k+1}[$ with $k \in \mathbb{N}$. By a simple differentiation of V and using the dynamic equation (4a) of the observation error, one can verify that $\dot{V}(t) = e^\top(t)\, \Gamma^{\sigma(t)}\, e(t)$, where

$$\Gamma^{\sigma(t)} = \left[He\left(P^{\sigma(t)}(t - \tau(t))\, A_{\sigma(t)}\right) + P_1^{\sigma(t)}\right].$$

Since $t - \tau(t) \in [0\,;\overline{\tau}]$, using condition (6) and convexity argument, one gets that $\Gamma^{\sigma(t)} \prec -\underline{\alpha}\, P^{\sigma(t)}(t - \tau(t))$ and consequently, $\dot{V}(\tau) \leq -\underline{\alpha} V(\tau)$. Integrating this differential inequality on $]\bar{\mathbf{t}}_k\,;\bar{\mathbf{t}}_{k+1}[$ yields:

$$V(t) \leq \exp\left(-\underline{\alpha}\, (t - \bar{\mathbf{t}}_k)\right) V\left(\bar{\mathbf{t}}_k^+\right),\ \forall \tau \in \,]\bar{\mathbf{t}}_k\,;\bar{\mathbf{t}}_{k+1}[. \tag{12}$$

Furthermore, since $e\left(s_k^+\right) = e\left(s_k^-\right)$, using (4b) one obtains $V\left(s_k^+\right) = e^\top\left(s_k^-\right)\, P^{\sigma\left(s_k^+\right)}\left(s_k - \tau\left(s_k^-\right)\right)\, e\left(s_k^-\right)$, $\forall k \in \mathbb{N}$. As $s_k - \tau\left(s_k^-\right) \in [0\,;\overline{\tau}]$, by a convexity argument, one can deduce from condition (8) that $P^{\sigma\left(s_k^+\right)}\left(s_k - \tau\left(s_k^-\right)\right) \prec \mu_s\, P^{\sigma\left(s_k^-\right)}\left(s_k - \tau\left(s_k^-\right)\right)$ and consequently, the relation

$$V\left(s_k^+\right) \leq \mu_s\, V\left(s_k^-\right), \quad \forall k \in \mathbb{N}, \tag{13}$$

holds. We will now establish a similar relation for the sampling times. Since $t_k^+ - \tau\left(t_k^+\right) = 0$ and $e\left(t_k^+\right) = e\left(t_k^-\right)$, using (4c), one can see that

$$V\left(t_k^+\right) = e^\top\left(t_k^-\right)\, \Omega_{\sigma(t_k)}^\top\, \left(P_0^{\sigma(t_k)}\right)^{-1}\, \Omega_{\sigma(t_k)}\, e\left(t_k^-\right)$$

where $\Omega_{\sigma(t_k)} = P_0^{\sigma(t_k)} - L_{\sigma(t_k)}\, C_{\sigma(t_k)}$. This implies that $V\left(t_k^+\right) - \underline{\mu}_c\, V\left(t_k^-\right) = -e\left(t_k^-\right) \widetilde{\Gamma}^{\sigma(t_k)}\, e(t_k)$ with $\widetilde{\Gamma}^{\sigma(t_k)} = \underline{\mu}_c P^{\sigma\left(t_k^-\right)}\left(t_k - \tau\left(t_k^-\right)\right) - \Omega_{\sigma(t_k)}^\top \left(P_0^{\sigma(t_k)}\right)^{-1} \Omega_{\sigma(t_k)}$. As $t_k - \tau\left(t_k^-\right) = t_k - t_{k-1} \in [\underline{\tau}\,;\overline{\tau}]$ (see Assumption 2) and $\sigma(t_k^-) = \sigma(t_k)$, then by also applying a convex argument and the Schur complement to condition (7), one gets that $\widetilde{\Gamma}^{\sigma(t_k)} \succ 0$. Therefore, $\forall k \in \mathbb{N}$, the relation $V\left(t_k^+\right) \leq \underline{\mu}_c\, V\left(t_k^-\right)$ holds for the sampling times. This relation and that established in (13) imply that

