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MIMO Feedback Linearization of Redundant Robotic Systems using Task-Priority Operational Space Control ⋆

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Abstract: Redundant robotic systems are designed to accomplish multiple tasks simultaneously. Task-priority control schemes exploit system redundancy by arranging tasks in priority and ensuring strict prioritization between tasks at different priority levels. This paper investigates the relationship between task-priority operational space control and feedback linearization of multiple-input-multiple-output (MIMO) systems. We derive sufficient conditions for input-output feedback linearization and input-to-state feedback linearization of a redundant robotic system influenced by a task-priority operational space pre-feedback control law. Moreover, we analyze the effect of incompatible tasks and provide sufficient conditions for input-output and input-to-state feedback linearizability of the controllable dimensions of incompatible lower-priority tasks. These conditions can be employed when designing the operational space tasks in order to guarantee both task space and joint space stability.

Keywords: Application of nonlinear analysis and design, Lagrangian and Hamiltonian systems, stability of nonlinear systems, redundancy resolution, task-priority control

1. INTRODUCTION

A robotic systems is termed kinematically redundant when it has more degrees of freedom (DOFs) than those strictly required to execute a given task. For such systems, additional lower-priority tasks can be executed by utilizing the redundant DOFs. Redundancy can be resolved at the velocity, acceleration or force level and typically employs some form of Jacobian pseudoinverse defining null-space operators for each task. These null-space operators ensure strict prioritization between tasks when two or more tasks cannot be achieved simultaneously.

Kinematic task-priority control resolves redundancy at the velocity or acceleration level by generating velocity or acceleration references for some dynamic controller to follow. The method was introduced in Hanafusa et al. (1981), further developed in Nakamura et al. (1987) and generalized to any number of tasks in Siciliano and Slotine (1991). An alternative to kinematic control is the operational space formulation introduced in Khatib (1987). The operational space formulation is a holistic approach that assigns joint torques directly by transforming the equations of motion from joint space into the operational space (also known as task space). Although mainly introduced for non-redundant systems, a dynamically consistent null space operator was defined in Khatib (1987), allowing two operational space tasks to be defined and controlled simultaneously. In Sentis and Khatib (2004); Sentis and Khatib (2006); Sentis (2007), the operational space formulation was extended to an arbitrary number of tasks by generalizing the dynamically consistent null space operator from Khatib (1987) to any number of priority levels. These null space operators ensure a prioritized hierarchy among tasks in the sense that torques generated by lower priority tasks do not produce accelerations or forces affecting the task dynamics of higher priority tasks.

Within various task-priority control schemes, the stability properties of lower-priority tasks, as well as joint space stability have been notoriously hard to analyze (Nakanishi et al., 2008). Stability of a kinematic task-priority control scheme was analyzed in Antonelli (2009), while a modified version of the extended operational space formulation from Sentis and Khatib (2004) investigated the stability properties of a lower-priority posture task in Sentis et al. (2013), where asymptotic stability of the controllable directions of the posture error was shown. In Dietrich et al. (2018), asymptotic stability was proven for the regulation case using a passivity-based operational space control law with an arbitrary number of priority levels with potential conflicting tasks.

The main contributions of this paper are sufficient conditions for input-output and input-to-state feedback linearizability of redundant robotic systems under the influence of a task-priority operational space pre-feedback control law. Furthermore, we analyze the case where tasks are incompatible and provide sufficient conditions for input-output and input-to-state feedback linearizability of the controllable dimensions of the incompatible lower-priority tasks. These

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conditions can be employed when designing the operational space tasks in order to guarantee stability in both task space and joint space. Stability can be ensured through input-output feedback linearization with asymptotically stable zero dynamics, or alternatively, by ensuring that the system is input-to-state feedback linearizable, which implies trivial zero dynamics.

This paper is organized as follows: After presenting relevant background material in Section 2, a state-space representation of the robotic equations of motion is derived under a task-priority operational space pre-feedback control law in Section 3. Moreover, the input-output dynamics from the virtual task control input to the output is also derived. Conditions for input-output and input-to-state feedback linearizability are given in Section 4. Additionally, we show how the control law from Sentis and Khatib (2004) can be seen as a special case of input-output feedback linearization with a PD controller for the linearized dynamics. Furthermore, we consider the case where tasks are incompatible and provide sufficient conditions for input-output and input-to-state feedback linearizability without requiring all tasks to be compatible. Section 5 verifies the theoretical results in simulation for a redundant underwater floating-base manipulator before Section 6 concludes the paper.

2. BACKGROUND THEORY

This section presents background material relevant to the rest of this paper.

2.1 Derivatives

For a mapping $A : \mathbb{R}^n \to \mathbb{R}^{l\times m}$, the partial derivative of $A(x)$ with respect to $x \in \mathbb{R}^n$ is written

$$\frac{\partial}{\partial x}A(x) = \left[\frac{\partial A}{\partial x_1}\ \frac{\partial A}{\partial x_2}\ \cdots\ \frac{\partial A}{\partial x_n}\right](x) \in \mathbb{R}^{l\times mn}, \qquad (1)$$

which reduces to the standard definition of the Jacobian matrix of a vector-valued function when $m = 1$. When x is a function of time $t \in \mathbb{R}_{\geq 0}$, the time derivative of $A(x)$ is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}A(x) = \sum_{i=1}^{n} \frac{\partial A}{\partial x_i}\dot{x}_i = \frac{\partial A}{\partial x}\left(\dot{x} \otimes I_m\right), \qquad (2)$$

where $\otimes$ denotes the Kronecker product. Note that when $m = 1$, we have $\dot{x} \otimes 1 = \dot{x}$ and hence (2) reduces to the familiar expression $\frac{\mathrm{d}}{\mathrm{d}t}A(x) = \frac{\partial A}{\partial x}\dot{x}$. Furthermore, we define the partial derivative of the product of two matrices $A : \mathbb{R}^n \to \mathbb{R}^{l\times m}$ and $B : \mathbb{R}^n \to \mathbb{R}^{m\times p}$ with respect to $x \in \mathbb{R}^n$ by

$$\frac{\partial}{\partial x}\left[A(x)B(x)\right] = \frac{\partial A(x)}{\partial x}\left(I_n \otimes B(x)\right) + A(x)\frac{\partial}{\partial x}B(x). \quad (3)$$

