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A Mathematical Model in Automatic Control Aerospace Engineering Education

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Abstract: The aerospace engineering educational system aims to create future professionals able to solve problems of high complexity, with time constraints and which solutions matches prescribed level of performance. In our past work, we introduced the innovative concept of the Professional Readiness Level (PRL) as a unique parameter to quantify how close the students are to the aerospace industry. In this paper we propose a dynamic model, of the PRL, capable to capture, in simple but effective way, the student behaviour we, as professors, observed in our educative experience.

Keywords: Education, Models

1. INTRODUCTION

The description and the analysis of the educational process through a NonLinear Differential System (NDS) constitutes a new way of thinking compared to traditional methods. The approaches that can be found in literature, indeed, are based more on the analysis of the educational results than on their modelling by means of Ordinary Differential Equations (ODEs). These ODE-based NDS allow us to formally derive how the students' abilities are expected to evolve over time and to investigate how to control the system by means of an education optimizing learning activity.

The educational process seen as a dynamic system has been implemented lagging behind other disciplines. Psychology, econometrics, theoretical biology have a long history in shaping their time-dependent processes of interest by means of ODEs. In the field of psychology, for example, there are interesting works as Abraham and Gilgen (1995); Guastello et al. (2008); Sulis et al. (1996), but there is much less literature on dynamic modelling in the education field. Noteworthy is the book by Koopmans and Stamovlasis (2016), where an Introduction to Education as a Complex Dynamical System can be found. Another approach using ODEs can be found in Makanda and Sypkens (2017), in which a mathematical model usually used to represent the spread of an epidemic disease is exploited.

Unlike the works mentioned above, the approach proposed in our manuscript is focused on the dynamics of a single student subject to teaching actions. It is worth observing that this model has been developed having in mind the Automatic Control Education for Aerospace Engineering (ACEAE). For this reason, Castaldi and Mimmo (2019) proposed a new transversal education classification, particularly suitable for ACEAE and based on the Professional Readiness Level (PRL). In this paper we provide, for the first time, the mathematical description of the PRL dynamics. In this work the PRL is exploited to describe the behavior of the individual student or the group of students and is defined as a composition of three factors: the Educational Level (EL), the Deadlines Meeting Level (DML) and the Practice Level (PL). The rest of this paper is organized as follows: this section ends with the introduction of the mathematical background necessary to understand the remaining of the work, the Section 2 describes the new taxonomy which defines the state of the students and the teaching control actions, the Section 3 presents the model of the dynamics of a single student belonging to a class whereas the section 4 proposes some simulations relative to a single student and, in conclusion, the final comments are addressed in Section 5.

1.1 Notation

This paper exploits a saturation function defined as $\mathtt{sat} : \mathbb{R} \mapsto [0, 1]$ with

$$\mathtt{sat}(s) := \begin{cases} 0 & s < 0 \\ s & s \in [0, 1] \\ 1 & s > 1 \end{cases} \quad (1)$$

Moreover, the absolute value function is represented by the operator $|\cdot| : \mathbb{R}^n \mapsto \mathbb{R}^n$ which is intended to be applied component-wise if $n > 1$. The function $\mathrm{sign}(\cdot) : \mathbb{R} \mapsto \{0, 1\}$ is defines as

$$\mathrm{sign}(s) := \begin{cases} 1 & s > 0 \\ 0 & s \leq 0 \end{cases}$$

Finally, the open ball centred at $x \in \mathbb{R}^n$ and with radius $\rho > 0$, namely $\mathcal{B}_\rho(x) \in \mathbb{R}^n$, is defined.

