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Guaranteeing the accuracy of digital control of a linear periodic object within the set of stabilizing regulators [1]

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Abstract: The article considers the task of optimization of a digital control system for a linear continuous periodic object. It is supposed that the system operates under the conditions of uncertainty of an external stationary stochastic disturbance applied directly to the periodic object. For the description of the system dynamics in continuous time the apparatus of the parametric transfer function is used. As an optimization criterion the score of guaranteed accuracy is used, which is an estimate of the H_2-norm of the system on the class of perturbations. The optimization is performed on the set of stabilizing causal controllers. An example is given.

INTRODUCTION

The problem of control of finite-dimensional continuous linear periodic objects (FDLCP-objects) is one of the actual problems in the modern automatic control theory. A considerable number of studies is devoted to various aspects of this problem (Richards (1983), Dugundji et al. (1983), Wiesel et al. (1983), Du Val et al. (1984), Liu et al. (1985), Allievi et al. (1996), Wereley (1991), Pandiyan et al. (1999), Mollerstedt (2000), Hansen et al. (2012), Karami et al. (2017) and literature cited there). Within this problem, it is of great interest to study digital control systems involving FDLCP objects, whereby these systems belong to the class of sampled-data systems (SD-systems).

In this paper, the problem of guaranteeing the accuracy of digital control of a FDLCP objects is considered. Hereby it is assumed that the FDLCP object of interest is affected by external stationary stochastic disturbances. It is assumed that there is a parametric and/or structural uncertainty of the spectral-correlation properties of the disturbance and only its belonging to the class $\mathcal{M}$, specified by generalized properties, is known. In solving the problem, the accuracy of the system function is not only guaranteed for a particular perturbation, but for the entire $\mathcal{M}$ class.

The general approach to solve the task of guaranteeing the accuracy for continuous control systems with LTI objects, based on the standard transfer function and the frequency characteristic concept, was presented in Nebylov et al. (2014). The use of the parametric transfer function (PTF) concept described in Rosenwasser et al. (2000) and Rosenwasser et al. (2006) allowed to extend this approach to a class of SD systems. Corresponding procedures for guaranteeing the accuracy of SD systems with LTI objects are described in Rosenwasser et al. (2005) and Rybinskii et al. (2013). The task of guaranteeing the accuracy of a SD control system containing an FDLCP object was first considere in Rybinskii et al. (2018). Here a numerical optimization of the estimation of the H_2 norm of the system was performed over all causal digital controllers. The stability of the system was checked by additional computations during the numerical minimization procedure. In the practical application of this optimization approach, problems occasionally arose because unstable solutions were not excluded from the solution set from the outset.

It is shown here that this difficulty can be overcome by combining the approach of Rybinskii et al. (2018) with the approach of causal stabilization of a FDLCP object considered in Lampe et al. (2007) and Rosenwasser et al. (2018). Here a characteristic equation for the SD control system with FDLCP object is constructed and the set of all causally stabilizing digital controllers is parameterized. The implementation of this approach and its integration into the solution algorithm ensures that only causal and stable controllers can enter the numerical optimization process from the outset. A positive effect besides the increased reliability of the algorithm is a reduced numerical effort.

This paper describes the method of numerical optimization of an SD control system for an FDLCP object according to the criterion of guaranteed accuracy over the set of causal and stabilizing controllers. The application of the method is illustrated by a numerical example.

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1. SYSTEM DESCRIPTION

This paper discusses the SD-control system $\mathcal{W}$ with the structure given in Fig. 1. The control object in the system $\mathcal{W}$ is the SISO FDLCP-object $\mathcal{P}$, whose dynamic is described by the K-order differential equation

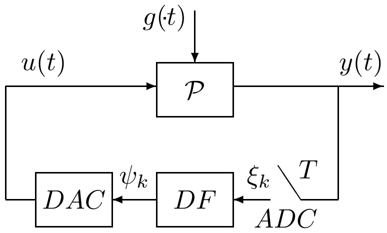

Fig. 1. The FDLCP-object SD control systems structure

$$\frac{d^K y(t)}{dt^K} + \sum_{k=1}^{K} a_k(t) \frac{d^{K-k} y(t)}{dt^{K-k}} = b_1(t)g(t) + b_2(t)u(t), \quad (1)$$

where $a_k(t) = a_k(t+T)$, $(k = 1, \ldots, K)$, $b_i(t) = b_i(t+T)$, $i = 1, 2$ – continuous periodic functions and T the objects period.

