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Game Theoretic Stochastic Energy Coordination under A Distributed Zeroth-order Algorithm ⋆

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Abstract: Dealing with the effects from uncertainties properly is a key problem in stochastic energy management problems to achieve safe and efficient operation of the system. In this paper, we study the problem of coordinating multi-period electric vehicles charging amidst uncertainty from the embedded renewable generation in a local distribution network under transformer capacity limits. A stochastic generalized game is presented to formulate the underlying electric vehicle coordination problem wherein the cost function of each player is affected by the intermittent renewable energy supply. Existing algorithms for seeking the equilibrium rely on conditions on the form of the cost functions. In our setting, however, stochastic effects are not known in advance which results in an unknown form of the cost functions. We propose a distributed iterative zeroth-order algorithm, which only relies on the observations of costs, to achieve a stochastic generalized Nash equilibrium of the game under the concept of Gaussian smoothing. Under certain mild assumptions, the proposed algorithm is guaranteed to converge to the neighborhood of the stochastic generalized Nash equilibrium. We demonstrate the algorithm for a distribution network energy management problem with 3 heterogeneous subgroups of electric vehicles.

Keywords: Energy coordination, random renewable generation, capacity limit, stochastic generalized Nash equilibrium, distributed zeroth-order algorithm.

1. INTRODUCTION

Trends in energy demand and environmental concerns have prompted the electric grid to evolve into a smart system incorporating distributed energy sources (including renewable energy) and electric vehicles (EVs) at the distribution level. Together with the advent of advanced information and communication systems, this enables the utilization of resources at the residential level of networks Stephens et al. (2015); Zipperer et al. (2013). This paper focuses on optimal energy management of a local distribution network under transformer capacity limits featuring EVs and uncertain local renewable supply. The users of such systems are usually modeled as self-interested agents with communication and control capabilities, who are able to control local generation and active loads to minimize their energy costs. Without a mechanism to coordinate the facilities, the overall performance of the power grid may deteriorate, due to the intermittent and volatile characteristics of EV charging and renewable generation.

In this paper, we analyze the optimal energy coordination of EV charging in a distribution network with embedded with renewable sources. Due to the limited capacity of the transformer connecting the distribution network to the grid, a generalized Nash equilibrium (GNE) model is used wherein individual users minimize their energy costs by adjusting their EV's charging strategy. The GNE model is similar to the formulation in Paccagnan et al. (2016), but is also popular in signal processing Yu et al. (2017) and cloud computing Ardagna et al. (2017). Motivated by this, many iterative algorithms, like the asymmetric projection method Paccagnan et al. (2016), ADMM/operator-splitting methods Yi and Pavel (2017), have been developed to solve GNE problems. Most of these focus on solving a subclass of GNEs called variational generalized Nash equilibrium Facchinei and Kanzow (2007); Facchinei and Pang (2003).

Note that all algorithms mentioned above require gradient information. However, due to the inevitable estimation error of renewable energy generation, it is more reasonable to formulate these problems as stochastic generalized Nash equilibrium (SGNE) problems Yu et al. (2017), where players minimize their expected cost functions. For SGNE problems, gradient-based algorithms are usually not applicable without some assumptions on the expectation and variance of the gradient information. Motivated by prob-

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lems like the one of interest here, where the expected cost of each player is easy to compute but that of the gradient is not, the so-called zeroth order (ZO) algorithms that do not require gradient information have been developed in optimization Liu et al. (2018); Balasubramanian and Ghadimi (2018).

In this work, we propose a distributed zeroth-order (DZO) algorithm that does not require any information about the stochastic gradient but only some noise-corrupted observations of players' cost. The algorithm originates from Thathachar and Sastry (2004); Nesterov and Spokoiny (2017). At each iteration, the system operator broadcasts information to every player used for their cost calculation. Usually, aggregation of all players' decision is chosen by the system operator for broadcasting to alleviate communicational burden Grammatico (2017). After that, each player updates their own decision based on several noisy cost observations and sends their new decision back to the system operator. In our mechanism, we assume the renewable estimation error is compensated by electricity purchase from the power grid, which means power flow through the transformer is also unpredictable. To ensure safe operation of the transformer, we design a robust coupling constraint set that takes any possible estimation error into accounts. We provide the condition for the convergence of our distributed zeroth-order algorithm, which works for broader classes of GNE problem than Tatarenko and Kamgarpour (2019a,b).

In Section 2 and Section 3, we formulate the energy coordination problem as a SGNE problem, where an extended game for SGNE is introduced for tackling coupling constraints and enforcing distributed optimization. Then, the details of our algorithm to solve the extended game is introduced in Section 4, where we also introduce some backgrounds for the ZO algorithm, and our modifications to ensure convergence to the neighborhood of the SGNE. Numerical results are provided in Section 5. Finally, Section 6 draws conclusions and future works.

