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Model Predictive Control with Forward-Looking Persistent Excitation

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Abstract: This work deals with the dual-control problem of simultaneous regulation and model parameter estimation in model predictive control. We propose an adaptive model predictive control which guarantees a persistently exciting closed loop sequence by only looking forward in time into the prediction horizon. Earlier works needed to look backwards and preserve prior regressor data. With the new approach, under the assumption of a known periodic persistently exciting reference trajectory around the equilibrium, we demonstrate exponential convergence of nonlinear systems under the influence of the adaptive model predictive control combined with a recursive least squares identifier with forgetting factor despite bounded noise. The results are, at this stage, local in state and parameter estimate space.

Keywords: Adaptive control, recursive least squares, closed-loop identification, model predictive control, persistence of excitation

1. INTRODUCTION

This paper revolves around a model predictive control (MPC) framework satisfying conditions for closed loop identification. If the system is such that the control input influences both the system state and its uncertainty (e.g. in the form of the covariance of the parameter or state estimate error), then the control inherits a dual function, see Feldbaum (1960-1961). In this case, on the one hand, the control objective involves the desire for regulation or trajectory tracking and hence a steady or slowly varying state. On the other hand, for identification purposes, to reduce the uncertainty and extract more information from the measurement the system is to be excited, see e.g. Bar-Shalom (1981). Anderson (1985), Mareels and Polderman (1996) and Brüggemann and Bitmead (2019) show that insufficient excitation may result in bursts of oscillatory behavior through parameter drift, singularity in the information matrix and an unobservable state even in the scalar case. Hence, the associated dual feedback control acts as an arbitrator between these antagonistic requirements. An overview of such dual problems can be found in Filatov and Unbehauen (2000).

Even though MPC is a widely used technique applied in various industries (e.g. Qin and Badgwell (2003)) the survey by Mayne (2014) points out that the field of adaptive MPC which relates to the dual problem has attracted relatively little interest in the controls community. Yet, the idea of using an MPC to fulfil the role of an arbitrator has been proposed in different publications. One common approach is to impose additional input constraints on the solution of the corresponding optimization problem. In this fashion, by including past information and thus *looking backwards* in time, the control directly ensures persistence of excitation of the initial step of the MPC solution. For instance, in Genceli and Nikolaou (1996), to identify FIR models and drive the related system to a given set point, additional periodic input constraints guarantee a periodic persistently exciting (PE) feedback control. In this way, past and future inputs are evaluated within the optimization. Similarly, Lu et al. (2019) propose a robust tube-based MPC for linear uncertain systems with an additional constraint to provide persistence of excitation. Though, the closed loop is not guaranteed to be PE. Instead of constraining the entire minimizng control sequence, Marafioti et al. (2014) suggest a *backward looking* memory-based MPC which only constrains the first control input as it is the only element of the sequence which is actually applied. The control strategy is analyzed for FIR and ARMA models. Feasibility and persistence of excitation can be guaranteed if, among other conditions, the initial control sequence is PE. In a backward looking fashion, the work by Larsson et al. (2015) takes into account the Fisher information matrix generated by past information as a further constraint for the optimization problem and focuses on the actual implementation.

Instead of modifying the constraints to achieve excitation, a number of authors (Hovd and Bitmead (2004), Heirung et al. (2015), Heirung et al. (2017)) adapt the cost function in that it also contains the parameter error covariance matrix as a proxy for uncertainty. In this way, the control is *looking forward* to seek persistence of excitation, although in the light of the MPC's receding horizon implementaion a PE property of the closed loop is not immediate. The article by Tanaskovic et al. (2014) takes a different path and splits the dual problem into two. Firstly, a nominal MPC ensures that the constraints hold for any element of a set of possible FIR models. Then, the second stage exhibits an exciting property by solving a optimization with the objective to reduce the size of the set. The idea of optimally selecting a model based on measurements is also pursued in Heirung et al. (2019) where the cost function incorporates an additional risk of choosing an incorrect model.

In this work, under the assumptions of full state feedback, no constraints and a periodic PE reference trajectory in the neighborhood of the equilibrium, rather than looking back using past information as done in Genceli and Nikolaou (1996) and Marafioti et al. (2014), we reformulate the requirement for a PE input as a *forward looking* condition on the reference trajectory and still guarantee the PE property of the closed loop driven by the MPC. The main contribution of this work is hence that persistence of excitation is guaranteed by solely looking forward in time despite the MPC's peculiarity of a receding horizon implementation. Further, the optimization problem solved online as part of the MPC framework neither complicates nor alters. Instead, the additional computation is moved offline. Moreover, exponential convergence of the closed loop as well as the parameter estimate is ensured for nonlinear systems of which the full state is available. Note that the strong assumption of a periodic PE reference trajectory is removed in Brüggemann and Bitmead (2020b) which is under review as an extension of this work.

The exposition of these results is structured as follows. After defining the system and posing the problem we recall the MPC from Köhler et al. (2018) and pick a reachable periodic reference trajectory in the neighborhood of the origin which is PE. For the actual parameter being known we show that given a continuity assumption on the plant and the cost function, if the initial state lies within a neighborhood of the PE reference trajectory at time zero the resulting closed loop is still PE. Then, it is demonstrated that if the initial estimate is close to the actual parameter the corresponding closed loop still establishes a PE closed loop sequence. This result is then used for a recursive least squares algorithm with forgetting factor for which exponential convergence is established. We concludingly present a simulation example.

