Fast interval estimation for discrete-time systems based on fixed-time convergence ⋆

Zhenhua Wang * **Thach Ngoc Dinh** ** **Qinghua Zhang** *** **Tarek Raïssi** ** **Yi Shen** *

* *School of Astronautics, Harbin Institute of Technology, Harbin 150001, China (e-mail: zhenhua.wang@hit.edu.cn).*
** *Conservatoire National des Arts et Métiers (CNAM), Cedric-Lab, Paris 75141, France (e-mail: ngoc-thach.dinh@lecnam.net, tarek.raissi@cnam.fr)*
*** *Univ. Gustave Eiffel, Inria, COSYS-SII, I4S, 35042 Rennes, France (e-mail: qinghua.zhang@inria.fr)*

Abstract: This paper studies interval estimation for discrete-time linear systems with unknown but bounded disturbance and measurement noise. Inspired by the well-known parity space approach in the field of fault diagnosis, we propose a fast interval estimation method with fixed-time convergence property. A singular value decomposition-based parameter optimization algorithm is used to attenuate the effect of uncertainties on the estimation error. Comparison study illustrates the superiority of the proposed method over existing technique.

Keywords: Interval estimation, discrete-time systems, fixed-time, optimization.

1. INTRODUCTION

Interval estimation is important in some applications such as model-predictive control and fault diagnosis (Bravo et al., 2006; Xu et al., 2013; Zhang & Yang , 2017). Although many state estimation methods have been proposed in the literature, most results on state estimation focus on point estimation. In practice, the accuracy of estimation is affected by the uncertainties such as unknown disturbance and measurement noise. Consequently, many robust methods have been proposed to attenuate the effect of uncertainties (Kalman , 1960; Xie & Souza , 1993; Zhang , 2002; Hammouri et al. , 2002). Among these methods, Kalman filtering and H_∞ observer design are two commonly used robust estimation techniques. In Kalman filtering, the uncertainties are assumed to be Gaussian noise, which may not be the case in practice. In the H_∞ design methods, it is assumed that the uncertainties are energy-bounded. However, most uncertainties are peak-bounded rather than energy-bounded. Compared with the Gaussian noise assumption in Kalman filtering and the energy-bounded assumption in H_∞ observer design, a more practical assumption is to consider that the uncertainties are unknown but bounded. Based on this assumption, interval estimation has been proposed and has received much attention in the past two decades (Gouze et al., 2000; Raïssi et al., 2012; Thabet et al., 2014; Wang et al., 2018).

Interval observer is a frequently used interval estimation method. An interval observer usually contains two traditional observers, one is the upper-bound observer and the other is the lower-bound observer. The interval observer is designed by constructing a stable and cooperative error dynamic system. Nevertheless, the design condition of interval observers is restrictive and limits the application scope of this technique. To relax the design condition of interval observers, interval observer design methods based on coordinate transformations have been proposed (Raïssi et al., 2012; Thabet et al., 2014). The coordinate transformations-based methods not only relax design constraints but also broaden the application scope. However, as pointed in Tang, Wang, Wang, Raïssi, & Shen (2019), the coordinate transformations may cause large conservatisms. Recently, Wang et al. have proposed a direct design method based on a new interval observer structure (Wang et al., 2018). Compared with the basic interval observer, the interval observer presented in Wang et al. (2018) has more degrees of design freedom, which can be optimized by robust design method to improve the estimation accuracy. Using the T-N-L observer structure proposed in Wang et al. (2018), Tang et al. have proposed a two-step interval estimation method based on zonotopic analysis (Tang, Wang, & Shen , 2019), which can obtain even better results than the interval observer presented in Wang et al. (2018).

Many state estimation methods usually consider the stability and robustness. Apart from stability and robustness, the property of fixed-time convergence is desired in some applications. Therefore, the fixed-time observer has attracted some attention (Engel & Kreisselmeier , 2002; Lopez-Ramirez et al. , 2016; Menard et al. , 2017; Rios & Teel , 2018). However, most existing results on fixed-time observers are studied in the continuous-time domain. To the best of our knowledge, only Zhang et al. (2019) and

⋆ This work was partially supported by National Natural Science Foundation of China (61973098, 61773145) and the fund from the National Defense Key Discipline of Space Exploration, Landing and Reentry in Harbin Institute of Technology under grant HIT.KLOF.2018.073.

