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Quaternion based Three-Dimensional Impact Angle Constrained Guidance

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Abstract: This paper proposes quaternion based three-dimensional guidance strategies, which ensure target interception with zero miss distance. Unlike most of the existing guidance strategies, the guidance command in this paper is derived using coupled engagement dynamics, which helps to maintain satisfactory performance for the engagements where decoupling is no longer valid. To avoid the well known possible singularities due to Euler's representation, a quaternion based representation of three-dimensional engagement is utilized. This facilitates the proposed guidance strategy to remain applicable and effective for wider range. In addition, guidance strategy is also derived to ensure target interception at a desired impact angle, which is expressed in terms of desired line-of-sight quaternion. Simulation results are shown to evaluate the efficacy of proposed guidance strategies for various engagement scenarios.

Keywords: Three-dimensional guidance, impact angle guidance, quaternion, coupled dynamics

1. INTRODUCTION

In order to counteract intelligent targets, armed with advanced interceptor defence systems, interceptor guidance laws with in-flight constraints might be effective. These in-flight constraints may include following a pre-defined trajectory, achieving a pre-defined impact time constraint or an attitude constraint, depending on mission requirements. Imposing constraint on interception angle leads to an effective and precise interception by maximizing penetrability and minimizing co-lateral damage. By approaching the target from a specific direction may also help to avoid the detection by enemy and a possible counterattack.

Owing to its numerous benefits, vast amount of research had been carried out to make impact angle constrained guidance more accurate, reliable, and robust. Kim and Grider (1973) made one of the initial efforts to design impact angle constrained sub-optimal guidance for a reentry vehicle. This work was further extended by York and Pastrick (1977) to account for first order autopilot. Few researchers, such as (Jeong et al., 2004; Kim et al., 1998; Lee et al., 2013; Ratnoo and Ghose, 2008, 2010), devised various guidance laws to fulfil impact angle constraints by making use of different supplementary bias terms with the conventional proportional navigation guidance (PNG) law.

Modern control techniques used for impact angle constrained guidance laws include work of (Song and Shin, 1999) where they proved that the time-optimal control results in a trajectory satisfying the predetermined impact angle with low maximum altitude. The works in (Feng et al., 2014; Ryoo et al., 2003, 2005; Shaferman and Shima, 2008), etc. also discussed optimal control based guidance laws using variety of performance index functions, which enable the target interception at predetermined impact angle. In (Ratnoo and Ghose, 2009) a state dependent Riccati equation (SDRE) method was proposed to solve impact angle problem against stationary target, by formulating it as nonlinear regulator problem.

Kumar et al. (2014); Rao and Ghose (2013) proposed sliding mode control based guidance law using nonlinear kinematics to intercept both maneuvering and non-maneuvering targets at a desired impact angle. However, the application of these guidance strategies were limited to the planar engagements. Later, Kumar and Ghose (2014, 2017) proposed 3D impact angle control guidance law using both conventional and non-singular SMC methods, which ensures asymptotic and finite time interception at desired impact angle. The interceptor's lateral accelerations in both pitch and yaw directions were used to enforce sliding mode on appropriately chosen switching surface. Maity et al. (2014) developed a 3D impact angle guidance based on generalised model predictive static programming technique. It is an iterative technique and a guess control history was required for the implementation of this guidance. Biswas et al. (2018) derived two guidance laws based on dynamic inversion (DI) for disturbance free system and SMC to provide robustness against uncertainties. Later in Biswas et al. (2019), another 3D guidance strategy was derived based on unique relation between LOS and impact angles, using sliding mode control. The 3D guidance strategies presented above were based on Euler's representations, which is known to encounter singularity during engagement, and this may limit the performance of proposed strategies.

Plenty of research has been done towards terminal phase constrained interceptor guidance laws, based on Euler angle representation. Although, Euler angles are directly available for computations and their dynamics is also comparatively easy to comprehend, these dynamics fail under gimbal-lock situations. To avoid such instances, engagement dynamics may be decoupled and then guidance strategies can be derived for planar engagements. However, it is well known that decoupling a coupled dynamics leads to a trade off between computational complexity and the overall effectiveness of the guidance law.