The FDLCP-object is controlled by a discrete controller whose sampling period coincides with the period T of the object. The controller includes an analog-to-digital converter (ADC), a digital filter (DF) and digital-to-analogue converter (DAC).

The ADC transforms the continuous signal $y(t)$ to a discrete sequence ξ_k

$$\xi_k = y(kT), \qquad k = 0, 1, 2, \ldots. \quad (2)$$

The algorithm forming the control sequence ψ_k, realized in DF, is given by the difference equation

$$\begin{aligned} \alpha_0 \psi_k + \alpha_1 \psi_{k-1} + \ldots + \alpha_R \psi_{k-R} = \\ = \beta_0 \xi_k + \beta_1 \xi_{k-1} + \ldots + \beta_R \xi_{k-R}, \end{aligned} \quad (3)$$

where $R > 0$ is integer, called the order of the controller and α_r, β_r $(r = 0, \ldots, R)$ are real constants. Using the backward operator $\zeta = e^{-sT}$ corresponding to a one-step backward shift, it is possible to specify for (3) the discrete transfer function (TF)

$$W_d(\zeta) = \frac{\beta(\zeta)}{\alpha(\zeta)} = \frac{\sum_{r=0}^{R} \beta_r \zeta^r}{\sum_{r=0}^{R} \alpha_r \zeta^r}, \quad (4)$$

here called controller TF. In (4) $\alpha(\zeta)$, $\beta(\zeta)$ are assumed as coprime polynomials and the causality condition, which in current case has the form

$$\alpha(0) = \alpha_0 \neq 0, \quad (5)$$

is fulfilled.

The control signal $u(t)$ is formed by the DAC, whose work is described by the equation

$$u(t) = h(t - kT)\psi_k, \qquad kT < t < (k+1)T, \quad (6)$$

where the modulation function $h(t)$ is a function of limited variation on the interval $0 < t < T$.

Let the considered system $\mathcal{W}$ be asymptotically stable. In addition, the external disturbance $g(t)$ acting on the system $\mathcal{W}$ will be considered as stochastic stationary centered. For this case, the output signal of $y(t)$ in steady state will be stochastic non-stationary periodic (Lampe et al. (2009, 2012)).

2. THE TASK STATEMENT

Let be Z the vector of design parameters of the system, formed in some way, which unambiguously defines TF (4) und let be $\mathcal{Z}$ the set of allowed values of this vector Z so that all controllers corresponding to vectors $Z \in \mathcal{Z}$ are at least stabilizing (providing asymptotic stability of the system $\mathcal{W}$) and causal. For a given spectral density $S_g(s)$ of the disturbance $g(t)$, the mean variance of the output signal $y(t)$ can be used as an indicator for the functional accuracy of the system $\mathcal{W}$ in steady state mode (Lampe et al. (2007, 2009, 2012, 2016); Rosenwasser et al. (2018)). This variance depends on vector Z and can be calculated by the expression

$$\bar{d}_y(Z) = \\ = \frac{1}{2T\pi i} \int_0^T \int_{-i\infty}^{i\infty} W_y(s, t, Z) S_g(s) W_y(-s, t, Z) ds dt. \quad (7)$$

In (7) $W_y(s, t, Z)$ is the PTF of the considered SD system from input $g(t)$ to output $y(t)$.

In the following it is assumed that the spectral density of $S_g(s)$ is unknown, but it is known that disturbance $g(t)$ belongs to a certain class $\mathcal{M}$. A model of such a class can be specified according to Nebylov et al. (2014) by the combination of $N + 1$ numbers

$$d_n = \frac{1}{\pi} \int_0^\infty \tilde{S}_g(\nu) \nu^{2n} d\nu, \qquad \begin{aligned} & n = 0, \ldots, N, \\ & \tilde{S}_g(\nu) = S_g(s)\Big|_{s=i\nu}, \end{aligned} \quad (8)$$

which have the meaning of the mean variance of the disturbance and its N derivatives.

