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Prescribed Performance Consensus of Heterogeneous Quantized High-Order Uncertain Multi-Agent Systems

Andreas P. Kechagias * and George A. Rovithakis *

* *Department of Electrical & Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece (e-mail: andreaskp@ece.auth.gr, robi@eng.auth.gr)*

Abstract: We consider the problem of guaranteeing output consensus, with prescribed transient and steady-state performance bounds, for a class of uncertain, heterogeneous, high-order, nonlinear multi-agent systems, in a leader-following scheme. The proposed control protocol is decentralized and of low-complexity. An interesting feature is that information (state measurements, control inputs) is exchanged only in quantized form. For that purpose, uniform-hysteric quantizers are utilized. Simulations illustrate the effectiveness of the approach.

Keywords: Multi-agent systems, Quantization, Performance

1. INTRODUCTION

Motivated by numerous applications, the problem of reaching consensus in a group of dynamical systems, which exchange information via an underline communication network (multi-agent system), has received significant attention. The problem is decomposed into achieving synchronization between specific variables of interest, irrespectively of the presence of nonlinear and possibly uncertain agent dynamics. Particularly, when heterogeneity is a leading property of agent dynamics, the focus is on reaching an agreement between the outputs of the agents. In this work, we consider heterogeneous high-order nonlinear agent dynamics in a leader-following scheme.

A large literature is available on the topic Mondal et al. (2017), Li et al. (2017), Zhang and Li (2017), incorporating various constraining assumptions, among which, the most significant one, concerns the level of knowledge required regarding the leader and the following agent dynamics, to develop a decentralized control solution. Even in cases where partial knowledge is available Xu et al. (2016), Yang et al. (2016), Ding (2015), the utilization of adaptive algorithms and/or neuro/fuzzy approximating structures to acquire such knowledge or to compensate for its absence, inevitably increase the complexity of the control solution. This observation stems from the fact that parameter estimates have to be updated on-line, increasing the number of nonlinear differential equations that have to be solved numerically, and additional calculations have to be conducted to produce the control signals; making difficult the distributed implementation of the proposed control scheme. Constraints in the computational power available on each agent platform, which typically applies, elevates further the problem.

In addition to the aforementioned, in a networked control systems framework, limitations apply with respect to the available bandwidth. To reduce the communication load, all signals in the closed-loop system are subject to quantization. However, the presence of quantization introduces discontinuities in the closed-loop, thus jeopardizing stability and degrading performance. In the literature of high-order nonlinear multi-agent systems, proposed solutions have been restricted in the homogeneous agent dynamics Li et al. (2019), Wang et al. (2017), addressing heterogeneity only in conjunction with linear agent dynamics Fu and Wang (2014), Zhu et al. (2015).

In the framework of multi-agent systems, enforcing consensus with guaranteed performance during transient and steady-state has received less attention despite its significance. In this direction, and exploiting the Prescribed Performance Control (PPC) methodology, first introduced by Bechlioulis and Rovithakis (2008), the work Wang et al. (2015) addresses nonlinear, uncertain, high-order, heterogeneous multi-agent systems. However, the utilization of fuzzy approximating structures to handle the uncertain nonlinearities and the associated adaptive laws, increase the complexity of the control solution. A low-complexity solution to the aforementioned problem was provided in Bechlioulis and Rovithakis (2016). Yet, none of the above-mentioned works discusses the incorporation of quantization in the closed-loop. A solution was provided in Liang et al. (2019), utilizing PPC methodology and fuzzy approximating structures. Only control input quantization was considered and the agent dynamics were homogeneous. Furthermore, the use of fuzzy systems led to a relatively complex control protocol.

From the aforementioned literature review becomes evident that currently, no approximation-free and low-complexity control protocol exists to guarantee consensus with prescribed performance for high-order, heterogeneous, nonlinear multi-agent systems, with all closed-loop signals (state measurements and control inputs) being quantized. A solution to fill this gap is proposed in this work.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider an heterogeneous multi-agent system consisting of a leader and N followers, with the leader acting as an external system that produces the desired reference trajectory for the group of followers. The dynamics of each agent satisfy an m_ith-order SISO nonlinear system:

$$\left.\begin{aligned} \dot{x}_{i,j} &= x_{i,j+1},\ j = 1, ..., m_i - 1 \\ \dot{x}_{i,m_i} &= f_i(\mathbf{x}_i) + g_i(\mathbf{x}_i) q(u_i) + d_i(t) \\ y_i &= x_{i,1} \end{aligned}\right\},\ i = 1, ..., N \quad (1)$$

where $\mathbf{x}_i = [x_{i,1}, ..., x_{i,m_i}]^T \in \mathbb{R}^{m_i}$, $i = 1, ..., N$ represent the state of each agent, $f_i : \mathbb{R}^{m_i} \to \mathbb{R}$, $i = 1, ..., N$ and $g_i \in \mathbb{R}^{m_i} \to \mathbb{R}$, $i = 1, ..., N$ are unknown locally Lipschitz functions, $u_i \in \mathbb{R}$, $i = 1, ..., N$ is the control input, $y_i \in \mathbb{R}$, $i = 1, ..., N$ is the output and $d_i : \mathbb{R}_+ \to \mathbb{R}, i = 1, ..., N$ represent piecewise continuous and bounded external disturbances.

Assumption 1. The function g_i is either strictly positive or strictly negative for all $\mathbf{x}_i \in \mathbb{R}^{m_i}$, $i = 1, ..., N$ and its sign denoted as sgn(g) is considered known.