Singularities can be dreadful for the overall system performance; and over the years, quaternions have been shown to be effective, as a rescue in such situations. Quaternions are widely used in spacecraft orientation control and unmanned aerial vehcile motion controls, because they require less computational power as compared to other conventional techniques.

In contrast to the existing works, this paper derives the interceptor-target engagement dynamics in quaternion form, and the same is used for designing the guidance strategies. The engagement parameters and their corresponding kinematics is first obtained in the quaternion form. The collision conditions for the Euler angle representation do not hold directly for the quaternions. Thus, we derived analogous necessary and sufficient collision condition for quaternion dynamics. The guidance strategies are derived based on dynamic inversion control technique for achieving interception of stationary and constant velocity targets. Also, the guidance strategy is extended to cater for the direction of impact in case of non-maneuvering targets. This is achieved by defining the interception angle as a function of LOS quaternion. As per the best of authors' knowledge, such guidance strategies using quaternion based dynamics are not presented earlier and is worth investigating here. The performance of these guidance laws are demonstrated with the numerical simulations, and shown to be satisfactory.

2. PROBLEM FORMULATION

Consider a 3D interceptor-target engagement geometry as shown in Fig. 1. The frames represented by axes $[X_I\ Y_I\ Z_I]$, $[X_M\ Y_M\ Z_M]$ and $[X_T\ Y_T\ Z_T]$ represent inertial frame of reference, the interceptor body frame and the target body frame, respectively. The relative distance between the interceptor and target is denoted by r in LOS frame, where the LOS frame is defined using azimuth angle ψ_L and elevation angle θ_L, with respect to the inertial frame. Similarly, the pairs (ψ_m, θ_m) and (ψ_t, θ_t) provide the orientation of the interceptor body frame and the target body frame with respect to inertial frame, respectively. The origin O of the inertial frame is considered to be at the launch point of the interceptor. Interceptor and target velocities in their corresponding body frames are represented as $(V_m, 0, 0)^T$ and $(V_t, 0, 0)^T$, which essentially implies that the velocity vectors are along x-direction of their respective frames. The lateral accelerations for the interceptor and the target, in the yaw and the pitch directions, are denoted by the pairs (A_{ym}, A_{zm}) and (A_{yt}, A_{zt}), respectively. The interceptor and target move with constant speed throughout the engagement, with the speed ratio $\nu = V_t/V_m < 1$. In 3D interceptor-target engagement, the velocity of the k^{th} vehicle in the inertial frame can be expressed as, for $k = \{m, t\}$,

$$V_{kI} = [V_k \cos\theta_k \cos\psi_k \ \ V_k \cos\theta_k \sin\psi_k \ \ V_k \sin\theta_k]^T .$$

The relative velocity components between interceptor and target are be obtained as

$$[V_r\ V_\theta\ V_\psi]^T = [\dot{r}\ r\dot{\theta}_L\ r\dot{\psi}_L \cos\psi_L]^T = [\mathbf{T}]_L^I\,[V_{tI} - V_{mI}],$$

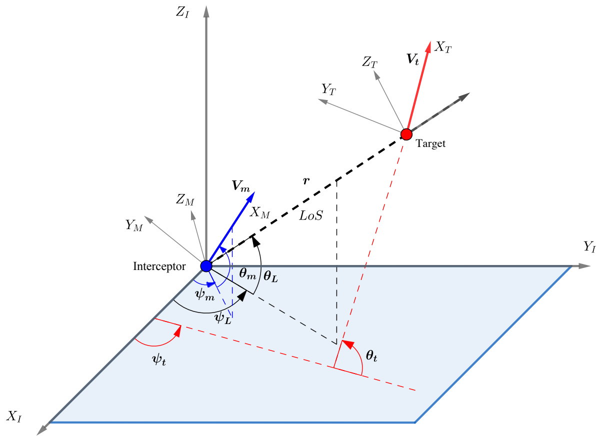


Fig. 1. Interceptor-target engagement geometry

where, $[\mathbf{T}]_I^L$ is a transformation matrix relating the vectors in the inertial frame and the LOS frame. The dynamics of elevation and azimuth angle for the k^{th} vehicle can be obtained using the expression,

$$\dot{\theta}_k = \frac{A_{zk}}{V_k};\ \dot{\psi}_k = \frac{A_{yk}}{V_k \cos\theta_k};\ \forall\ k = \{m, t\}.$$