$$V\left(\bar{\mathbf{t}}_k^+\right) \leq \tilde{\boldsymbol{\mu}}_k\, V\left(\bar{\mathbf{t}}_k^-\right), \quad \forall k \in \mathbb{N} \tag{14}$$

where constants $\tilde{\boldsymbol{\mu}}_k$, $k \in \mathbb{N}$, are defined by:

$$\tilde{\boldsymbol{\mu}}_k = \begin{cases} \mu_s & \text{if} \ \ \bar{\mathbf{t}}_k \in \{s_k\}_{k\in\mathbb{N}}, \\ \underline{\mu}_c & \text{if} \ \ \bar{\mathbf{t}}_k \in \{t_k\}_{k\in\mathbb{N}}. \end{cases} \tag{15}$$

Now passing to the limit in (12) when τ tends to $\bar{\mathbf{t}}_{k+1}$ from below and using (14) yields:

$$V\left(\bar{\mathbf{t}}_{k+1}^-\right) \leq \tilde{\boldsymbol{\mu}}_k \exp\left(-\underline{\alpha}\, (\bar{\mathbf{t}}_{k+1} - \bar{\mathbf{t}}_k)\right) V\left(\bar{\mathbf{t}}_k^-\right),\ \forall k \in \mathbb{N}. \tag{16}$$

Now, let $t \in [0\,;\infty)$ and denote by $N(0,t)$, the number of sampling and switching times in the interval $[0\,;t)$, that is $N(0,t) = \mathrm{N}_s(0,t) + \mathrm{N}_c(0,t)$. We recall that $\mathrm{N}_s(0,t)$ and $\mathrm{N}_c(0,t)$ are the number of switching times and sampling times in $[0\,;t)$, respectively. By iterating the relation (16) for $k = 0, 1, \dots N(0,t) - 1$, one gets:

$$V\left(\bar{\mathbf{t}}_{N(o,t)}^-\right) \leq \left(\prod_{k=0}^{N(0,t)-1} \tilde{\boldsymbol{\mu}}_k\right) \exp\left(-\underline{\alpha} \bar{\mathbf{t}}_{N(0,t)}\right) V(0). \tag{17}$$

Furthermore, as $t \in \,]\bar{\mathbf{t}}_{N(0,t)}\,;\bar{\mathbf{t}}_{N(0,t)+1}[$, by setting $\tau = t$ and $k = \bar{\mathbf{t}}_{N(0,t)}$ in (12) and using (14), one obtains $V(t) \leq \tilde{\boldsymbol{\mu}}_{N(0,t)} \exp\left(-\underline{\alpha}\, \left(t - \bar{\mathbf{t}}_{N(0,t)}\right)\right) V\left(\bar{\mathbf{t}}_{N(0,t)}^-\right)$, which implies according to (17) that:

$$V(t) \leq \left(\prod_{k=0}^{N(0,t)} \tilde{\boldsymbol{\mu}}_k\right) \exp\left(-\underline{\alpha}\, t\right) V(0). \tag{18}$$

It follows from the definition (15) of constants $\tilde{\boldsymbol{\mu}}_k$ and the definition of $N(0,t)$ that $\prod_{k=0}^{N(0,t)} \tilde{\boldsymbol{\mu}}_k = \mu_s^{\mathrm{N}_s(0,t)}\, \underline{\mu}_c^{\mathrm{N}_c(0,t)}$. From Assumption 1, $\mathrm{N}_s(0,t) \leq N_0 + \frac{t}{\underline{s}}$ and from Remark 3, $\mathrm{N}_c(0,t) \geq \frac{t}{\overline{\tau}} - 1$. So, one has $\prod_{k=0}^{N(0,t)} \tilde{\boldsymbol{\mu}}_k \leq \frac{\mu_s^{N_0}}{\underline{\mu}_c} \exp\left\{\left(\frac{\ln \mu_s}{\underline{s}} + \frac{\ln \underline{\mu}_c}{\overline{\tau}}\right) t\right\}$. Therefore:

$$V(t) \leq \frac{\mu_s^{N_0}}{\underline{\mu}_c} \exp\left(-2\,\beta\, t\right) V(0). \tag{19}$$

where β is defined by (11). Moreover, one can see that:

$$V(0) \leq \theta_{\max} \|e(0)\|^2, \quad V(t) \geq \theta_{\min} \|e(t)\|^2, \tag{20}$$

with the constants $\theta_{\min} = \min_{i\in\mathscr{S}} \min_{\tau\in[0;\overline{\tau}]} \lambda_{\min}\left(P^i(\tau)\right)$ and $\theta_{\max} = \max_{i\in\mathscr{S}} \lambda_{\max}\left(P_0^i\right)$. In the definition of $\theta_{\min}$, the continuity of the eigenvalue function as well as the Weirestrass extreme value theorem guarantee the existence of $\min_{\tau\in[0;\overline{\tau}]} \lambda_{\min}\left(P^i(\tau)\right)$, $\forall i \in \mathscr{S}$. Finally, combining (19) and (20) yields $\|e(t)\| \leq \sqrt{\dfrac{\mu_s^{N_0}\,\theta_{\max}}{\underline{\mu}_c\,\theta_{\min}}} \exp(-\beta t) \|e(0)\|$. This concludes the proof.

Remark 9. Note that (9) gives a trade-off between the convergence rate in continuous time $\underline{\alpha}$, the discrete time convergence rate $\underline{\mu}_c$ introduced by G_σ, and the parameters μ_s that accounts for instability introduced by the switching. As it is the case for systems without switching (Raff and Allgower, 2007; Etienne et al., 2017), no subsystem A_i, $i \in \mathscr{S}$, has to be stable to ensure the converge of the observer. However in order to verify the LMIs conditions, every pair (A_i, C_i), $i \in \mathscr{S}$, must be detectable.

3.2 *Case of asynchronous switching signals*

We assume now that the active mode is not detected instantaneously but with a delay. Such an assumption is coherent with the fact that the output $y(t)$ is not continuously monitored. Therefore looking at σ as an output it is clear that some delay in the active mode reconstruction/measurement is reasonable. Considering x given by (1) and $\hat{x}$ defined by (2). We define in this case the extended state $z(t) = \left[x^\top(t)\ e^\top(t)\right]^\top$. We also define the observer based feedback $u(t) = K_{\hat{\sigma}(t)}\hat{x}(t)$. Then the following extended dynamical system is defined:

$$\begin{cases} \dot{z}(t) = \mathcal{A}_{\sigma\hat{\sigma}} z(t), \quad t \neq s_k, & \text{(21a)} \\ z(s_k^+) = z(s_k), & \text{(21b)} \\ z(t_k^+) = J_{\sigma(t_k)\hat{\sigma}(t_k)} z(t_k), & \text{(21c)} \end{cases}$$

where:

$$\mathcal{A}_{\sigma\hat{\sigma}} = \begin{bmatrix} A_\sigma + B_\sigma K_{\hat{\sigma}} & -B_\sigma K_{\hat{\sigma}} \\ A_{\sigma\hat{\sigma}} + B_{\sigma\hat{\sigma}} K_{\hat{\sigma}} & A_{\hat{\sigma}} - B_{\sigma\hat{\sigma}} K_{\hat{\sigma}} \end{bmatrix}, \tag{22a}$$

$$J_{\sigma\hat{\sigma}} = \begin{bmatrix} I & 0 \\ G_{\hat{\sigma}(t_k)} C_{\sigma\hat{\sigma}} & I - G_{\hat{\sigma}} C_{\hat{\sigma}} \end{bmatrix}, \tag{22b}$$

with:

$$A_{\sigma\hat{\sigma}} = A_\sigma - A_{\hat{\sigma}},\ B_{\sigma\hat{\sigma}} = B_\sigma - B_{\hat{\sigma}},\ C_{\sigma\hat{\sigma}} = C_\sigma - C_{\hat{\sigma}}. \tag{23}$$