Dinh et al. (2019) have considered fixed-time observer design for discrete-time systems. Zhang et al. (2019) proposes a fixed-time unknown input observer design method for discrete-time singular systems. Zhang et al. (2019) only considers disturbance that can be decoupled. In practice, however, most of disturbances and measurement noise cannot be decoupled. Dinh et al. (2019) studies fixed-time interval estimation for discrete-time systems with undecoupled disturbances. Note that the performance of the method proposed in Dinh et al. (2019) depends on the observer gain. Unfortunately, Dinh et al. (2019) does not provide a principle on how to optimize the parameter matrices.

In this paper, we propose a new fixed-time interval estimation method. The proposed method is inspired by the parity space approach, which is well-known in the field of fault diagnosis (Ding , 2008). The main advantage of the proposed method is that the effect of the uncertainties can be attenuated by parameter optimization. To the best of our knowledge, this is the first fixed-time estimation method that can attenuate the effect of uncertainties.

Notation. In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ stand for n and $m \times n$ dimensional real Euclidean space, respectively, I and 0 represent the identity matrix and zero matrix with appropriate dimensions, respectively, and $\mathrm{tr}(\cdot)$ and $\|\cdot\|_F$ are the trace operator and the Frobenius norm, respectively. For a matrix M, M^T denotes the transpose of M. If M is of full column rank, we use $M^\dagger$ to denote its generalized inverse $(M^TM)^{-1}M^T$ and use $M^\perp$ to represent its orthogonal complement $I - MM^\dagger$. In addition, the comparison operators $\geq$ and $\leq$ on vectors and matrices should be understood element-wise.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following system

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + D_1 d_k \\ y_k = Cx_k + D_2 d_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state vector, $u_k \in \mathbb{R}^{n_u}$ denotes the known input, $y_k \in \mathbb{R}^{n_y}$ is the measurement, and $d_k \in \mathbb{R}^{n_d}$ a vector concentrates the process disturbance and measurement noise.

The aim of this paper is to estimate the intervals of x_k. In the following, it is assumed that (C, A) is observable and A is invertible. In addition, d_k is assumed to be unknown but bounded as follows:

$$\underline{d} \leq d_k \leq \bar{d} \tag{2}$$

Without loss of generality, we can assume that $\underline{d} = -\bar{d}$. If this is not the case, we can rewrite d_k as

$$d_k = \frac{\bar{d} + \underline{d}}{2} + \tilde{d}_k$$

and treat $\frac{\bar{d} + \underline{d}}{2}$ as a known input. Consequently, the $\tilde{d}_k$ is an unknown input satisfying

$$-\frac{\bar{d} - \underline{d}}{2} \leq \tilde{d}_k \leq \frac{\bar{d} - \underline{d}}{2}.$$

Remark 1. If A is not invertible, we can rewrite the system in (1) as follows

$$\begin{cases} x_{k+1} = (A - LC)x_k + Bu_k + D_1 d_k + Ly_k - LD_2 d_k \\ y_k = Cx_k + D_2 d_k \end{cases}$$

which is equivalent to the original system in (1) . It is not difficult to find a matrix L such that $A - LC$ is invertible and $(C, A - LC)$ is observable. By letting $A' = A - LC$, we can obtain a system with invertible A'. Therefore, it is not restrictive to assume that A is invertible. In this paper, we only consider the case that A is invertible.

The following definitions and lemma will be used in this paper.

Definition 1. The Minkowski sum of two sets $\mathcal{X}$ and $\mathcal{Y}$ is

$$\mathcal{X} \oplus \mathcal{Y} = \{x + y : x \in \mathcal{X}, y \in \mathcal{Y}\}.$$

Definition 2. Given a set $\mathcal{X} \subset \mathbb{R}^n$, its linear image associated with a matrix $L \in \mathbb{R}^{m\times n}$ is defined as

$$L\mathcal{X} = \{Lx : x \in \mathcal{X}\}.$$

Definition 3. An m-order zonotope $\mathcal{Z}$ in the n-dimensional space is defined as

$$\mathcal{Z} = p \oplus H\mathbb{B}^m = \{p + Hz : z \in \mathbb{B}^m\}$$

where $p \in \mathbb{R}^n$ is the center of $\mathcal{Z}$, $H \in \mathbb{R}^{n\times m}$ is the shape matrix of $\mathcal{Z}$, and $\mathbb{B}^m = [-1, 1]^m$ is a hypercube. In this paper, $\mathcal{Z}$ is also denoted as $\langle p, H\rangle$ for simplicity.