2. PROBLEM FORMULATION

Let the system be

$$x_{k+1} = f(x_k, u_k) + w_k, \tag{1}$$

with $x_k \in \mathbb{R}^n$ being the state, $u_k \in \mathbb{R}^m$ the input and $w_k \in \mathbb{R}^n$ the disturbance at time k. Suppose the following.

Assumption 1. $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is twice continuously differentiable. △

Moreover, the state is accessible for measurement and for some $\bar{w} > 0$ the disturbance satisfies

$$|w_k| < \bar{w}. \tag{2}$$

Assume further that some system parameters $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_S]^\top \in \mathbb{R}^S$ are unknown. By assuming that f is linear in those θ, one may reformulate

$$f(x_k, u_k,) = f_0(x_k, u_k) + \sum_{i=1}^{S} \theta_i f_i(x_k, u_k),$$

where $f_i : \mathbb{R}^n \times R^m \to \mathbb{R}^n$ are basis functions so that the state recursion in (1) can be written as

$$x_{k+1} = f_0(x_k, u_k) + \varphi_k^T \theta + w_k, \tag{3}$$

where the regressor

$$\varphi_k^\top = [f_1(x_k, u_k) \ f_2(x_k, u_k) \ \dots \ f_S(x_k, u_k)] \in \mathbb{R}^{n \times S}.$$

In order to present concisely the main problem to be solved we first require a definition of a PE sequence.

Definition 2. The sequence $\{x_k, u_k\}$ is said to be persistently exciting (PE) if for some constant M and all j there exist positive constants α and β such that

$$0 < \alpha I \leq \sum_{i=j}^{j+M} \varphi_i \varphi_i^\top \leq \beta I < \infty.$$

△

The concise formulation of the main problem follows.

Problem 3. Regulate the state x_k in (1) while guaranteeing that the closed loop sequence $\{x_k, u_k\}$ is PE. △

In order to solve this problem we subsequently present an MPC to track a periodic reference trajectory around the origin rendering the closed loop sequence PE, while a recursive least squares identifier with forgetting factor ensures an accurate parameter estimate of θ.

3. PRELIMINARIES

The preliminary results are arranged as follows. After introducing assumptions on the existence of a reachable periodic PE reference trajectory we present the MPC framework based on the nonlinear MPC in Köhler et al. (2018) using the notion of incremental stability. Hereby, we include an assumption on the solution of the corresponding optimization problem which entails a continuous feedback law. The section concludes with an assumption on the system being incrementally stabilizable.

3.1 Reachable periodic PE reference trajectory

Define the following feasible trajectory and consider the related assumptions for system (1).

Definition 4. A sequence $\{x_r(k), u_r(k)\}$ for system (1) with $w_k = 0$ is said to be reachable period-M if it satisfies

$$x_r(k+1) = f(x_r(k), u_r(k)),$$
$$x_r((k+1)M) = x_r(kM),$$
$$u_r((k+1)M) = u_r(kM),$$

for all k. △

For ease of notation, this period, M, coincides with that in Definition 2 for persistence of excitation.

Assumption 5. A reachable period-M sequence $\{x_r(k), u_r(k)\}$ exists for system (1) with $w_k = 0$. △

Assumption 6. The particular reachable period M sequence $\{x_r(k), u_r(k)\}$ is PE. △

Remark 7. Note that Assumption 5 is an algebraic condition on $\{x_r(\cdot), u_r(\cdot), \theta\}$, which, subject to the conditions of the Implicit Function Theorem (Rudin (1986)), yields $\{x_r(\cdot)\}$ as a continuous function of $\{u_r(\cdot), \theta\}$. This analysis is under review, see Brüggemann and Bitmead (2020b) in which we replace both Assumptions 5 and 6. △

3.2 The model predictive control framework

We apply the trajectory in Assumption 6 to an MPC problem with related finite-horizon cost function

$$J_N(x_k, u_{\cdot|k}, k) = \sum_{i=0}^{N-1} l(x_{i|k}, u_{i|k}, k),$$

where $x_{i|k}$ represents the state prediction at time instant $k+i$ given the current state x_k. The control input $u_{i|k}$ is denoted accordingly, so that the control sequence $u_{\cdot|k} = \{u_{i|k}\}_{i=0}^{N-1}$. The running cost is defined as

$$l(x_{i|k}, u_{i|k}, k) = |x_{i|k} - x_r(k+i)|_Q^2 + |u_{i|k} - u_r(k+i)|_R^2,$$

where $Q = Q^\top > 0, R = R^\top > 0$. The MPC framework solves the optimization problem

$$\begin{aligned} V_N(x_k, k) = \min_{u_{\cdot|k}} & J_N(x_k, u_{\cdot|k}, k) \quad (4)\\ s.t. \quad & x_{0|k} = x_k, \\ & x_{i+1|k} = f(x_{i|k}, u_{i|k}) \end{aligned}$$

at every time instant k and applies the first control input $\hat{u}^\star_{0|k}$ of the minimizing sequence $\hat{u}^\star_{\cdot|k}$ to the system in (1).