Theorem 10. Consider the hybrid system (21) with switching times $\{s_k\}_{k\in\mathbb{N}}$ and sampling sequences $\{t_k\}_{k\in\mathbb{N}}$ satisfying Assumptions 1 and 2. If there exist symmetric matrices $Q_m^i \in \mathbb{S}_{2n}$, $m \in \{0,1\}$, $i \in \mathscr{S}$, and scalars $\underline{\alpha}$, $\overline{\alpha} \in \mathbb{R}$ and $\mu_s, \underline{\mu}_c, \overline{\mu}_c \in \mathbb{R}^+$ with $\mu_s, \overline{\mu}_c \geq 1$ and $\underline{\mu}_c \leq 1$ such that the following conditions are verified:

$$\forall i \in \mathscr{S},\ \forall \tilde{\tau} \in \{0, \overline{\tau}\}, \quad Q^i(\tilde{\tau}) \succ 0, \tag{24}$$

$$\forall i,j \in \mathscr{S},\ \forall \tilde{\tau} \in \{0,\overline{\tau}\},\ He\left(Q^i(\tilde{\tau})\ \mathcal{A}_{ji}\right) + Q_1^i + \alpha_{ij}\, Q^i(\tilde{\tau}) \prec 0, \tag{25}$$

$$\forall i,j \in \mathscr{S},\ \forall \tilde{\tau} \in \{\underline{\tau},\overline{\tau}\}, \quad \begin{bmatrix} \mu_{ij}\, Q^i(\tilde{\tau}) & Q_0^i J_{ji} \\ * & Q_0^i \end{bmatrix} \succ 0, \tag{26}$$

$$\forall i,j \in \mathscr{S},\ \forall \tilde{\tau} \in \{0,\overline{\tau}\},\ Q^i(\tilde{\tau}) \prec \mu_s Q^j(\tilde{\tau}), \tag{27}$$

$$-\underline{\alpha}[1 - \frac{\delta}{\underline{s}}] + \overline{\alpha}\frac{\delta}{\underline{s}} + \frac{ln\mu_s}{\underline{s}} + \frac{ln\underline{\mu}_c}{\overline{\tau}}[1 - \frac{\delta}{\underline{s}}] + \frac{\delta\, ln\overline{\mu}_c}{\underline{\tau}\,\underline{s}} < 0. \tag{28}$$

where:

$$Q^i(\tilde{\tau}) = Q_0^i + \tilde{\tau}\, Q_1^i, \tag{29}$$

and

$$(\alpha_{ij}, \mu_{ij}) = \begin{cases} (\underline{\alpha}, \underline{\mu}_c) \text{ if } i = j, \\ (-\overline{\alpha}, \overline{\mu}_c) \text{ if } i \neq j, \end{cases} \tag{30}$$

then the system (21) is GUES (i.e. the OBSAP is solved) with the convergence rate:

$$\beta = \frac{1}{2}\left(\underline{\alpha}[1 - \frac{\delta}{\underline{s}}] - \overline{\alpha}\frac{\delta}{\underline{s}} - \frac{ln\mu_s}{\underline{s}} - \frac{ln\underline{\mu}_c}{\overline{\tau}}[1 - \frac{\delta}{\underline{s}}] - \frac{\delta\, ln\overline{\mu}_c}{\underline{\tau}\,\underline{s}}\right) \tag{31}$$

Proof 2. Steps of the proof are similar to that followed in the proof of Theorem 8. The main difference lies in the treatment of existing delays in the active mode detection. In this case, we consider the piecewise continuous Lyapunov function candidate $V(t) = z^\top(t)\, Q^{\hat{\sigma}(t)}(t - \tau(t))\, z(t)$, $t \in [0;\infty)$ where $\tau(t) = \max\{t_k\colon t \geq t_k\}$ and $Q^{\hat{\sigma}(t)}(t - \tau(t)) = Q_0^{\hat{\sigma}(t)} + (t - \tau(t))\, Q_1^{\hat{\sigma}(t)}$. Note that condition (24) implies that V is positive definite. Let's also consider $\{\bar{\mathbf{t}}_k\}_{k\in\mathbb{N}}$, the strictly increasing sequence associated to the sequence $\{t_k\}_{k\in\mathbb{N}} \cup \{s_k\}_{k\in\mathbb{N}}$. Let $t \in\ ]\bar{\mathbf{t}}_k\,;\bar{\mathbf{t}}_{k+1}[$ with $k \in \mathbb{N}$.