We now assume that as an indicator of the accuracy of the system operating on the class $\mathcal{M}$ an estimate of the guaranteed accuracy is chosen - the number

$$\bar{D}_y(Z) \geq \bar{d}_y(Z), \qquad \forall g(t) \in \mathcal{M}. \quad (9)$$

The way it is calculated is described below.

So the objective considered in this paper is as follows: Let for the system $\mathcal{W}$ the equation (1) of the dynamics of the FDLCP-object $\mathcal{P}$ be given. Also let the disturbanc class of $\mathcal{M}$ be given by the numbers $\{d_n\}$.

In addition, the greatest admissible order $R_{\max}$ of the controller und the modulation function $h(t)$ are specified.

It is necessary

(1) to create a vector Z of the systems design parameters that unambiguously defines the controller TF $W_d(\zeta)$ (4) and to construct the set $\mathcal{Z}$ so that each controller corresponding to a vector $Z \in \mathcal{Z}$ will be causal and stabilizing, and its order does not exceed the number $R_{\max}$;
(2) to create the calculation method for the guaranteed accuracy estimation $\bar{D}_y(Z)$ by (9);

(3) to choose the vector $Z = Z_{gar}$ from the set $\mathcal{Z}$ for which corresponding controller provides the minimal estimation value (9) on the class $\mathcal{M}$

$$\bar{D}_y(Z_{gar}) = \min_Z \bar{D}_y(Z). \tag{10}$$

If the estimation $\bar{D}_y(Z_{gar})$ will not exceed the maximum permissible value of the average signal variance of $y(t)$, the operation of system $\mathcal{W}$ will be guaranteed successful for any disturbance in the class of $\mathcal{M}$.

3. PTF OF THE SYSTEM $\mathcal{W}$

The presented solution method is based on the use of the PTF and the parametric amplitude frequency response (PAFR) of the system $\mathcal{W}$. The general approach for constructing the PTF for SD-systems containing an FDLCP control object is described in Lampe et al. (2009, 2012, 2016). To apply this approach to the study of the system under consideration $\mathcal{W}$ we introduce the notation

$$v(t) = \left[\, y(t) \;\; \frac{dy(t)}{dt} \;\ldots\; \frac{dy^{K-1}(t)}{dt} \,\right]' \tag{11}$$

as the $K \times 1$ state vector of the object $\mathcal{P}$, the apostrophe denotes the transposing operation. This allows us to describe the dynamics of object $\mathcal{P}$ by the state equation

$$\frac{v(t)}{dt} = A(t)v(t) + B_1(t)g(t) + B_2(t)u(t) \tag{12}$$

and the output equation

$$y(t) = C(t)v(t). \tag{13}$$

In (12) and (13)

$$A(t) = \begin{bmatrix} 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 \\ -a_K(t) & -a_{K-1}(t) & \ldots & -a_1(t) \end{bmatrix} = A(t+T) \tag{14}$$

is a $K \times K$ periodic matrix,

$$B_1(t) = \begin{bmatrix} 0 \\ \ldots \\ b_1(t) \end{bmatrix} = B_1(t+T)$$
$$B_2(t) = \begin{bmatrix} 0 \\ \ldots \\ b_2(t) \end{bmatrix} = B_2(t+T) \tag{15}$$

are $K \times 1$ periodic column matrices and

$$C(t) = [\, 1 \;\ldots\; 0 \,] = C \tag{16}$$

is a $1 \times K$ row matrix.

Using the matrixes $A(t)$, $B_1(t)$, $B_2(t)$, $C(t)$ the PTF $W_y(s,t,Z)$ can be built in the form

$$W_y(s,t,Z) = CW_v(s,t,Z), \tag{17}$$

(Lampe et al. (2016, 2009)) where the PTF $W_v(s,t,Z)$ is defined by the expression

$$\begin{aligned} W_v(s,t,Z) = e^{-st}H(t)\,[V_0(s) + \\ + \int_0^t H^{-1}(\nu)B_1(\nu)e^{s\nu}d\nu + \\ +\psi_0(s)\int_0^t H^{-1}(\nu)B_2(\nu)h(\nu)d\nu \Bigg], \end{aligned} \tag{18}$$

in which

$$\begin{aligned} V_0(s) &= e^{-sT}\tilde{V}^o(s)M\sigma(s,T), \\ \psi_0(s) &= e^{-sT}\tilde{\psi}^o(s)M\sigma(s,T). \end{aligned} \tag{19}$$