2. SYSTEM MODEL

We consider a local distribution network, depicted in Fig. 1, including a group of electricity users $\mathcal{N} = \{1, 2, \cdots, N\}$ who may have distributed renewable generation, an electric vehicle (EV) and inelastic demand Stephens et al. (2015) and are jointly subject to the transformer capacity limit. The objective of this paper is to carry out an optimization coordination of energy consumption over a finite time horizon, indexed by $\mathcal{T} = \{0, 1, \cdots, T-1\}$. We assume that the time steps are of unit length, so that energy and power quantities are interchangeable.

2.1 Individual Model for Each User

Suppose that for each user $n \in \mathcal{N}$, the inelastic demand is known private information, and denote the inelastic demand over the horizon of interest as a vector $\boldsymbol{d}_n = [d_{n,t}, t \in \mathcal{T}]^\top \in \mathbb{R}^T$, where $y^\top$ represents the transpose of y. The individual renewable generation is not controllable but we assume that it comes at zero cost so that users would like to consume renewable generation first. Due to

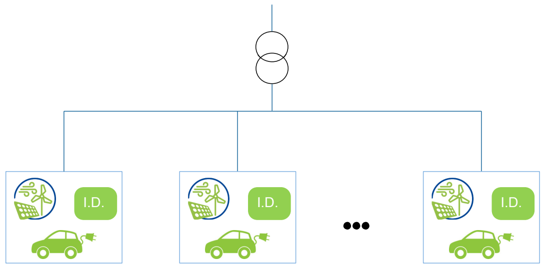

Fig. 1. Topology of a local distribution network.

the intermittent nature of renewable resources, generation forecasts are usually inaccurate to some degree. We denote renewable generation for each user $n \in \mathcal{N}$ by $\boldsymbol{g}_n = [g_{n,t}, t \in \mathcal{T}]^\top \in \mathbb{R}^T$, and express it as the sum

$$\boldsymbol{g}_n = \boldsymbol{r}_n + \boldsymbol{\delta}_n \,,$$

where $\boldsymbol{r}_n = [r_{n,t}, t \in \mathcal{T}]^\top \in \mathbb{R}^T$ is the forecast and $\boldsymbol{\delta}_n = [\delta_{n,t}, t \in \mathcal{T}]^\top \in \mathbb{R}^T$ is a random forecast error. Here we do not have assumptions on $\boldsymbol{\delta}_n$.

Hence, the control variable of each user $n \in \mathcal{N}$ is its EV's charging demand over the time horizon $\mathcal{T}$. In this paper, only grid to vehicle operations are considered, i.e. EVs are not allowed to give feedback to the grid. Denote by $\boldsymbol{u}_n = [u_{n,t}, t \in \mathcal{T}]^\top \in \mathbb{R}^T$ a charging decision of its EV, with $u_{n,t}$ satisfying

$$u_{n,t} \in [0, u_n^+], \quad \forall t \in \mathcal{T}, \tag{1}$$

where $u_n^+ > 0$ is the maximum charging rate. Let $\boldsymbol{x}_n = [x_{n,t}, t \in \mathcal{T}]^\top \in \mathbb{R}^T$ denote the normalized state of charge (SoC) trajectory for user n's EV. This must satisfy

$$x_{n,t} \in [x_n^-, x_n^+], \quad \forall t \in \mathcal{T},$$

where x_n^- and x_n^+ represent the minimal and maximal SoC values respectively. The case that user n does not have an EV can be modelled by setting u_n^+, x_n^-, and x_n^+ to zero.

The state dynamics governing $\boldsymbol{x}_n$ are

$$x_{n,t+1} = x_{n,t} + s_n u_{n,t}, \tag{2}$$

where s_n is a constant that depends on the charging efficiency and the battery capacity of n.

Suppose that at the beginning of the time horizon, the initial state of user n's EV is $x_{n,0}$ with $x_{n,0} \in [x_{n,0}^-, x_{n,0}^+]$. Then constraint (2) is equivalent to:

$$s_n \sum_{\tau=0}^{t} u_{n,\tau} \geq x_n^- - x_{n,0}, \quad \forall t \in \mathcal{T}\,, \tag{3a}$$

$$s_n \sum_{\tau=0}^{t} u_{n,\tau} \leq x_n^+ - x_{n,0}, \quad \forall t \in \mathcal{T}\,. \tag{3b}$$

The set of user n's feasible actions can then be written

$$\mathcal{U}_n \triangleq \{\boldsymbol{u}_n \,|\, \text{ s.t. (1) and (3)}\}\,. \tag{4}$$