One has $\dot{V}(t) = z^\top(t)\Upsilon^{\hat{\sigma}(t)} z(t)$ with $\Upsilon^{\hat{\sigma}(t)}$ the matrix defined by $\Upsilon^{\hat{\sigma}(t)} = He\left(Q^{\hat{\sigma}(t)}(t - \tau(t))\, \mathcal{A}_{\sigma\hat{\sigma}}\right) + Q_1^{\hat{\sigma}(t)}$. Using (25), one can verify that $\Upsilon^{\hat{\sigma}(t)} \prec -\alpha_{\hat{\sigma}(t)\,\sigma(t)}\, Q^{\hat{\sigma}(t)}(t - \tau(t))$ and consequently, $\dot{V}(t) \leq -\alpha_{\hat{\sigma}(t)\,\sigma(t)}\, V(t)$. Integrating this differential inequality and using the definition (30) of constant $\alpha_{\hat{\sigma}(t)\,\sigma(t)}$, one gets:

$$V(t) \leq \exp\{-\underline{\alpha}\, M_{atch}(\bar{\mathbf{t}}_k, \tau) + \overline{\alpha}\, M_{atch}^{iss}(\bar{\mathbf{t}}_k, \tau)\}\, V(\bar{\mathbf{t}}_k^+)$$

where $M_{atch}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \mathbb{1}_{\sigma(s)=\hat{\sigma}(s)} \mathrm{d}s$ and $M_{atch}^{iss}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \mathbb{1}_{\sigma(s)\neq\hat{\sigma}(s)} \mathrm{d}s$, $\forall \tau_1, \tau_2 \in [0;\infty)$ with $\mathbb{1}$, the indicator function. Since $M_{atch}(\tau_1, \tau_2) = \tau_2 - \tau_1 - M_{atch}^{iss}(\tau_1, \tau_2)$, then

$$V(\tau) \leq \exp\{\gamma(\bar{\mathbf{t}}_k, \tau)\}\, V(\bar{\mathbf{t}}_k^+), \quad \forall \tau \in\ ]\bar{\mathbf{t}}_k\,;\bar{\mathbf{t}}_{k+1}[\tag{32}$$

where the notation:

$$\gamma(\tau_1, \tau_2) = -\underline{\alpha}(\tau_2 - \tau_1) + (\underline{\alpha} + \overline{\alpha})\ M_{atch}^{iss}(\tau_1, \tau_2) \tag{33}$$

is introduced for simplicity in the presentation. Note that $M_{atch}^{iss}(\tau_1, \tau_2)$ satisfies:

$$M_{atch}^{iss}(\tau_1, \tau_2) \leq \delta\, \mathrm{N}_s(\tau_1, \tau_2) \leq \delta\left(N_0 + \frac{\tau_2 - \tau_1}{\underline{s}}\right). \tag{34}$$

Now, we will also determine in this case an upper bound of $V(\bar{\mathbf{t}}_k^+)$ similar to that obtained in (14). For the switching times, one can see that $V(s_k^+) = z^\top(s_k^-) Q^{\hat{\sigma}(s_k^+)}(s_k - \tau(s_k^-)) z(s_k^-)$. Moreover, it follows from (27) that $Q^{\hat{\sigma}(s_k^+)}(s_k - \tau(s_k^-)) \prec \mu_s Q^{\hat{\sigma}(s_k^-)}(s_k - \tau(s_k^-))$. Consequently:

$$V(s_k^+) \leq \mu_s\, V(s_k^-), \quad \forall k \in \mathbb{N}. \tag{35}$$

For the sampling times, using (21c), one can verify that $V(t_k^+) - \mu_{\hat{\sigma}(t_k^-)\sigma(t_k^-)}\, V(t_k^-) = -z^\top(t_k^-)\, \widetilde{\Upsilon}^{\hat{\sigma}(t_k)}\, z(t_k^-)$ with

