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State Predictive Control with Multiple Modification Terms and Robust Stability Analysis Based on Complementary Sensitivity Functions

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Abstract: As an effective control method for systems with time delay in the input, state predictive control is well known. An idea of adding a single modification term to its control law was presented recently, and it was suggested that an appropriate modification term could contribute to improving robust stability of the control system to some parametric uncertainties. This extended control method is called modified state predictive control. Motivated by the preceding study, this paper considers introducing multiple modification terms into the control law of state predictive control, aiming at improving robust stability for non-parametric uncertainties. We first derive the characteristic equation of the modified state predictive control systems with multiple modification terms, and give a necessary and sufficient condition for stability. We then derive an explicit representation of the complementary sensitivity function associated with the robust stability analysis problem for multiplicative uncertainties. Finally, we demonstrate through numerical examples that state predictive control with appropriate multiple modification terms could be useful in improving robust stability compared with that with a single modification term or no such a term.

Keywords: continuous-time systems, time delay, state predictive control, robust stability, complementary sensitivity function.

1. INTRODUCTION

As an effective control method for systems with input delay, the Smith method (Smith, 1959) and finite spectrum assignment, also known as state predictive control (Manitius and Olbrot, 1979) is well known. The key idea of the latter method is to virtually remove the effect of input delay from the closed-loop system by predicting the future state $x(t+h)$, where t is the current time and h denotes the input delay. Recently, an idea of adding a single modification term to its control law was introduced, and such a control method was called modified state predictive control (Masui et al., 2017). It was then suggested that using an appropriate modification term could improve robust stability compared with the conventional state predictive control. This paper considers modified state predictive control with multiple modification terms and studies its effectiveness in improving robust stability from a perspective different from the one taken in Masui et al. (2017).

This paper is organized as follows. As a preliminary, we first review the conventional state predictive control as well as modified state predictive control with a single modification term. Then, we consider introducing multiple modification terms and derive the characteristic equation of the closed-loop system. We next study robust stability for the multiplicative uncertainties of the plant. To this end, we derive an explicit representation of the complementary sensitivity function of the nominal closed-loop system. Finally, through numerical examples, we demonstrate that state predictive control with appropriate multiple modification terms could be useful in improving robust stability compared with that with a single modification term or no such a term.

The following notation is used in this paper. $\mathbf{R}$ and $\mathbf{N}$ denote the set of real numbers and that of positive integers, respectively. $|\cdot|$ denotes the determinant of a matrix. $\mathcal{RH}_\infty$ denotes the set of matrices whose entries are proper stable real-rational functions, and the H_∞ norm of $F(s) \in \mathcal{RH}_\infty$ is denoted by $\|F(s)\|_\infty$.

2. CONVENTIONAL STATE PREDICTIVE CONTROL AND MODIFIED STATE PREDICTIVE CONTROL WITH A SINGLE MODIFICATION TERM

This section first reviews the conventional state predictive control method (Manitius and Olbrot, 1979). Then, to motivate the study in the present paper, a modified state predictive control method with a single modification term studied in Masui et al. (2017) is reviewed. This modified control method is based on an idea of adding a single modification term to the input of the conventional state predictive control so that some sort of freedom in the locations of the closed-loop poles can be obtained.

2.1 State Predictive Control

Consider the continuous-time plant with input delay given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) \\ v(t) &= u(t-h), \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $v(t) \in \mathbf{R}^m$, $A \in \mathbf{R}^{n\times n}$ and $B \in \mathbf{R}^{n\times m}$, and $u(t)$ denotes the plant input at time t. Assuming that the pair (A, B) is stabilizable, let F be a stabilizing state feedback gain. A basic idea of the conventional state predictive control is to virtually apply the state feedback through this gain "on a future state," i.e., to virtually apply the control input given by

$$u(t) = Fx(t+h) \tag{2}$$

so that the effect of input delay can be canceled and the closed-loop system would behave virtually in the same way as

$$\dot{x}(t) = (A + BF)x(t). \tag{3}$$

Obviously, however, (2) cannot be implemented directly as a control law even if the state is accessible, because $x(t+h)$ is a future state. This issue is readily circumvented by solving the state equation (1) for $x(t+h)$ with the (present) state $x(t)$ and $v(\tau),\ \tau \in [t, t+h)$ (or equivalently, the past plant input $u(\tau),\ \tau \in [t-h, t)$) as