Given a real valued function $\lambda : \mathbb{R}^n \to \mathbb{R}$ and vector field $f : \mathbb{R}^n \to \mathbb{R}^n$. The Lie derivative of λ along f is given by

$$L_f\lambda(x) = \sum_{i=1}^{n} \frac{\partial \lambda}{\partial x_i} f_i(x) = \frac{\partial \lambda}{\partial x} f(x). \qquad (4)$$

2.2 Modeling of robotic systems

The system configuration of an n degree of freedom (DOF) robotic system can be expressed by the joint variables $q = \mathrm{col}\,(q_1, q_2, \ldots, q_n) \in \mathbb{R}^n$. The dynamic equations of motion for a robotic manipulator are given by (Siciliano and Khatib, 2016)

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau, \qquad (5)$$

where $M(q) \in \mathbb{R}^{n\times n}$ is the manipulator inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal matrix, $G(q) \in \mathbb{R}^n$ is the gravity torque vector, and $\tau \in \mathbb{R}^n$ is the joint torque vector.

A task is defined as a generic m-dimensional control objective, specified as a function of the system configuration. The relationship between the joint space and task space variables are given by the direct kinematics equation (Siciliano and Khatib, 2016)

$$\sigma = f_\sigma(q), \qquad (6)$$

where $f_\sigma : \mathbb{R}^n \to \mathbb{R}^m$ is a mapping from joint space to task space, which is nonlinear in general. Differentiating (6) with respect to time once and twice, yields the first- and second-order differential kinematics equation

$$\dot{\sigma} = J(q)\dot{q}, \qquad (7)$$

$$\ddot{\sigma} = J(q)\ddot{q} + \dot{J}(q, \dot{q})\dot{q}, \qquad (8)$$

where $J = \frac{\partial f_\sigma(q)}{\partial q} \in \mathbb{R}^{m\times n}$ is the configuration-dependent task Jacobian matrix, $\dot{q} \in \mathbb{R}^n$ is the system velocity vector and $\ddot{q} \in \mathbb{R}^n$ is the system acceleration vector. A robotic system is kinematically redundant when it has more DOFs than those strictly required to execute a given task (Siciliano and Khatib, 2016), which requires the dimension of the system configuration q to be larger than the dimension of the task variable σ.

2.3 Operational Space Control

The operational space dynamics are found by solving (5) for $\ddot{q}$, which combined with (8) yields

$$\ddot{\sigma} = JM^{-1}\left(\tau - C\dot{q} - G\right) + \dot{J}\dot{q}. \qquad (9)$$

By mapping the generalized torque into a generalized force through the relation

$$\tau = J^T F, \qquad (10)$$

defining the inertia matrix associated with the task variable σ as

$$\Lambda = \left(JM^{-1}J^T\right)^{-1} \in \mathbb{R}^{m\times m}, \qquad (11)$$

and pre-multiplying both sides of (9) by Λ, the operational space dynamics are obtained as

$$\Lambda\ddot{\sigma} + \Lambda\left(JM^{-1}C\dot{q} - \dot{J}\dot{q}\right) + \Lambda JM^{-1}G = F, \qquad (12)$$

which can be written as

$$\Lambda\ddot{\sigma} + d + p = F, \qquad (13)$$

where $d = \Lambda\left(JM^{-1}C\dot{q} - \dot{J}\dot{q}\right)$ and $p = \Lambda JM^{-1}G$.

If the system is redundant with respect to σ, we may decompose the torque vector into a torque corresponding to the primary task and another torque acting in the null-space of the primary task as follows (Khatib, 1987)

$$\tau = J^T F + N\tau_0, \qquad (14)$$

where τ_0 is an arbitrary torque acting in the null-space of J. The null-space operator N satisfies $JM^{-1}N = 0$ and is given by

$$N = I_n - J^T\bar{J}^T, \qquad (15)$$

with

$$\bar{J} = M^{-1}J^T\left(JM^{-1}J^T\right)^{-1} \in \mathbb{R}^{n\times m}. \tag{16}$$

The matrix $\bar{J}$ is known as the dynamically consistent pseudoinverse of J, which is a weighted pseudoinverse of J where the weight is the inverse of the inertia matrix (Khatib, 1987).

2.4 Extension to k tasks

By defining the task specific inertia matrix and dynamically consistent pseudoinverse of task i as

$$\Lambda_i = \left(J_iN_i^TM^{-1}N_iJ_i^T\right)^{-1} = \left(J_iM^{-1}N_iJ_i^T\right)^{-1}, \tag{17}$$

$$\bar{J}_i = M^{-1}J_i^T\Lambda_i, \tag{18}$$

the null-space operator in (15) can be extended to an arbitrary number of priority levels as follows (Sentis and Khatib, 2004)

$$N_1 = I, \tag{19a}$$

$$N_{k+1} = \left(I - N_kJ_k^T\bar{J}_k^T\right)N_k. \tag{19b}$$

A pre-feedback control law for k tasks arranged in priority is then given by (Sentis and Khatib, 2004)

$$\tau = J_1^Tu_1 + N_2J_2^Tu_2 + \cdots + N_kJ_k^Tu_k, \tag{20}$$

with

$$u_i = \Lambda_ia_i + d_i + p_i, \tag{21}$$

$$a_i = \mu_i - J_iM^{-1}\sum_{j=1}^{i-1}N_jJ_j^T\Lambda_ja_j, \tag{22}$$

$$\mu_i = \ddot{\sigma}_{d,i} - K_{d,i}\dot{\tilde{\sigma}}_i - K_{p,i}\tilde{\sigma}_i, \tag{23}$$

$$d_i = \Lambda_i\left(J_iM^{-1}C\dot{q} - \dot{J}_i\dot{q}\right) \tag{24}$$

$$p_i = \Lambda_iJ_iM^{-1}G, \tag{25}$$

where $\tilde{\sigma}(x,t) = \sigma(x) - \sigma_d(t)$ represents the task error and $K_{d,i}$ and $K_{p,i}$ are derivative and proportional gains, respectively.

2.5 Input-output feedback linearization of MIMO systems

This section is based upon Isidori (1995) and Sastry (1999).