In turn

$$\begin{aligned} \tilde{V}^o(s) &= \left[E - e^{-sT}M - e^{-sT}MD\tilde{W}_d(s)C(0)\right]^{-1}, \\ \tilde{\psi}^o(s) &= W_d(s)C(0)\tilde{V}^o(s), \\ D &= \int_0^T H^{-1}(\nu)B_2(\nu)h(\nu)d\nu \\ \sigma(s,T) &= \int_0^T H^{-1}(\nu)B_1(\nu)e^{s\nu}d\nu. \end{aligned} \tag{20}$$

In (20) appears the function

$$\tilde{W}_d(s) = \left.\frac{\beta(\zeta)}{\alpha(\zeta)}\right|_{\zeta=e^{-sT}} \tag{21}$$

and the monodromy matrix for FDLCP-object $\mathcal{P}$

$$M = H(T). \tag{22}$$

In turn, $H(t)$ appearing in (22) is the fundamental Cauchy's matrix of the FDLCP-object $\mathcal{P}$, which can be found by integrating the differential equation

$$\frac{dH(t)}{dt} = A(t)H(t) \tag{23}$$

with the initial condition $H(0) = E$. Here and in (20) E is the unit matrix of appropriate dimension.

As matrices $A(t)$, $B_1(t)$, $B_2(t)$ and $C(t)$ have the form (14)-(16), the PTF $W_y(s,t,Z)$ is scalar. The PAFR $A_y(\nu,t,Z)$ of the system $\mathcal{W}$ can be calculated by the formula

$$A_y(\nu,t,Z) = \left.|W_y(s,t,Z)|\right|_{s=i\nu}, \tag{24}$$

where ν is the frequency. It is obvious, that owing to periodicity of the system $\mathcal{W}$ (and PTF $W_y(s,t,Z)$) for the PAFR the relation $A_y(\nu,t,Z) = A_y(\nu,t+T,Z)$ is valid.

In the following the function $\bar{A}_y(\nu,Z)$, found as the average PAFR over time

$$\bar{A}_y(\nu,Z) = \frac{1}{T}\int_0^T A_y(\nu,t,Z)dt, \tag{25}$$

will be used.

4. CONSTRUCTING THE SET $\mathcal{Z}$

Below we will assume that the set of $\mathcal{Z}$ is formed by vectors of type Z, each of which defines a causal stabilizing regulator of the type (4) and the order of which is $R \leq R_{\max}$, where $R_{\max}$ is given. This set $\mathcal{Z}$ can be constructed using the results in Lampe et al. (2007, 2016). In these works the general algorithm of construction the set of all causal stabilizing controllers for SD-control systems for FDLCP objects of type (12), (13) is formulated. It is shown that the task of causal stabilization can be reduced to the solution of the determinant polynomial equation (DPE)

$$\det \begin{bmatrix} E-\zeta M & O & -\zeta MD \\ -C(0) & E & O \\ O & -\beta(\zeta) & \alpha(\zeta) \end{bmatrix} \sim \Delta(\zeta), \tag{26}$$

where O are zero matrices of corresponding dimension, the D matrix is determined by (20) and $\Delta(\zeta)$ is a stable polynom (without roots inside and on the unit circle). For properties of DPE (26) and ways of solution refer to Rosenwasser et al. (2006); Lampe et al. (2007, 2016). It is shown in Lampe et al. (2007) that if the pair $[M, D]$ is completely controllable and the pair $[M, C]$ is completely observable, DPE (26) is equivalent to the equation

$$\det \begin{bmatrix} d(\zeta) & -\zeta m(\zeta) \\ -\beta(\zeta) & \alpha(\zeta) \end{bmatrix} \sim \Delta(\zeta), \tag{27}$$

where $d(\zeta)$ and $m(\zeta)$ are polynomials in the given case. The polynomial $d(\zeta)$ could be found using