Assumption 2. The output of the leader $x_0(t) : \mathbb{R}_+ \to \mathbb{R}$ and its first-order derivative $\dot{x}_0$ are assumed to be bounded. However, only x_0 is available for measurement at each time instant and is considered unknown in advance.

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to model the communication among the followers, where $\mathcal{V} = \{v_1, ..., v_N\}$ indicates the set of vertices that represent the followers and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ indicates the set of edges. The graph is assumed as simple ($(v_i, v_i) \notin \mathcal{E}$). The adjacency matrix is denoted as $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ with $a_{ij} \in \{0, 1\}$, $i, j = 1, ..., N$. If $a_{ij} = 0$ then the agent i does not obtain information regarding the state of the agent j ($(v_i, v_j) \notin \mathcal{E}$), whereas if $a_{ij} = 1$ there is an information-sharing from agent j to agent i ($(v_i, v_j) \in \mathcal{E}$). State information regarding the leader agent (labeled as v_0) is only provided to a subgroup of the N agent. This information-sharing is modeled by a diagonal matrix $B = diag([b_1, b_2, ..., b_n]) \in \mathbb{R}^{N \times N}$. If $b_i = 1$, then agent i obtains information regarding the state of the leader node, if there is no such information-sharing, then $b_i = 0$. The set of neighbors of an agent v_i is denoted by $N_i = \{v_j : (v_i, v_j) \in \mathcal{E}\}$. The augmented graph is defined as $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \{v_0, v_1, ..., v_N\}$ and $\bar{\mathcal{E}} = \mathcal{E} \cup \{(v_i, v_0) : b_i = 1\} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$.
We pose the following assumption for the graph.

Assumption 3. The augmented graph $\bar{\mathcal{G}}$ is a directed acyclic graph composed of layers and each agent in a layer receives information only from an agent at a higher-level. The highest level contains only the leader [1].

The signals $u_i(t), x_i(t), x_0(t)$, $i = 1, ..., N$ enter the closed-loop in quantized form. In this work we consider uniform-hysteric quantizers (Ceragioli et al. (2011)) of the form:

$$q(z) = \begin{cases} q^-(z) - \delta & , if\ z \leq q^-(z) - \delta \\ q^-(z) & , if\ q^-(z) - \delta < z < q^-(z) + \delta. \\ q^-(z) + \delta & , if\ z > q^-(z) + \delta \end{cases} \quad (2)$$

In (2), the step size of the quantizer is denoted by $\delta > 0$ and the value of $q(z)$ at the time instant before t is denoted by $q^-(z)$. The output of the quantizer takes values from $\mathbb{Q} = \{w\delta \mid w = 0, \pm 1, \pm 2, ...\}$. Even though in generality we could employ quantizers (2) possessing different step-size for each signal, to simplify notation, in this work we shall consider identical step-sizes. It is true (Bikas and Rovithakis (2019a)):

$$q(z) = z + D(z),\ \forall t \geq 0, \quad (3)$$

with $D(z)$ satisfying $|D(z)| \leq \delta$, $\forall t \geq 0$.

[1] An example graph satisfying Assumption 3 is presented in Fig. 1

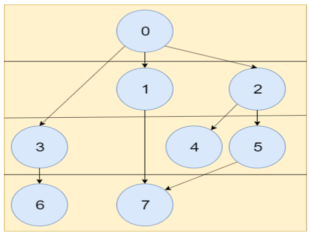

Fig. 1. Communication graph that obeys Assumption 3

Signal quantization introduces discontinuities in the closed loop. This leads to problems concerning the existence of solutions. Additionally, the chattering phenomenon must be prevented. Hysteric quantizers are employed to effectively avoid these problems. Such quantizers demonstrate a positive dwell time (Bikas and Rovithakis (2019a)) that guarantee the existence of solutions over a maximal time interval and the chattering-free performance.
Our objective is to design a low-complexity, distributed control protocol for the heterogeneous multi-agent system (1) satisfying Assumptions 1, 2, under a directed communication graph obeying Assumption 3, when all signals in the closed-loop are quantized, such that the output disagreement errors

$$\varepsilon_i(t) = x_{i,1}(t) - x_0(t) \in \mathbb{R},\ i = 1, ..., N \quad (4)$$

are driven to predefined and arbitrarily small neighborhoods of the origin, with prespecified minimum convergence rate, keeping all signals in the closed-loop bounded.

Remark 1. Besides being decentralized, the proposed control protocol should be of low-complexity as well. The latter, which is a highly desirable implementation property, should be attributed to *i)* no a priori knowledge regarding the agent nonlinearities should be required, *ii)* no approximating structures (i.e., neural networks, fuzzy systems) should be incorporated to acquire such knowledge, *iii)* no hard calculations (analytic or numerical) should be required to produce the control protocol and *iv)* the controller should be static, thus avoiding the expansion of the dynamic order of the closed-loop.

3. MAIN RESULTS

The following theorem, whose proof is presented in Section 4, summarizes the main results of this work.