2. THE PROFESSIONAL READINESS LEVELS TAXONOMY

The involvement of students in practical activities can happen in different ways such as term papers, internship or master theses. In presence of funded projects, the practical activities are oriented to the solution of a specific problem and are always associated with quite hard constraints in terms of expected performance and deadlines. On the other hand, in absence of funded projects, the practical activities are mostly related to:

- the assessment of the students' capacity of translation of the theoretical knowledge into the solution of practical control problems;
- the improvement of the facilities of the laboratory (hopefully part of a long-term planning);
- the application of the last scientific research results;

The topics are then classified in levels of difficulty-complexity in a scale compose by seven different degrees:

(0) **Class.** No problem to be solved. The student receive the base knowledge to deal with the following points;
(1) **Bachelor term papers.** Solve an academic "ideal" problem without time of performance constraints;
(2) **Master term papers.** Solve an academic "ideal" problem without time constraints but guaranteeing the prescribed level of performance;
(3) **Bachelor internship**. Investigate a real system of low complexity and learn how to manage it. No deadlines are foreseen;
(4) **Master internship.** Investigate a real system of high complexity and learn how to manage it. No deadlines are foreseen;
(5) **Bachelor thesis.** Implement the theoretical knowledge to real but not complex systems making the control system working in the laboratory. Time deadlines are given;
(6) **Master thesis.** Implement the theoretical knowledge to real, complex systems making the control system working in the laboratory and assessing the obtained performances. Time deadlines are given.

Students able to solve a problem rates at sixth level show a good propensity and aptitude to demanding works to demonstrating a good Level of Professional Readiness (PRL). At the opposite, students able to solve only problems rated at the first complexity level are good students but not yet ready for being engineers.

The PRL can be seen as a composition of three factors which are the Educational Level (EL), the Deadlines Meeting Level (DML) and the Practice Level (PL). The ELs describe the level of understanding (consciousness) that the student have on the taught topics (either a specific topic or the set of topics). The ELs can be classified in six levels, as suggested by Krathwohl (2002), ranging from the bare capacity of memorizing to the ability to create. This work introduces a floor level (0-EL) for those students who approach to a given topic for the first time:

(0) **Fresh Memory:** no information available;
(1) **Remember:** be able to recognize or remember facts, terms, basic concepts, or answers without necessarily understanding what they mean;
(2) **Understand:** be able to organize, compare, translate, interpret, give descriptions, and state the main ideas;
(3) **Apply:** be able to use prior knowledge to solve problems, identify connections and relationships and how they apply in new situations;
(4) **Analyse:** be able to break information into component parts, determine how the parts relate to one another, identify motives or causes, make inferences, and find evidence to support generalizations;
(5) **Evaluate:** be able to present and defend opinions by making judgments about information, the validity of ideas, or quality of work based on a set of criteria
(6) **Create:** be able to build a structure or pattern from diverse elements and to put parts together to form a whole;

This work introduces two further evaluation scales to assess the capacity of meeting deadlines and to work in practical context. The DMLs are then classified in four levels:

(0) **No time limits:** no time limits by which the assigned task must be accomplished;
(1) **Homework:** soft deadline that is identified by the time needed by the student to conclude that task. In general, a delay in the accomplishment of the task does not imply any consequence (maybe a delayed exam);
(2) **Thesis/Journal:** medium-hardness deadline which could be postponed only if strictly necessary;
(3) **Project/Conference:** hard-deadline which could be postponed only in exceptional cases.

Finally, the students' ability to deal with practical problems which implies their ability in translating the theoretical concepts in practice is classified in the following five PLs:

(0) **Conceptual:** solution not given or given only with conceptual paradigms without solid formalisms;
(1) **Theory:** symbolic formulations, solutions given in terms of existence;
(2) **Academic:** simple examples (not necessarily realistic, maybe linear), solution not necessarily feasible (too strong assumptions);
(3) **Realistic:** quasi-real complex examples (realistic, non linear), solution feasible but not necessarily implementable (does not take into account for the plant limitation such as the band of actuators and sensors);
(4) **Practice:** real systems (really smart approximations), solution implementable with few practical modifications.