Consider an input affine nonlinear control system of the form

$$\begin{aligned}\dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)u_i \\ y_j &= h_j(x), \quad 1 \le j \le m\end{aligned} \tag{26}$$

where $x \in D \subset \mathbb{R}^n$ is the state vector, $f, g_i : D \to \mathbb{R}^n$ are smooth vector fields, and $h_i : D \to \mathbb{R}$ are smooth functions. Differentiating the ith output y_i with respect to time yields

$$\dot{y}_i = L_fh_i + \sum_{j=1}^{m}\left(L_{g_j}h_i\right)u_j. \tag{27}$$

Observe that if $L_{g_j}h = 0$ for all $j = 1, \ldots, m$, then the input does not appear in $\dot{y}_i$. Assume that y_i has to be differentiated with respect to time r_i times before at least one component of the control input vector u explicitly appears in a time derivative of y_i, then the r_ith derivative of y_i is given by

$$y_i^{(r_i)} = L_f^{r_i}h_i + \sum_{j=1}^{m}L_{g_j}\left(L_f^{r_i-1}h_i\right)u_j. \tag{28}$$

The integer r_i is defined as the smallest integer such that

$$L_{g_j}L_f^kh_i(x) = 0, \quad 1 \le j \le m,\ k \le r_i - 2 \tag{29a}$$

$$L_{g_j}L_f^{r_i-1}h_i(x) \ne 0, \quad \text{for at least one } 1 \le j \le m. \tag{29b}$$

For single-input single-output (SISO) systems with $m = 1$, (29) is the definition of the relative degree of $y = h(x)$, with $h : \mathbb{R}^n \to \mathbb{R}$. The concept of relative degree is extended to multiple-input multiple-output (MIMO) systems as follows (Isidori, 1995; Sastry, 1999):

Definition 1. (Vector relative degree). The system (26) has a vector relative degree $\{r_1, \ldots, r_m\}$ at a point x_0 if

(i)
$$L_{g_j}L_f^kh_i(x) = 0, \quad 0 \le k \le r_i - 2, \tag{30}$$
for all $1 \le j \le m$, for all $1 \le i \le m$, and for all x in a neighborhood of x_0.

(ii) The $m \times m$ matrix
$$A(x) = \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \ldots & L_{g_m}L_f^{r_1-1}h_1(x) \\ L_{g_1}L_f^{r_2-1}h_2(x) & \ldots & L_{g_m}L_f^{r_2-1}h_2(x) \\ \vdots & \ddots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \ldots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix}, \tag{31}$$
is nonsingular at $x = x_0$.

Whenever the system (26) has a well-defined vector relative degree $\{r_1, \ldots, r_m\}$ at x_0, we say that the system is input-output feedback linearizable at x_0 since the control law

$$u = A^{-1}(x)\left(\mu - b(x)\right), \tag{32}$$

where $\mu = \mathrm{col}\left(\mu_1, \ldots, \mu_m\right)$ and

$$b(x) = \begin{bmatrix} L_f^{r_1}h_1(x) \\ L_f^{r_2}h_2(x) \\ \vdots \\ L_f^{r_m}h_m(x) \end{bmatrix}, \tag{33}$$

yields the linear and decoupled system

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix}. \tag{34}$$

Furthermore, if $r_1 + r_2 + \cdots + r_m = n$, we say that the system is input-to-state or full-state feedback linearizable since the set of functions

$$\phi_k^i(x) = L_f^{k-1}h_i(x), \quad i \le k \le r_i, \quad 1 \le i \le m, \tag{35}$$

completely define a local coordinate transformation at x_0.

3. STATE-SPACE REPRESENTATION

In this section, we derive a state-space representation of the robotic equations of motion under a task-priority operational space pre-feedback control law for an arbitrary number of tasks. Moreover, the input-output dynamics from the virtual task control inputs to the output task errors are also derived.

3.1 Equations of motion

The task-priority operational space control law in (20) can be rewritten as

$$\tau = J_1^Tu_1 + N_2J_2^Tu_2 + \cdots + N_kJ_k^Tu_k \tag{36}$$

$$= \begin{bmatrix} J_1^T & N_2 J_2^T & \dots & N_k J_k^T \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_k \end{bmatrix} \tag{37}$$

$$= T(q)u, \tag{38}$$

where $u_i \in \mathbb{R}^{m_i}$, $u \in \mathbb{R}^m$ with $m = \sum_{i=1}^k m_i$, and where the null space operators N_i are given by (19). Define the equality task variables as $\sigma_i(x) \in \mathbb{R}^{m_i}$ and the corresponding task error as

$$y_i(x,t) = \sigma_i(x) - \sigma_{i,d}(t), \quad i = 1, \dots, k, \tag{39}$$

where $\sigma_{i,d}(t)$ is the desired task value. By applying the pre-control law in (38), the dynamic equations of motion (5) can be expressed in state-space form as a nonlinear affine control system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x,t), \end{aligned} \tag{40}$$

where $x = \mathrm{col}\,(x_1, x_2) = \mathrm{col}\,(q, \dot{q}) \in D \subset \mathbb{R}^{2n}$ is the state vector of joint angles and joint velocities and

$$f(x) = \begin{bmatrix} x_2 \\ -M(x_1)^{-1}\big(C(x)x_2 + G(x_1)\big) \end{bmatrix} \in \mathbb{R}^{2n}, \tag{41}$$

$$g(x) = \begin{bmatrix} 0_{n \times m} \\ M(x_1)^{-1}T(x_1) \end{bmatrix} \in \mathbb{R}^{2n \times m}, \tag{42}$$

$$h(x,t) = \begin{bmatrix} y_1(x,t) \\ \vdots \\ y_k(x,t) \end{bmatrix} \in \mathbb{R}^m. \tag{43}$$