The objective of this paper is to derive engagement parameters and the interceptor-target engagement dynamics in quaternion form, develop the necessary and sufficient conditions for target interception, and design of guidance strategies with and without imposing impact angle constraints. The guidance strategies should be derived without decoupling the engagement dynamics. This will facilitate the proposed guidance strategy to remain applicable for a large domain.

3. DERIVATION OF QUATERNION DYNAMICS

In this section, interceptor - target engagement parameters are obtained in quaternion form, followed by the derivation of quaternion dynamics.

3.1 Engagement Parameters in Quaternion Form

It can be observed from 3D interceptor-target engagement shown in Fig. 1, the components of LOS vector in the inertial frame may be obtained by first rotating it along Y_I axis by an angle θ_L, followed by a rotation along Z_I axis with an angle ψ_L. Let the quaternion $[Y_r]$ and $[Z_r]$ represent the rotations of LOS along Y_I axis with elevation angle θ_L and Z_I axis with azimuth angle ψ_L, respectively. The LOS vector in the inertial frame can be given by

$$r_I = [Z_r][Y_r]r[Y_r]^\star[Z_r]^\star = [Z_rY_r]r[Z_rY_r]^\star = [Q_L]r[Q_L]^\star$$

where $[Q_L] = [Z_rY_r]$, represents the composite rotation of LOS vector to transform it from LOS frame to the inertial frame. Note that, ψ_L is rotation in positive sense, while θ_L is a rotation in negative sense. Thus, the corresponding quaternions $[Z_r]$ and $[Y_r]$ can be obtained by,

$$[Z_r] = \cos\frac{\psi_L}{2} + \hat{\mathbf{k}}\sin\frac{\psi_L}{2}, \quad [Y_r] = \cos\frac{\theta_L}{2} - \hat{\mathbf{j}}\sin\frac{\theta_L}{2}. \quad (1)$$

Taking the quaternion product of $[Z_r]$ and $[Y_r]$ results into equivalent quaternion $[Q_L]$ as

$$[Q_L] = q_{0r} + q_{1r}\hat{i} - q_{2r}\hat{j} + q_{3r}\hat{k}$$

where, the components of quaternions are,

$$q_{0r} = \cos\frac{\psi_L}{2}\cos\frac{\theta_L}{2}, \quad q_{1r} = \sin\frac{\psi_L}{2}\sin\frac{\theta_L}{2}, \quad q_{2r} = \cos\frac{\psi_L}{2}\sin\frac{\theta_L}{2}, \quad q_{3r} = \sin\frac{\psi_L}{2}\cos\frac{\theta_L}{2}. \tag{2}$$

Similarly, as seen from Fig. 1, the interceptor velocity vector V_m and the target velocity vector V_t also undergo a sequential rotation along Y_I and Z_I axes with their corresponding elevation angles (θ_m, θ_t) and azimuth angles $(\psi_m,\ \psi_t)$ respectively. Thus, in the inertial frame, the interceptor velocity (V_{mI}) and the target velocity (V_{tI}) may also be obtained as

$$V_{mI} = [Q_m]V_m[Q_m]^\star, \quad V_{tI} = [Q_t]V_t[Q_t]^\star; \tag{3}$$

where, the quaternions $[Q_m] = [Z_mY_m]$ and $[Q_t] = [Z_tY_t]$, are the equivalent quaternions representing composite rotations of the interceptor and the target velocities along Y_I and Z_I axes, respectively. Based on these engagement parameters, the derivation of dynamics of interceptor - target engagement is discussed next.