Remark 8. For clarity in our development, we do not include state or input constraints in our formulation here. They can be added within the local stability framework, c.f. Köhler et al. (2018), but would require tracking their associated assumptions connected with evolution within the interior of the feasible set. △

In order to render the closed loop convergent to the given reference trajectory and thus induce persistence of excitation a continuous feedback law as well as a stabilizability condition on the system are required.

3.3 Continuous feedback law

Similarly to Mayne and Michalska (1990), we therefore assume the Hessian matrix of the cost function to be positive definite.

Assumption 9. The minimizing control sequence $u^\star_{\cdot|k}$ satisfies

$$\left.\frac{\partial^2 J_N(x_k, u_{\cdot|k}, k)}{\partial u_{\cdot|k}^2}\right|_{x_r(k), u^\star_{\cdot|k}, k} > 0. \quad (5)$$

△

Remark 10. As an explanatory remark on the notation used in Assumption 9 above, the left hand side of (5) reads out as the second partial derivative of J_N with respect to the sequence $u_{\cdot|k}$ holding x_k and k constant, and evaluated at $x_r(k), u^\star_{\cdot|k}$ and k. △

The continuity property of the feedback control generated by the MPC follows.

Lemma 11. Under Assumption 1 and 9 the feedback control $u^\star_{0|k}$ related to the MPC in (4) is continuous in θ and x_k for a neighborhood of $(x_r(k), \theta)$. △

Proof. By Assumption 1, J_N is twice continuous differentiable. Then, with Assumption 9, continuity follows from (Johansen, 2011, Theorem 5.1). □

Lemma 11 ensures that a small change in the parameter or state results only in a small change in the generated control sequence. This relation is essential for the local analysis in the next section.

3.4 Local incremental stabilizability

The next assumption describes a local incremental stabilizability condition, which is similar to local exponential stabilizability around a given reference trajectory.

Assumption 12. (Köhler et al., 2019, Assumption 1) There exist a control law $\kappa : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$, a δ-Lyapunov function $V_\delta : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_{\geq 0}$ that is continuous in the first argument and satisfies $V_\delta(x', x', u') = 0 \ \forall \ (x', u')$, and parameters $c_{\delta,l}, c_{\delta,u}, \delta_{loc}, k_{max} > 0, \rho \in (0,1)$, such that the following properties hold for all (x, x', u') with $V_\delta(x, x', u') \leq \delta_{loc}$:

$$\begin{aligned} c_{\delta,l}|x - x'|^2 \leq V_\delta(x, x', u') &\leq c_{\delta,u}|x - x'|^2 \\ |\kappa(x, x', u') - u'| &\leq k_{max}|x - x'| \\ V_\delta(x^+, x'^+, u'^+) &\leq \rho V_\delta(x, x', u'), \end{aligned}$$

where $x^+ = f(x, \kappa(x, x', u'))$ and $x'^+ = f(x', u')$. △

Note that neither the δ-Lyapunov function V_δ nor the control law κ is required for the implementation of the MPC but instead is used for the stability analysis. The exposition continues with the main results.

4. PERSISTENTLY EXCITING REFERENCE TRACKING WITH NOISE

The results on persistence of excitation of the closed loop in this section rely on practical stability of the tracking error. As a short intermezzo, we thus first adapt Köhler et al. (2018) related to state regulation to the case of trajectory tracking. Then, we analyze persistence of excitation of the closed loop sequence. It is shown that under the assumption of a given period-M PE reference trajectory, knowing the true parameter and a sufficiently small disturbance, the closed loop sequence is PE for all initial conditions x_0 within a neighborhood of the initial reference trajectory. Then, an equivalent guarantee is obtained when additionally the uncertain parameter lies within a neighborhood of the actual parameter.

4.1 Practically stable tracking error

The following lemma shows exponential convergence of the closed loop to a neighborhood of the reference trajectory, where the size of the neighborhood depends on the bound on the disturbance w_k.

Lemma 13. Suppose that Assumption 12 is satisfied. For any $c_x > 0$ there exist $\bar{w} > 0$ and a sufficiently large horizon N, such that for all initial conditions $|x_0 - x_r(0)| \leq c_x$ and all disturbances $|w_k| \leq \bar{w}$, the perturbed closed loop converges exponentially to the set $\mathcal{Z}_{RPI} := \{x_k - x_r(k) : V_N(x_k, k) \leq V_{RPI}(\bar{w}; N, c_x)\}$, where V_{RPI} is a $\mathcal{K}$-function in $\bar{w}$ which depends on N, c_x. △

Proof. This lemma is concise version of (Köhler et al., 2018, Theorem 8) related to state regulation applied to the case of reference tracking. The proof is analogous considering the δ-Lyapunov function from Assumption 12. □

4.2 Persistently exciting perturbed solution

Lemma 13 above establishes practical stability of the tracking error in the presence of a bounded disturbance. If the neighborhood of the reachable PE reference trajectory to which the closed loop converges is sufficiently small the corresponding closed loop is PE in finite time.