Theorem 1. Consider the heterogeneous multi-agent system (1) satisfying Assumptions 1, 2, under a directed communication graph obeying Assumption 3. Given any initial conditions $\mathbf{x}_i(0) \in \mathbb{R}^{m_i}$ and for all $i = 1, ..., N$, the distributed control protocol:

$$\psi_{i,0} \triangleq \frac{\sum_{l\in N_i} a_{il} q(x_{l,1}) + b_i q(x_0))}{\sum_{l\in N_i} a_{il} + b_i}, \tag{5a}$$

$$\xi_{i,j} = \frac{q(x_{i,j}) - \psi_{i,j-1}}{\rho_{i,j} - \delta} \in \mathbb{R},\ j = 1, ..., m_i, \tag{5b}$$

$$\psi_{i,j} = -k_{i,j} ln(\frac{1+\xi_{i,j}}{1-\xi_{i,j}}) \in \mathbb{R},\ k_{i,j} > 0\ , j = 1, ..., m_i - 1, \tag{5c}$$

$$u_i = -sgn(g_i) k_{i,m_i} ln(\frac{1+\xi_{i,m_i}}{1-\xi_{i,m_i}}) \in \mathbb{R},\ k_{i,m_i} > 0, \tag{5d}$$

where

$$\rho_{i,j}(t) = (\rho^0_{i,j} - \rho^\infty_{i,j}) e^{-l_{i,j} t} + \rho^\infty_{i,j} \tag{5e}$$

with

$$0 < l_{i,j}, \rho^0_{i,j} \equiv \rho_{i,j}(0) > |q(x_{i,j}(0)) - \psi_{i,j-1}(0)| + \delta \tag{5f}$$

$$\rho^0_{i,j} > \rho^\infty_{i,j} > \delta,\ j = 1, ..., m_i \tag{5g}$$

guarantees:

(i) $$|\varepsilon_i(t)| < (N+1)(\frac{N^3+N^2-N}{N-1})^{\frac{N-1}{2}} (\rho_{i,1}(t) + \delta) \tag{6}$$

(ii) the boundedness of all signals in the closed-loop

Remark 2. Careful inspection of (6) reveals that all output disagreement errors $\varepsilon_i(t)$ present a predefined minimum convergence rate equal to $l_{i,1}$. Furthermore, the minimum output synchronization accuracy at steady state is prescribed to be $2\delta(N+1)(\frac{N^3+N^2-N}{N-1})^{\frac{N-1}{2}}$. The latter is true owing to (6), (5g) and the exponentially decaying structure of (5e). However, following closely the line of proof of Theorem 1, it is not difficult to verify that the minimum output synchronization accuracy can be reduced by 50%, if we assume that each agent measures its own state without the intervention of quantization. Nevertheless, as anticipated, state quantization constraints the achievable prescribed output synchronization accuracy, whose effect on performance is effectively shrunk, only if we select the quantization step-size δ so as $2\delta(N+1)(\frac{N^3+N^2-N}{N-1})^{\frac{N-1}{2}}$ becomes less than the accuracy of the corresponding measurement device, thus achieving practical convergence of $\varepsilon_i(t)$ to zero.

Remark 3. As already stated in Remark 2, only $\rho_{i,1}$ is related, via the selection of $l_{i,1}$ and $\rho^\infty_{i,1}$, to the introduction of prescribed performance on the output disagreement error $\varepsilon_i(t)$. The rest of the $\rho_{i,j}$-functions, $j = 2, ..., m_i$ comprise together with $k_{i,j}$ control elements can be freely chosen provided they satisfy (5f), (5g).

Remark 4. The proposed control protocol (5) is decentralized and of low-complexity as it satisfies all qualitative properties stated in Remark 1.

4. PROOF OF THEOREM 1

Let the strictly increasing function $T : (-1, 1) \to \mathbb{R}$ with $T(*) = ln((1+*)/(1-*))$ and for all $i = 1, ..., N$, define the normalized errors:

$$\xi^\sigma_{i,j} = \frac{x_{i,j} + \gamma\delta - \psi^\sigma_{i,j-1}}{\rho_{i,j} - \delta} \in \mathbb{R},\ j = 1, ..., m_i, \tag{7}$$

where $\sigma = \{min, max\}$,

$$\gamma = \begin{cases} 1 & , if\ \sigma = max, \\ -1 & , if\ \sigma = min, \end{cases}$$

and

$$\psi^\sigma_{i,0} = \frac{\sum_{l\in N_i} a_{il}(x_{l,1} - \gamma\delta) + b_i(x_0 - \gamma\delta)}{\sum_{l\in N_i} a_{il} + b_i} \tag{8a}$$

$$\psi^\sigma_{i,j} = -k_{i,j} T(\xi^\sigma_{i,j}),\ j = 1, ..., m_i - 1 \tag{8b}$$

Taking into account (3), (8b), and the strictly increasing property of T, it is concluded that for all $t \geq 0$,

$$\xi^{min}_{i,j} \leq \xi_{i,j} \leq \xi^{max}_{i,j},\ i = 1, ..., N\ , j = 1, ..., m_i \tag{9}$$

Define $r_{i,j} = \rho_{i,j} - \delta > 0$, $i = 1, ..., N$, $j = 1, ..., m_i$. Differentiating (7) with respect to time and using (3), it is obtained for all $i = 1, ..., N$ and $j = 1, ..., m_i - 1$

$$\dot{\xi}^\sigma_{i,j} = \frac{1}{r_{i,j}} [\xi^\sigma_{i,j+1} r_{i,j+1} - \gamma\delta + \psi^\sigma_{i,j} - \dot{\psi}^\sigma_{i,j-1} - \xi^\sigma_{i,j} \dot{r}_{i,j}] \tag{10a}$$

$$\dot{\xi}^\sigma_{i,m_i} = \frac{1}{r_{i,m_i}} [f_i(\mathbf{x}_i) + g_i(\mathbf{x}_i) u_i + g_i(\mathbf{x}_i) D(u_i) + d_i(t) - \dot{\psi}^\sigma_{i,m_i-1} - \xi^\sigma_{i,m_i} \dot{r}_{i,m_i}] \tag{10b}$$