$\widetilde{\Upsilon}^{\hat{\sigma}(t_k)}$ defined by $\widetilde{\Upsilon}^{\hat{\sigma}(t_k)} = \mu_{\hat{\sigma}(t_k^-)\sigma(t_k^-)} Q^{\hat{\sigma}(t_k^-)}(t_k - t_{k-1}) - J^{\top}_{\hat{\sigma}(t_k^-)} Q^{\hat{\sigma}(t_k^-)}(0) J_{\hat{\sigma}(t_k^-)}$. Applying the Schur complement to (26), one can show that $\widetilde{\Upsilon}^{\hat{\sigma}(t_k)}$ is positive definite and, consequently, $V(t_k^+) \leq \mu_{\hat{\sigma}(t_k^-)\sigma(t_k^-)} V(t_k^-)$, $\forall k \in \mathbb{N}$. This inequality and (35) imply that

$$V(\bar{\mathbf{t}}_k^+) \leq \tilde{\boldsymbol{\mu}}_k V(\bar{\mathbf{t}}_k^-), \quad \forall k \in \mathbb{N}, \tag{36}$$

where, in this case, $\tilde{\boldsymbol{\mu}}_k$ is the scalar defined by:

$$\tilde{\boldsymbol{\mu}}_k = \begin{cases} \mu_s & \text{if } \bar{\mathbf{t}}_k \in \{s_k\}_{k\in\mathbb{N}}, \\ \underline{\mu}_c & \text{if } \bar{\mathbf{t}}_k \in \{t_k\}_{k\in\mathbb{N}} \text{ and } \sigma(\bar{\mathbf{t}}_k) = \hat{\sigma}(\bar{\mathbf{t}}_k), \\ \overline{\mu}_c & \text{if } \bar{\mathbf{t}}_k \in \{t_k\}_{k\in\mathbb{N}} \text{ and } \sigma(\bar{\mathbf{t}}_k) \neq \hat{\sigma}(\bar{\mathbf{t}}_k). \end{cases} \tag{37}$$

By passing to the limit in (32) when τ tends to $\bar{\mathbf{t}}_{k+1}$ from below and using (36), one obtains:

$$V\left(\bar{\mathbf{t}}_{k+1}^-\right) \leq \tilde{\boldsymbol{\mu}}_k \exp\{\gamma\left(\bar{\mathbf{t}}_k, \bar{\mathbf{t}}_{k+1}\right)\} V\left(\bar{\mathbf{t}}_k^-\right), \quad \forall k \in \mathbb{N}. \tag{38}$$

Now, let $t \in [0\,;\infty)$ and denote by $N(0,t)$ the number of sampling times and switching times in $[0\,;t)$, i.e. $N(0,t) = \mathrm{N}_s(0,t) + \mathrm{N}_c(0,t)$. Similarly to the proof of (18), by iterating relation (38) for $k = 0,1,\ldots,N(0,t)$, and using after (32) and (36) rewritten for $\tau = t$ and $k = \bar{\mathbf{t}}_{N(0,t)}$, one gets:

$$V(t) \leq \left(\prod_{k=0}^{N(0,t)} \tilde{\boldsymbol{\mu}}_k\right) \exp\{\gamma(0,t)\} V(0)\,. \tag{39}$$