2.2 Electricity Purchases

Since the system is connected to the electricity market via a transformer, matching the net demand of the local distribution system after accounting for local generation and consumption is subject to the transformer capacity limits. We use $\boldsymbol{b}_n = [b_{n,t}, t \in \mathcal{T}]^\top \in \mathbb{R}^T$ to denote the vector of user n's purchases by time step. Then, the total energy purchase should satisfy the transformer limit, which brings coupling among users:

$$\sum_{n\in\mathcal{N}} \boldsymbol{b}_n \leq \boldsymbol{c}^a, \tag{5}$$

where $\boldsymbol{c}^a = [c_t^a, t \in \mathcal{T}]^\top \in \mathbb{R}^T$ is the transformer limit. The power balance that should be satisfied at all times leads to:

$$\boldsymbol{b}_n(\boldsymbol{\delta}_n) \triangleq \boldsymbol{u}_n + \boldsymbol{d}_n - \boldsymbol{r}_n - \boldsymbol{\delta}_n \triangleq \boldsymbol{u}_n - \boldsymbol{\delta}_n + \boldsymbol{w}_n, \tag{6}$$

where $\boldsymbol{w}_n = \boldsymbol{d}_n - \boldsymbol{r}_n$ is the nominal net demand. Substituting (6) into (5) gives a coupling constraint set for all the users:

$$\mathcal{Q} \triangleq \left\{ \boldsymbol{u} \in \mathcal{U} \Big| \sum_{n\in\mathcal{N}} (\boldsymbol{u}_n - \boldsymbol{\delta}_n + \boldsymbol{w}_n) \leq \boldsymbol{c}^a \right\}, \tag{7}$$

where $\boldsymbol{u}$ represents the charging profile of all the users and $\mathcal{U} = \mathcal{U}_1 \times \cdots \times \mathcal{U}_n$.

Since renewable generation $\boldsymbol{\delta}_n$ is unpredictable, it introduces uncertainty into the energy management system. We therefore have to design a mechanism that ensures any deviation of renewable generation can be compensated by the grid. Since the constraint coupling the actions of the users only imposes an upper bound, to simplify the problem, we design a new robust coupling constraint set:

$$\mathcal{Q}_s \triangleq \left\{ \boldsymbol{u} \in \mathcal{U} \Big| \sum_{n\in\mathcal{N}} (\boldsymbol{u}_n + \boldsymbol{w}_n) \leq \boldsymbol{c}^a - \boldsymbol{\delta}^s \right\}, \tag{8}$$

where $\boldsymbol{\delta}^s$ is the worst underestimation of renewable generation that we expect to face across the distribution network. Since the amount of renewable generation in the distribution network is relatively small compared with the capacity limit of a transformer, the robust constraint above is not very conservative. Moreover, the conservativeness can be reduced if we have more accurate prediction for the renewable generation.

The common electricity price billed to all users consists of a baseline value derived from a day-ahead market price, varying by time step t, plus a real-time component that depends on the total energy purchase, *after* uncertainty values have been revealed. We use $\boldsymbol{b}(\boldsymbol{\delta})$ as shorthand for all users' purchases and, following Bompard et al. (2007), assume the real-time price component is linearly increasing in $\boldsymbol{b}(\boldsymbol{\delta})$:

$$\boldsymbol{p}(\boldsymbol{b}(\boldsymbol{\delta})) = \alpha^p \sum_{n\in\mathcal{N}} \boldsymbol{b}_n(\boldsymbol{\delta}_n) + \beta^p,$$

where α^p is a positive parameter, and $\beta^p = [\beta_t^p, t \in \mathcal{T}]^\top \in \mathbb{R}^T$ is a given time-varying tariff set in the day-ahead market.

3. ENERGY COORDINATION GAME

We focus on a distributed and non-cooperative scheme to coordinate user actions where users care only about optimizing individual objectives. Due to the existence of the coupling constraint in (7), we develop a stochastic generalized game which admits a stochastic generalized Nash equilibrium (SGNE).

For a given realized value of $\boldsymbol{\delta}$, the cost to user n is the sum of the power purchases and the battery cost,

$$J_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}; \boldsymbol{\delta}) = \boldsymbol{p}(\boldsymbol{b}(\boldsymbol{\delta}))^\top \boldsymbol{b}_n(\boldsymbol{\delta}_n) + h_n(\boldsymbol{u}_n), \tag{9}$$

where $h_n(\boldsymbol{u}_n)$ is the tradeoff between the battery degradation and the charging benefit, which we assume to be a quadratic function,

$$h_n(\boldsymbol{u}_n) = \sum_{t\in\mathcal{T}} (\alpha_n^f u_{n,t}^2 + \beta_n^f u_{n,t}) + \alpha_n^s \Big(\sum_{t\in\mathcal{T}} u_{n,t} - \Gamma_n \Big)^2,$$

where $\alpha_n^f > 0$ and $\beta_n^f \geq 0$ are coefficients of the degradation cost depending on battery characteristics, and $\Gamma_n \leq \frac{1}{s_n}(x_{n,T}^+ - x_{n,0})$ is the desired charging energy over the time horizon. As usual, u_n denotes the decisions of player n whereas $\boldsymbol{u}_{-n}$ those of all other players. We note that under our assumptions, for given $\boldsymbol{u}_{-n}$ and $\boldsymbol{\delta}$, $J_n(\cdot, \boldsymbol{u}_{-n}; \boldsymbol{\delta})$ is quadratic.