Lemma 14. Suppose Assumption 1, 5, 6, 9 and 12 hold and the parameter θ is known. Then, there exists a reachable PE reference trajectory and for any $c_x > 0$ there exist $\bar{w} > 0$ and a sufficiently large horizon N, such that for all $|x_0 - x_r(0)| \leq c_x$, $|w_k| \leq \bar{w}$ and horizon N, the closed loop sequence is PE for all $k \geq k_{PE}$ for some $k \in \mathbb{N}$. If $x_0 = x_{\bar{k}}$, where $\bar{k} \in \{k \geq k_{PE} \wedge k \mod M = 0\}$, then this holds for all k. △

Proof. The proof is based on continuity arguments and divided into three steps. Step I shows the existence of a PE sequence in the neighborhood of the PE reference trajectory. Step II relates $\mathcal{Z}_{RPI}$ to this neighborhood and step III proves the statement using Lemma 13.

Step I: By Assumption 1 and 9 (via Lemma 11), f and $u^\star_{0|k}$ are continuous. Thus, with Assumption 5 and 6 there exists a PE sequence $\{x_r(k), u_r(k)\}$, for which there exists a positive ϵ_{PE} such that $|x_r(k) - x_k| \leq \epsilon_{PE}$ implies $\{x_k, u^\star_{0|k}\}$ is PE.

Step II: Note that $V_N(x_k, k) < \epsilon$ implies that $|x_k - x_r(k)|^2_Q < \epsilon$ for all $\epsilon > 0$. Hence, by continuity of V_{RPI}, for any ϵ_{PE} from step I there exist ϵ_1 and $\bar{w}_1 > 0$ such that $\min_{\bar{x}\in\mathcal{Z}_{RPI}} |x - \bar{x}|^2_Q < \epsilon_1$ implies that $|x - x_r(k)| < \epsilon_{PE}$.

Step III: For any $c_x > 0$, let N and $\bar{w}_2$ satisfy Lemma 13 (requiring Assumption 12), and define $\bar{w} = \min\{\bar{w}_1, \bar{w}_2\}$. The statement to be proven holds for $c_x, N, \bar{w}$ and k_{PE}, where k_{PE} is such that $\min_{\bar{x}\in\mathcal{Z}_{RPI}} |x_{k_{PE}} - \bar{x}|^2 < \epsilon_1$, by using step I and II. The related guarantee for all k is a direct consequence. □

Lemma 14 establishes that the closed loop sequence can be chosen as PE given a known parameter θ. As this feature is only of interest if the parameter θ is unknown and thus to be estimated we aim to establish similar results for the case of an estimate $\hat{\theta}$ within the neighborhood of the true parameter.

4.3 Persistently exciting perturbed uncertain solution

Theorem 15. Suppose Assumption 1, 5, 6, 9 and 12 hold. Let the control input be derived by the optimization problem in (4) with θ substituted by some $\hat{\theta}$. Then, for any reachable PE reference trajectory there exist $c_x, \bar{w}, c_\theta > 0$ and a sufficiently large horizon N such that for all $|x_0 - x_r(0)| \leq c_x$, $|w_k| \leq \bar{w}$ and $|\theta - \hat{\theta}| \leq c_\theta$, the closed loop sequence is PE for all $k \geq k_{PE}$. If $x_0 = x_{\bar{k}}$, where $\bar{k} \in \{k \geq k_{PE} \wedge k \mod M = 0\}$, then this holds for all k. △

Proof. Let $\hat{u}^\star_{0|k}$ be the MPC feedback control for some $\hat{\theta}_k$, and $u^\star_{0|k}$ that corresponding to θ. Then

$$\begin{aligned} x_{k+1} &= f(x_k, \hat{u}^\star_{0|k}) + w_k \\ &= f(x_k, u^\star_{0|k}) + \hat{w}_k, \end{aligned}$$

where

$$\hat{w}_k = \Big(f(x_k, \hat{u}^\star_{0|k}) - f(x_k, u^\star_{0|k})\Big) + w_k.$$

By continuity of the MPC feedback via Lemma 11 and continuity of f through Assumption 1, the disturbance $\hat{w}_k$ can be bounded by having $\hat{\theta}_k$ sufficiently close to θ for all k. Thus, the result follows from Lemma 14. □

Theorem 15 demonstrates that given sufficient assumptions the closed loop under the influence of the MPC delivers a PE closed loop sequence. It is now time to elaborate why persistence of excitation is desired and therefore introduce the estimation algorithm.

5. RECURSIVE LEAST SQUARES WITH FORGETTING FACTOR

In order to estimate the unknown parameter θ we select a recursive least squares algorithm with forgetting factor. Therefore, define

$$\tilde{x}_{k+1} = x_{k+1} - f_0(x_k, u_k),$$

and consider the corresponding recursive algorithm

$$\hat{\theta}_{k+1} = \hat{\theta}_k + P_{k-1}\varphi_k D_k^{-1}\left(\tilde{x}_{k+1} - \varphi_k^\top \hat{\theta}_k\right), \tag{6}$$

where $D_k = \lambda T + \varphi_k^\top P_{k-1}\varphi_k$ with $T = T^\top > 0 \in \mathbb{R}^{n\times n}$, and

$$P_{k+1} = \lambda^{-1}\Big(I - P_k\varphi_{k+1}D_{k+1}^{-1}\varphi_{k+1}^\top\Big)P_k, \tag{7}$$

where the forgetting factor $\lambda \in (0, 1)$ is constant and $P_{-1} \in \mathbb{R}^{S\times S}$ is symmetric positive definite. The matrix T is related to the weight associated with the prediction error of each element of the state, see the following lemma.