With Definition 3, (2) can be reformulated as follows:

$$d_k \in \mathcal{Z}_d = \langle 0, W_d\rangle \tag{3}$$

where

$$W_d = \mathrm{diag}\left(\bar{d}\right).$$

Lemma 1 (Wang et al., 2015). Given two matrices $\mathscr{Y} \in \mathbb{R}^{b\times c}$ and $\mathscr{Z} \in \mathbb{R}^{a\times c}$, if the matrix $\mathscr{Y}$ is of full column rank, then the general solution of $\mathscr{X}\mathscr{Y} = \mathscr{Z}$ is

$$\mathscr{X} = \mathscr{Z}\mathscr{Y}^\dagger + \mathscr{S}\mathscr{Y}^\perp$$

where $\mathscr{S} \in \mathbb{R}^{a\times b}$ is a freely chosen matrix.

3. MAIN RESULTS

3.1 The proposed estimator

Since A is invertible, the system in (1) is equivalent to the following *backward propagation equation*:

$$\begin{cases} x_{k-1} = A^{-1}x_k - A^{-1}Bu_{k-1} - A^{-1}D_1 d_{k-1} \\ y_k = Cx_k + D_2 d_k \end{cases} \tag{4}$$

For simplicity, we denote

$$\tilde{A} = A^{-1},\ \tilde{B} = -A^{-1}B,\ \tilde{D}_1 = -A^{-1}D_1 \tag{5}$$

and

$$\mathbf{y}_k = \begin{bmatrix} y_k \\ \vdots \\ y_{k-s} \end{bmatrix},\ \mathbf{u}_k = \begin{bmatrix} u_k \\ \vdots \\ u_{k-s} \end{bmatrix}$$

Herein, $s \geq n_x - 1$ is a integer related to the convergence time.

Based on (4), it is not difficult to express $\mathbf{y}_k$ as follows:

$$\mathbf{y}_k = M_x x_k + M_u \mathbf{u}_k + M_d \mathbf{d}_k \tag{6}$$

where

$$M_x = \begin{bmatrix} C \\ C\tilde{A} \\ \vdots \\ C\tilde{A}^s \end{bmatrix},\ \mathbf{d}_k = \begin{bmatrix} d_k \\ \vdots \\ d_{k-s} \end{bmatrix},$$

$$M_u = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & C\tilde{B} & 0 & \cdots & 0 \\ 0 & C\tilde{A}\tilde{B} & C\tilde{B} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & C\tilde{A}^{s-1}\tilde{B} & C\tilde{A}^{s-2}\tilde{B} & \cdots & C\tilde{B} \end{bmatrix}$$

$$M_d = \begin{bmatrix} D_2 & 0 & 0 & \cdots & 0 \\ 0 & C\tilde{D}_1 + D_2 & 0 & \cdots & 0 \\ 0 & C\tilde{A}\tilde{D}_1 & C\tilde{D}_1 + D_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & C\tilde{A}^{s-1}\tilde{D}_1 & C\tilde{A}^{s-2}\tilde{D}_1 & \cdots & C\tilde{D}_1 + D_2 \end{bmatrix}$$

Based on (6), we propose the following estimator for system (1):

$$\hat{x}_k = T(\mathbf{y}_k - M_u \mathbf{u}_k), \ k > s \tag{7}$$

where $T \in \mathbb{R}^{n_x \times (s+1)n_y}$ is a matrix satisfying

$$TM_x = I \tag{8}$$

Remark 2. The proposed method is inspired by the parity space approach in the field of fault diagnosis, but the unknown state is eliminated in the parity space approach while the state is reconstructed in the proposed method.

The following lemma is proposed to analyse the solvability of (8).

Lemma 2. The equation in (8) is solvable if (C, A) is observable and A is invertible.