3.2 Input-output dynamics

If the task error given by (39) is only a function of the configuration of the robotic system, then $y_i(x_1)$ has to be differentiated with respect to time twice for the input to appear. The time derivative of $y_i(x_1)$ is given by

$$\dot{y}_i(x,t) = \frac{\partial y_i}{\partial x}\dot{x} + \frac{\partial y_i}{\partial t} \tag{44}$$

$$= \frac{\partial \sigma_i(x_1)}{\partial x}\big(f(x) + g(x)v\big) - \dot{\sigma}_{i,d}(t), \tag{45}$$

where

$$\frac{\partial \sigma_i(x_1)}{\partial x} f(x) = \begin{bmatrix} \frac{\partial \sigma_i(x_1)}{\partial x_1} & \frac{\partial \sigma_i(x_1)}{\partial x_2} \end{bmatrix} f(x) \tag{46}$$

$$= \frac{\partial \sigma_i(x_1)}{\partial x_1} x_2 \tag{47}$$

$$= J_i(x_1)x_2, \tag{48}$$

and

$$\frac{\partial \sigma_i(x_1)}{\partial x} g(x) = \begin{bmatrix} \frac{\partial \sigma_i(x_1)}{\partial x_1} & \frac{\partial \sigma_i(x_1)}{\partial x_2} \end{bmatrix} \begin{bmatrix} 0 \\ M(x_1)^{-1}T(x_1) \end{bmatrix} \tag{49}$$

$$= 0. \tag{50}$$

As expected, the control input does not appear in $\dot{y}_i$. Differentiating the task error with respect to time once more yields

$$\ddot{y}_i = \frac{\partial \dot{y}_i}{\partial x}\dot{x} + \frac{\partial \dot{y}_i}{\partial t} \tag{51}$$

$$= \frac{\partial}{\partial x}\big(J_i(x_1)x_2\big)\, g(x)u + \frac{\partial}{\partial x}\big(J_i(x_1)x_2\big)\, f(x) - \ddot{\sigma}_{i,d}(t) \tag{52}$$

$$= A_i(x)u + b_i(x) - \ddot{\sigma}_{i,d}(t), \tag{53}$$

where

$$A_i(x) = \frac{\partial}{\partial x}\big(J_i(x_1)x_2\big)\, g(x) \tag{54}$$

$$= \begin{bmatrix} \frac{\partial J_i(x_1)x_2}{\partial x_1} & \frac{\partial J_i(x_1)x_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} 0 \\ M(x_1)^{-1}T(x_1) \end{bmatrix} \tag{55}$$

$$= J_i(x_1)M(x_1)^{-1}T(x_1), \tag{56}$$

and $b_i(x)$ is obtained from (57)-(60). Since two arbitrary vectors $a, b \in \mathbb{R}^n$ satisfy

$$(I_n \otimes b)\, a = (a \otimes I_n)\, b, \tag{61}$$

it follows that $(I_n \otimes x_2)\, x_2 = (x_2 \otimes I_n)\, x_2$, which together with (2) and $\dot{x}_1 = x_2$ implies that

$$\dot{J}_i(x_1) = \frac{\partial J_i(x_1)}{\partial x_1}(\dot{x}_1 \otimes I_n). \tag{62}$$

Consequently, (60) can be rewritten as

$$b_i(x) = \dot{J}_i x_2 - J_i M^{-1}(C x_2 + G). \tag{63}$$

The input-output dynamics of all tasks can therefore be expressed as

$$\underbrace{\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \vdots \\ \ddot{y}_k \end{bmatrix}}_{\ddot{y}} = \underbrace{\begin{bmatrix} A_1(x) \\ A_2(x) \\ \vdots \\ A_k(x) \end{bmatrix}}_{A(x)} u + \underbrace{\begin{bmatrix} b_1(x) \\ b_2(x) \\ \vdots \\ b_k(x) \end{bmatrix}}_{b(x)} - \underbrace{\begin{bmatrix} \ddot{\sigma}_{1,d}(t) \\ \ddot{\sigma}_{2,d}(t) \\ \vdots \\ \ddot{\sigma}_{k,d}(t) \end{bmatrix}}_{\ddot{\sigma}_d(t)}, \tag{64}$$

where

$$b(x) = \begin{bmatrix} \dot{J}_1 x_2 - J_1 M^{-1}(C x_2 + G) \\ \dot{J}_2 x_2 - J_2 M^{-1}(C x_2 + G) \\ \vdots \\ \dot{J}_k x_2 - J_k M^{-1}(C x_2 + G) \end{bmatrix}, \tag{65}$$

and

$$A(x) = \begin{bmatrix} J_1 M^{-1} J_1^T & 0_{m_1 \times m_2} & 0_{m_1 \times m_3} & \cdots & 0_{m_1 \times m_k} \\ J_2 M^{-1} J_1^T & J_2 M^{-1} N_2 J_2^T & 0_{m_2 \times m_3} & \cdots & 0_{m_2 \times m_k} \\ J_3 M^{-1} J_1^T & J_3 M^{-1} N_2 J_2^T & J_3 M^{-1} N_3 J_3^T & \ddots & 0_{m_3 \times m_k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ J_k M^{-1} J_1^T & J_k M^{-1} N_2 J_2^T & J_k M^{-1} N_3 J_3^T & \dots & J_k M^{-1} N_k J_k^T \end{bmatrix}, \tag{66}$$

since $A_i = J_i M^{-1} T$ and $J_i M^{-1} N_k = 0$ for $i < k$.

4. MIMO FEEDBACK LINEARIZATION

This section presents the main results of this paper; namely, sufficient conditions for input-output and input-to-state feedback linearizability of a redundant robotic system under the influence of a task-priority operational space pre-feedback control law. Moreover, the results are extended to the case where tasks are incompatible.

4.1 Input-output feedback linearization

We want to show that the system (5) is input-output feedback linearizable under the influence of the task-priority operational space control law (38) whenever the operational space tasks are chosen such that they are non-conflicting and kinematic singularities are avoided.

Kinematic singularities are all points x for which

$$\mathrm{rank}\,\big(J_i(x)\big) < m_i, \tag{67}$$

$$b_i(x) = \frac{\partial}{\partial x}\left(J_i(x_1)x_2\right) f(x) = \begin{bmatrix} \frac{\partial J_i(x_1)x_2}{\partial x_1} & \frac{\partial J_i(x_1)x_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} x_2 \\ -M(x_1)^{-1}\left(C(x)x_2 + G(x_1)\right) \end{bmatrix} \tag{57}$$

$$= \frac{\partial J_i(x_1)x_2}{\partial x_1} x_2 - \frac{\partial J_i(x_1)x_2}{\partial x_2} M(x_1)^{-1}\left(C(x)x_2 + G(x_1)\right) \tag{58}$$

$$= \frac{\partial J_i(x_1)}{\partial x_1}\left(I_n \otimes x_2\right) x_2 + J_i(x_1)\frac{\partial x_2}{\partial x_1} - \left(\frac{\partial J_i(x_1)}{\partial x_2}\left(I_n \otimes x_2\right) + J_i(x_1)\frac{\partial x_2}{\partial x_2}\right) M(x_1)^{-1}\left(C(x)x_2 + G(x_1)\right) \tag{59}$$

$$= \frac{\partial J_i(x_1)}{\partial x_1}\left(I_n \otimes x_2\right) x_2 - J_i(x_1) M(x_1)^{-1}\left(C(x)x_2 + G(x_1)\right). \tag{60}$$

with $J_i \in \mathbb{R}^{m_i \times n}$ and $m_i \leq n$. Furthermore, all tasks are compatible around some point x_0 if

$$\operatorname{rank}\left(N_i(x_0)J_i^T(x_0)\right) = m_i, \quad i = 1, \ldots, k, \tag{68}$$

where $N_i \in \mathbb{R}^n$ and $n \geq m_i$. Note that (68) implies that $\operatorname{rank}(J_i(x_0)) = m_i$, i.e. that kinematic singularities are avoided.