Lemma 16. The algorithm in (6) and (7) converges to the value θ which minimizes

$$\lambda^k|\hat{\theta}_0 - \theta|^2_{P_{-1}^{-1}} + \sum_{i=1}^{k} \lambda^{k-i}|\tilde{x}_i - \varphi_{i-1}^\top\theta|_{T^{-1}}.$$

. △

Proof. The proof is analogous to that of (Islam and Bernstein, 2019, Theorem 2) and hence omitted for brevity. □

We also wish to obtain convergence of the estimate to (a neighborhood of) the true parameter, i.e.

$$\tilde{\theta}_k = \theta - \hat{\theta}_k. \tag{8}$$

This is achieved by the next lemma whose sufficient condition underpins our desire for a PE closed loop. The result is an extension of Johnstone et al. (1982) adapted to multiple output systems.

Lemma 17. Suppose the sequence $\{x_k, u_k\}$ is PE and w_k satisfies (2). Then, for any initial condition $\tilde{\theta}_0$ the estimation error $\tilde{\theta}_k$ converges exponentially to a ball centered on θ with a radius proportional to the bound on w, i.e. for any $\tilde{\theta}_0$ there exist $\gamma_1, \gamma_2 > 0$ such that for all $k \geq M$

$$|\tilde{\theta}_k| \leq \gamma_1\lambda^{k/2}|\tilde{\theta}_0| + \gamma_2\frac{\lambda^{k/2} - 1}{\lambda^{1/2} - 1}\bar{w}.$$

△

Proof. It is shown in Johnstone et al. (1982) that the result holds for SISO systems and no disturbance. An equivalent result for the multiple output case is under review, see Brüggemann and Bitmead (2020a). Exponential convergence of the linear error dynamics implies BIBO stability, which gives the desired result. □

We have thus shown that under the assumption of a bounded disturbance and a PE sequence, the estimate converges exponentially to the actual parameter without noise, or in the case of a bounded disturbance, to a neighborhood whose size depends on the bound of the disturbance. Exponential convergence is decisive for the preservation of a PE closed loop sequence, as disclosed in the next section, where we combine results from this section and those of previous ones.

6. PERIODIC ADAPTIVE MODEL PREDICTIVE CONTROL

All the local results above share a common concept. That is, in a utopian world with suitable initial conditions, perfect knowledge of the uncertainty and under sufficient conditions a PE closed loop is guaranteed. Gradually watering down these conditions by contemplating sufficiently small neighborhoods has been shown not to affect the substance of the initial statement about the PE closed loop sequence; provided we carry along suitable smoothness and regularity assumptions. Consistent with this strategy, this section focuses on the the estimation error and its interplay with the neighborhoods introduced before. In this fashion, by noting that if the bound on the estimation error implies a neighborhood for which a PE closed loop sequence exists, then we achieve a PE closed loop sequence despite uncertainty.

6.1 Convergence under bounded noise

The following theorem states that under sufficient conditions, if the disturbance is bounded and the initial state and the initial parameter estimate are within a neighborhood of the periodic PE reference trajectory and the true parameter, respectively, then the estimation error and the closed loop tracking error exponentially converge to a neighborhood around the reference trajectory and the true parameter, respectively.

Theorem 18. Suppose Assumption 1, 5, 6, 9 and 12 hold. Let the control input be derived by the optimization problem in (4) with θ substituted by $\hat{\theta}_0$ for $k < M$ and $\hat{\theta}_k$ given by the recursion in (6) for $k \geq M$. Then, for any reachable PE reference trajectory there exist $c_{w,k}, c_\theta, \bar{w}$ and a sufficiently large horizon N such that for all $|x_0 - x_r(0)| \leq c_{w,k}$, $|w_k| \leq \bar{w}, |\tilde{\theta}_0| \leq c_\theta$

$$x_k - x_r(k) \to \mathcal{Z}_{RPI},$$
$$|\tilde{\theta}_k| \to \frac{\gamma_2}{1-\lambda^{1/2}}\bar{w},$$

as $k \to \infty$. △

Proof. Let $c_{x,1}, \bar{w}_1, c_{\theta,1} > 0$ such that Theorem 15 holds. Then, with Lemma 17, let $\bar{w}_2, c_{\theta,2} > 0$ such that $|\tilde{\theta}_0| \leq c_{\theta,2}$ and $|w_k| < \bar{w}_2$ imply $|\tilde{\theta}_k| \leq c_{\theta,1}$ for all $k \geq M$. By selecting $\bar{w} = \min\{\bar{w}_1, \bar{w}_2\}$ and $c_\theta = \min\{c_{\theta,1}, c_{\theta,2}\}$, the conclusion follows from Theorem 15 and Lemma 17. □

The convergence result for the uncertain and perturbed system is numerically demonstrated in the next section for a non-infinitesimal neighborhood of initial conditions about their nominal values. Observe that the aforementioned theorem relies on a periodic PE reference trajectory, also depending on the uncertain parameter. By continuity arguments, an equivalent statement holds for a periodic PE reference generated with an initial parameter estimate in a sufficiently small neighborhood of the true parameter. However, persistence of excitation and feasibility of the reference trajectory is generally not ensured for all initial estimates which may deviate substantially from the true parameter. The same obstacle may occur if the reference trajectory is updated online using the current estimate. Furthermore, note that the convergence result is only local with respect to the uncertain parameter and the initial state.