Proof. Denote the observability matrix of (C, A) as

$$\mathscr{O}_{CA} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n_x - 1} \end{bmatrix}.$$

Since (C, A) is observable, we have

$$\text{rank}\mathscr{O}_{CA} = n_x$$

Note that

$$\mathscr{O}_{CA} A^{-(n_x-1)} = \begin{bmatrix} CA^{-(n_x-1)} \\ CA^{-(n_x-2)} \\ \vdots \\ C \end{bmatrix}$$

Using the Sylvester's inequality (Koenig , 2006)

$$\text{rank}M + \text{rank}N - p \leq \text{rank}(MN) \leq \min\{\text{rank}M, \text{rank}N\}$$

where $M \in \mathbb{R}^{m \times p}$ and $N \in \mathbb{R}^{p \times n}$, we have

$$\text{rank} \begin{bmatrix} CA^{-(n_x-1)} \\ CA^{-(n_x-2)} \\ \vdots \\ C \end{bmatrix} = n_x$$

Since $s \geq n_x - 1$, we have

$$\text{rank}M_x = \text{rank} \begin{bmatrix} CA^{-(n_x-1)} \\ CA^{-(n_x-2)} \\ \vdots \\ C \end{bmatrix} = n_x$$

It is known from Lemma 1 that (8) is solvable, and the general solution is

$$T = M_x^\dagger + SM_x^\perp$$

where S is a freely chosen matrix. □

Remark 3. When $k \geq s$, we have

$$\hat{x}_k = T(\mathbf{y}_k - M_u \mathbf{u}_k) = x_k + TM_d \mathbf{d}_k. \tag{9}$$

If there is no disturbance in (1), we have

$$x_k = \hat{x}_k$$

which implies that (7) is a fixed-time convergence estimator in the disturbance-free situation. If the disturbance d_k is considered, the $\hat{x}_k$ will not exactly converge to x_k, but the estimator still has a fixed-time convergence property.

Based on (9), it is easy to obtain that

$$x_k \in \mathcal{X}_k = \langle \hat{x}_k, H \rangle \tag{10}$$

where $H \in \mathbb{R}^{sn_x \times (s+1)n_d}$ is

$$H = -TM_d \begin{bmatrix} W_d & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & W_d \end{bmatrix}.$$

Based on (10), we can estimate the intervals of x_k as follows:

$$\hat{x}_k - \begin{bmatrix} \sum_{j=1} |H_{1j}| \\ \vdots \\ \sum_{j=1} |H_{n_x j}| \end{bmatrix} \leq x_k \leq \hat{x}_k + \begin{bmatrix} \sum_{j=1} |H_{1j}| \\ \vdots \\ \sum_{j=1} |H_{n_x j}| \end{bmatrix} \tag{11}$$

Herein, H_{ij}, $i = 1, \cdots, n_x$ denotes the element of H in the ith row, jth column.

3.2 Optimal Design

In this subsection, the matrix T is designed to attenuate the effect of disturbance on estimation error. To this end, we denote the following estimation error

$$e_k = \hat{x}_k - x_k = TM_d \mathbf{d}_k.$$

To attenuate the effect of $\mathbf{d}_k$ on e_k, the matrix T is designed by minimizing the Frobenius norm of TM_d, i.e. T is obtained by solving the following constrained optimization problem

$$\begin{aligned} &\min_T \|TM_d\|_F \\ &\text{s.t. } TM_x = I \end{aligned} \tag{12}$$

The following theorem is proposed to design T.

Theorem 1. The matrix T minimizing $\|TM_d\|_F$ while satisfying $TM_x = I$ is given by

$$T = M_x^\dagger - M_x^\dagger M_d V_1^T (S_1)^{-1} U_1^T M_x^\perp \tag{13}$$

where U_1, S_1 and V_1 are the matrices associated with the non-zero singular values of $M_x^\perp M_d$. More specifically, the singular value decomposition of $M_x^\perp M_d$ is

$$M_x^\perp M_d = USV$$

and matrices $U \in \mathbb{R}^{(s+1)n_y \times (s+1)n_y}$, $S \in \mathbb{R}^{(s+1)n_y \times (s+1)n_d}$ and $V \in \mathbb{R}^{(s+1)n_d \times (s+1)n_d}$ are rewritten as

$$U = [U_1 \ U_2], \ S = \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix}, \ V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where $U_1 \in \mathbb{R}^{(s+1)n_y \times r}$, $S_1 \in \mathbb{R}^{r\times r}$ and $V_1 \in \mathbb{R}^{r\times(s+1)n_d}$. Herein, r is used to denote the rank of S, which equals to the number of non-zero singular values in S.