3.2 Quaternion Interceptor-Target Dynamics

This section derives quaternion based interceptor-target dynamics. Note that a unit quaternion $Q(t)$ describes the variation of orientation of some moving object, represented by its body frame; relative to a fixed frame.

Let $\omega(t)$ be the angular velocity of the body frame with respect to the inertial frame. Then, the derivative of $Q(t)$, as given in (Jia, 2008), can be represented by,

$$\dot{Q} = \frac{1}{2}\omega Q. \tag{4}$$

The angular velocity ω of any vector may be derived from the Euler angle velocities such as

$$\omega = R_z\, R_y\, R_x\, \dot{\phi} + R_z\, R_y\, \dot{\theta} + R_z\, \dot{\psi}, \tag{5}$$

where, R_x, R_y, and R_z represent the Euler rotation matrices along X, Y, and Z axis, respectively. The dynamics of angular velocity is given by Euler's equation (Hoffman et al., 2018), as

$$J\dot{\omega} = -\omega^\times J\omega + u, \tag{6}$$

where the matrix $\omega^\times$ represents the skew-symmetric matrix, while the terms J and u denote positive definite inertia matrix and control torque, respectively. Thus, the quaternion system dynamics can be expressed by,

$$\dot{Q} = \frac{1}{2}\omega\, Q \quad \text{and} \quad J\dot{\omega} = -\omega^\times J\omega + u. \tag{7}$$

Using Eq. (7), we develop the collision condition for a interceptor-target engagement in the next subsection.

3.3 Collision Conditions for Quaternion Dynamics

For a guaranteed target interception, it is necessary that the LOS does not rotate in space with time, and the closing velocity of interceptor-target engagement must be positive $(V_c = -\dot{r})$. When the interceptor and the target satisfy these conditions, they are considered to be on collision course and the interception is inevitable. In Euler angle dynamics, these conditions are given by (Zarchan, 2012),

$$\dot{\theta_L} = \dot{\psi_L} = 0; \quad \dot{r} < 0. \tag{8}$$

For quaternion dynamics, we can borrow positive closing velocity condition, that is, $(V_c = -\dot{r})$ straight from Euler dynamics. However, in case of the quaternion dynamics, the orientation of LOS is given by the quaternion $[Q_L]$. Thus, the rate of rotation of LOS is controlled by the rate of change of $[Q_L]$, as given by Eq. (4). Clearly, achieving $\dot{Q}_L = 0$ ensures that the LOS orientation $[Q_L]$ remains constant in the space. Hence, the necessary and sufficient condition for interception in the quaternion dynamics can be expressed mathematically as

$$[\dot{Q_L}] = \frac{1}{2}\omega_L[Q_L] = 0; \quad \dot{r} < 0. \tag{9}$$

The required angular velocity of LOS vector, ω_L in Eq. (9) can be obtained from Eq. (5), and is equal to

$$\omega_L = \left[\dot{\theta_L}\sin\psi_L,\ -\dot{\theta_L}\cos\psi_L,\ \dot{\psi_L}\right]^T. \tag{10}$$

Using this collision condition, the guidance law will now be derived for the interceptor to achieve a target interception in the subsequent section.

4. DERIVATION OF GUIDANCE STRATEGY

In this section, the derivation of guidance strategy using dynamic inversion control technique is presented. It is important to observe that, to satisfy collision conditions mentioned in Eq. (9), either $[Q_L]$ or ω_L has to be equal to zero. However, it is also clear from Eq. (2) that there are no real values of θ_L and ψ_L, which make $[Q_L] = 0$. Hence, to achieve successful interception, ω_L needs to be controlled to zero.