Theorem 1. Consider the system (40), obtained from (5) under the influence of the task-priority pre-feedback control law (38). If the task Jacobians $J_i \in \mathbb{R}^{m_i}$ and the null-space operators $N_i \in \mathbb{R}^n$ given by (19) satisfy

$$\operatorname{rank}\left(N_i(x)J_i^T(x)\right) = m_i, \quad i = 1, \ldots, k, \tag{69}$$

for all $x \in U$, where U is a neighborhood of x_0. Then, the system (40) is input-output feedback linearizable in U with a vector relative degree $\{r_1, r_2, \ldots, r_m\} = \{2, 2, \ldots, 2\}$.

Proof. Eq. (69) implies that the matrix $N_i J_i^T$ has full rank and hence that (17) is well-defined for all $i = 1, \ldots, k$ since the inertia matrix $M(x_1)$ has full rank for all x. Consequently, every submatrix in (66) has full rank, which further implies that $A(x)$ has full rank for $x \in U$ and thus that the system in (40) has a vector relative degree

$$\{r_1, r_2, \ldots, r_m\} = \{2, 2, \ldots, 2\}, \tag{70}$$

at x_0. □

When Theorem 1 is satisfied, the control input

$$u = A^{-1}(x)\left(\mu - b(x) + \ddot{\sigma}_d(t)\right), \tag{71}$$

renders the input-output dynamics (64) equivalent to m fully linearized and independent single-input single-output subsystems

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \vdots \\ \ddot{y}_k \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_k \end{bmatrix}. \tag{72}$$

Moreover, we can show that the inverse of $A(x)$ is given by the closed-form expression

$$A^{-1} = \begin{bmatrix} \Lambda_1 & 0_{12} & 0_{13} & \ldots & 0_{1k} \\ -\Lambda_2 J_2 M^{-1}\Gamma_{21} & \Lambda_2 & 0_{23} & \ldots & 0_{2k} \\ -\Lambda_3 J_3 M^{-1}\Gamma_{31} & -\Lambda_3 J_3 M^{-1}\Gamma_{32} & \Lambda_3 & \ddots & 0_{3k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\Lambda_k J_k M^{-1}\Gamma_{k1} & -\Lambda_k J_k M^{-1}\Gamma_{k2} & -\Lambda_k J_k M^{-1}\Gamma_{k3} & \ldots & \Lambda_k \end{bmatrix}, \tag{73}$$

where $0_{kp} \in \mathbb{R}^{k \times p}$ is the $k \times p$ zero matrix, Λ_j is given by (17) and for each $j = 1, 2, \ldots, k$

$$\Gamma_{ij} = \begin{cases} N_{i-1}J_{i-1}^T M_{i-1}, & i = j+1, \\ \left(I - N_{i-1}J_{i-1}^T \Lambda_{i-1} J_{i-1} M^{-1}\right)\Gamma_{(i-1)j}, & i > j+1, \end{cases} \tag{74}$$

for all $i = j+1, \ldots, k$.

Remark 1. Since (72) is a linear system, μ can easily be designed such that the input-output dynamics of every task is exponentially stable. However, the zero dynamics must be asymptotically stable in order to guarantee that (40) is minimum phase, and hence asymptotically stable (Isidori, 1995).

4.2 Equivalence with a traditional task-priority operational space control law

The task-priority operational space control law in Sentis and Khatib (2004) can be seen as a special case of input-output feedback linearization with a PD controller for the linearized dynamics. Specifically, by defining the vectors $\xi_i := \operatorname{col}(y_i, \dot{y}_i)$, Eq. (72) yields the following differential equations

$$\dot{\xi}_i = F_i \xi_i + G_i \mu_i, \tag{75}$$

where

$$F_i = \begin{bmatrix} 0_{m_i \times m_i} & I_{m_i \times m_i} \\ 0_{m_i \times m_i} & 0_{m_i \times m_i} \end{bmatrix}, \quad G_i = \begin{bmatrix} 0_{m_i \times m_i} \\ I_{m_i \times m_i} \end{bmatrix}, \tag{76}$$

for $i = 1, \ldots, k$. By employing the PD control law

$$\mu_i = \begin{bmatrix} -K_{p,i} & -K_{d,i} \end{bmatrix} \xi_i, \tag{77}$$

the closed-loop system for each task becomes

$$\ddot{y}_i + K_{d,i}\dot{y}_i + K_{p,i} y_i = 0, \tag{78}$$

under the assumption that all tasks are compatible, i.e. the conditions of Theorem 1. The control law given by (38), (71) and (77) is exactly equal to (20), which was introduced in Sentis and Khatib (2004).

4.3 Input-to-state feedback linearization

In order to avoid analyzing complicated zero dynamics, we can obtain trivial internal dynamics by designing the tasks such that the system has a vector relative degree at a point x_0 satisfying $r_1 + r_2 + \cdots + r_m = 2n$.

Theorem 2. The system (40), obtained from (5) under the influence of the task-priority pre-feedback control law (38), is input-to-state feedback linearizable in a neighborhood U of x_0 if the tasks are chosen such that the system is input-output feedback linearizable in U with a vector relative degree $\{r_1, r_2, \ldots, r_m\} = \{2, 2, \ldots, 2\}$ with $m = n$.