Proof. From Lemma 2, it is known that $TM_x = I$ is solvable, and the generalized solution of T is

$$T = M_x^{\dagger} + XM_x^{\perp} \tag{14}$$

Substituting (14) into (12), the problem in (12) is converted as follows:

$$\min_X \|(M_x^{\dagger} + XM_x^{\perp})M_d\|_F \tag{15}$$

Note that

$$\begin{aligned}
&\min_X \|(M_x^{\dagger} + XM_x^{\perp})M_d\|_F \\
&= \min_X \operatorname{tr}\{(M_x^{\dagger} + XM_x^{\perp})M_dM_d^T(M_x^{\dagger} + XM_x^{\perp})^T\} \\
&= \min_X \operatorname{tr}\{M_x^{\dagger}M_dM_d^TM_x^{\dagger T} + M_x^{\dagger}M_dM_d^TM_x^{\perp T}X^T \\
&\quad + XM_x^{\perp}M_dM_d^TM_x^{\dagger T} + XM_x^{\perp}M_dM_d^TM_x^{\perp T}X^T\}
\end{aligned} \tag{16}$$

This is non-trivial problem since $M_x^{\perp}M_dM_d^TM_x^{\perp T}$ may be a singular matrix.

To solve the optimization problem in (16), $M_x^{\perp}M_d$ is decomposed as

$$M_x^{\perp}M_d = USV = [U_1\ U_2]\begin{bmatrix} S_1 & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix} V_1 \\ V_2\end{bmatrix}.$$

It follows that

$$M_x^{\perp}M_d = U_1S_1V_1 \tag{17}$$

Substituting (17) into (16) gives

$$\begin{aligned}
&\min_X \|(M_x^{\dagger} + XM_x^{\perp})M_d\|_F \\
&= \min_X \operatorname{tr}\{M_x^{\dagger}M_dM_d^TM_x^{\dagger T} + M_x^{\dagger}M_dV_1^TS_1^TU_1^TX^T \\
&\quad + XU_1S_1V_1M_d^TM_x^{\dagger T} + XU_1S_1S_1^TU_1X^T\}
\end{aligned} \tag{18}$$

Note that $V_1V_1^T = I$ is used here.

By letting

$$Y = XU_1 \tag{19}$$

the optimization problem in (18) becomes

$$\begin{aligned}
&\min_X \|(M_x^{\dagger} + XM_x^{\perp})M_d\|_F \\
&= \min_X \operatorname{tr}\{M_x^{\dagger}M_dM_d^TM_x^{\dagger T} + M_x^{\dagger}M_dV_1^TS_1^TY^T \\
&\quad + YS_1V_1M_d^TM_x^{\dagger T} + YS_1S_1^TY^T\}
\end{aligned} \tag{20}$$

Denote

$$\begin{aligned}
J = &\operatorname{tr}\{M_x^{\dagger}M_dM_d^TM_x^{\dagger T} + M_x^{\dagger}M_dV_1^TS_1^TY^T \\
&+ YS_1V_1M_d^TM_x^{\dagger T} + YS_1S_1^TY^T\}
\end{aligned}$$

and let

$$\frac{\partial J}{\partial Y} = 0$$

we have

$$2M_x^{\dagger}M_dV_1^TS_1^T + 2YS_1S_1^T = 0 \tag{21}$$

The solution to (21) is

$$Y = -M_x^{\dagger}M_dV_1^T(S_1)^{-1}.$$

Using (19) gives

$$X = YU_1^{\dagger} = YU_1^T.$$

Substituting X into (14), we obtain (13). □

Remark 4. Although we only consider linear time-invariant systems in this paper, the proposed method can be easily extended to linear time-varying systems.

4. SIMULATIONS

In this section, a numerical example from Dinh et al. (2019) is used to illustrate the performance of the proposed method. The system has the form of (1) with the following parameters:

$$A = \begin{bmatrix} 5/4 & 1 \\ -3/8 & 1/8\end{bmatrix},\ B = \begin{bmatrix} 1/4 & 0 \\ 0 & 1/8\end{bmatrix},\ C = [1\ 0],$$

$$D_1 = \begin{bmatrix} 1/9 & 0 \\ 1/9 & 0\end{bmatrix},\ D_2 = [0\ 1/9],$$

$$x_0 = \begin{bmatrix} 2.3 \\ 1\end{bmatrix},\ u_k = \begin{bmatrix} 1 \\ 1\end{bmatrix},\ d_k = \begin{bmatrix} \sin(k) \\ \sin(k^2)\end{bmatrix}.$$