In a stochastic game, each player wishes to minimize the expected cost over possible realizations of $\boldsymbol{\delta}$. Let f_n denote the cost function of player n, then

$$\begin{aligned} & f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}) \triangleq \mathbb{E}_{\boldsymbol{\delta}}[J_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}; \boldsymbol{\delta})] \\ &= \Big(\alpha^p \sum_{i\in\mathcal{N}} (\boldsymbol{u}_i + \boldsymbol{w}_i)^\top + (\beta^p)^\top \Big) (\boldsymbol{u}_n + \boldsymbol{w}_n) + h_n(\boldsymbol{u}_n) \\ &\quad - \Big(\alpha^p \sum_{i\in\mathcal{N}} (\boldsymbol{u}_i + \boldsymbol{w}_i)^\top + (\beta^p)^\top \Big) \mathbb{E}[\boldsymbol{\delta}_n] \\ &\quad - \alpha^p \sum_{i\in\mathcal{N}} \mathbb{E}[\boldsymbol{\delta}_i]^\top (\boldsymbol{u}_n + \boldsymbol{w}_n) + \alpha^p \sum_{i\in\mathcal{N}} \mathbb{E}[\boldsymbol{\delta}_i^\top \boldsymbol{\delta}_n]. \end{aligned} \tag{10}$$

Hence, the stochastic generalized game can be written as

$$\mathcal{G} = \langle \mathcal{N}, \{\boldsymbol{u}_n\}_{n\in\mathcal{N}}, \{f_n\}_{n\in\mathcal{N}} \rangle,$$

and the individual best response problem for each player by

$$\boldsymbol{u}_n^* = \underset{\boldsymbol{u}_n \in \mathcal{Q}_n(\boldsymbol{u}_{-n})}{\arg\min} \; f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}),$$

where $\mathcal{Q}_n(\boldsymbol{u}_{-n})$ is the strategy set of player n such that

$$\mathcal{Q}_n(\boldsymbol{u}_{-n}) = \{\boldsymbol{u}_n \in \mathcal{U}_n \,|\, \boldsymbol{u}(\boldsymbol{u}_{-n}) \in \mathcal{Q}_s\}.$$

Definition 1. (Stochastic Generalized Nash Equilibrium). A strategy $\boldsymbol{u}^* \in \mathcal{Q}$ is called a SGNE of the game $\mathcal{G}$, if the following holds

$$f_n(\boldsymbol{u}_n^*, \boldsymbol{u}_{-n}^*) \leq f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}^*), \quad \forall \boldsymbol{u}_n \in \mathcal{Q}_n(\boldsymbol{u}_{-n}^*), \forall n \in \mathcal{N}.$$

We denote the set of SGNEs for game $\mathcal{G}$ as SGNE($\mathcal{G}$). It is easy to verify that the local constraint sets $\mathcal{U}_n$ and the coupling constraint set $\mathcal{Q}$ are compact and convex, and satisfy the Slater's constraint qualification. Moreover, the expected cost $f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n})$ is quadratic on $\boldsymbol{u}_n$ for a given $\boldsymbol{u}_{-n}$.

For game $\mathcal{G}$, its corresponding variational inequality (VI) problem is denoted by VI($\mathcal{Q}, M$), where $M(\boldsymbol{u}) = [M_n(\boldsymbol{u}), n \in \mathcal{N}] : \mathbb{R}^{NT} \to \mathbb{R}^{NT}$ is the game mapping to represent the stacked gradients of the cost functions of each player with respect to their own strategies, that is,

$$M_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}) = \frac{\partial f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n})}{\partial \boldsymbol{u}_n}.$$

As $f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n})$ is quadratic, it is easy to verify that $M(\boldsymbol{u})$ is τ-strongly monotone with $\tau = \alpha^p + 2\min_{n\in\mathcal{N}} \alpha_n^f$ and admits a Lipschitz constant L_1 of it thanks to the continuity and the fact that $\mathcal{U}$ is compact. Under these conditions, and denoting the solutions to VI($\mathcal{Q}, M$) by SOL($\mathcal{Q}, M$), Facchinei and Kanzow (2007)[Theorem 5] and Facchinei and Pang (2003)[Theorem 2.3.3] then lead to that a solution to VI($\mathcal{Q}, M$) is also a SGNE of $\mathcal{G}$ and the game $\mathcal{G}$ has a unique equilibrium.