$$x(t+h) = e^{Ah}x(t) + \int_t^{t+h} e^{A(t+h-\tau)}Bv(\tau)d\tau. \tag{4}$$

This corresponds to the prediction law for $x(t+h)$, by which (4) leads equivalently to the control law

$$u(t) = F\left\{e^{Ah}x(t) + \int_t^{t+h} e^{A(t+h-\tau)}Bu(\tau-h)d\tau\right\}, \tag{5}$$

which is implementable if the state is accessible. This control method virtually removing the effect of delay by using the above state prediction is called state predictive control. It is well known that the characteristic equation of the closed-loop system given by (1) and (5) is given by

$$|sI - A - BF| = 0 \tag{6}$$

as expected by the virtual equivalence of this closed-loop system and (3). Hence, the state predictive control system is stable if and only if $A + BF$ is Hurwitz.

2.2 Output Feedback Case

When the state $x(t)$ is not accessible, we can introduce an observer in a mostly usual fashion. Suppose that $y(t) = Cx(t)$ is measurable, where $C \in \mathbf{R}^{l\times n}$, and (C, A) is detectable. Then, a full-order observer in the context of state predictive control is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t-h) + L\left(C\hat{x}(t) - y(t)\right), \tag{7}$$

where $\hat{x}(t) \in \mathbf{R}^n$ is the estimate of $x(t)$, and $L \in \mathbf{R}^{n\times l}$ is an observer gain such that $A + LC$ is Hurwitz. It would be natural to consider replacing $x(t)$ with $\hat{x}(t)$ in (5) when the state is not accessible. It is well known that under the modified control law, the characteristic equation of the closed-loop system is given by

$$|sI - A - BF||sI - A - LC| = 0. \tag{8}$$

This implies that the so-called separation principle between the feedback gain and the observer gain holds also in state predictive control.

2.3 Modified State Predictive Control with a Single Modification Term

This section reviews the modified state predictive control method with a single modification term studied in Masui et al. (2017). The idea of this method corresponds to virtually applying a modified form of (2) given by

$$u(t) = Fx(t+h) + M_0(u(t-h) - Fx(t)), \tag{9}$$

where the coefficient matrix $M_0 \in \mathbf{R}^{m\times m}$ for the modification term is Schur for the reason stated later. The actual control law is readily given by

$$\begin{aligned} u(t) = F&\left(e^{Ah}x(t) + \int_t^{t+h} e^{A(t+h-\tau)}Bv(\tau)d\tau\right) \\ &+ M_0(u(t-h) - Fx(t)) \end{aligned} \tag{10}$$

where the first term is nothing but the control law of the conventional state predictive control. The second term could be interpreted as reflecting (through the matrix M_0) the deviation of the past input $u(t-h)$ from what is considered to be desirable in the sense of (2).

Remark 1. Such a deviation arises actually by the use of the modification term itself, but introducing the term is considered to be useful if the coefficient matrix M_0 is chosen appropriately. The present paper aims at confirming that this is indeed the case from a perspective different from the one taken in Masui et al. (2017) (see Section 4 for more details), and further developing extended arguments on exploiting the freedom obtained by a more generalized modification of the control law.

The characteristic equation of the closed-loop system given by (1) and (10) has been studied in Masui et al. (2017) and is given by

$$|sI - A - BF|\left|I - M_0e^{-hs}\right| = 0. \tag{11}$$

Now, the set of the roots of $\left|I - M_0e^{-sh}\right| = 0$ is given by

$$\left\{(1/h)\ln \nu : |\nu I - M_0| = 0,\ \nu \neq 0\right\}. \tag{12}$$

Hence, we see that the closed-loop system is stable if and only if $A + BF$ is Hurwitz (i.e., stable in the continuous-time sense) and M_0 is Schur (i.e., stable in the discrete-time sense).

When a full-order observer is used, the control law is modified in an obvious fashion, in which case the characteristic equation of the closed-loop system is given by

$$|sI - A - BF|\left|I - M_0e^{-hs}\right||sI - A - LC| = 0. \tag{13}$$

We see that introducing the modification term does not affect the validity of the so-called separation principle.