7. SIMULATION EXAMPLE

Consider the nonlinear scalar system from Hovd and Bitmead (2004),

$$x_{k+1} = f_0(x_k, u_k) + \varphi^\top \theta + w_k,$$

where

$$\begin{aligned} f_0(x_k, u_k) &= u_k, \\ \theta &= [\theta_1 \;\; \theta_2]^\top, \\ \varphi_k &= [f_1(x_k) \;\; f_2(x_k, u_k)]^\top, \end{aligned} \tag{9}$$

with

$$f_1(x_k) = x_k, \quad f_2(x_k, u_k) = x_k u_k. \tag{10}$$

The parameters $\theta_1 = 1$ and $\theta_2 = 0.1$ are unknown and the noise $|w_k| \leq 0.07$, uniformly distributed. The main objective is to regulate the state which presumes an accurate estimate of the unknown parameter θ. We therefore generate a reachable periodic PE reference trajectory around the origin and use the MPC in (4) with an estimate given by the recursive least squares in (6) - (7) with a forgetting factor $\lambda = 0.7$ and weight $T = 1$. Assumption 1 clearly holds. In order to satisfy Assumption 5 and 6, we generate a reference trajectory by solving

$$\begin{aligned} (\bar{x}_r, \bar{u}_r) = \underset{\substack{\{x_0,x_1,\ldots,x_{N-1}\} \\ \{u_0,u_1,\ldots,u_{N-1}\}}}{\arg\min} \; & \frac{1}{N}\sum_{i=0}^{N-1} |x_{i|k}|_Q^2 + |u_{i|k}|_R^2 \\ s.t.\; x_{k+1} &= f(x_k, u_k), \\ x_N &= x_0, \\ \alpha I &\leq \sum_{i=0}^{N-1} \varphi_i^\top \varphi_i \leq \beta I, \end{aligned}$$

with $\alpha = 0.1, \beta = 0.3$ and $Q = 6, R = 0.1$ for J_N, and defining

$$x_r(k) = \bar{x}_r(k \mod 2)$$
$$u_r(k) = \bar{u}_r(k \mod 2).$$

We chose $N = 2$ since it results in a low cost relative to other small integer values. The solution of the minimization problem guarantees a PE period-2 reference trajectory. It also ensures that the reference values are in the neighborhood of the origin to which we want to steer the system. The optimization problem is solved within seconds on a regular laptop, by the interior point algorithm in MATLAB. Next, Assumption 12 holds by letting

$$\begin{aligned} \kappa(x_k, x_r(k), u_r(k)) = \frac{1}{\theta_2 x_k + 1} & K(x_k - x_r(k)) \\ & + u_r(k)(\theta_2 x_r(k) + 1), \\ V_\delta = |x_k - x_r(k)|_P^2&, \end{aligned}$$

where P and K relate to the discrete-time infinite-horizon linear quadratic regulator using common notation. Lastly, Assumption 9 is numerically verified.

The figures below are based on an initial estimate $\hat{\theta}_0 = [1.5 \ \ -0.4]^\top$. Figure 1 depicts a fast convergence of the closed loop to the reference trajectory despite noise and parameter uncertainties. The effect of the noise on the parameter estimate is more significant, as illustrated in Figure 2. Although the estimate converges to a ball centered on the true parameter, the estimates continuously move around within this ball. A similar pattern can be observed in Figure 3 in which the norm of the estimation error is plotted. The estimation error converges exponentially to a neighborhood of the origin and remains there.

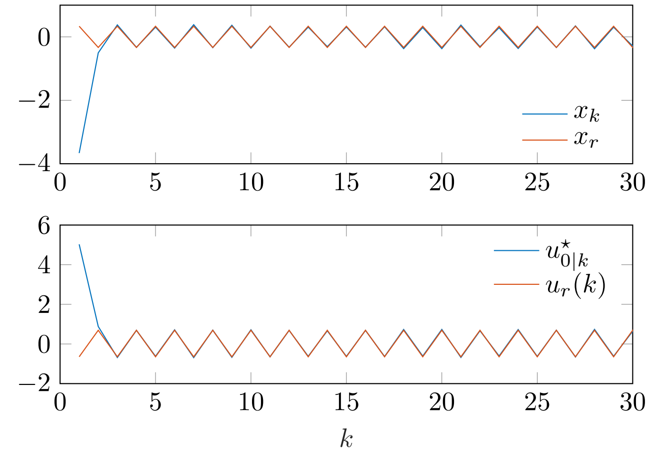

Fig. 1. Closed loop with bounded noise $|w_k| \le 0.07$ versus PE reference trajectory

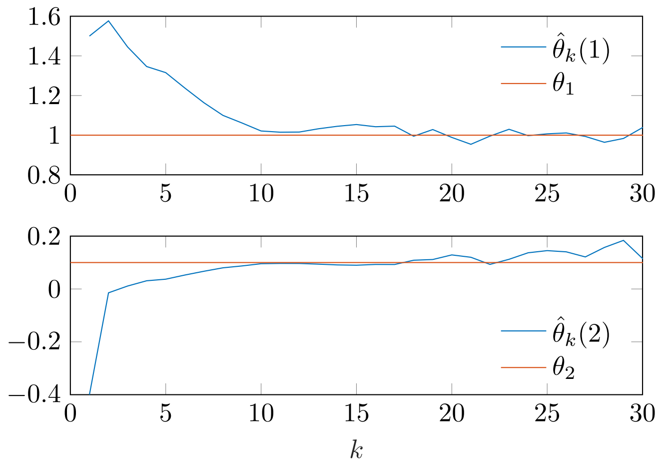

Fig. 2. Parameter estimate given $|w_k| \le 0.07$.