Proof. When $m = n$ we have $r_1 + \cdots + r_n = 2n$, hence the local coordinate transformation $z(x,t) = \Phi(x,t) = \operatorname{col}(y_1, \ldots, y_k, \dot{y}_1, \ldots, \dot{y}_k) \in \mathbb{R}^{2n}$ is a local diffeomorphism. The control input $u = A^{-1}(x)\left(\mu - b(x) - \ddot{\sigma}_d(t)\right) \in \mathbb{R}^n$, transforms (64) into the linear and decoupled controllable system

$$\dot{z} = Fz + G\mu, \tag{79}$$

with

$$\left[\frac{\partial \Phi}{\partial x}\Big(f(x)-g(x)A^{-1}(x)\big(b(x)+\ddot{\sigma}_d(t)\big)\Big)+\frac{\partial \Phi}{\partial t}\right]_{x=\Phi^{-1}(z)} = Fz, \quad (80)$$

$$\left[\frac{\partial \Phi}{\partial x} g(x)A^{-1}(x)\right]_{x=\Phi^{-1}(z)} = G, \quad (81)$$

where

$$F = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}, \quad G = \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \end{bmatrix}, \quad (82)$$

and

$$\frac{\partial \Phi}{\partial t} = -\begin{bmatrix} \ddot{\sigma}_{1,d}(t) \\ \vdots \\ \ddot{\sigma}_{k,d}(t) \end{bmatrix} \in \mathbb{R}^{2n}, \quad (83)$$

$$\frac{\partial \Phi}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x} \\ \vdots \\ \frac{\partial y_k}{\partial x} \\ \frac{\partial \dot{y}_1}{\partial x} \\ \vdots \\ \frac{\partial \dot{y}_k}{\partial x} \end{bmatrix} = \begin{bmatrix} J_1 & 0_{m_1\times n} \\ \vdots & \vdots \\ J_k & 0_{m_k\times n} \\ \dot{J}_1 & J_1 \\ \vdots & \vdots \\ \dot{J}_k & J_k \end{bmatrix} \in \mathbb{R}^{2n\times 2n}. \quad \square \quad (84)$$

Remark 2. If the tasks are designed such that Theorem 2 holds, then the virtual control input μ in (79) can be designed using linear control techniques to achieve exponential stability of $z = 0$, which is equivalent to exponential stability of $x = \Phi^{-1}(0)$. Moreover, if $U = \mathbb{R}^{2n}$ then global exponential stability can be guaranteed.

4.4 Incompatible tasks

If the jth task becomes incompatible with one or more higher priority tasks at some point x_0 we have

$$\mathrm{Null}\left(N_j(x_0)J_j^T(x_0)\right) \neq \varnothing, \quad (85)$$

which implies that $\mathrm{rank}\left(N_j(x_0)J_j^T(x_0)\right) = \breve{m}_j < m_j$ and hence that Λ_j^{-1} and $A(x_0)$ are singular and that the system does not have a vector relative degree. However, as long as each of the higher priority tasks are still compatible with one another, the submatrix of $A(x_0)$ containing the first $\sum_{i=1}^{j-1} m_i$ rows and columns will still be nonsingular. Consequently, the first $j-1$ tasks are still input-output feedback linearizable.

As for the jth task, if $\breve{m}_j \geq 1$, we can show that the $\breve{m}_j$ controllable dimensions are still input-output feedback linearizable. To characterize the controllable dimensions, the linearly dependent columns of the matrix $N_j(x)J_j^T(x)$ are removed. A permutation vector of indices $E_j \in \mathbb{R}^{\breve{m}_j}$ corresponding to the linearly independent columns of $N_jJ_j^T$ can be computed from the QR decomposition of $J_jN_j^T$. This permutation vector is then utilized to obtain a reduced task Jacobian $\breve{J}_j \in \mathbb{R}^{\breve{m}_j\times n}$. The controllable dimensions $\breve{y}_j \in \mathbb{R}^{\breve{m}_j}$ of y_j are found by selecting the components of y_j contained in the permutation vector E_j, i.e. $\breve{y}_j = y_j(E_j)$ in MATLAB colon notation. The computational procedure is summarized in Algorithm 1.

Define $\breve{m} = \sum_{i=1}^{k} \breve{m}_i$ as the sum the controllable task dimensions. By computing the task Jacobians and null-space operators according to Algorithm 1 and applying the task-priority operational space pre-feedback control law

Algorithm 1 Computing controllable task dimensions, Jacobians and null-space operators

Input: $J_i(x_1), y_i(x_1)$, $1 \leq i \leq k$.
Output: $\breve{J}_i, \breve{y}_i, \breve{\Lambda}_i$ and $\breve{N}_i$, $1 \leq i \leq k$.

```
for i = 1 to k do
    Compute the permutation vector E_i corresponding to linearly
      independent rows of N_i J_i^T from a QR factorization of J_i N_i^T.
    Set J̆_i = J_i(E_i, :) (In MATLAB colon notation).
    Obtain the controllable dimensions from y̆_i = y_i(E_i).
    Compute Λ̆_i according to (17) with J_i = J̆_i.
    Compute N̆_i according to (19) with J_i = J̆_i.
end for
return J̆_i, y̆_i, Λ̆_i and N̆_i, 1 ≤ i ≤ k.
```

$$\tau = \breve{J}_1^T\breve{u}_1 + \breve{N}_2\breve{J}_2^T\breve{u}_2 + \cdots + \breve{N}_k\breve{J}_k^T\breve{u}_k = \breve{T}\breve{u}, \quad (86)$$

the system (5) becomes

$$\begin{aligned} \dot{x} &= f(x) + \breve{g}(x)\breve{u}, \\ \breve{y}_i &= \breve{\sigma}_i(x_1) - \breve{\sigma}_{i,d}(t), \quad i = 1, \ldots, k, \end{aligned} \quad (87)$$

where $\breve{g}(x) \in \mathbb{R}^{2n\times\breve{m}}$ is given by (42) with $T(x_1) = \breve{T}(x_1)$.

Corollary 1. The system (87), obtained from (5) under the influence of the task-priority pre-feedback control law (86), is input-output feedback linearizable in a neighborhood U around x_0.

Proof. The proof follows immediately from Theorem 1 since the matrix $\breve{N}_i(x_0)\breve{J}_i^T(x_0)$ has full rank, which implies that $\breve{A}(x_0) \in \mathbb{R}^{\breve{m}\times\breve{m}}$ is non-singular. $\square$

Corollary 2. The system (87) obtained from (5) under the influence of the task-priority pre-feedback control law (86) is input-to-state feedback linearizable in a neighborhood U around x_0 if the sum of controllable task dimensions is equal to the dimension of the configuration space, i.e. $\breve{m} = n$.