Define $\xi_\sigma = [\xi^\sigma_{1,1}, ..., \xi^\sigma_{N,m_N}] \in \mathbb{R}^M$, where $M = \sum_i^N m_i$. Moreover, define:

$$\dot{\xi}_\sigma = h_\sigma(\xi_\sigma, t) = [h^\sigma_{1,1}, ..., h^\sigma_{N,m_N}]^T \tag{11}$$

where the h_σ functions are the right-hand sides of (10). Further, the $\psi^\sigma_{i,j}$ have been substituted by (8b). Defining $\xi = [\xi_{min}, \xi_{max}]^T$, we can write (10) in compact form as:

$$\dot{\xi} = [h^T_{min} h^T_{max}] \in \mathbb{R}^{2M} \tag{12}$$

Additionally, let us define the open set $\Omega_\xi = (-1, 1)^{2M} \subset \mathbb{R}^{2M}$. To prove Theorem 1, it is sufficient to show that the controller (5), guarantees that the functions $T(\xi^\sigma_{i,j})$, $i = 1, ..., N$, $j = 1, ..., m_i$, appearing in (10) remain bounded. Since the function T is strictly increasing, it leads to proving that ξ evolves strictly within a subset of Ω_ξ for all $t \geq 0$. Indeed, the latter and (9), leads us to the conclusion that $\xi_{i,j}$, $i = 1, ..., N$, $j = 1, ..., m_i$ will also evolve strictly within $(-1, 1)$ for all $t \geq 0$. Therefore, concluding that all closed-loop signals $u_i, \xi_{i,j}, \psi_{i,j}$, $i = 1, ..., N$, $j = 1, ..., m_i - 1$ remain bounded for all $t \geq 0$.

The proof of Theorem 1 consists of two phases. First, a unique maximal solution $\xi(t)$ of (12) over the set Ω_ξ for a time interval $[0, \tau_{max})$ is ensured. Then, in the second phase we show the validity of statements (i), (ii).

Phase A: The set Ω_ξ is open and non-empty. Also the performance functions $\rho_{i,j}(t)$, $i = 1, .., N$, $j = 1, ..., m_i$ are selected so that $\xi(0) \in \Omega_\xi$. System (12) consists of discontinuous dynamics owing to the quantized feedback in u_i. Therefore, it is not clear whether for any initial condition $\xi(0) \in \Omega_\xi$, there exist a unique maximal solution of $\xi(t)$ in $[0, \tau_{max})$, with $\tau_{max} \in [0, +\infty]$. However, following standard arguments (Bikas and Rovithakis, 2019a, Lemma A.1), we can readily show the correctness of the latter statement. Hence, $\xi(t) \in \Omega_\xi$ for all $t \in [0, \tau_{max})$ with $\tau_{max} \in [0, +\infty]$. Additionally, the chattering phenomenon is prevented.

Phase B: Owing to (9) and the results of Phase A, we conclude that $\xi_{i,j} \in (-1, 1)$, $i = 1, .., N$, $j = 1, ..., m_i$, for all $t \in [0, \tau_{max})$. Therefore,

$$\epsilon_{i,j} = T(\xi_{i,j}),\ i = 1, .., N,\ j = 1, ..., m_i \tag{13a}$$

$$\epsilon^\sigma_{i,j} = T(\xi^\sigma_{i,j}),\ i = 1, .., N,\ j = 1, ..., m_i \tag{13b}$$

are well-defined for all $t \in [0, \tau_{max})$.

Let L_κ, $\kappa = 1, 2, ...$ denoting the layers of the communication graph stated in Assumption 3, excluding the highest (layer 0), which contains only the leader. In what follows, we shall restrict our attention to the agents comprising L_1, utilizing a recursive step-like procedure. The analysis for all subsequent graph layers is similar and it is thus omitted.
Define $\underline{m} = min_{i\in L_1} m_i$ and $\overline{m} = max_{i\in L_1} m_i$.
Step 1 ($j = 1$ and $i \in L_1$): Define the positive definite functions:

$$V_{1,\sigma} = \sum_{i\in L_1} (\epsilon^\sigma_{i,1})^2. \tag{14}$$

Differentiating (14) with respect to time and using (10a), it is obtained for all $t \in [0, \tau_{max})$:

$$\dot{V}_{1,\sigma} = \sum_{i\in L_1} \phi^\sigma_{i,1}\epsilon^\sigma_{i,1}(\xi^\sigma_{i,2}r_{i,2} - \gamma\delta + \psi^\sigma_{i,1} - \dot{\psi}^\sigma_{i,0} - \xi^\sigma_{i,1}\dot{r}_{i,1}), \tag{15}$$