As the analysis in the previous sections is of local nature, the convergence results are not expected to hold for a more potent noise or large deviations either of the initial state or initial estimate. Such a scenario is observed for a noise of $|w_k| < 0.42$, uniformly distributed. Figure 4 displays a closed loop that still follows the reference trajectory albeit the tracking error has increased. Consistent with the previous simulation, one sees in Figure 5 that the estimates are more affected and vastly vary without indicating converging tendencies. As we numerically verify that the closed loop is PE, the increased estimation error is regarded as a result of the noise and not of the tracking error.

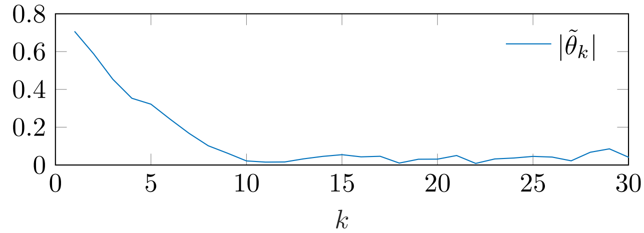

Fig. 3. Estimation error given $|w_k| \le 0.07$.

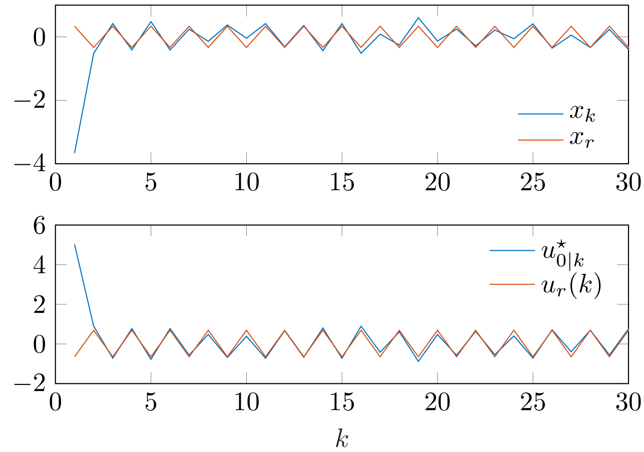

Fig. 4. Closed loop with $|w_k| \le 0.42$ and reference trajectory.

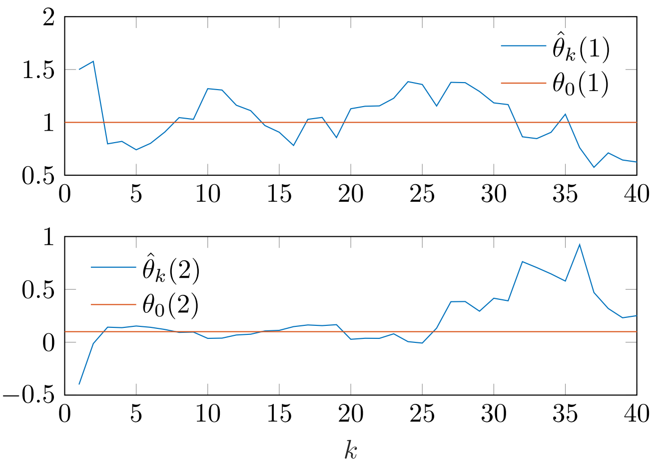

Fig. 5. Parameter estimate given $|w_k| \le 0.42$.

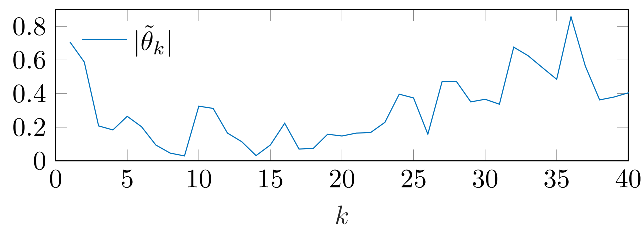

Fig. 6. Estimation error given $|w_k| \le 0.42$.

8. CONCLUSION

This work achieves a PE closed loop sequence by only *looking forward* in time given a known periodic PE reference trajectory, despite disturbances, uncertainties and the MPC's nature of a receding horizon implementation, without complicating the optimization problem solved online. The theory is supported by two simulation examples which illustrate the effect of noise on the closed loop performance and parameter estimation and thus go beyond our local analysis. The extension of our results will include an analysis of the construction of a periodic PE reference trajectory and is currently under review, see Brüggemann and Bitmead (2020b).

REFERENCES

Anderson, B.D. (1985). Adaptive systems, lack of persistency of excitation and bursting phenomena. *Automatica*, 21(3), 247 – 258. doi:10.1016/0005-1098(85)90058-5.

Bar-Shalom, Y. (1981). Stochastic dynamic programming: Caution and probing. *IEEE Transactions on Automatic Control*, 26(5), 1184–1195. doi:10.1109/TAC.1981.1102793.