4.1 Target Interception

To achieve target interception, the error in angular velocity can be defined as $e = \omega_L$. Consider a first order error dynamics given by

$$\dot{e} + K\, e = 0, \tag{11}$$

where $K \in \mathbb{R}^{3\times3}$ is a diagonal positive definite matrix, with diagonal entries as the time constant of desired error dynamics for each component. The error and its dynamics reduce to $e = \omega_L$, $\dot{e} = \dot{\omega}_L$, which on substitution in Eq.(11) results in

$$\dot{\omega}_L + K\,\omega_L = 0. \tag{12}$$

Pre-multiplying both sides of Eq.(12) by inertia matrix J and using Eq.(6), one may obtain

$$-\omega_L^{\text{x}} J\omega_L + u + J\, K\, \omega_L = 0,$$

which, in turn, gives the designed control torque as

$$u = \omega_L^{\text{x}} J\omega_L - J\, K\, \omega_L, \tag{13}$$

to ensure $\omega_L \to \omega_{Ld} = 0$ asymptotically. Note that when the desired control objective is achieved, $\omega_L = 0$, and thus the control effort u also becomes zero. Subsequently, the interceptor with this guidance strategy intercepts stationary, moving, as well as maneuvering targets. Although, the guidance law, given by Eq. (13), ensures target interception, it does not guarantee the achievement of interception from a desired specific direction.

4.2 Target Interception at Desired Impact Angle

As discussed in Kumar and Ghose (2017), impact angle and LOS angle have one-to-one correspondence, and in fact, for stationary target, desired impact angle can be expressed in terms of desired LOS orientation at the time of interception, if interceptor is on collision course with target. Thus, to achieve interception at pre-defined angle, the LOS angle at the time of interception needs to be controlled.

Let us assume that $[Q_{Ld}]$ denotes the quaternion, representing desired LOS orientation at the time of impact. To achieve interception from a desired impact direction, in addition to ensuring collision conditions, the guidance strategy must also ensure that the following relation holds

$$[Q_L] = [Q_{Ld}]. \tag{14}$$

The difference between two quaternions, $[Q_L]$ and $[Q_{Ld}]$, is given as $[Q_L][Q_{Ld}]^\star$, where $[Q_{Ld}]^\star$ denotes the conjugate of $[Q_{Ld}]$. Hence, post-multiplying Eq. (14) by $[Q_{Ld}]^\star$ on both sides, we get

$$[Q_L][Q_{Ld}]^\star = [Q_{Ld}][Q_{Ld}]^\star = 1. \tag{15}$$

In other words, the constraint on guidance design also needs to account for the relation $[Q_L][Q_{Ld}]^\star - 1 = 0$. To do so, let us define error of the system given by $e = [Q_L][Q_{Ld}]^\star - 1$. Note that, in order to satisfy the collision conditions given in Eq.(9), $\dot{Q}_{Ld} = 0$ and $\ddot{Q}_{Ld} = 0$. Therefore, on differentiating e one may get,

$$\dot{e} = \dot{Q}_L[Q_{Ld}]^\star, \quad \ddot{e} = \ddot{Q}_L[Q_{Ld}]^\star.$$

By differentiating the quaternion $[Q_L]$ twice, we get

$$\ddot{Q}_L = \frac{1}{2}\left[\dot{\omega}_L Q_L + \omega_L \dot{Q}_L\right]. \tag{16}$$

Pre-multiplying both sides of this equation by inertia matrix J results in,

$$J\ddot{Q}_L = \frac{1}{2}\left[J\dot{\omega}_L Q_L + J\omega_L \dot{Q}_L\right]. \tag{17}$$

Substituting the value of $J\dot{\omega}_L$ from Eq. (6) in Eq. (17) results into

$$J\ddot{Q}_L = \frac{1}{2}\left[(-\omega_L^\times J\omega_L + u)Q_L + J\omega_L \dot{Q}_L\right]. \tag{18}$$

Consider the second order error dynamics, given by

$$\ddot{e} + K_v\dot{e} + K_p e = 0, \tag{19}$$

where $K_p, K_v \in \mathbb{R}^{3\times 3}$ are diagonal, and positive definite matrices, representing the damping factor and natural frequencies of the second order error dynamics, of each component. Substituting values of e, $\dot{e}$ and $\ddot{e}$ in the error dynamics, we get

$$0 = [\ddot{Q}_L][Q_{Ld}]^\star + K_v[\dot{Q}_L][Q_{Ld}]^\star + K_p\left([Q_L][Q_{Ld}]^\star - 1\right)$$
$$[\ddot{Q}_L] = -K_v[\dot{Q}_L] - K_p[Q_L] - K_p[Q_{Ld}].$$