where $\phi^\sigma_{i,j} = 2/(1 + (\xi^\sigma_{i,j})^2)r_{i,j}$. Notice that $\phi^\sigma_{i,j} > 0$ for all $t \in [0, \tau_{max})$, since $\xi_\sigma \in (-1, 1)^M$ for all $t \in [0, \tau_{max})$ and $r_{i,j} > 0$ for all $t \geq 0$. Furthermore, since $i \in L_1$, it is concluded that $\psi^\sigma_{i,0} = x_0 - \gamma\delta$. Differentiating $\psi^\sigma_{i,0}$ with respect to time, yields $\dot{\psi}^\sigma_{i,0} = \dot{x}_0$, which are bounded owing to Assumption 2. Also, $r_{i,2}$ and $\dot{r}_{i,1}$ are bounded by construction. Additionally, define $S^\sigma_{i,1} = |\xi^\sigma_{i,2}r_{i,2}| + \delta + |\dot{x}_0| + |\xi^\sigma_{i,1}\dot{r}_{i,1}| > 0$. Using the aforementioned arguments, we conclude the existence of positive constants $\bar{S}^\sigma_{i,1}$, such that $S^\sigma_{i,1} \leq \bar{S}^\sigma_{i,1}$ for all $t \in [0, \tau_{max})$. Thus, for all $t \in [0, \tau_{max})$:

$$\begin{aligned}\dot{V}_{1,\sigma} &\leq \sum_{i\in L_1} \phi^\sigma_{i,1}|\epsilon^\sigma_{i,1}|\bar{S}^\sigma_{i,1} + \phi^\sigma_{i,1}\epsilon^\sigma_{i,1}\psi^\sigma_{i,1}\\ &= \sum_{i\in L_1} \phi^\sigma_{i,1}|\epsilon^\sigma_{i,1}|(\bar{S}^\sigma_{i,1} - k_{i,1}|\epsilon^\sigma_{i,1}|),\end{aligned} \tag{16}$$

where we have also employed (8b) and (13b).
Consequently, $\dot{V}_{1,\sigma} < 0$ when $|\epsilon^\sigma_{i,1}| > \bar{S}^\sigma_{i,1}/k_{i,1}$. Therefore, $|\epsilon^\sigma_{i,1}| \leq \bar{\epsilon}^\sigma_{i,1} \triangleq max\{|\epsilon^\sigma_{i,1}(0)|, \bar{S}^\sigma_{i,1}/k_{i,1}\}$ for all $t \in [0, \tau_{max})$. Using the inverse logarithmic function we conclude that

$$-1 < T^{-1}(-\bar{\epsilon}^\sigma_{i,1}) \leq \xi^\sigma_{i,1} \leq T^{-1}(\bar{\epsilon}^\sigma_{i,1}) < 1 \tag{17}$$

for all $t \in [0, \tau_{max})$. In addition, owing to (8b) and (13b), $\psi^\sigma_{i,1}$ are also bounded for all $t \in [0, \tau_{max})$. Combining the latter with (9), it is obtained that for all $t \in [0, \tau_{max})$,

$$-1 < T^{-1}(-\bar{\epsilon}^{min}_{i,1}) \leq \xi_{i,1} \leq T^{-1}(\bar{\epsilon}^{max}_{i,1}) < 1 \tag{18}$$

Finally, notice that owing to the continuity of $h^\sigma_{i,1}$ in (11), there exists positive constants $\bar{h}^\sigma_{i,1}$ that satisfy $|h^\sigma_{i,1}| \leq \bar{h}^\sigma_{i,1}$ for all $t \in [0, \tau_{max})$. The latter and (8b) lead to $|\dot{\psi}^\sigma_{i,1}| \leq 2k_{i,1}\bar{h}^\sigma_{i,1}/(1 - (\xi^\sigma_{i,1})^2)$ for all $t \in [0, \tau_{max})$ and so $\dot{\psi}^\sigma_{i,1}$ are bounded.

To continue, let us define the neighborhood synchronization error:

$$e_i = \frac{\sum_{l\in N_i} a_{il}(q(x_{i,1}) - q(x_{l,1})) + b_i(q(x_{i,1}) - q(x_0))}{\sum_{l\in N_i} a_{il} + b_i} \tag{19}$$

and the neighborhood errors:

$$e^\sigma_i = \frac{\sum_{l\in N_i} a_{il}(x_{i,1} - x_{l,1} + 2\gamma\delta) + b_i(x_{i,1} - x_0 + 2\gamma\delta)}{\sum_{l\in N_i} a_{il} + b_i}, \tag{20}$$

It is not difficult to verify that $\xi^\sigma_{i,1} = e^\sigma_i/r_{i,1}$ and $e^{min}_i \leq e_i \leq e^{max}_i$. Further, after performing some straightforward algebraic manipulations, it yields:

$$e^\sigma_i = \frac{(L + B)\varepsilon_i}{\sum_{l\in N_i} a_{il} + b_i} + 2\gamma\delta \tag{21}$$

By construction, the maximum value of $\sum_{l\in N_i} a_{il} + b_i$ is $N + 1$ and owing to Assumption 3 the L+B matrix is nonsingular. Utilizing the aforementioned, we conclude:

$$|\varepsilon_i(t)| \leq \frac{(N + 1)(|e^\sigma_i| + 2\delta)}{\vartheta_{min}(L + B)}, \forall t \in [0, \tau_{max}) \tag{22}$$

where $\vartheta_{min}(*)$ denotes the minimum singular value of a matrix. However, $\vartheta_{min}(L+B)$ is a global topology variable and therefore cannot be employed in distributed control schemes. To relax this issue, the conservative lower bound $\vartheta_{min}(L+B) \geq \frac{N-1}{N^3+N^2-N}^{\frac{N-1}{2}}$, Hong and Pan (1992), that depends solely on the the number of agents N and not the graph is utilized. Also, using (17) and the definition of $r_{i,j}$, we conclude that $|e^\sigma_i(t)| < \rho_{i,1}(t) - \delta$. Thus, for all $t \in [0, \tau_{max})$:

$$|\varepsilon_i(t)| < (N + 1)(\frac{N^3 + N^2 - N}{N - 1})^{\frac{N-1}{2}}(\rho_{i,1}(t) + \delta), \tag{23}$$

$$-\rho_{i,1}(t) < q(x_{i,1})(t) - q(x_0)(t) = e_i < \rho_{i,1}(t). \tag{24}$$

Step j ($j = 2, .., \underline{m} - 1$ and $i \in L_1$): Using the positive definite functions $V_{j,\sigma} = \sum_{i\in L_1}(\epsilon^\sigma_{i,j})^2$, employing the boundness of $\dot{\psi}^\sigma_{i,j-1}$ from the previous step, and using the same line of analysis, we conclude for all $t \in [0, \tau_{max})$:

$$-1 < T^{-1}(-\bar{\epsilon}^{min}_{i,j}) \leq \xi_{i,j} \leq T^{-1}(\bar{\epsilon}^{max}_{i,j}) < 1, \tag{25}$$

$|h^\sigma_{i,j}| \leq \bar{h}^\sigma_{i,j}$ and the boundness of $\dot{\psi}^\sigma_{i,j}$.
Step $\underline{m}$ ($j = \underline{m}$ and $i \in L_1$): Let w be the agent, whose dynamic model is of order $\underline{m}$. For all agents of L_1, we follow the same line of analysis as in the previous steps using the positive definite function $V_{\underline{m},\sigma} = \sum_{i\in L_1}(\epsilon^\sigma_{i,\underline{m}})^2$, with the exception of agent w, which is provided below. In this direction define:

$$V_{w,\underline{m},\sigma} = (\epsilon^\sigma_{w,\underline{m}})^2. \tag{26}$$

Differentiating (26) with respect to time and using (10b), it is obtained:

$$\begin{aligned}\dot{V}_{w,\underline{m},\sigma} = \phi^\sigma_{w,\underline{m}}\epsilon^\sigma_{w,\underline{m}}(&f_w(\mathbf{x}_w) + g_w u_w + g_w D(u_w) + d_w(t)\\ &- \dot{\psi}^\sigma_{w,\underline{m}-1} - \xi^\sigma_{w,\underline{m}}\dot{r}_{w,\underline{m}}), \forall t \in [0, \tau_{max}).\end{aligned} \tag{27}$$

Notice that owing to the continuity of f, g, application of the extreme value theorem guarantees the existence of positive constants $\overline{f}, \underline{g}, \overline{g}$ such that $|f(*)| \leq \overline{f}$ and $0 < \underline{g} \leq |g(*)| \leq \overline{g}$. Additionally, define the functions: $S^\sigma_{w,\underline{m}} = |f_w(\mathbf{x}_w)| + |g_w D(u_w)| + |d_w(t)| + |\dot{\psi}^\sigma_{w,\underline{m}-1}| + |\xi^\sigma_{w,\underline{m}}\dot{r}_{w,\underline{m}}| > 0$. The use of the above analysis guarantees the existence of positive constants $\bar{S}^\sigma_{w,\underline{m}}$, such that $S^\sigma_{w,\underline{m}} \leq \bar{S}^\sigma_{w,\underline{m}}$ for all $t \in [0, \tau_{max})$. Therefore, for all $t \in [0, \tau_{max})$

$$\begin{aligned}\dot{V}_{w,\underline{m},\sigma} &\leq \phi^\sigma_{w,\underline{m}}|\epsilon^\sigma_{w,\underline{m}}|\bar{S}^\sigma_{w,\underline{m}} + \phi^\sigma_{w,\underline{m}}\epsilon^\sigma_{w,\underline{m}}g_w u_w\\ &= \phi^\sigma_{w,\underline{m}}|\epsilon^\sigma_{w,\underline{m}}|\bar{S}^\sigma_{w,\underline{m}}\\ &\quad - \phi^\sigma_{w,\underline{m}}\epsilon^\sigma_{w,\underline{m}}g_w sgn(g_w)k_{w,\underline{m}}\epsilon_{w,\underline{m}}\\ &= \phi^\sigma_{w,\underline{m}}|\epsilon^\sigma_{w,\underline{m}}||g_w|\Big(\frac{\bar{S}^\sigma_{w,\underline{m}}}{\underline{g}_w}\\ &\quad - sgn(\epsilon^\sigma_{w,\underline{m}})sgn(\epsilon_{w,\underline{m}})k_{w,\underline{m}}|\epsilon_{w,\underline{m}}|\Big).\end{aligned} \tag{28}$$

Proposition 1. Bikas and Rovithakis (2019b), it holds that $sgn(\epsilon^{\sigma}_{i,m_i})sgn(\epsilon_{i,m_i}) = 1$ and $|\epsilon_{i,m_i}| = c_i(t)|\epsilon^{\sigma}_{i,m_i}|$ where $c_i(t) \in [\underline{c}_i, +\infty]$ for positive constants $\underline{c}_i$ and for all $\epsilon_{i,m_i} \notin \mathbb{D}_i$, where $\mathbb{D}_i$ are compact subsets of $\mathbb{R}$. Furthermore, $\epsilon^{\sigma}_{i,m_i}$ remain bounded if $\epsilon_{i,m_i} \in \mathbb{D}_i$.