The above mentioned taxonomy represents a high-level classification in which the bounds between two adjacent categories are barely identifiable. Indeed, the students pass, with continuity over time, from one level to the next by experimenting intermediate values between the origin and the destination level. For this reason, this paper models the students levels as continuous time functions. Furthermore, to uniform the domains and to make the levels independent of the full scale (represented by the number of levels in that category), this paper normalize both the educational actions and the student levels to belong in the compact real domain $[0,\,1]$. Mathematically, given the time span $[t_0,\,t]$ with $t > t_0 \geq 0$, the student levels are collected into the continuous time function

$$x : [t_0,\, t] \mapsto [0,\, 1]^3 \tag{2}$$

where $x(\tau) = \mathrm{col}\left(\overline{\mathrm{EL}}(\tau),\, \overline{\mathrm{DML}}(\tau),\, \overline{\mathrm{PL}}(\tau)\right)$ represents the vector of the student's normalized levels at time τ. The variable x is called *state* of the student.

In analogy, also the normalized educative control action is captured by the continuous time variable

$$u : [t_0,\, t] \mapsto [0,\, 1]^3. \tag{3}$$

The control action is modelled as a three dimensional vector because each educational activity influences all the components of x and, in particular, this work assumes a uniform impact so that the variable u assumes the following form

$$u(\tau) := \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \bar{u}(\tau) \tag{4}$$

with $\bar{u} : [t_0,\, t] \mapsto [0,\, 1]$ corresponding to the normalized educational action.

Finally, the normalized PRL is defined as function of x which, in first instance, can be modelled as a linear combination of the student normalized levels. Formally, the normalized PRL is described by:

$$y : [0,\, 1]^3 \mapsto [0,\, 1] \tag{5}$$

where $y = Cx$ with $\|C\|_1 = 1$. Moreover, given the i-th entry of C as C_i, then $C_i > 0$ for $i = 1, 2, 3$.

3. PRL DYNAMICS

The PRL of a student varies over time and is influenced by several factors, principally related to the nature of the student. As example, students have their own inertia in learning or forgetting concepts, they react in different ways to the same educational stimuli, they can be more or less efficient in self-learning activities or they can influence each other with personal preferences.

Given a set of n students, possibly interacting each other, and m educational actions with $n, m \in \mathbb{Z}$, this paper models the PRL dynamics of the i-th student as

$$\begin{aligned} \dot{x}_i &= \alpha_i(x_i, \upsilon_i)\,(\upsilon_i(x_i, u) - x_i) \\ &\qquad + \sum_{j=1}^{n} \beta_{ij}\mu_i(x_j - x_i)(x_j - x_i) \\ y_i &= Cx_i \end{aligned} \tag{6}$$

with initial conditions $x_i(t_0) = x_{0_i}$ and where $u = \mathrm{col}(u_1(\tau), \ldots, u_m(\tau))$ for $\tau \in [t_0,\, t]$

$$\upsilon_i(x_i, u) = \mathtt{sat}\left(\sum_{k=1}^{m} \eta_i(u_k(\tau) - x_i)u_k(\tau) + \delta_i(x_i) \right). \tag{7}$$

The remainder of this section describes the terms appearing in the equations (6) and (7).

3.1 Student Activity $\alpha_i(x_i, \upsilon_i)$

The student ability to keep its state, *i.e.* its levels of education, deadline meeting and practice, is captured by means of the function $\alpha_i(x_i, \upsilon_i)$. The function is representative of both the natural susceptibility to new information and to the inevitable forgetting ratio. We observed that, student characterized by lower values of the state (closer to 0) demonstrate higher inertia to learn new concepts whereas they show a more rapid forgetting of their knowledge. At the opposite, higher state values (closer to 1) are normally associated smart students able to rapidly catch new teachings and capable to keep their state for longer times. As example, focusing on the $\overline{\mathrm{EL}}$ only, a student with an $\overline{\mathrm{EL}}$ close to zero is just able to remember but, since the concepts are not well understood are subject to a rapid forgetting process. Instead, a student who deeply understood the theory and is able to criticize or to create, usually learn more rapidly while remembering the concepts for a longer time. For any fixed υ_i, the function $\alpha_i(x_i, \upsilon_i)$ is continuous and monotonically increasing ($w.r.t.\ x_i$) in case of learning whereas is monotonically decreasing in case of memorizing. The function image is bounded from below by 0 which, in case of learning, means that the subject does not learn whereas, in case of memorizing, means that the student does not forget. To make the formulation as generic as possible, the function $\alpha(x_i, \upsilon_i)$ is designed to assign, to each entry of the state x_i, an independent dynamics. Thus, the function $\alpha_i(x_i, \upsilon_i)$ depends on the state x_i and on the *task* which is *to learn* or *to remember*. This paper classifies the task by means of the sign of the term $\upsilon_i(x_i, t)$. Indeed, the latter term has the meaning of the educational action and is equal to zero if and only if the student does not learn by means of neither external, $u_k(\tau)$, nor self–actions, $\delta_i(x_i)$.