Pre-multiplying both sides of above equation by inertia matrix J leads to

$$J[\ddot{Q}_L] = -JK_v[\dot{Q}_L] - JK_p[Q_L] - JK_p[Q_{Ld}]. \tag{20}$$

By substituting Eq. (18) in Eq. (20), the control torque, u, is given by,

$$\begin{aligned} u = \omega_L^\times J\omega_L - J(2K_v + \omega)[\dot{Q}_L][Q_L^*] \\ + 2K_p J([Q_{Ld}] - [Q_L])[Q_L^*]. \end{aligned} \tag{21}$$

Equation (21) represents the necessary control torque required to intercept a target at pre-defined angle. This is achieved by reducing the error between actual LOS quaternion $[Q_L]$ and desired LOS quaternion $[Q_{Ld}]$ to zero. In this case, proposed guidance strategy ensures the alignment of interceptor to collision course from desired direction.

Remark : This law may also work for moving targets and achieve interception at pre-specified LOS quaternion $[Q_{Ld}]$, however, owing to the target motion, the interception angle may not be expressed directly in terms of LOS angle at the time of impact. In other words, guidance command, given by Eq. (21), can be used to intercept moving targets at pre-defined impact angles, provided that the desired impact angle is given in terms of desired LOS orientation at the time of impact. For better understanding, a typical interceptor-target engagement scenario is presented in following section for constant velocity targets.

5. SIMULATIONS

In this section, we evaluate the performance of proposed guidance laws, derived in previous section, to validate their efficacy for various engagement geometries. The simulation parameters and initial conditions used are listed in Table 1. The inertia matrix J is $diag(5.7\times 10^3, 5.7\times 10^3, 28.224)$. Also, the maximum allowable interceptor lateral acceleration is assumed to be limited to 400 m/s^2 in each direction.

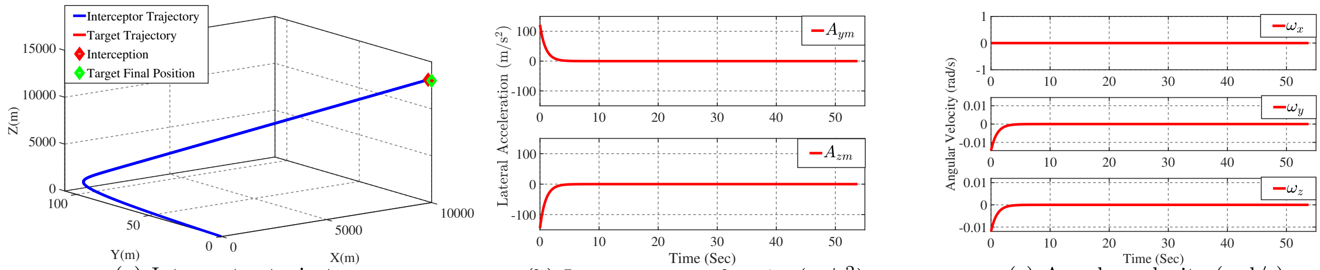
Table 1. Simulation parameters.

Symbol	Values	Symbol	Values
M	$(0, 0, 0)$	θ_m	22°
T	$(10000, 0, 13000)$	ψ_m	15°
V_m	500m/s	θ_t	0°
V_t	250m/s	ψ_t	0°