Brüggemann, S. and Bitmead, R.R. (2019). Surrogate problems for tractable excitation management in stochastic mpc. In *2019 IEEE Conference on Decision and Control (CDC)*.

Brüggemann, S. and Bitmead, R.R. (2020a). Exponential convergence of recursive least squares with forgetting factor for multiple-output systems. *arXiv preprint*, arXiv:2003.07334 [math.OC].

Brüggemann, S. and Bitmead, R.R. (2020b). Model predictive control with forward-looking persistent excitation. *arXiv preprint*, arXiv:2004.01625 [eess.SY].

Feldbaum, A.A. (1960-1961). Dual control theory. I-IV. *Automation Remote Control*, 21,22.

Filatov, N.M. and Unbehauen, H. (2000). Survey of adaptive dual control methods. *IEE Proceedings - Control Theory and Applications*, 147(1), 118–128. doi:10.1049/ip-cta:20000107.

Genceli, H. and Nikolaou, M. (1996). New approach to constrained predictive control with simultaneous model identification. *AIChE Journal*, 42(10), 2857–2868. doi:10.1002/aic.690421015.

Heirung, T.A.N., Foss, B., and Ydstie, B.E. (2015). MPC-based dual control with online experiment design. *Journal of Process Control*, 32, 64 – 76. doi:10.1016/j.jprocont.2015.04.012.

Heirung, T.A.N., Santos, T.L., and Mesbah, A. (2019). Model predictive control with active learning for stochastic systems with structural model uncertainty: Online model discrimination. *Computers & Chemical Engineering*, 128, 128 – 140. doi:10.1016/j.compchemeng.2019.05.012.

Heirung, T.A.N., Ydstie, B.E., and Foss, B. (2017). Dual adaptive model predictive control. *Automatica*, 80, 340 – 348. doi:10.1016/j.automatica.2017.01.030.

Hovd, M. and Bitmead, R.R. (2004). Interaction between control and state estimation in nonlinear MPC. *IFAC Proceedings Volumes*, 37(9), 119 – 124. doi:10.1016/S1474-6670(17)31803-7. 7th IFAC Symposium on Dynamics and Control of Process Systems 2004 (DYCOPS -7), Cambridge, USA, 5-7 July, 2004.

Islam, S.A.U. and Bernstein, D.S. (2019). Recursive least squares for real-time implementation [lecture notes]. *IEEE Control Systems Magazine*, 39(3), 82–85. doi:10.1109/MCS.2019.2900788.

Johansen, T.A. (2011). Introduction to nonlinear model predictive control and moving horizon estimation. In M. Huba, S. Skogestad, M. Fikar, M. Hovd, T.A. Johansen, and B. Rohal'-Ilkiv (eds.), *Selected Topics on Constrained and Nonlinear Control*. STU Bratislava – NTNU Trondheim.

Johnstone, R.M., Johnson, C.R., Bitmead, R.R., and Anderson, B.D. (1982). Exponential convergence of recursive least squares with exponential forgetting factor. *Systems & Control Letters*, 2(2), 77 – 82. doi:10.1016/S0167-6911(82)80014-5.

Köhler, J., Müller, M.A., and Allgöwer, F. (2019). Nonlinear reference tracking: An economic model predictive control perspective. *IEEE Transactions on Automatic Control*, 64(1), 254–269. doi:10.1109/TAC.2018.2800789.

Köhler, J., Müller, M.A., and Allgöwer, F. (2018). A novel constraint tightening approach for nonlinear robust model predictive control. In *2018 Annual American Control Conference (ACC)*, 728–734. doi:10.23919/ACC.2018.8431892.

Larsson, C.A., Rojas, C.R., Bombois, X., and Hjalmarsson, H. (2015). Experimental evaluation of model predictive control with excitation (MPC-X) on an industrial depropanizer. *Journal of Process Control*, 31, 1 – 16. doi:10.1016/j.jprocont.2015.03.011.

Lu, X., Cannon, M., and Koksal-Rivet, D. (2019). Robust Adaptive Model Predictive Control: Performance and Parameter Estimation. *arXiv e-prints*, arXiv:1911.00865.

Marafioti, G., Bitmead, R.R., and Hovd, M. (2014). Persistently exciting model predictive control. *International Journal of Adaptive Control and Signal Processing*, 28(6), 536–552. doi:10.1002/acs.2414.

Mareels, I. and Polderman, J. (1996). *Adaptive Systems: An Introduction*. Birkhauser.

Mayne, D.Q. and Michalska, H. (1990). Receding horizon control of nonlinear systems. *IEEE Transactions on Automatic Control*, 35(7), 814–824. doi:10.1109/9.57020.

Mayne, D.Q. (2014). Model predictive control: Recent developments and future promise. *Automatica*, 50(12), 2967 – 2986. doi:10.1016/j.automatica.2014.10.128.

Qin, S. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7), 733 – 764. doi:10.1016/S0967-0661(02)00186-7.

Rudin, W. (1986). *Principles of Mathematical Analysis*. McGraw - Hill Book C.

Tanaskovic, M., Fagiano, L., Smith, R., and Morari, M. (2014). Adaptive receding horizon control for constrained mimo systems. *Automatica*, 50(12), 3019 – 3029. doi:10.1016/j.automatica.2014.10.036.