With these concepts in hand, this paper denotes with $\mathcal{A}$ the set of sufficiently continuous functions which verify the following properties:

$$\begin{aligned} \mathcal{A} := \Bigg\{ & f : [0,\, 1] \times \{0, 1\} \mapsto R_{>0}, \\ & \frac{\partial f}{\partial s}(s, w) := \begin{cases} \leq 0 & w = 0 \\ \geq 0 & w = 1 \end{cases}, \ \forall\, s \in [0,\, 1] \Bigg\}. \end{aligned} \tag{8}$$

Finally, the student inertia $\alpha_i(x_i, \upsilon_i)$ results to be the vectorial composition of three function a_{i_1}, a_{i_2} and $a_{i_3} \in \mathcal{A}$ as follows:

$$\alpha_i(x_i, \upsilon_i) := \mathrm{diag}\left(\begin{bmatrix} a_{i_1}(x_i^{(1)}, w_i^{(1)}) \\ a_{i_2}(x_i^{(2)}, w_i^{(3)}) \\ a_{i_3}(x_i^{(2)}, w_i^{(3)}) \end{bmatrix} \right) \tag{9}$$

where $w_i^{(j)} = \mathrm{sign}(\upsilon_i^{(j)} - x_i^{(j)})$.

3.2 Efficiency of the Educative Action $\eta_i(u_k(\tau) - x_i)$

The educational action $u_k(\tau)$ is adopted by the students in different ways and in function of several factors such as the personal feeling and the way the education is given. In particular, the students are more likely to recognize better educative actions which are close to their current level. As example, the level of the notions given in a class increase gradually since the beginning of the course because, at the beginning, the students are not supposed to be able to catch the deep meaning of argument too far from their current basic knowledge. This attitude is also verified when a student with high values of state (close to 1) is subject to teaching actions of low level. Indeed, if the teaching action is too basic for the current student state it is felt as a lack of new relevant information (condition of no-learning). Roughly speaking, the efficiency of the education action, $\eta(u_k(\tau) - x_i)$, capture the *scepticism* vs. *belief* concepts: too radical ideas create more concerns than information whereas the slightly different ideas are more likely to be understood and then appreciated.

In agreement with the standard concept of efficiency, this paper defines $\mathcal{H}$ as the set of sufficiently continuous functions which posses the following properties:

$$\mathcal{H} := \Big\{ f : [0,\,1] \mapsto [0,\,1], \\ f(0) = 1,\, f(1) = 0,\, \frac{\partial f}{\partial s}(s) \le 0 \Big\}. \tag{10}$$

Thanks to this definition, the education efficiency $\eta_i(x_i - u_i)$ is given by the vectorial composition of three functions belonging to $\mathcal{H}$, namely h_{i_1}, h_{i_2} and h_{i_3}:

$$\eta_i(x_i - u_i) := \operatorname{diag}\left(\begin{bmatrix} h_{i_1}(|x_i^{(1)} - u_i^{(1)}|) \\ h_{i_2}(|x_i^{(2)} - u_i^{(2)}|) \\ h_{i_3}(|x_i^{(3)} - u_i^{(3)}|) \end{bmatrix} \right). \tag{11}$$

3.3 Self-Learning Ability $\delta_i(x_i)$

The self-learning capacity is one of the inherent abilities which can be correlated to the student state. Usually, students described by states close to 0 do not posses a sufficient level of knowledge and spirit of criticism to learn autonomously. Indeed, the self-learning action is a complex task requiring the capacity to identify the right references, to understand and criticism them possibly without the support of teachers. For this reason, the self-learning ability can be thought as a continuous monotonically increasing function of the student state.