Obviously, introducing a modification term generally leads to the presence of new closed-loop poles that were absent in the conventional state predictive control (unless $M_0 \neq 0$ is a nilpotent). Roughly speaking, this could be interpreted as some of the invisible closed-loop poles at $-\infty$ being shifted to the right, and one might thus argue that the use of the modification term is pointless. Instead, the basic idea of introducing the modification term actually lies in exploiting its ability in giving some freedom in the locations of the closed-loop poles. The idea could be restated as follows: with the difference in the closed-loop poles in mind, the conventional state predictive control might correspond to high-gain feedback whereas the modification term could possibly lead to reducing the controller gain and thus improving some performance of the closed-loop system while maintaining fundamental features of state predictive control. This standpoint for the use of the modification term motivates us to consider exploiting more freedom, i.e., introducing multiple modification terms to discuss further possible improvement of the closed-loop performance.

3. MODIFIED STATE PREDICTIVE CONTROL WITH MULTIPLE MODIFICATION TERMS

This section considers introducing multiple modification terms into the control law of state predictive control. As mentioned above, the modified state predictive control with a single modification term in Masui et al. (2017) is based on the deviation of the past control input $u(t-h)$ from that given by (2) with t shifted to the past by h. The key idea in the present paper is to generalize the modification term by considering similar deviations for other values of the shift in the interval $(0, h]$, and actually to consider the corresponding multiple modification terms.

3.1 Modified State Predictive Control with Multiple Modification Terms and the Characteristic Equation of the Control System

First, suppose for simplicity that the state is accessible (if this is not the case, we can employ an observer, and such a case will also be covered later). Now, we take N $(\in \mathbf{N})$ values of shift by which we consider shifting t in (2) to the past. They are denoted by $\mu_i h$ with $\mu_0 < \mu_1 < \cdots < \mu_{N-1} \leq 1$. This implies that we consider $u(t-\mu_i h) - Fx(t-\mu_i h + h)$ in modifying the control law (5) for the conventional state predictive control. In other words, we consider virtually applying the control input given by

$$u(t) = Fx(t+h) + \sum_{i=0}^{N-1} M_i \left\{u(t-\mu_i h) - Fx(t-\mu_i h + h)\right\} \tag{14}$$

where $M_i \in \mathbf{R}^{m\times m}$ $(i = 0,\ldots,N-1)$ are the coefficient matrices satisfying the condition given later (to avoid possible ambiguity, we basically assume that $M_i \neq 0$ $(i = 0,\ldots,N-1)$ without loss of generality, while $N = 0$ is allowed so that the conventional state predictive control can be covered as a special case). However, not only the first term but also the second term is not implementable because $x(t-\mu_i h+h)$ at time t (corresponding to the left-hand side) is a future state for each $i = 0,\ldots,N-2$ and $i = N-1$ (unless $\mu_{N-1} = 1$). This issue can also be circumvented by solving the state equation (1) for $x(t-\mu_i h + h)$, whereby we obtain the prediction formula

$$x(t-\mu_i h+h) = e^{A(-\mu_i h+h)}x(t) + \int_t^{t-\mu_i h+h} e^{A(t-\mu_i h+h-\tau)}Bu(\tau-h)d\tau. \tag{15}$$

This readily leads to a generalized version of (10) corresponding to an implementable form of (14).

The characteristic equation of the closed-loop system is given by

$$|sI - A - BF| \left| I - \sum_{i=0}^{N-1} M_i e^{-s\mu_i h}\right| = 0. \tag{16}$$

Hence, F together with M_i $(i = 0,\ldots,N-1)$ and μ_i $(i = 0,\ldots,N-1)$ must be chosen in such a way the all the roots of this characteristic equation lie in the open-left half plane. For general values of μ_i $(i = 0,\ldots,N-1)$, it is difficult to obtain the roots of the second factor in the left-hand side of (16). However, when these values have mutually rational ratios, we can determine the closed-loop poles easily. In the following subsection, we first describe the closed-loop poles for the case when $\mu_i h$ $(i = 0,\ldots,N-1)$ are equally spaced in the interval $(0, h]$ (more precisely, when $\mu_i = (i+1)/N$ $(i = 0,\ldots,N-1)$). This corresponds to the case when the deviations from what (2) would imply are monitored for N equally spaced values of past t in the interval of width h ending at the present time. The closed-loop poles for the case when μ_i $(i = 0,\ldots,N-1)$ have mutually rational ratios can readily be obtained through the results for this special case.