To deal with the global coupling constraint, we introduce an additional player indexed by 0, whom we refer to as the dual player, to minimize the cost.

$$f_0^e(\boldsymbol{\lambda}, \boldsymbol{u}) \triangleq \boldsymbol{\lambda}^\top \Big(\boldsymbol{c}^a - \boldsymbol{\delta}^s - \sum_{n \in \mathcal{N}} (\boldsymbol{u}_n + \boldsymbol{w}_n) \Big), \tag{11}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^T_{\geq 0}$ is the strategy of player 0. Correspondingly, the cost functions of the original N players, whom we refer to as the primal players, incorporate the coupling constraints and construct the individual optimization problem:

$$\min_{\boldsymbol{u}_n \in \mathcal{U}_n} f_n^e(\boldsymbol{u}_n, \boldsymbol{u}_{-n}, \boldsymbol{\lambda}) \triangleq f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}) - f_0^e(\boldsymbol{\lambda}, \boldsymbol{u}).$$

The resulting game, denoted by $\mathcal{G}^e$, has $N+1$ players but no coupling constraints. The corresponding operator $M^e(\boldsymbol{u}, \boldsymbol{\lambda}) = [M_0^e(\boldsymbol{u}, \boldsymbol{\lambda}); M_1^e(\boldsymbol{u}, \boldsymbol{\lambda}); \cdots ; M_N^e(\boldsymbol{u}, \boldsymbol{\lambda})] : \mathbb{R}^{(N+1)T} \to \mathbb{R}^{(N+1)T}$, is defined by

$$M_0^e(\boldsymbol{u}, \boldsymbol{\lambda}) = \boldsymbol{c}^a - \boldsymbol{\delta}^s - \sum_{n \in \mathcal{N}} (\boldsymbol{u}_n + \boldsymbol{w}_n),$$

$$M_n^e(\boldsymbol{u}, \boldsymbol{\lambda}) = \frac{\partial f_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n})}{\partial \boldsymbol{u}_n} + \boldsymbol{\lambda}, \quad \forall n \in \mathcal{N}$$

and can be verified to be a monotone operator. Then Paccagnan et al. (2016) ensures that a solution $(\boldsymbol{u}^*; \boldsymbol{\lambda}^*)$ to $\text{VI}(\mathcal{U} \times \mathbb{R}^T_{\geq 0}, M^e)$ is a stochastic Nash equilibrium (SNE) of game $\mathcal{G}^e$ and $\boldsymbol{u}^*$ is a SGNE of game $\mathcal{G}$.

For notation simplicity, we denote the strategy profile in the extended game by $\boldsymbol{\eta}$, i.e. $\boldsymbol{\eta} = (\boldsymbol{u}; \boldsymbol{\lambda}) \in \mathbb{R}^{NT+T}$.

4. DISTRIBUTED ZEROTH-ORDER ALGORITHM

In distributed zeroth-order (DZO) algorithms, an iterative algorithm is typically implemented to estimate the gradients. Suppose that the index k denotes the iteration of the DZO algorithms and vectors with subscript k represent the corresponding values of the vectors at iteration k.

For each player n at iteration k, an extra vector $\boldsymbol{x}_{n,k}$ is generated as follows,

$$\boldsymbol{x}_{n,k} = \boldsymbol{u}_{n,k} + \sigma_k \cdot \boldsymbol{v}_{n,k}$$

where $\boldsymbol{v}_{n,k}$ is generated from a Gaussian distribution with the identity covariance matrix. The Gaussian approximation, see Nesterov and Spokoiny (2017), of $f_n^e(\cdot)$ with parameter σ_k is defined as

$$\begin{aligned} & f^e_{n,\sigma_k}(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k) \\ & \triangleq \int f_n^e(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)\, p(\boldsymbol{x}_{n,k} | \boldsymbol{u}_{n,k}, \sigma_k^2 \mathbf{I}_T) d\boldsymbol{x}_{n,k} \\ & = \int f_n^e(\boldsymbol{u}_{n,k} + \sigma_k \boldsymbol{v}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)\, p(\boldsymbol{v}_{n,k} | \mathbf{0}, \mathbf{I}_T) d\boldsymbol{v}_{n,k}. \end{aligned}$$

If σ_k is sufficiently small, the Gaussian approximation of a function is almost the same as the original one. Taking the partial derivative of $f^e_{n,\sigma_k}(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)$ with respect to $\boldsymbol{u}_{n,k}$, denoted by $\tilde{M}_n^e(\boldsymbol{\eta}_k)$, it gives

$$\begin{aligned} \tilde{M}_n^e(\boldsymbol{\eta}_k) & \triangleq \frac{\partial f^e_{n,\sigma_k}(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)}{\partial \boldsymbol{u}_{n,k}} \\ & = \int (f_n^e(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k) - f_n^e(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)) \\ & \quad \frac{\boldsymbol{x}_{n,k} - \boldsymbol{u}_{n,k}}{\sigma_k^2} \cdot p(\boldsymbol{x}_{n,k} | \boldsymbol{u}_{n,k}, \sigma_k^2 \mathbf{I}_T) d\boldsymbol{x}_{n,k}. \end{aligned} \tag{12}$$