Furthermore, students which have a sufficiently strong self-learning ability are able to auto-sustain and improve their status. This phenomenon is particularly suitable to describe the Ph.D. students which, after a training phase, are able to become and make progresses as researchers. More in detail, in absence of academic educative actions ($u_k(\tau) = 0$, $k = 1, \ldots, m$) and without any interaction with other subjects ($n = 1$ or $\beta_{ij} = 0$, for $i, j \in \{1, \ldots, n\}$), the dynamics of the state is described by

$$\dot{x}_i = \alpha_i(x_i, \upsilon_i)\,(\delta_i(x_i) - x_i)\,. \tag{12}$$

Thus, the self-learning ability $\delta_i(x_i)$ is said to be *poor*, if $\delta_i(x_i) - x_i < 0$, *sufficient* if $\delta_i(x_i) - x_i = 0$ and rich if $\delta_i(x_i) - x_i > 0$. We observed that there exists a direct correlation between the self-learning ability and the state and, in particular, the self-learning abilities are sufficient and rich for high values of the state.

This paper defines by $\mathcal{D}$ the set of sufficiently continuous functions which verify the following properties:

$$\mathcal{D} := \Big\{ f : [0,\,1] \mapsto [0,\,1], \\ f(0) = 0,\, f(1) = 1,\, \frac{\partial f}{\partial s}(s) \ge 0, \\ \exists\, s^\star : (f(s) - s^\star)\,(s - s^\star) > 0 \;\forall s \in (0,\,1),\, s \ne s^\star \Big\}. \tag{13}$$

Given three functions, namely d_{i_1}, d_{i_2} and $d_{i_3} \in \mathcal{D}$, the self-learning ability $\delta_i(x_i)$ is defined as the following vectorial composition

$$\delta_i(x_i) := \begin{bmatrix} d_{i_1}(x_i^{(1)}) \\ d_{i_2}(x_i^{(2)}) \\ d_{i_3}(x_i^{(3)}) \end{bmatrix}. \tag{14}$$

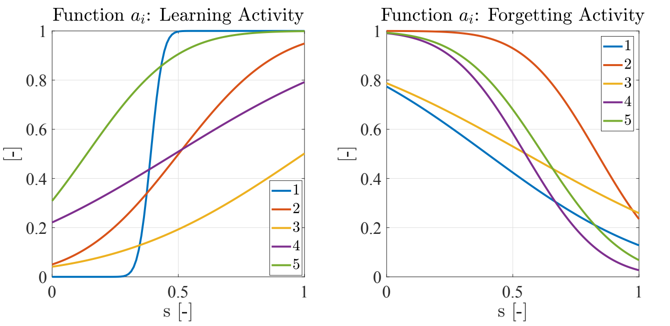
4. SIMULATION EXPERIMENTS

4.1 Simulator set-up

The simulation reported in this section have been obtained by means of a particular choice of the functions α_i, δ_i, and η_i. These functions have been designed to posses features as continuity (at least $\mathcal{C}^0$), monotonicity, non linearity and boundedness. This paper adopts the so called *error function* to build a function, namely f, which, based on its parameters, can belong to each of the classes $\mathcal{A}$ and $\mathcal{H}$. In detail, the function $f : [0,\,1] \mapsto [0,\,1]$ is defined as

$$f(s) := b_o + g_o \mathtt{erf}((s - b_a)/g_s),\; s \in [0,\,1] \tag{15}$$

where $b_o = g_o = 1/2$, b_a and g_s are tunable real parameters and $\mathtt{erf}(\cdot)$ is the *error function*. The terms a_i and h_i are defined as vectorial composition of functions f whose parameters g_s and b_a are stochastic variable, realizations of random processes characterized by a uniform probability in the set $[0,\,1]$ for b_a and $[-1,\,1]$ for g_s, see Figures 1 and 2.



(a) $b_a \in [0,\,1]$ and $g_s \in [0,\,1]$ (b) $b_a \in [0,\,1]$ and $g_s \in [-1,\,0]$

Fig. 1. Function a_i with b_a and g_s randomly generated.