Let $\mathrm{N}_c^{match}(0,t)$ be the number of sampling times t_k in $[0\,;t)$ with no error in the estimation of the switching signal (i.e. $\sigma(t_k) = \hat{\sigma}(t_k)$) and let $\mathrm{N}_c^{\overline{match}}(0,t) = \mathrm{N}_c(0,t) - \mathrm{N}_c^{match}(0,t)$. It follows from the definition (37) of $\tilde{\boldsymbol{\mu}}_k$, that $\prod_{k=0}^{N(0,t)} \tilde{\boldsymbol{\mu}}_k = \mu_s^{\mathrm{N}_s(0,t)}\, \underline{\mu}_c^{\mathrm{N}_c^{match}(0,t)}\, \overline{\mu}_c^{\mathrm{N}_c^{\overline{match}}(0,t)}$. Since $\mathrm{N}_s(0,t) \leq N_0 + \frac{t}{\underline{s}}$, $\mathrm{N}_c^{\overline{match}}(0,t) \leq \frac{M_{\overline{atch}}^{iss}(0,t)}{\underline{\tau}} \leq \frac{\delta}{\underline{\tau}}\left(N_0 + \frac{t}{\underline{s}}\right)$ and $\mathrm{N}_c^{match}(0,t) \geq \frac{t - M_{\overline{atch}}^{iss}(0,t)}{\overline{\tau}} \geq \frac{1}{\overline{\tau}}\left[t\left(1 - \frac{\delta}{\underline{s}}\right) - \delta\, N_0\right]$, then:

$$\prod_{k=0}^{N(0,t)} \tilde{\boldsymbol{\mu}}_k \leq \mu_s^{N_0}\, \overline{\mu}_c^{\frac{N_0\,\delta}{\underline{\tau}}}\, \underline{\mu}_c^{-\frac{\delta\, N_0}{\overline{\tau}}} \exp\{\tilde{\beta}\, t\}, \tag{40}$$

where $\tilde{\beta} = \frac{ln\underline{\mu}_c}{\overline{\tau}}[1 - \frac{\delta}{\underline{s}}] + \frac{\delta\, ln\overline{\mu}_c}{\underline{\tau}\,\underline{s}} + \frac{ln\mu_s}{\underline{s}}$. Furthermore, from (34) and (33),

$$\gamma(0,t) \leq (\underline{\alpha} + \overline{\alpha})\; \delta\, N_0 + t\left[-\left(1 - \frac{\delta}{\underline{s}}\right)\underline{\alpha} + \overline{\alpha}\frac{\delta}{\underline{s}}\right].$$

Then combining (39) and (40) gives the inequality $V(t) \leq M \exp\{-2\,\beta\, t\}\, V(0)$, with β the constant introduced in (31) and $\tilde{M}$ the constant defined by $\tilde{M} = \exp\{(\underline{\alpha} + \overline{\alpha})\; \delta\, N_0\}\, \mu_s^{N_0}\, \overline{\mu}_c^{\frac{N_0\,\delta}{\underline{\tau}}}\, \underline{\mu}_c^{-\frac{\delta\, N_0}{\overline{\tau}}}$. Finally, using the relations $V(0) \leq \bar{\theta}_{\max} \|z(0)\|^2$ and $V(t) \geq \bar{\theta}_{\min} \|z(t)\|^2$ with constants $\bar{\theta}_{\max} = \max_{i\in\mathscr{S}} \lambda_{\max}\left(Q_0^i\right)$ and $\bar{\theta}_{\min} = \min_{i\in\mathscr{S}} \min_{\tau\in[0;\overline{\tau}]} \lambda_{\min}\left(Q^i(\tau)\right)$, one obtains

$$\|z(t)\| \leq \sqrt{\frac{\tilde{M}\,\bar{\theta}_{\max}}{\bar{\theta}_{\min}}} \exp\{-\beta\, t\}\, \|z(0)\|\,,$$

which concludes the proof.

Remark 11. The parameter (28) gives a trade off between admissible degradation of convergence of the Lyapunov functions due to both δ and μ_s and the convergence ensured by $\underline{\alpha}$ and $\underline{\mu}_c$. Interestingly, when $\delta = 0$ one recovers (9). Note that, while the existence of parameters $\underline{\alpha}$ and $\overline{\alpha}$ is classical for switched systems with delays in the active mode detection, the presence of $\overline{\mu}_c$ is not. This is due to the fact that this parameter accounts for impulsive observation when the system's and the observer's active modes mismatch.