3.2 Roots of the Characteristic Equation

In this subsection, we assume that μ_i $(i = 0,\ldots,N-1)$ have mutually rational ratios and describe the roots of the characteristic equation. We begin with the special case with $\mu_i = (i+1)/N$ $(i = 0,\ldots,N-1)$, when the characteristic equation leads to

$$|sI - A - BF| \left| I - \sum_{i=0}^{N-1} M_i e^{-s(\frac{i+1}{N})h}\right| = 0. \tag{17}$$

It suffices to consider the roots of

$$\left| I - \sum_{i=0}^{N-1} M_i e^{-s(\frac{i+1}{N})h}\right| = 0, \tag{18}$$

for which we have the following theorem.

Theorem 1. The set of the roots of (18) is given by

$$\left\{(N/h)\ln \nu : |\nu I - M| = 0,\ \nu \neq 0\right\}, \tag{19}$$

where M is given by

$$M = \begin{bmatrix} 0 & I_m & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & I_m \\ M_{N-1} & M_{N-2} & \cdots & M_0 \end{bmatrix}. \tag{20}$$

It readily follows for the above special case that our modified state predictive control system is stable if and only if $A + BF$ is Hurwitz and M is Schur.

When μ_i $(i = 0, \ldots, N-1)$ have mutually rational ratios, consider the smallest positive integer N_0 such that $\mu_i h = (n_i/N_0)\alpha$ $(i = 0, \ldots, N-1)$ for some positive constant α, where $n_i \le N_0$ $(i = 0, \ldots, N-1)$ are positive integers and thus $\alpha \le h$. By taking such N_0, the general case here can be viewed as a particular situation with "$N = N_0$" and "$h = \alpha$" in the special case studied above. More specifically, we can regard the N terms in the second factor of (16) to be equal to N_0 terms with the coefficient matrices being zero except for N of them. More details can be described as follows.

For each μ_i $(i = 0, \ldots, N-1)$, take $j =: j(i)$ such that $\mu_i = (j+1)\alpha/N_0 h$, where $0 \le j \le N_0 - 1$. Furthermore, for each $j = 0, \ldots, N_0 - 1$, redefine "M_j" under the situation "$N = N_0$" as the original M_i when $j = j(i)$, and as 0 when none of $i = 0, \ldots, N-1$ satisfies $j = j(i)$. Under the redefined "M_j $(j = 0, \ldots, N_0 - 1)$" in this way, consider the associated M in (20). Then, the set of the roots of the second factor in (16) is given by (19) with N/h replaced by N_0/α.

3.3 Characteristic Equation for the Output Feedback Case

It is not hard to show that when the state is not accessible, an observer may be introduced in an obvious fashion without essentially affecting the above arguments. Furthermore, we can readily show that the characteristic equation of the closed-loop system is given by

$$|sI - A - BF| \left| I - \sum_{i=0}^{N-1} M_i e^{-s\mu_i h} \right| |sI - A - LC| = 0. \tag{21}$$

4. COMPLEMENTARY SENSITIVITY FUNCTION AND ROBUST STABILITY OF MODIFIED STATE PREDICTIVE CONTROL SYSTEMS

This section is devoted to establishing a theoretical framework through which we can study an advantage of introducing a modification term into the control law of the conventional state predictive control, as well as exploiting more freedom by increasing the number of the modification terms.

The use of a modification term was first discussed in Masui et al. (2017), where only a single term was actually used. In that study, the role of the term was analyzed through a robust stability perspective of (modified) state predictive control systems. More precisely, the plant was assumed to have parametric uncertainties in its delay h and (steady-state) gain. The robust stability problems in such a situation for the conventional state predictive control have been studied in Bao and Araki (1988) and Furutani and Araki (1998). These studies were extended in Masui et al. (2017) for the case of a single modification term, and it was suggested that the modification term could contribute to enhancing robustness with respect to such parametric uncertainties.

The present paper adopts a somewhat different perspective to the robust stability problem of (modified) state predictive control, where we consider non-parametric uncertainties of the plant. More specifically, we consider the multiplicative uncertainties in the output side of the plant. As is well known, the associated complementary sensitivity function of the (modified) state predictive control systems will then be quite important in the theoretical treatment of the robust stability problem. After quickly reviewing fundamentals of such an analysis method for robust stability, this section is mainly interested in explicitly describing the complementary sensitivity function of the modified state predictive control systems with multiple modification terms.