Let $s = 2$, we have

$$M_x = \begin{bmatrix} 1 & 0 \\ 0.2353 & -1.8824 \\ -1.2734 & -4.8720\end{bmatrix},$$

$$M_u = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0588 & 0.2353 & 0 & 0 \\ 0 & 0 & 0.3183 & 0.6090 & -0.0588 & 0.2353\end{bmatrix},$$

$$M_d = \begin{bmatrix} 0 & 0.1111 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1830 & 0.1111 & 0 & 0 \\ 0 & 0 & 0.6828 & 0 & 0.1830 & 0.1111\end{bmatrix}.$$

It follows that

$$M_x^{\dagger} = \begin{bmatrix} 0.6848 & 0.4334 & -0.1674 \\ -0.1446 & -0.1605 & -0.1432\end{bmatrix},$$

$$M_x^{\perp} = \begin{bmatrix} 0.3152 & -0.4334 & 0.1674 \\ -0.4334 & 0.5959 & -0.2302 \\ 0.1674 & -0.2302 & 0.0890\end{bmatrix}.$$

Using the singular value decomposition of $M_x^{\perp}M_d$, we obtain

$$U_1 = \begin{bmatrix} -0.5614 \\ 0.7719 \\ -0.2982\end{bmatrix},\ S_1 = 0.1386,$$

$$V_1 = [0\ -0.45\ -0.45\ 0.6187\ -0.3937\ -0.2391].$$

Then we have

$$T = \begin{bmatrix} 0.7975 & 0.2784 & -0.1076 \\ 0.1290 & -0.5367 & 0.0021\end{bmatrix}.$$

Using the proposed method, we get the interval estimation results in Fig. 1 and Fig. 2. The interval estimation results obtained by the method proposed in Dinh et al. (2019) are also depicted for comparison. In Fig. 1 and Fig. 2, the red solid lines are used to denote the upper and lower bounds estimated by the proposed method while the blue dashed lines are used to represent the interval estimations obtained by the method in Dinh et al. (2019). It can be seen that the proposed method provides more accurate interval estimations than the one in Dinh et al. (2019). We obtain more accurate results because the gain matrix T can be optimized while the method in Dinh et al. (2019) did not have an explicit way to optimize the parameter matrix.

5. CONCLUSION

In this paper, we propose a fixed-time interval estimation method for discrete-time linear systems. Based on the backward propagation equation of the estimated system, an estimator combined with interval analysis is used to

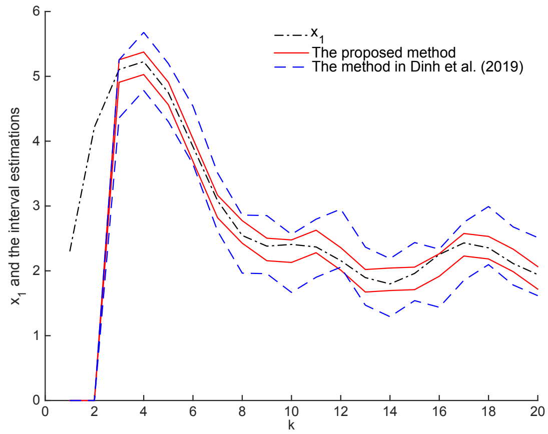

Fig. 1. The first component of x_k and the interval estimation results

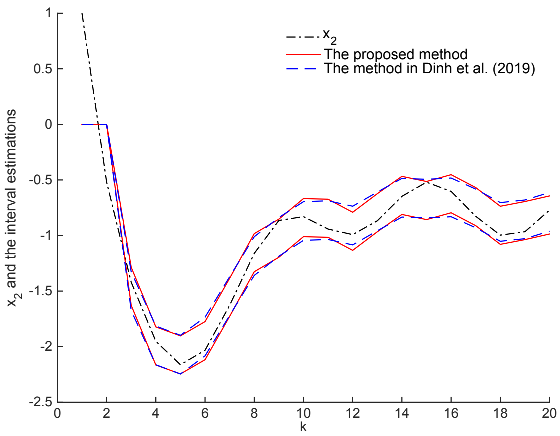

Fig. 2. The second component of x_k and the interval estimation results

provide interval estimation with fixed-time convergence. Moreover, the proposed method uses parameter optimization to attenuate the effect of uncertainties so as to achieve accurate estimations. Simulation results show the effectiveness of the proposed method.