The last equality relies on the following equation, which is also the critical point to design the zeroth-order algorithm,

$$\int \frac{\boldsymbol{x}_{n,k} - \boldsymbol{u}_{n,k}}{\sigma_k^2} \cdot p(\boldsymbol{x}_{n,k} | \boldsymbol{u}_{n,k}, \sigma_k^2 \mathbf{I}_T) d\boldsymbol{x}_{n,k} = 0.$$

For each player $n \in \mathcal{N}$, we define

$$\begin{aligned} g_n^e(\boldsymbol{\eta}_k, \boldsymbol{x}_{n,k}) = & \frac{f_n^e(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k) - f_n^e(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)}{\sigma_k} \\ & \cdot \boldsymbol{v}_{n,k}. \end{aligned} \tag{13}$$

Then, (12) can be rewritten as

$$\tilde{M}_n^e(\boldsymbol{\eta}_k) = \int g_n^e(\boldsymbol{\eta}_k, \boldsymbol{x}_{n,k}) \cdot p(\boldsymbol{x}_{n,k} | \boldsymbol{u}_{n,k}, \sigma_k^2 \mathbf{I}_T) d\boldsymbol{x}_{n,k}.$$

This form suggests that $g_n^e(\cdot)$ can be interpreted as the stochastic gradient descent of the Gaussian approximation of the original objective function.

Since the cost function f_n is quadratic with respect to $\boldsymbol{u}_n$, we could obtain the following lemma.

Lemma 1. For each $n \in \mathcal{N}$, the following holds

$$\tilde{M}_n^e(\boldsymbol{\eta}_k) = \int M_n^e(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k) p(\boldsymbol{x}_{n,k} | \boldsymbol{u}_{n,k}, \sigma_k^2 \mathbf{I}_T) d\boldsymbol{x}_{n,k}$$

For the uncertainty of renewable generation, we make the following assumption on the second moment of stochastic effects.

Assumption 1. The variance of the function value on δ is finite, i.e., there exists a constant D_1, such that

$$\text{Var}[J_n^e(\cdot; \boldsymbol{\delta})] = \mathbb{E}[(J_n^e(\cdot; \boldsymbol{\delta}) - f_n^e(\cdot))^2] \leq D_1, \forall n \in \mathcal{N}. \tag{14}$$

We compute an approximation $\hat{g}_n^e$ of the pseudo-gradient (13),

$$\begin{aligned} \hat{g}_n^e(\boldsymbol{\eta}_k, \boldsymbol{x}_{n,k}) = & \frac{\tilde{f}^e_{n,k}(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k) - \hat{f}^e_{n,k}(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)}{\sigma_k} \\ & \cdot \boldsymbol{v}_{n,k} \end{aligned} \tag{15}$$

where $\tilde{f}^e_{n,k}(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)$, $\hat{f}^e_{n,k}(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)$ are estimations of $f^e_{n,k}(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)$ and $f^e_{n,k}(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k)$ respectively, based on some noisy observations,

$$\tilde{f}^e_{n,k}(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k) = \frac{1}{T_1} \sum_{t_1=1}^{T_1} J_n^e(\boldsymbol{x}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k; \boldsymbol{\delta}_{k,t_1}),$$

$$\hat{f}^e_{n,k}(\boldsymbol{u}_{n,k}, \boldsymbol{u}_{-n,k}, \boldsymbol{\lambda}_k) = \frac{1}{T_2} \sum_{t_2=1}^{T_2} J_n^e(\boldsymbol{u}_{n,k}, \cdot, \cdot; \boldsymbol{\delta}_{k,T_1+t_2}),$$

where T_1, T_2 are the counts of noisy observations at point $\boldsymbol{x}_{n,k}$ and $\boldsymbol{u}_{n,k}$ respectively per iteration.

Hence, the update of the primal players is given as follows:

$$\boldsymbol{u}_{n,k+1} = \Pi_{\mathcal{U}_n} \| \boldsymbol{u}_{n,k} - h_k \cdot \hat{g}_n^e(\boldsymbol{\eta}_k, \boldsymbol{x}_{n,k}) \|, \quad \forall n \in \mathcal{N}, \tag{16}$$

where h_k is the step-size at iteration k.

To recover strong monotonicity, we apply the so-called Tikhonov regularization Facchinei and Pang (2003) to the dual update,

$$\begin{aligned} \boldsymbol{\lambda}_{k+1} = \Pi_{\mathbb{R}^T_{\geq 0}} \| \boldsymbol{\lambda}_k + h_k \cdot (& \sum_{n \in \mathcal{N}} (\boldsymbol{u}_n + \boldsymbol{w}_n) - (\boldsymbol{c}^a - \boldsymbol{\delta}^s) \\ & - r_k \boldsymbol{\lambda}_k) \|, \end{aligned} \tag{17}$$

where $r_k > 0$ is the regularization parameter that decreases along the iterations.

Our algorithm is summarized in Algorithm 1. The convergence properties of this algorithm are demonstrated in the simulation results.