In Figure 1a are depicted 5 samples of the function a_{i_j} obtained by a random generation of the parameters b_a and g_s. If each a_{i_j} represents the first entry of the function α_i of each student (*i.e.* by imposing $j = 1$), this figure depicts the learning ability (or equivalently the inverse of the learning inertia) of five students where among them, the student 5 behaves much better than the student 3 whereas the student 1 is characterized by a quasi-instantaneous change of mindset (probably he/her discovered the way to study better). Anyway, at the beginning the student 5 is barely dynamic and the educative actions take effects in a time much longer than for the rest of the students.

Figure 1b shows the typical evolution of the function a_{i_j} when relative to the student ability to memorize (or the easiness of forgetting). The student who better behaves, in the mean, could be considered the number 1 whereas, at the beginning, the numbers 2, 4 and 5 are scarcely able to remember the acquired informations. Furthermore, the student 4 is who shows the best ability to not forget high-level notions.

With respect to the Figure 2, first the plots are considered as representative of the function h_i. Furthermore, each h_i is thought as first entry of the function η_i of the i-th student. Thus, the student 4 is highly influenced by the educative actions, for a good range of differences between the current state of the student and the level of educational

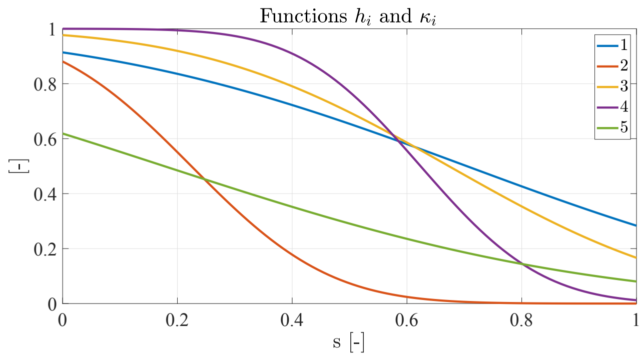


Fig. 2. Functions h_i with $b_a \in [0, 1]$ and $g_s \in [-1, 0]$ randomly generated.

action. As consequence, for the student 4 it is possible to provide teaching actions which may substantially differ from his/her state to make the student learning rapidly. In comparison, the student 2 shows a steep decrease in efficiency so that, if stimulated by the same educative actions given to the student 4, improves much less.

Furthermore, the function d_i is defined as a vector composition of three functions g defined for $s \in [0, 1]$ as:

$$g(s) := \frac{\mathtt{erf}((s - b_a)/g_s) - \mathtt{erf}((-b_a)/g_s)}{\mathtt{erf}((1 - b_a)/g_s) - \mathtt{erf}((-b_a)/g_s)}. \tag{16}$$

The parameters g_s and b_a are set as $g_s = 1/2$ and $b_a \in [0.55, 0.65]$. Figure 3 depicts five samples of the function

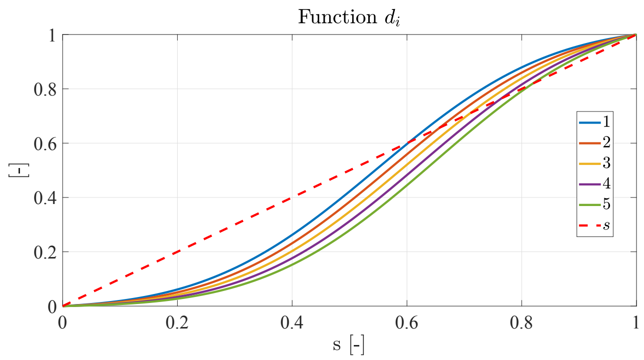


Fig. 3. Function d_i with $g_s = 1/3$ and $b_a \in [0.55, 0.65]$ randomly generated.