5.1 Target Interception

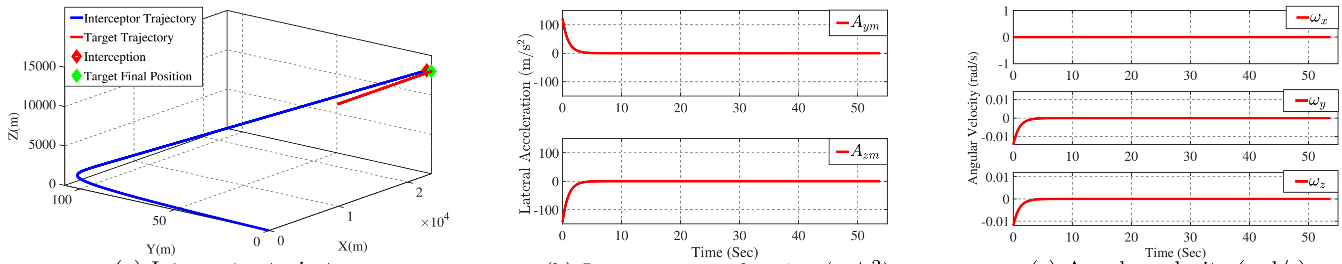
In this subsection, we evaluate the performance of the guidance law designed for target interception. We first consider a typical engagement scenario with initial conditions same as in Table 1. The simulation results depicted in Fig. 2, show the trajectories of interceptor, lateral acceleration profiles in both pitch and yaw directions, and the variations of components of angular velocity with respect to time. As can be seen from Fig. 2(a), the interceptor intercepts the target successfully. The lateral acceleration requirement of the interceptor is higher during the initial phase of engagement, as shown in Fig. 2(b). But towards the interception, acceleration demand reduces to zero, which is desirable feature for any guidance. The reason behind initial high acceleration demand is due to the requirement of interceptor to align on a collision course, that is, to make $[\dot{Q}_L]$ equal to zero. Once, the LOS quaternion attains a constant value, the control requirement goes to zero. Clearly, when there is no change in rotation of LOS, angular velocity, which is also defined as the error in this system, goes to zero as evident from Fig. 2(c).

Next we perform simulation for moving target, where the target is moving in horizontal direction, receding away from interceptor, with constant velocity V_t. The initial conditions are the same as in Table 1, and the results are plotted in Fig. 3. The interceptor is still able to intercept the target, as shown in Fig. 3(a). As soon as the angular velocity of LOS reaches to zero as shown in Fig. 3(c), $[Q_L]$ attains a constant value, making the acceleration requirement equal to zero, as presented in 3(b). Other behaviours are similar to those in the previous case.



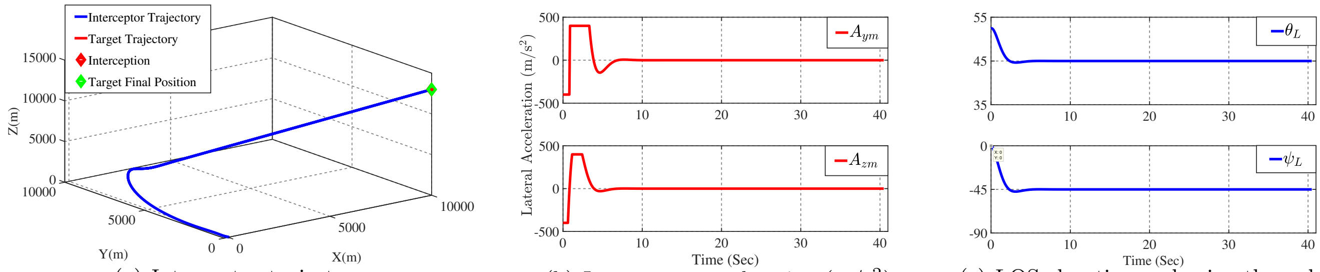
(a) Interceptor trajectory (b) Interceptor acceleration$(\mathrm{m/s^2})$ (c) Angular velocity (rad/s)

Fig. 2. Engagement scenario of interception of a stationary target.