Proof. The proof follows immediately from Theorem 2 and Corollary 1. $\square$

Remark 3. A consequence of Corollary 2 is that exponential stability of the controllable directions $\breve{z}(x,t) = \breve{\Phi}(x,t) = \mathrm{col}\left(\breve{y}_1, \ldots, \breve{y}_k, \dot{\breve{y}}_1, \ldots, \dot{\breve{y}}_k\right)$ or equivalently, exponential stability of $x = \Phi^{-1}(0)$ can be guaranteed by simply defining the task $\sigma_k(x_1) = x_1$ with task error

$$y_k(x_1) = x_1 - x_{1,d}, \quad (88)$$

at the lowest priority level, thereby ensuring that $\breve{m} = n$. Moreover, if $U = \mathbb{R}^{2n}$ and the permutation vectors of indices $E_1, \ldots, E_k$ are constant, then global exponential stability can be guaranteed.

5. SIMULATIONS

This section applies the theoretical results of this paper to a simulation study of an articulated intervention-AUV (AIAUV) based on the Eelume robot (Schmidt-Didlaukies et al., 2018; Liljebäck and Mills, 2017) depicted in Fig. 1.

The system configuration is described by $\xi = \mathrm{col}\left(p_{ib}^i, q, \theta\right)$, where $p_{ib}^i \in \mathbb{R}^3$ is the position of the base in an inertial frame, $q = \mathrm{col}\,(\eta, \epsilon) \in \mathbb{R}^4$ is a unit quaternion describing the orientation of the base and $\theta \in \mathbb{R}^n$ are the joint angles. The joint velocities are given by $\dot{\theta}$ and the linear and angular

Fig. 1. The Eelume AIAUV (Courtesy of Eelume)

velocities of the base frame with respect to an inertial frame are denoted v^i_{ib} and ω^i_{ib}, respectively. These velocities are collected in the velocity vector $\zeta = \mathrm{col}\left(v^i_{ib}, \omega^i_{ib}, \dot{\theta}\right) \in \mathbb{R}^{6+n}$. The equations of motion are given by (Schmidt-Didlaukies et al., 2018)

$$\dot{\xi} = J_\xi(q)\zeta, \tag{89a}$$

$$M(\theta)\dot{\zeta} + C(\theta,\zeta)\zeta + D(\theta,\zeta)\zeta + G(\xi) = \tau, \tag{89b}$$

where $M(\theta)$ is the inertia matrix including hydrodynamic added mass, $C(\theta,\zeta)$ is the Coriolis-centripetal matrix including hydrodynamic added mass, $D(\theta,\zeta)$ is the damping matrix, $G(\xi)$ is the vector of gravitational and buoyancy forces and moments, and τ is the control input vector of generalized forces and torques. Moreover, the kinematic transformation matrix is given by

$$J_\xi(q) = \begin{bmatrix} R^i_b(q) & 0_{3\times 3} & 0_{3\times n} \\ 0_{4\times 3} & T_q(q) & 0_{4\times n} \\ 0_{n\times 3} & 0_{n\times 3} & I_n \end{bmatrix}, \quad T_q(q) = \frac{1}{2}\begin{bmatrix} -\epsilon^T \\ \eta I_3 + [\epsilon]_\times \end{bmatrix}, \tag{90}$$

where $R^i_b(q) \in \mathrm{SO}(3)$ is a rotation matrix representing the orientation of the base relative to the inertial frame and $[\cdot]_\times : \mathbb{R}^3 \to \mathfrak{so}(3) \subset \mathbb{R}^{3\times 3}$ denotes the skew symmetric map.

The task with the highest priority is the end-effector configuration task. The task error is defined by

$$y_1(\xi) := \begin{bmatrix} R_d^T\left(p^i_{ie} - p^i_{d,e}\right) \\ \left[\log\left(R_d^T R^i_e\right)\right]_\vee \end{bmatrix} \in \mathbb{R}^6, \tag{91}$$

where p^i_{ie} and $p^i_{d,e}$ are the measured and desired positions of the end-effector in the inertial frame, respectively. Moreover, $R^i_e(q,\theta) = R^i_b(q)R^b_e(\theta) \in \mathrm{SO}(3)$ is the rotation matrix from the end-effector frame to the inertial frame and $R_d \in \mathrm{SO}(3)$ is the desired orientation of the end-effector. Additionally, $\log : \mathrm{SO}(3) \to \mathfrak{so}(3)$ is the matrix logarithm from the special orthogonal group of dimension three to its Lie algebra and $\vee : \mathfrak{so}(3) \to \mathbb{R}^3$ is the "vee" operator identifying the Lie algebra with $\mathbb{R}^3$ (Chirikjian, 2011).

We exploit system redundancy by defining a positioning task for the base. The task error is defined by $y_2 := p^i_{ib} - p^i_{d,b}$, where $p^i_{d,b}$ is the desired position of the AIAUV base frame in the inertial frame.

Because y_1 and y_2 consume at most 9 DOFs, we have at least 5 uncontrolled DOFs at all times. Consequently, stability of (89) can only be guaranteed if the resulting zero dynamics is asymptotically stable. As an alternative to performing a complicated analysis of the zero dynamics, a joint angle regulation task $y_3 = \theta$ is designed to eliminate the residual DOFs of the system.

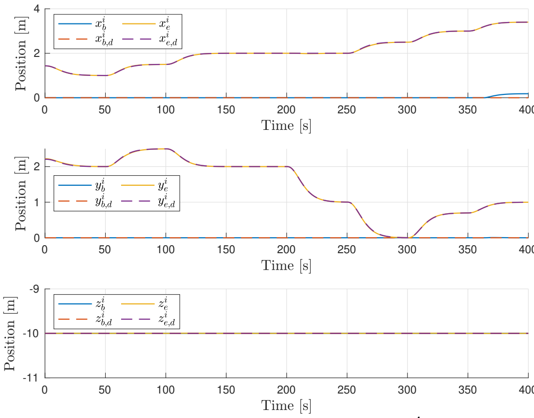

Fig. 2. The position of the end-effector p^i_{ie} and base p^i_{ib}.