Remark 12. The conditions given in both Theorems 8 and 10 amount to solve a set of bilinear matrix inequalities. This is in general a hard problem. However, by choosing *a priori* a small amount of parameters (namely $(\underline{\alpha}, \overline{\alpha}, \underline{\mu}_c, \overline{\mu}_c, \mu_s)$ for Theorem 10 when $\underline{\tau}$, $\overline{\tau}$ and $\underline{s}$ are given), the problem at hand becomes LMIs that can be solved for large scale systems. Furthermore, note that this number of parameters is independent on the dimension of the system.

Remark 13. Note that here the result obtained do not assume a link between t_k, s_k and $\hat{s}_k$ if one assume that the switching signal is transmitted along the output, simpler condition might be derived, furthermore if one assume that $C_\sigma = C$, then no attention has to be paid to potential error when triggering the impulsive observation and less conservative results shall be obtained as well.

4. SIMULATION RESULT

Consider the following academic example (also considered in Zhang and Gao, 2010; Zhang and Xiang, 2016; Etienne et al., 2018) given by:

$$A_1 = \begin{bmatrix} 0.2 & -0.5 \\ 0.5 & -0.3 \end{bmatrix}, B_1 = \begin{bmatrix} -0.4 \\ 1.8 \end{bmatrix}, C_1 = [1, 0],$$

$$A_2 = \begin{bmatrix} 0.2 & 0.3 \\ -1 & 0.4 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ 1.5 \end{bmatrix}, C_2 = [1, 0],$$

with the parameters $\underline{\tau} = 0.3$, $\overline{\tau} = 0.4$ and $\underline{s} = 7$.

4.1 Syntheses

In this part, with the parameters $\underline{\alpha} = 0.2$ and $\mu_s = 5$ and $\underline{\mu}_c = 0.98$, we synthesis an impulsive observer without delay in the active mode detection. As is clear in Theorem 8 it is not necessary for the system to be stable (stabilized Fig.1.a) for the observer to converge (Fig.1.b). Furthermore, the switching of the original system does not lead to instability of the observer. In Fig.1.c one can see that $t_{k+1} - t_k$ is randomly drown in the interval $[0.3., 0.4]$.

Analyse

Considering a known (emulation based) stabilizing controller $K_1 = [2.1124, -0.9336], K_2 = [-1.3111, -1.3158]$. Taking the same parameters as before with furthermore $\delta = 0.15$, $\overline{\mu}_c = 5$ and $\overline{\alpha} = 3$, we found that the condition of Theorem 10 are verified. We can see in Fig.2 that with the proposed controller and observer gains the overall extended system is asymptotically stable event with delay in the active mode detection.

5. CONCLUSION

In this work we have studied the impulsive observer synthesis problem for switched system with synchronous and

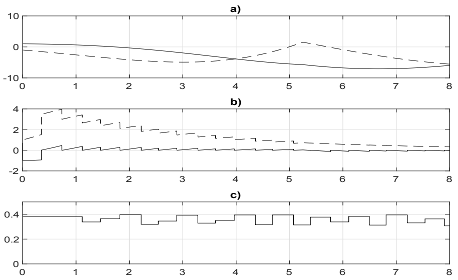

Fig. 1. a)Trajectory of the system. b) Trajectory of the observation error (plain line measured states dotted lined unmeasured state). c)Inter sampling time.

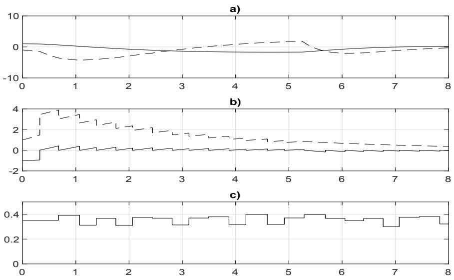

Fig. 2. a)Trajectory of the system. b) Trajectory of the observation error (plain line measured states dotted lined unmeasured state). c)Inter sampling time.

asynchronous switching signals. First impulsive observer gain synthesis have been obtained in the synchronous case. Then, an emulation based approach has been used to establish asymptotic stability of an observer based controller in the asynchronous situation. Both results are provided in terms of LMIs and allow for effective computation of the controller action or the convergence rate.

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