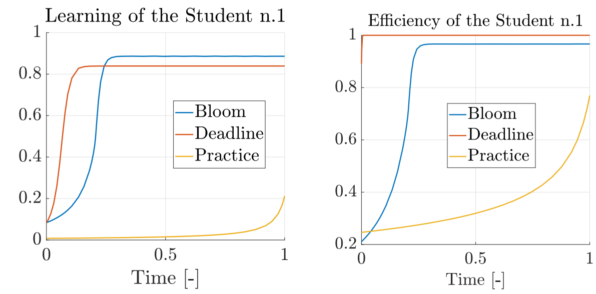
d_i which, for simplicity of analysis, are associated to the first entry of the δ_i of five different students. The lower is the value of the b_a the higher is the self-learning ability thus, the student 1 performs better than the second and so on. This picture reports also the function s to make graphically appreciable the difference between $d_i(s)$ and s. It is worth observing that, for $d_i(s) - s < 0$, the students are unable to self-improve theyr state. In this context, the student 1 performs better because becomes self-sufficient before the remaining part of the students.

4.2 Simulation results

Single Student With Constant Education Actions. The simulation proposed in this section describes the behaviour, over time, of a single student subject to a teaching action. The simulated student is described by the parameters listed in the Table 1 in which the columns *Bloom*, *Deadline* and *Practice* indicate the values of the parameters respectively associated to the first, second and third functions of α, η and δ. The entries of the function $\alpha_i(x)$ have been magnified respectively by (3.3279, 2.8517, 2.2867) (randomly generated) whereas the simulation time, fixed to 20 seconds, has been normalized to 1. The educative actions are set to be constant and equal to 1 to simulate an inefficient system in which too complex notions are provided to a freshmen whose initial state is equal to $x(0) = (0.0146, 0.0759, 0.0407)$.

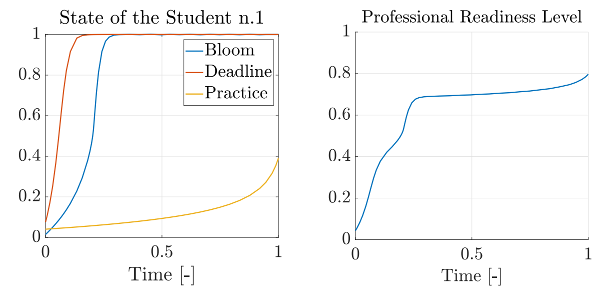
Function	Param.	Bloom	Deadline	Practice
Learning Act.	g_s	0.7339	0.7422	0.5539
Learning Act.	b_s	0.5405	0.6140	0.5637
Forgett. Act.	$-g_s$	0.5665	0.7477	0.6398
Forgett. Act.	b_s	0.7120	0.5815	0.5588
Self Learning	b_s	0.6056	0.5970	0.5765
Efficiency	$-g_s$	0.7265	0.1377	0.5890
Efficiency	b_s	0.6854	0.9406	0.7909

Table 1. Parameters of the student n.1



(a) Learning Activity: $\alpha_i(x(t), v(t))$ for $v(t) > 0$. (b) Efficiency: $\eta(u(t) - x(t))$.

Fig. 4. Student Learning Activity and Efficiency over time: case of constant teaching.

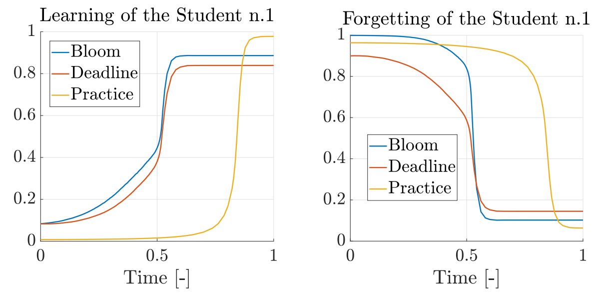


(a) Student State: $x(t)$ (b) Student PRL: $PRL(t)$

Fig. 5. Student State and PRL over time: case of constant teaching.

The combination of a poor learning ability and an inefficient teaching action leads to a slow and unsatisfactory increment of the practice level. Indeed, as shown in Figures 4a-4b, the student does not react, as he does for the case of the $\overline{\mathtt{EL}}$ and $\overline{\mathtt{DML}}$, to the teaching stimulus. The next paragraph shows that a gradual educative action can lead to better results because of a more efficient teaching. In particular, the final PRL reached in Figure 5b is overcome by that reached in Figure 8b.