$$d(\zeta) = \det(E - \zeta M), \tag{28}$$

and polynomial $m(\zeta)$ follows from representation

$$W_{\mathcal{P}}(\zeta) = \frac{\zeta m(\zeta)}{d(\zeta)}, \tag{29}$$

where

$$W_{\mathcal{P}}(\zeta) = \zeta C(E - \zeta M)^{-1} MD \tag{30}$$

is the discrete model of the continous FDLCP-object $\mathcal{P}$ given by (12), (13). Thus, the problem of constructing the set $\mathcal{Z}$ is reduced to the question of parameterizing the solution set of the scalar polynomial equation

$$d(\zeta)\alpha(\zeta) + \zeta m(\zeta)\beta(\zeta) = \Delta(\zeta). \tag{31}$$

The polynomials $m(\zeta)$ and $\alpha(\zeta)$ are coprime under the assumptions made, therefore the equation (31) for any polynomial $\Delta(\zeta)$ is solvable. The whole set of polynomials $\alpha(\zeta)$, $\beta(\zeta)$, which is the solution to the equation (31), can be parameterized in the form

$$\alpha(\zeta) = \alpha_0(\zeta) + \xi(\zeta)\zeta m(\zeta),\ \beta(\zeta) = \beta_0(\zeta) - \xi(\zeta)d(\zeta), \tag{32}$$

where $\xi(\zeta)$ is null or any other polynomial, and $\alpha_0(\zeta)$, $\beta_0(\zeta)$ is a particular solution of (31). As a particular solution the minimal solution of equation (31) can be chosen. If the polynomial $\Delta(\zeta)$ was chosen such that

$$\deg \Delta(\zeta) < \deg m(\zeta) + 1 + \deg d(\zeta), \tag{33}$$

the equation (31) has only the minimal solution and for this minimal solution applies

$$\deg \alpha_0 \le R_0, \qquad \deg \beta_0 \le \deg d(\zeta) - 1, \tag{34}$$

where the designation

$$R_0 = \deg m(\zeta) \tag{35}$$

is used. Then the following cases are possible:

(1) Let $R_{\max} = R_0$. In this case the set $\mathcal{Z}$ is defined by the set of polynomials $\Delta(\zeta)$ satisfying (34).
This set unambiguously corresponds to the minimal solution $\alpha_0(\zeta)$, $\beta_0(\zeta)$, satisfying (33). According to terminology in Rybinskii et al. (2014) this set is called the reference controller set and the number R_0 the reference order. The monic polynomial $\Delta(\zeta)$ can be specified by the roots ζ_j $(j = 1, \dots, N_\zeta)$ where N_ζ = deg $m(\zeta)$deg $d(\zeta)$-1). For the vector Z in this case we have

$$Z = \{\zeta_j\}, \qquad j = 1, \dots, N_\zeta, \tag{36}$$

and the set $\mathcal{Z}$ can be given by the relation

$$\mathcal{Z}: \quad |\zeta_j| > 1, \quad \forall j = 1, \dots, N_\zeta. \tag{37}$$

(2) Let $R_{\max} > R_0$. In this case, the set $\mathcal{Z}$ corresponds to the set of polynomial pairs $\alpha(\zeta)$, $\beta(\zeta)$, satisfying (32) with

$$\deg \xi(\zeta) < R_{\max} - \deg d(\zeta). \tag{38}$$

Since the polynomial $\xi(\zeta)$ can be specified by the coefficients $\tilde{\xi}_i$ $(i = 1, \dots, N_\xi)$, where $N_\xi = R_{\max} -$ deg $d(\zeta)$, for vector Z in this case we have

$$Z = \{\zeta_j, \tilde{\xi}_i\}, \qquad j = 1, \dots, N_\zeta, \quad i = 1, \dots, N_\xi, \tag{39}$$

and the set $\mathcal{Z}$ can be given by

$$\mathcal{Z}: \quad \begin{cases} |\zeta_j| > 1, & \forall j = 1, \dots, N_\zeta, \\ \tilde{\xi}_i \in \mathcal{R}, & \forall j = 1, \dots, N_i \xi \end{cases} \tag{40}$$

where $\mathcal{R}$ is the set of real numbers.

(3) If $R_{\max} < R_0$, the solution of the problem is possible only with a special polynomial selection of $\Delta(\zeta)$. The ways of such a choice are described in Rybinskii et al. (2014).