To proceed, we distinguish two cases.
Case ($\epsilon_{w,\underline{m}} \notin \mathbb{D}_w$): Using Proposition 1, $\dot{V}_{w,\underline{m},\sigma}$ becomes

$$\dot{V}_{w,\underline{m},\sigma} \leq \phi^{\sigma}_{w,\underline{m}}|\epsilon^{\sigma}_{w,\underline{m}}||g_w|\Big(\frac{\bar{S}^{\sigma}_{w,\underline{m}}}{\underline{g}_w} - k_{w,\underline{m}}\underline{c}_w|\epsilon^{\sigma}_{w,\underline{m}}|\Big), \quad (29)$$

which is negative provided that $|\epsilon^{\sigma}_{w,\underline{m}}| > \bar{S}^{\sigma}_{w,\underline{m}}/k_{w,\underline{m}}\underline{g}_w\underline{c}_w$. Thus, $|\epsilon^{\sigma}_{w,\underline{m}}| \leq \bar{\epsilon}^{\sigma}_{1,w,\underline{m}} \triangleq max\{|\epsilon^{\sigma}_{w,\underline{m}}(0)|, \bar{S}^{\sigma}_{w,\underline{m}}/k_{w,\underline{m}}\underline{g}_w\underline{c}_w\}$.
Case ($\epsilon_{w,\underline{m}} \in \mathbb{D}_w$): Since $\epsilon_{w,\underline{m}} \in \mathbb{D}_w$, there exists constants $\bar{\epsilon}^{\sigma}_{2,w,\underline{m}} > 0$ such that $|\epsilon^{\sigma}_{w,\underline{m}}| \leq \bar{\epsilon}^{\sigma}_{2,w,\underline{m}}$.
Combining the two cases, we conclude that $\dot{V}_{w,\underline{m},\sigma} < 0$ for $|\epsilon^{\sigma}_{w,\underline{m}}| \leq \bar{\epsilon}^{\sigma}_{w,\underline{m}} \triangleq max\{\bar{\epsilon}^{\sigma}_{1,w,\underline{m}}, \bar{\epsilon}^{\sigma}_{2,w,\underline{m}}\}$ for all $t \in [0, \tau_{max})$. Hence, as in Step 1, we show that $-1 < T^{-1}(-\bar{\epsilon}^{\sigma}_{i,\underline{m}}) \leq \xi^{\sigma}_{i,\underline{m}} \leq T^{-1}(\bar{\epsilon}^{\sigma}_{i,\underline{m}}) < 1$. Owing to the aforementioned results and (9), we conclude for all $t \in [0, \tau_{max})$:

$$-1 < T^{-1}(-\bar{\epsilon}^{min}_{i,\underline{m}}) \leq \xi_{i,\underline{m}} \leq T^{-1}(\bar{\epsilon}^{max}_{i,\underline{m}}) < 1. \quad (30)$$

Step j ($j = \underline{m}+1,..,\overline{m}$ and $i \in L_1$): Using the positive definite functions $V_{j,\sigma} = \sum_{i\in L_1}(\epsilon^{\sigma}_{i,j})^2$, employing the boundness of $\dot{\psi}^{\sigma}_{i,j-1}$ from the previous step, and using the same line of analysis, we conclude for all $t \in [0, \tau_{max})$:

$$-1 < T^{-1}(-\bar{\epsilon}^{min}_{i,j}) \leq \xi_{i,j} \leq T^{-1}(\bar{\epsilon}^{max}_{i,j}) < 1, \quad (31)$$

$|h^{\sigma}_{i,j}| \leq \bar{h}^{\sigma}_{i,j}$ and the boundness of $\dot{\psi}^{\sigma}_{i,j}$. Furthermore, owing to (30), (31) and (5d), we conclude the boundness of u_i for all $t \in [0, \tau_{max})$.
Finally, from (18), (25), (30) and (31), we conclude that ξ evolves strictly within a compact subset of Ω_{ξ}. Therefore, by (Khalil, 2001, Theorem 3.3), we can extend the solution to $\tau_{max} = +\infty$. Consequently, from (7) we conclude the boundness of $x_{i,j}$, $i \in L_1$, $j = 1, ..., m_i$.

5. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed controller, we consider an heterogeneous multi-agent system consisted of $N = 5$ agents and a leader. For simulation purposes, we assume that the leader dynamics satisfy:

$$\ddot{x}_0 = -cos(t/3) \quad (32)$$

The dynamics of the agents obey:

$$\left.\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= 2x^2_{i,2} + g(\mathbf{x}_i)q(u) + d_i(t) \\ y_i &= x_{i,1} \end{aligned}\right\} \; i \in \{1,3,4\} \quad (33)$$

and

$$\left.\begin{aligned} \dot{x}_{i,1} &= (x_{i,1}+3)^2 + g(\mathbf{x}_i)q(u) + d_i(t) \\ y_i &= \; x_{i,1} \end{aligned}\right\} \; i \in \{2,5\} \quad (34)$$

where $q(u)$ represents the quintized control input, $d_i(t)$ are external disturbances with $d_i(t) = 0.2cos(3t)$, $i \in \{1,2,3,4,5\}$ and $g(\mathbf{x}_i) = (1+(x_{i,1}+3)^2)$, $i \in \{1,2,3,4,5\}$. Furthermore, the communication topology is described by a the following augmented neighboring sets $N_1 = \{0\}$, $N_2 = \{1\}$, $N_3 = \{1\}$, $N_4 = \{3\}$, $N_5 = \{4\}$. For control input quantization, uniform hysteric quantizers (2) are employed having step-size $\delta_u = 1$.