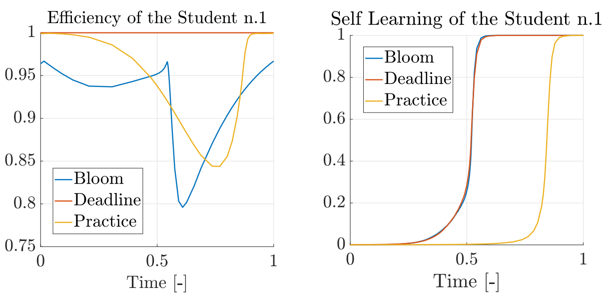
Single Student With Linearly Time Varying Education Actions. Taking the same student of the previous simulation case and assuming a teaching law which increases linearly from `0` to `1` over the time span, the student evolution is depicted in Figures 6-8.



(a) Learning Activity: $\alpha_i(x(t), v(t))$ for $v(t) - x(t) > 0$.

(b) Forgetting Activity: $\alpha_i(x(t), v(t))$ for $v(t) - x(t) \leq 0$.

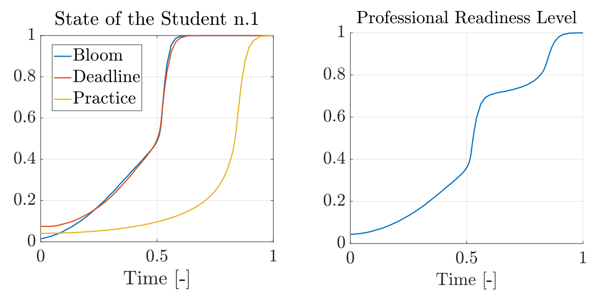
Fig. 6. Student Activity over time: case of linearly time varying teaching.



(a) Efficiency: $\eta_i(u(t) - x(t))$

(b) Self Learning: $\delta_i(x(t))$

Fig. 7. Student Efficiency and Self Learning over time: case of linearly time varying teaching.



(a) Student State: $x(t)$

(b) Student PRL: $PRL(t)$

Fig. 8. Student Efficiency and Self Learning over time: case of linearly time varying teaching.

The parameters describing the student are listed in Table 1. Thanks to the teaching action the student improves both his capacity of learning and his ability to memorize. Indeed, as depicted in Figure 6a and 6b the student shows an improvement of the entries of the function $\alpha(x(t), v(t))$ for $v(t) - x(t) > 0$ and, simultaneously a decrease of the functions composing $\alpha(x(t), v(t))$ for $v(t) - x(t) =\leq 0$. With reference to the Figures 6-8, the teaching actions, which are equally distributed on the three element of the state, have three different effects due to the different attitudes of the student. Since the student shows difficulties in learning and memorize how to deal with practical activities, the teaching actions, even if provided with a good efficiency, result in a slow increment of the third entry of the state, *i.e.* $x^{(3)}(t)$, over time. For this reason, the PRL shows two increasing phases, the first relative to the improvement of the $\overline{\text{EL}}$ and $\overline{\text{DML}}$ and the second due to the improvement of the $\overline{\text{PL}}$. Moreover, at time $t \approx 0.5$, the self learning ability of the student is sufficiently high to make the student self improving his states $x^{(1)}(t)$ and $x^{(2)}(t)$. Indeed, in Figure 8a, after time $t \approx 0.5$ the educational level, $\overline{\text{EL}}$, and the deadline meeting level, $\overline{\text{DML}}$, increase with a higher rate.

5. CONCLUSIONS AND FUTURE PERSPECTIVES

This paper presented a new mathematical model which describes the dynamics of a student subject to educative actions in the context of Aerospace Engineering. After the introduction of a suitable taxonomy, this work details each single term appearing into the proposed model. Finally, the correctness of the model is confirmed by the simulation results which offer also a first sight on the design of effective educative actions. Future works will investigate the model of the group as a whole and will propose new verification criteria to measure the state of the students, as defined in the present manuscript.

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