4.1 Robust Stability Radius under Multiplicative Uncertainties

Let us consider the multiplicative uncertainties in the output side of the nominal plant G, and suppose that the actual plant lies in the set

$$\mathcal{G} = \{(I + \Delta W)G : \Delta \in \mathcal{RH}_\infty, \|\Delta\|_\infty \le 1\}. \tag{22}$$

Here, the weight W is a fixed stable transfer matrix such that ΔW with $\|\Delta\|_\infty \le 1$ expresses the uncertainties dependent on the angular frequencies. When the actual plant is in the set (22), the (modified) state predictive control system, assuming the output feedback case, can be expressed as the block diagram in Fig. 1, where K is the the (modified) state predictive controller (and r is the reference).

It is well known that the associated complementary sensitivity function plays a key role in the robust stability analysis for multiplicative uncertainties (Doyle et al., 2009; Zhou and Doyle, 1998). It corresponds to the transfer matrix from r to y under $\Delta = 0$, which is given by

$$T(s) = (I + G(s)K(s))^{-1} G(s)K(s), \tag{23}$$

where $G(s)$ and $K(s)$ denote the transfer matrices of G and K, respectively.

The following theorem is well known.

Theorem 2. Suppose that the system in Fig. 1 is stable for the nominal plant G (i.e., under $\Delta = 0$). Then, it is robustly stable with respect to the plant set $\mathcal{G}$ if and only if

$$\|WT\|_\infty < 1. \tag{24}$$

On the basis of the above theorem, we consider the robust stability radius of the perturbed closed-loop system by introducing the set of perturbed plants

$$\mathcal{G}_\beta = \{(I + \Delta W)G : \Delta \in \mathcal{RH}_\infty, \|\Delta\|_\infty \le \beta\} \tag{25}$$

parameterized through $\beta > 0$. The robust stability radius β_{sup} is then defined as the supremum of β for which the control system is robustly stable with respect to $\mathcal{G}_\beta$. It readily follows from the above theorem that

$$\beta_{\text{sup}} = 1/\|WT\|_\infty. \tag{26}$$

It is often the case that the multiplicative uncertainties mostly increase as the angular frequency increases and thus $\|W(j\omega)\|$ is roughly an increasing function of ω. Hence, it is generally considered that the complementary

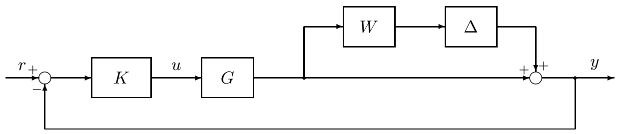

Fig. 1. Control system with multiplicative uncertainties

sensitivity function with small gains in a high frequency range mostly contributes to improving robust stability or the robust stability radius.

4.2 *Complementary Sensitivity Function of Modified State Predictive Control Systems*

In this subsection, we consider the modified state predictive control system with multiple modification terms. More precisely, we consider the output feedback case with a full-order observer, and derive the associated complementary sensitivity function $T(s)$. Specifically, we give an explicit representation of the transfer matrix $K(s)$ of the modified state predictive controller K (consisting of the state feedback gain, the full-order observer and the state prediction mechanisms relevant to (14)) as well as that of the nominal plant G, so that $T(s)$ in (23) can readily be obtained.

First, it is obvious that

$$G(s) = C(sI - A)^{-1}Be^{-sh}. \tag{27}$$

On the other hand, $K(s)$ can be obtained by applying the Laplace transformation to (7), as well as (14) and (15) with x replaced by $\hat{x}$, and computing the transfer matrix from $-Y(s)$ to $U(s)$, where $Y(s)$ and $U(s)$ denote the Laplace transforms of y and u, respectively. This procedure leads to

$$K(s) = -K_{\mathrm{d}}(s)^{-1}K_{\mathrm{n}}(s) \tag{28}$$

where

$$\begin{aligned} K_{\mathrm{d}}(s) = I &- e^{-sh}Fe^{Ah}(sI - A - LC)^{-1}B \\ &- FZ(s)B - E(s) + e^{-sh}J(s)B \end{aligned} \tag{29}$$