REFERENCES

J. M. Bravo, T. Alamo, and E. F. Camacho. Robust MPC of constrained discrete-time nonlinear systems based on approximated reachable sets. *Automatica*, 42: 1745-1751, 2006.

F. Xu, V. Puig, C. Ocampo-Martinez, F. Stoican, and S. Olaru. Actuator-fault detection and isolation based on interval observers and invariant sets. *The 52nd IEEE Conference on Decision and Control (CDC)*, 2013.

Z. H. Zhang, G. H. Yang. Fault detection for discrete-time LPV systems using interval observers. *International Journal of Systems Science*, 48: 2921-2935, 2017.

R. E. Kalman. A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, 82: 35-45, 1960.

L. Xie and C.E. de Souza. H_∞ state estimation for linear periodic systems. *IEEE Transactions on Automatic Control*, 38: 1704-1707, 1993.

Q. Zhang. Adaptive observer for multiple-input-multiple-output (MIMO) linear parameter-varying systems. *IEEE Transactions on Automatic Control*, 47: 525-529, 2002.

H. Hammouri, B. Targui, and F. Armanet. High-gain observer based on a triangular structure. *International Journal of Robust and Nonlinear Control*, 12:497-518, 2002.

J. L. Gouze, A. Rapaport, M. Z. Hadj-Sadok. Interval observers for uncertain biological systems. *Ecological Modelling*, 133: 45-56, 2000.

T. Raïssi, D. Efimov, A. Zolghadri. Interval state estimation for a class of nonlinear systems. *IEEE Transactions on Automatic Control*, 57: 260-265, 2012.

R. E. H. Thabet, T. Raïssi, C. Combastel, D. Efimov, and A. Zolghadri. An effective method to interval observer design for time-varying systems. *Automatica*, 50: 2677-2684, 2014.

Z. Wang, C.-C. Lim, and Y. Shen. Interval observer design for uncertain discrete-time linear systems. *Systems & Control Letters*, 116: 41-46, 2018.

W. Tang, Z. Wang, Y. Wang, T. Raïssi, and Y. Shen. Interval estimation methods for discrete-time linear time-invariant systems. *IEEE Transactions on Automatic Control*, 2019, 64(11): 4717-4724.

W. Tang, Z. Wang, and Y. Shen. Interval estimation for discrete-time linear systems: A two-step method. *Systems & Control Letters*, 123: 69-74, 2019.

R. Engel, G. Kreisselmeier, A continuous-time observer which converges in finite time. *IEEE Transactions on Automatic Control*, 47, pp. 1202-1204, 2002.

F. Lopez-Ramirez, A. Polyakov, D. Efimov, W. Perruquetti, Finite-time and fixed-time observers design via implicit Lyapunov function, *2016 European Control Conference (ECC)*, pp. 289-294, 2016.

T. Menard, E. Moulay, W. Perruquetti, Fixed-time observer with simple gains for uncertain systems, *Automatica*, 81, pp. 438-446, 2017.

H. Rios , A. R. Teel, A hybrid fixed-time observer for state estimation of linear systems, *Automatica*, 87, pp. 103-112, 2018.

J. Zhang, M. Chadli, Y. Wang, A fixed-time observer for discrete-time singular systems with unknown inputs, *Applied Mathematics and Computation*, 363, pp. 1-12, 2019.

T. Dinh, F. Mazenc , T. Raïssi, Finite-time guaranteed state estimation for discrete-time systems with disturbances, *The 4th conference on Control and Fault-Tolerant Systems (SysToL19)*, June 2019, pp. 342-347, Casablanca, Morocco,

S. X. Ding. *Model-based Fault Diagnosis Techniques: Design Schemes Algorithms and Tools*. Springer, 2008.

Z. Wang, M. Rodrigues, D. Theilliol, and Y. Shen. Actuator fault estimation observer design for discrete-time linear parameter-varying descriptor systems. *International Journal of Adaptive Control and Signal Processing*, 29: 242-258, 2015.

D. Koenig. Observer design for unknown input nonlinear descriptor systems via convex optimization. *IEEE Transactions on Automatic Control*, 51: 1047-1052, 2006.