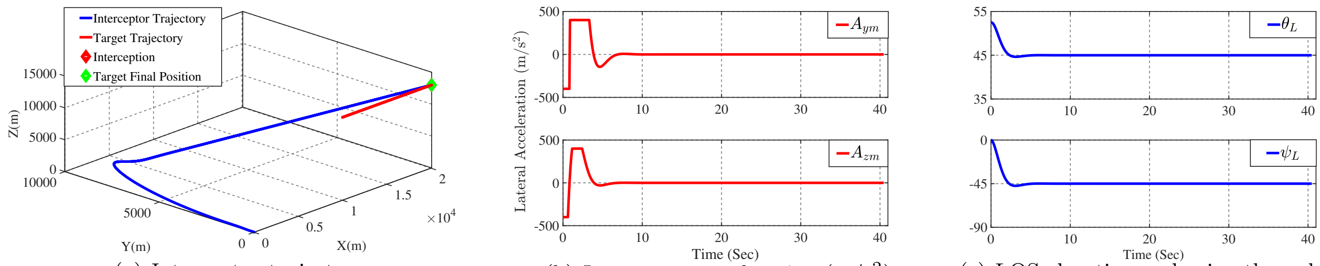
(a) Interceptor trajectory (b) Interceptor acceleration $(\mathrm{m/s^2})$ (c) Angular velocity (rad/s)

Fig. 3. Engagement scenario of interception of a moving target.



(a) Interceptor trajectory (b) Interceptor acceleration $(\mathrm{m/s^2})$ (c) LOS elevation and azimuth angles

Fig. 4. Engagement scenario of interception of a stationary target at pre-defined impact angle.



(a) Interceptor trajectory (b) Interceptor acceleration $(\mathrm{m/s^2})$ (c) LOS elevation and azimuth angles

Fig. 5. Engagement scenario of interception of a moving target at pre-defined impact angle.

5.2 Target Interception at Pre-defined Impact Angle

To evaluate the performance of proposed guidance strategy, simulation is performed with a desired impact angle of $\theta_d = -45°$ and $\psi_d = 45°$, leading to a desired quaternion

$$[Q_{Ld}] = 0.8531 - (0.1464\hat{i} + 0.3536\hat{j} + 0.3536\hat{k})$$

where $\hat{i}, \hat{j}$, and $\hat{k}$ are the unit vectors along x, y and z directions, respectively. The initial conditions and simulation parameters used are the same as those listed in Table 1. As can be seen from Fig. 4(a), the interceptor performs a significant maneuver to satisfy impact angle constraint, also reflected from its trajectory, to approach target from desired direction, which was absent in Fig. 2(a). The requirement on interceptor's maneuver is shown in Fig. 4(b). For the purpose of visualization, the orientation of LOS throughout the engagement is also shown in the form of Euler angles in Fig. 4(c). Note that, θ_L and ψ_L become constant, once the interceptor orients itself in the desired direction.

To test the performance of proposed impact angle guidance, simulation is also performed against a moving target, with the initial conditions as listed in Table 1, and results are shown in Fig. 5. The interceptor is required to intercept the target at a specified impact angle, given in terms of LOS orientation (θ_d,ψ_d), with desired quaternion [Q_{Ld}]. Fig. 5(a) shows the evolution of trajectories of interceptor and target, which confirms the successful target interception, with interceptor lateral acceleration profiles as shown in Fig. 5(b). From Fig. 5(c), it is clear that at the time of impact, the LOS orientation is same as the pre-specified values of θ_d and ψ_d. However, owing to the target motion, this orientation does not depict the value of actual impact angles, but it is representative of them.

6. CONCLUSION

Most of the researchers prefer using planar engagements for developing guidance laws, as occurrence of singularities is a serious issue in nonlinear engagement kinematics. However, this reduces overall effectiveness of guidance laws with in-flight constraints. In this paper, guidance strategies are derived after obtaining 3D interceptor-target engagement dynamics in quaternion form, which allows us to avoid the possible singularities in case of gimbal-lock situations, and also reduces computational burden on the system. Based on this newly devised quaternion dynamics, the guidance laws for unconstrained interception and impact angle constrained interception are derived. These guidance strategies were derived using dynamic inversion control technique. Both the guidance laws were shown to perform satisfactorily, with the help of numerical simulations. Simulations also reveal that the impact angle constrained guidance law can intercept a moving target at a pre-defined direction of impact. Future work involves consideration of multiple in-flight constraints to the existing guidance law along with investigating the practical implementability of the proposed dynamics.

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