$$K_{\mathrm{n}}(s) = -Fe^{Ah}(sI - A - LC)^{-1}L + J(s)L \tag{30}$$

with

$$E(s) = \sum_{i=0}^{N-1} M_i e^{-s\mu_i h}(I - FZ_i(s)B) \tag{31}$$

$$J(s) = \sum_{i=0}^{N-1} M_i F e^{Ah(1-\mu_i)}(sI - A - LC)^{-1} \tag{32}$$

$$Z(s) = (I - e^{Ah}e^{-sh})(sI - A)^{-1}$$

$$Z_i(s) = (I - e^{Ah(1-\mu_i)}e^{-s(1-\mu_i)h})(sI - A)^{-1}. \tag{33}$$

Substituting (28) into (23) and rearranging the result, we see that the complementary sensitivity function of the modified state predictive control system is given by

$$T(s) = -G(s)\left\{K_{\mathrm{d}}(s) - K_{\mathrm{n}}(s)G(s)\right\}^{-1}K_{\mathrm{n}}(s). \tag{34}$$

5. NUMERICAL EXAMPLES AIMING AT COMPARING THE FREQUENCY RESPONSES OF THE COMPLEMENTARY SENSITIVITY FUNCTIONS AND ROBUST STABILITY RADII

In this section, we show the effectiveness of introducing multiple modification terms through numerical examples by comparing the frequency responses of the complementary sensitivity functions and the robust stability radii.

5.1 *Frequency Response of the Complementary Sensitivity Function of Modified State Predictive Control Systems*

This subsection gives numerical examples showing that introducing appropriate multiple modification terms could contribute to reducing, over some (high) frequency ranges, the gains of the complementary sensitivity function, compared with introducing a single modification term or no such a term.

Consider the SISO system given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
$$C = [3\ 2\ 3] \tag{35}$$

and the delay $h = 1$. Let the state feedback gain F be such that $A + BF$ has the eigenvalues -3, -2 and -1. Suppose that the state is not accessible and take the full-order observer gain L such that $A + LC$ has the same eigenvalues as the above. In the following, we fix F and L, and consider different modification.

We first consider the conventional state predictive control (i.e., without any modification term), for which the complementary sensitivity function $T(s)$ is denoted by $T_0(s)$. We next consider the case of a single modification term (i.e., $N = 1$) with $\mu_0 = 1$ and $M_0 = 0.5$. This value of M_0 was taken from the range $(-1, 1)$ (because M_0 must be Schur) as a value for which the gain of $T(s)$ becomes comparatively smaller than that of $T_0(s)$ defined above. The corresponding $T(s)$ is denoted by $T_1(s)$.

We then consider the use of multiple modification terms with $N = 3$. The following two cases are considered:

(a) $\mu_0 = 1/8$, $\mu_1 = 1/4$, $\mu_2 = 1$, $M_0 = -0.1$, $M_1 = 0.65$, $M_2 = 0.1$;
(b) $\mu_0 = 1/10$, $\mu_1 = 1/5$, $\mu_2 = 1$, $M_0 = 0.45$, $M_1 = 0.45$, $M_2 = -0.1$.

In both cases, the closed-loop stability is ensured for the nominal plant. The complementary sensitivity functions corresponding to (a) and (b) are denoted by $T_{2\mathrm{a}}(s)$ and $T_{2\mathrm{b}}(s)$, respectively.

The gain plots of the frequency responses of $T_0(s)$, $T_1(s)$, $T_{2\mathrm{a}}(s)$ and $T_{2\mathrm{b}}(s)$ are shown in Fig. 2.

From Fig. 2, we see the gains of $T_{2\mathrm{a}}(s)$ and $T_{2\mathrm{b}}(s)$ are smaller than those of $T_0(s)$ and $T_1(s)$ in a high frequency