During the numerical optimization, it is necessary to choose a vector $Z \in \mathcal{Z}$ that the guaranteed accuracy estimation on class $\mathcal{M}$ reaches the minimal value. It is shown in Rybinskii et al. (2018) that it is possible to use number $\bar{D}_y$ as such estimation. This number is calculated as follows: Let the disturbances class $\mathcal{M}$ be given by the combination of $N+1$ numbers (8). I addition, let the function $\bar{A}_y(\nu, Z)$ be constructed for the system $\mathcal{W}$ with a fixed vector Z, the function $\bar{A}_y(\nu, Z)$ is built and function $C_y(\nu, Z)$ is found. This function has the form

$$C_y(\nu, Z) = \sum_{n=0}^{N} c_n(Z)\nu^{2n} \tag{41}$$

and conditions

$$C_y(\nu, Z) \approx \bar{A}_y(\nu, Z), \qquad C_y(\nu, Z) \ge \bar{A}_y(\nu, Z). \tag{42}$$

are satisfied. Then the guaranteed quality estimation for system $\mathcal{W}$ can be calculated by the formula

$$\bar{D}_y = \sum_{n=0}^{N} c_n(Z) d_n. \tag{43}$$

Numerous practical experiments have shown that for the numerical optimization of the guaranteed accuracy estimation $\bar{D}_y$ on the set $\mathcal{Z}$ the application of genetic algorithms is very effective.

5. NUMERICAL EXAMPLE

As an example let's consider the optimization by the guaranteed accuracy criterium for the system $\mathcal{W}$ with the object $\mathcal{P}$ of second order, for which

$$a_1(t) = \frac{2\cos\omega t + 4(\sin\omega t + 2)}{\sin\omega t + 2};$$
$$a_2(t) = \frac{4\cos\omega t - 4\sin\omega t + 7(\sin\omega t + 2)}{\sin\omega t + 2}; \quad (44)$$
$$b_1(t) = \frac{1}{\sin\omega t + 2}; \qquad b_2(t) = 2.15 = const = b_2,$$

with the period $T = 0.1$ sec, $\omega = \frac{2\pi}{T}$. For the DAC a zero order hold was assumed, for which $h(t) = 1$.

For the given object the vector $v(t)$ (11) and matrices (14)-(16) have a form

$$v(t) = \left[\, y(t) \;\; \frac{dy(t)}{dt} \,\right]', \quad (45)$$

$$A(t) = \begin{bmatrix} 0 & 1 \\ -a_2(t) & -a_1(t) \end{bmatrix},$$
$$B_1(t) = \begin{bmatrix} 0 \\ b_1(t) \end{bmatrix}, \qquad B_2(t) = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}, \qquad C = [\,1\;\;0\,]. \quad (46)$$

The Cauchy's matrix $H(t)$ for this object and the monodromy matrix M (22) will have dimension 2×2, the matrix of D (20) will have dimension 2×1.

Further let's use the notations

$$H(t) = \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{bmatrix},$$
$$G(t) = H^{-1}(t) = \begin{bmatrix} g_{11}(t) & g_{12}(t) \\ g_{21}(t) & g_{22}(t) \end{bmatrix}, \quad (47)$$

Then

$$M = \begin{bmatrix} h_{11}(T) & h_{12}(T) \\ h_{21}(T) & h_{22}(T) \end{bmatrix}, \qquad D = b_2 \int_0^T \begin{bmatrix} g_{12}(t) \\ g_{22}(t) \end{bmatrix} dt, \quad (48)$$

and from relations (29), (30) it is possible to get

$$m(\zeta) = [\,1\;\;0\,]\, adj(E - \zeta M)MD,$$
$$d(\zeta) = \det(E - \zeta M) \quad (49)$$

and it is obvious, that $\deg m(\zeta) = 1$ and $\deg d(\zeta) = 2$. Therefore according to (35) for the controllers reference order we have R_0 =1. Let the object $\mathcal{P}$ work under the condition of an external stationary centered disturbance $g(t)$, belonging to the class $\mathcal{M}$, given by the combination of the numbers (8)

$$d_0 = 0.3162 \qquad d_1 = 1.5705, \quad (50)$$

which are the variance of disturbance and the variance of their first derivative. Different spectral densities for disturbances in class $\mathcal{M}$ are shown in Fig. 2.