Algorithm 1 DZO Algorithm for Energy Coordination

Require:
Initialize $k \leftarrow 0$, $\boldsymbol{u}_{n,k}, \forall n \in \mathcal{N}$ and $\boldsymbol{\lambda}_k$;
The number of iterations K;

```
while k < K do
    η_k = [u_{1,k}; ··· ; u_{N,k}; λ_k];
    h_k = 1/(k+1)^a, σ_k = 1/(k+1)^b, r_k = 1/(k+1)^c;
    for n ∈ N do
        Choose a v_{n,k} (Gaussian) to get x_{n,k}
        Generate
            J_n^e(x_{n,k}, u_{-n,k}, λ_k; δ_{k,t_1}), ∀t_1 ∈ {1, ..., T_1}
            J_n^e(u_{n,k}, u_{-n,k}, λ_k; δ_{k,T_1+t_2}), ∀t_2 ∈ {1, ..., T_2}
        Update u_{n,k+1} by (16);
    end for
    Update λ_{k+1} by (17);
    k ← k + 1
end while
return u_{n,k}, ∀n ∈ N and λ_k
```

5. SIMULATION RESULTS

In this section, we provide numerical results to illustrate the effectiveness of our algorithm. Assume that in the distribution network, there are N groups of households within each the homes have the same EVs and renewables.

Besides the terms defined in (9), we add one more quadratic cost for coupling constraints' violation into the cost function $J_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}; \boldsymbol{\delta})$:

$$J_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}; \boldsymbol{\delta}) = \boldsymbol{p}(\boldsymbol{b}(\boldsymbol{\delta}))^\top \boldsymbol{b}_n(\boldsymbol{\delta}_n) + h_n(\boldsymbol{u}_n) + \rho \left\| \max\left\{0, \sum_{n\in\mathcal{N}} (\boldsymbol{u}_n + \boldsymbol{w}_n) - (\boldsymbol{c}^a - \boldsymbol{\delta}^s)\right\} \right\|^2 .$$

After adding the new penalty above, the mapping $M(\boldsymbol{u})$ remains τ-strongly monotone and locally Lipschitz continuous. By this small change, the optimal solution will not change but the algorithm becomes more robust as otherwise the stochastic generation deviation introduces inaccuracy into the gradient estimation $\hat{g}_n^e(\boldsymbol{\eta}_k, \boldsymbol{x}_{n,k})$ and causes coupling constraint violation.

We select the transformer capacity to be large enough to serve all inflexible demand, but small enough so that not all EV demand can be served in addition at all times; this ensures that some of the coupling constraints will be active. For the initialization of this simulation, we set the number of the types of the players $N = 3$, and the length of the time horizon $T = 12$, so that each time slots is equivalent to 2 hours. We consider each type of players as an aggregate player. Without loss of generality, we set $s_n = 1$. The capacity of the EV of each player is $B_n = 100$kWh and the upper bound for the charging decision is $u_n^+ = 0.4B_n \in \mathbb{R}^d$. Assume that $x_n^+ = 0.9B_n$, $x_n^- = x_{n,0}$ and $x_{n,0}$ is uniformly chosen from the interval $[0, 0.2B_n]$ for each player. The limited capacity of the transformer is $\boldsymbol{c}^a = 0.26NB_n$. Cost parameters are $\alpha^p = \frac{0.1}{N}, \beta^p = 0.3, \alpha_n^s = 100, \alpha_n^f = 1, \beta_n^f = 0, \rho = 100$. For the renewable generation, we keep the maximum entry