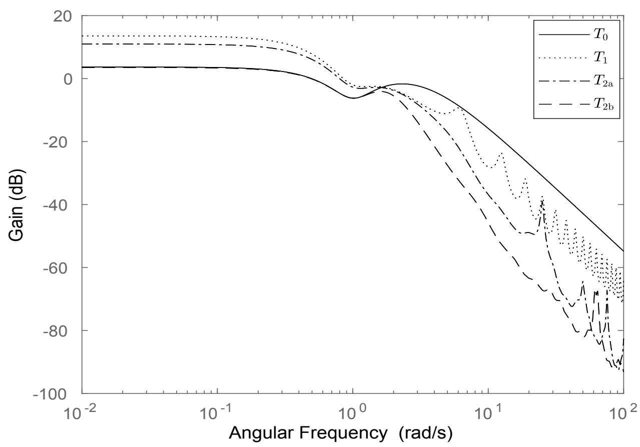

Fig. 2. Frequency responses of $T_0(s)$, $T_1(s)$, $T_{2\mathrm{a}}(s)$ and $T_{2\mathrm{b}}(s)$

range. As mentioned in the preceding section, this feature is generally considered to contribute to improving robust stability. Thus, it is suggested that state predictive control with appropriate multiple modification terms could be useful in improving robust stability compared with that with a single modification term or no such a term.

5.2 Improvement of the Robust Stability Radius

We next consider confirming the effectiveness of appropriately introduced modification terms in a more quantitative manner. More specifically, the qualitative effectiveness suggested in the preceding subsection is reinforced by further computing the robust stability radius numerically, where we assume that

$$W(s) = \frac{10(s+0.1)}{s+40}. \tag{36}$$

The associated gain plots for the frequency responses of $WT_0(s)$, $WT_1(s)$, $WT_{2\mathrm{a}}(s)$ and $WT_{2\mathrm{b}}(s)$ are shown in Fig. 3, by which the H_∞ norms of these transfer functions and the associated robust stability radius (given as the reciprocals of the H_∞ norms) are as shown in Table 1.

From numerical example, we can see that the robust stability radii under multiple modification terms are larger than those without a modification term or with a single modification term. We have thus confirmed that introducing appropriate multiple modification terms actually contributes to improving the robust stability radius in this numerical example.

6. CONCLUSIONS

In this paper, we first introduced multiple modification terms into the control law of state predictive control. Next, we derived the characteristic equation of the closed-loop systems and gave a necessary and sufficient condition for its stability. We further derived the complementary sensitivity function of the closed-loop system so that robust stability for multiplicative uncertainties can be analyzed. Finally, we demonstrated through numerical examples that introducing appropriate multiple modification terms could contribute to improving robust stability and the robust stability radius compared with those with a single modification term or no such a term. The numerical examples used multiple modification terms chosen by trial and error, and the effectiveness of using multiple modification terms was manifested even under such a procedure. This would imply that optimizing the parameters of the modification terms under a given N, the number of the terms, could lead to promising improvement of robust stability. Unfortunately, however, this may not be an easy issue, and further investigation on the issue remains our future studies.

Before closing the paper, we remark that the arguments in this paper up to the introduction of multiple modification terms and the derivation of the characteristic equation can be regarded as a sort of counterpart of the relevant study for discrete-time systems with input delay (Hagiwara and Araki, 1988). In this sense, optimization of the modification terms is also an interesting topic for discrete-time systems and it might be a simpler problem to start with.

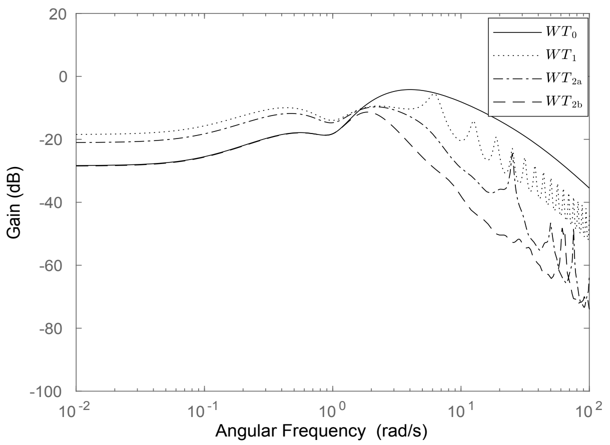

Fig. 3. Frequency responses of weighted complementary sensitivity functions

Table 1. The H_∞ norms of the weighted complementary sensitivity functions and the robust stability radii

	$T_0(s)$	$T_1(s)$	$T_{2\mathrm{a}}(s)$	$T_{2\mathrm{b}}(s)$
weighted H_∞ norm	0.62	0.53	0.33	0.27
robust stability radius	1.61	1.89	3.03	3.70

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