Following the technique stated above, let's perform the following synthesis of the digital controller of reference order R_0 for the given object $\mathcal{P}$.

In this case by (33) we will get $\deg \Delta(\zeta) = 3$, therefore the optimized parameter vector Z, formed with use (36), takes the form

$$Z = [\,\zeta_1\;\;\zeta_2\;\;\zeta_3\,], \quad (51)$$

where numbers ζ_k, $(k = 1, \ldots, 3)$ are chosen during the optimization procedure according to (40). The controller

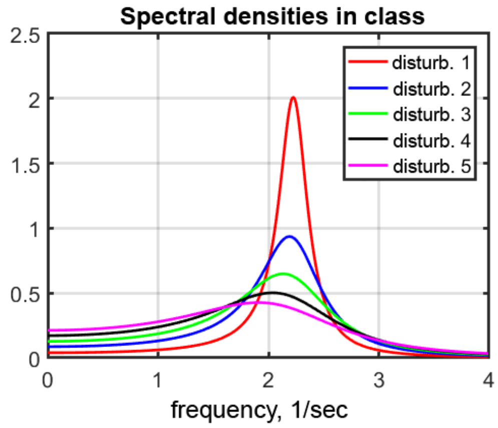

Fig. 2. Different spectral density realizations in class $\mathcal{M}$

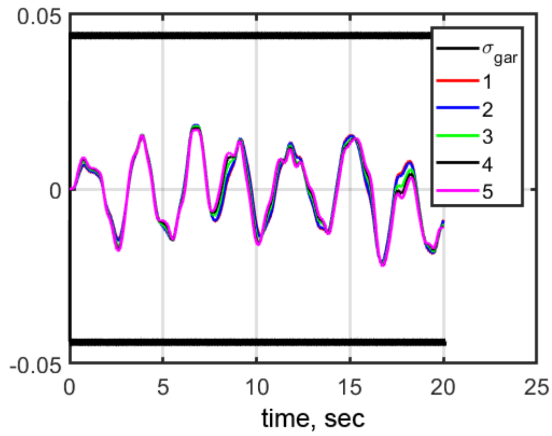

Fig. 3. Simulation results for SD-system $\mathcal{W}$ for different disturbances inside the class $\mathcal{M}$

TF (polynomials $\alpha_0(\zeta)$, $\beta_0(\zeta)$), corresponding to the vector (51) can be found as the minimal solution of the polynomial equation (31) in which $\Delta(\zeta) = \prod_{k=1}^{3}(\zeta - \zeta_k)$.

As a result of numerical optimization using the genetic algorithm, the vector

$$Z_{gar} = [\,1.406\;\;1.740\;\;1.469\,], \quad (52)$$

was found. The causal controllers TF, corresponding to vector (52) is

$$W_d(\zeta) = \frac{5.043\zeta - 105.3}{\zeta 1.231}. \quad (53)$$

The controller for the system $\mathcal{W}$ with TF (53) provides for any disturbance within class $\mathcal{M}$ a standard deviation of the output signal not greater than $\bar{\sigma}_{gar} = \sqrt{\bar{D}_y} \leq 0.043$.

This fact is illustrated in Fig. 3, where simulation examples for systems $\mathcal{W}$ with controller (53) under the conditions of disturbances from class $\mathcal{M}$ are shown. In the same figure the borders, determined by the standard deviation $\bar{\sigma}_{gar}$ are given. It is obvious, that the output signal does not exceed the borders. Controller (53) is the guaranteed accuracy controller for the system $\mathcal{W}$ on disturbance class $\mathcal{M}$ (50).

CONCLUSION

The optimization technique by the criterion of the guaranteed accuracy on the set of discrete causal stabilizing controllers for SISO continuous linear periodic object SD control system is considered. The concept of parametric transfer function is the cornerstone for this technique. To construct the set of stabilizing controllers the apparatus of determinant polynomial equations is used. Moreover, to solve the causality problem, an approach based on the use of the backward shift operator ζ was used. An estimate of the worst mean variance of the system output for an acting disturbance within a given class is used as optimization criterium. The efficiency of the proposed technique is illustrated by a numerical example.

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