By defining $x = \mathrm{col}(x_1, x_2) = \mathrm{col}(\xi, \zeta)$ and employing the task-priority operational space pre-feedback control law in (86), restated in terms of the case study as

$$\tau = \breve{J}_1(\xi)^T\breve{u}_1 + \breve{N}_2(\xi)\breve{J}_2^T(\xi)\breve{u}_2 + \breve{N}_3(\xi)\breve{J}_3^T(\xi)\breve{u}_3 \tag{92}$$

$$= \breve{T}(\xi)\breve{u}, \tag{93}$$

the equations of motion (89) together with the task error outputs can be rewritten as the nonlinear system

$$\dot{x} = f(x) + \breve{g}(x)\breve{u}, \tag{94a}$$

$$\breve{y}_i = \breve{h}_i(x_1), \quad 1 \le i \le 3 \tag{94b}$$

where

$$f(x) = \begin{bmatrix} J_\xi(x_1)x_2 \\ -M(x_1)^{-1}\left(C(x)x_2 + D(x)x_2 + G(x_1)\right) \end{bmatrix}, \tag{95}$$

$$\breve{g}(x) = \begin{bmatrix} 0 \\ M(x_1)^{-1}\breve{T}(x_1) \end{bmatrix}. \tag{96}$$

By deriving the input-output dynamics as in Section 3.2 and applying Algorithm 1 we obtain the reduced input-output dynamics

$$\mathrm{col}\left(\ddot{\breve{y}}_1, \ddot{\breve{y}}_2, \ddot{\breve{y}}_3\right) = \breve{A}(x)\breve{u} + \breve{b}(x) - \ddot{\breve{\sigma}}_d(t), \tag{97}$$

which satisfies $\breve{m}_i = n+6$ for all x such that $\mathrm{tr}(R_d^T R^i_e) \neq -1$ since y_1 and y_3 are always compatible and $m_1 + m_3 = 14$. By Corollary 2, the system (94) is input-to-state feedback linearizable in a neighborhood U around $x(t_0) = x_0$ where the permutation vectors E_1, E_2 and E_3 obtained from Algorithm 1 are constant. Hence, the feedback linearizing control input

$$\breve{u} = \breve{A}^{-1}(x)\left(\breve{\mu} - \breve{b}(x) + \ddot{\breve{\sigma}}_d(t)\right), \tag{98}$$

with the virtual control input $\breve{\mu} = \mathrm{col}(\breve{\mu}_1, \breve{\mu}_2, \breve{\mu}_3)$ given by

$$\breve{\mu}_i = -K_{p,i}\breve{y}_i - K_{d,i}\dot{\breve{y}}_i, \quad 1 \le i \le 3, \tag{99}$$

results in the following decoupled and linear systems

$$\ddot{\breve{y}}_i + K_{d,i}\dot{\breve{y}}_i + K_{p,i}\breve{y}_i = 0, \quad 1 \le i \le 3, \tag{100}$$

where $K_{p,i} = 0.04 I_{\breve{m}_i}$ and $K_{d,i} = 0.4 I_{\breve{m}_i}$ for $1 \le i \le 3$.

Simulation results are presented in Figs. 2 to 4. Whenever a change in compatibility occurs between the tasks, the permutation vectors E_2 and E_3 will change, which results in discontinuities in (93), as observed in Fig. 4 at $t \simeq 363\,\mathrm{s}$. From $t \ge 350\,\mathrm{s}$, the desired end-effector position moves outside of the manipulator workspace as defined by the desired base position. As a result, the lower-priority base position task is no longer controllable in the x-direction, as observed from Fig. 2. Moreover, from Fig. 3 it is clear

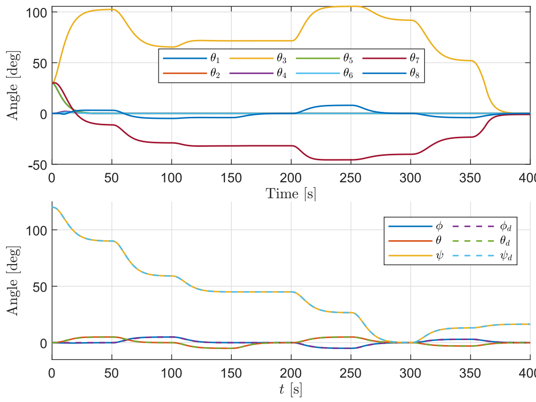

Fig. 3. The joint angles θ and the measured and desired orientation of the end-effector represented by the roll-pitch-yaw Euler angles ϕ, θ and ψ.

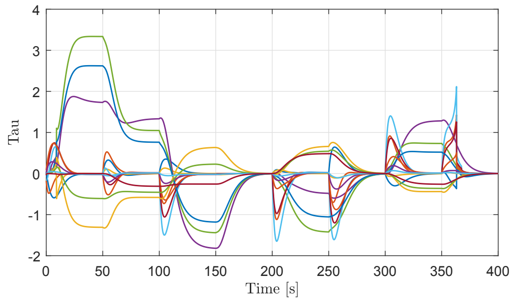

Fig. 4. The control input τ of generalized forces and torques.

that the joint regulation task becomes fully compatible for $t \geq 350\,\mathrm{s}$, since every component of θ converges to zero. Finally, we observe from Fig. 4 that the commanded forces and torques are well within the physical limits of the Eelume robot.

6. CONCLUSIONS AND FUTURE WORK

In this paper we have shown how a well-known task-priority operational space control law can be formulated in the standard framework of feedback linearization of nonlinear MIMO control systems. Through this formulation, we obtain sufficient conditions for the input-output and input-to-state feedback linearizability of a redundant robotic system influenced by a task-priority operational space pre-feedback control law. These conditions can thus be employed when designing the operational space tasks, and it is shown that under these conditions both task space and joint space stability can be guaranteed. Moreover, we have shown that in the case where the tasks are not compatible, certain components of the incompatible lower-priority tasks may still be input-output linearizable, and we have provided sufficient conditions for input-to-state linearizability when compatibility between tasks cannot be guaranteed.

Future work should investigate the effect of discontinuities in the control law as a result of changes in the controllable dimensions of lower-priority tasks. In order to obtain a continuous control law, smoothing techniques such as the one in Moe et al. (2018) could be employed.

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