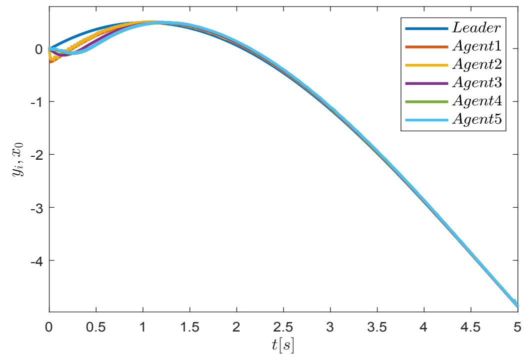

Fig. 2. Evolution of the agents output y_i, $i \in \{1,2,3,4,5\}$ alongside of leaders output x_0

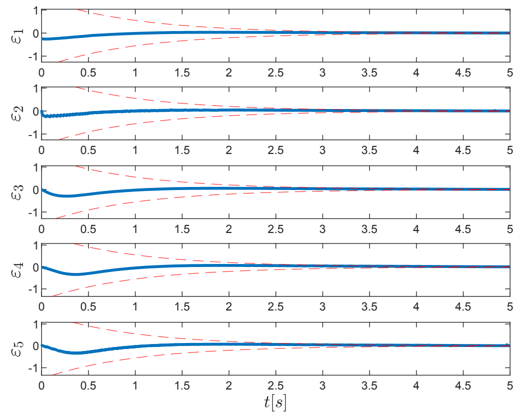

Fig. 3. Evolution of the output disagreement errors ε_i, $i \in \{1,2,3,4,5\}$ along with the prescribed performance specifications

For the output disagreement errors, a maximum steady state error of 0.1 and minimum convergence rate e^{-t} are requested. To a priori satisfy that demand, we utilize (6) and conclude that state quantization should be performed by uniform hysteric quantizers having step-size $\delta_x = 10^{-7}$. In addition, we select $\rho^{\infty}_{i,1} = 1.2\ 10^{-5} > \delta_x$, $i \in \{1,2,3,4,5\}$. Furthermore, we set $l_{i,1} = 1$ and $\rho^0_{i,1} = 2|e_i(0)|+0.5$ for all $i \in \{1,2,3,4,5\}$. The rest of the control elements are chosen as $\rho_{i,2}(t) = e^{-0.3t}+3$, $i \in \{1,3,4\}$, $k_{1,1} = 1.8$, $k_{2,1} = 8$, $k_{3,1} = 3$, $k_{4,1} = 1.8$, $k_{5,1} = 8$, $k_{1,2} = 10$, $k_{3,2} = 15$, $k_{4,2} = 15$.
Simulation results are presented in Figs. 2-5. Specifically, in Fig. 2 the output of all agents and the leader are plotted, illustrating the achievement of output consensus. As clearly presented in Fig. 3, all output disagreement errors ε_i converge to a neighborhood of zero of size significantly less than the predefined value of 0.1 with a minimum convergence rate e^{-t}. The requested control effort in its quantized form is plotted in Fig. 4. Fig. 5 presents the bounded evolution of the rest of the system states, i.e., $x_{i,2}$, $i \in \{1,3,4\}$

6. CONCLUSION

The problem of achieving consensus with prescribed transient and steady-state bounds, for high-order, heteroge-

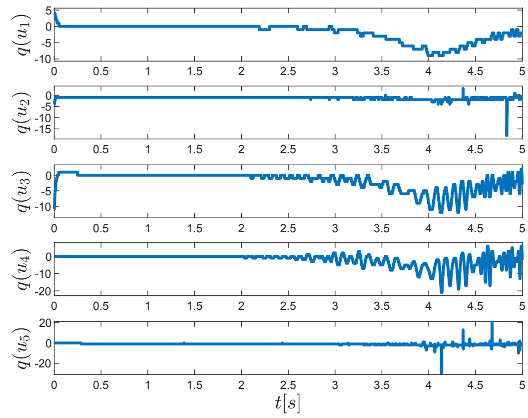

Fig. 4. The required quantized control inputs $q(u_i)$, $i \in \{1, 2, 3, 4, 5\}$

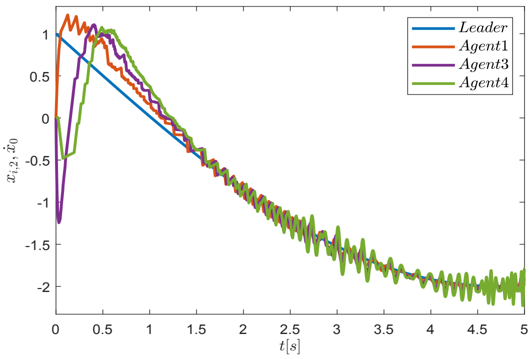

Fig. 5. Evolution of states $x_{i,2}$, $i = \{1, 3, 4\}$ along with $\dot{x}_0$

neous, uncertain, multi-agent systems, utilizing a low-complexity control protocol and quantized signals (state measurements, control inputs), was not considered in the past. A solution to fill this gap was proposed in this work. We used uniform-hysteric quantizers owing to their simplicity. The low-complexity property of the proposed solution is attributed to the fact that it is approximation-free, requiring simple calculations and it is static, thus preventing the expansion of the dynamic order of the closed-loop. Simulation results clarify and verify the approach.

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