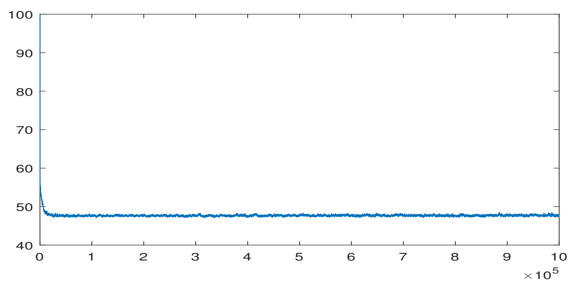

Fig. 2. Norm of primal variables, $T_1 = T_2 = 10$

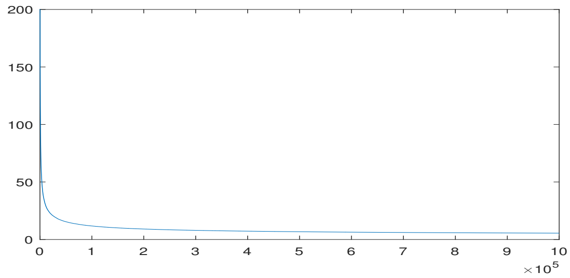

Fig. 3. Norm of dual variables, $T_1 = T_2 = 10$

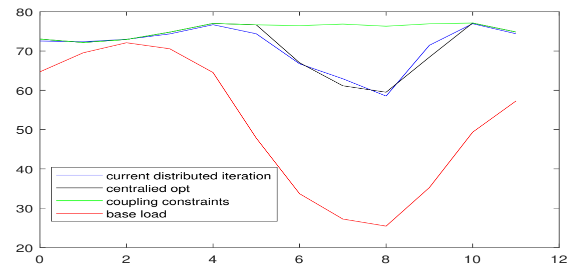

Fig. 4. Aggregate load, $T_1 = T_2 = 10$

of $\sum_{n\in\mathcal{N}} \boldsymbol{g}_n$ to be 20% of $\boldsymbol{c}^a$ and the renewable energy generation comes from solar panel. The base load shown in Fig. 4 is tuned to ensure the coupling constraints are active. In the end, we set the number of iterations to $K = 10^6$ and the stochastic effect is set to be a Gaussian distribution with zero means and variance is 10% of the magnitude of $\boldsymbol{g}_n$. We set $\boldsymbol{\delta}^s$ to be 3 times of the sum of variance, i.e. we ignore those stochastic effects beyond the range of 3 times of the variance.

Firstly, we set $T_1 = T_2 = 10$. Fig. 2 shows the convergence of players' actions to the SGNE while Fig. 3 the corresponding values of the dual λ that seems to be gradually decreasing. This can be explained by the slowly decreasing rate r_k, which affects the convergence of $y_{\lambda,k}$ to λ^* and consequently the convergence of λ_k.

Figure 4 shows the aggregate load. The green line corresponds to the capacity of the transformer whereas the red line to the base load. Fig. 4 shows the so-called valley-filling property of the Nash equilibrium. We note that result of our regularized DZO is very close to the "Social welfare" solution where the sum of the cost functions for all players is minimized in a centralized way.

When we decrease the numbers of observations to $T_1 = T_2 = 2$, the solution again approaches the SGNE for the

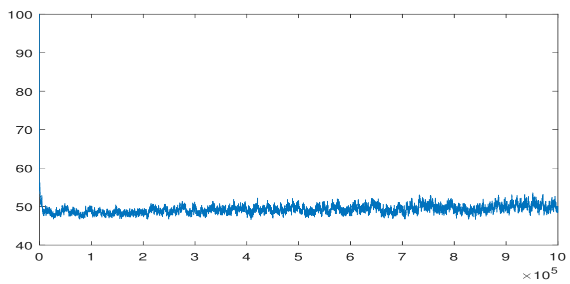

Fig. 5. Norm of primal variables, $T_1 = T_2 = 2$

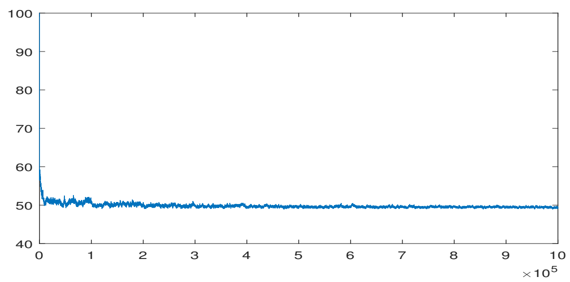

Fig. 6. Norm of primal variables (gradually increasing times of averaging)

first few iterations. However, as the number of iterations increases, the solution appears to drift, as shown in Fig. 5, the iterates seem to deviate from SGNE albeit slowly. The reason for this behavior is the absence of an averaging step, since the gradient estimation (15) involves only two points at $\boldsymbol{x}_{n,k}$ and $\boldsymbol{u}_{n,k}$. Although the convergence under diminishing step size is theoretically correct, the gradient estimation becomes ineffective if the magnitude of the stochastic effects is comparable to the difference of expected cost at $\boldsymbol{x}_{n,k}$ and $\boldsymbol{u}_{n,k}$. Therefore, if the averaging number is fixed, the diminishing smoothing parameter σ_k would gradually magnify the stochastic effects and cause the drifting in Fig. 5.

To solve this problem, we can either fix the value of the smoothing parameter σ_k after a certain number of iterations, or increase the averaging number as the number of iterations increases. Though a formal way for doing this is beyond the scope of this paper, we give a simple implementation for the latter method where the number of averaging step per iteration is increased for every 10^5 iterations, leading to an averaging number of 10 by the end of the iteration $K = 10^6$. As shown in Fig. 6, this resolves the drifting issue observed in Fig. 5 and the iterates stay near the SGNE. We note that this issue is not uncommon when the cost function f_n is very smooth near the SGNE.

6. CONCLUSION

We analyzed the properties of a zeroth-order algorithm in distributed case and implemented two variants to deal with the issue of drifting due to inadequate averaging. Moreover, a regularization term was added to the dual update in the DZO algorithm to ensure the convergence to the unique SGNE of the strongly monotone game. Future work will focus on finding a better method to regularize stochastic effects that does not require an increasing number of observations per iteration.

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