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Adaptive Sliding Mode Control for Cholera Epidemic Model

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Abstract: Cholera is an acute diarrhoeal infection caused by bacteria *Vibrio cholerae*. The SIQRB (Susceptible-Infected-Quarantined-Recovered-Bacteria) epidemic model with a control function is studied to analyze the dynamics of cholera. The control function represents the fraction of infected individuals that are submitted to treatment in quarantine until complete recovery. One of the drawbacks of mathematical modeling is the presence of parametric uncertainties. Designing a control strategy used in accommodating these uncertainty factors drives the development of robust control. In this case, the sliding mode control is applied to handle parametric uncertainties. The sliding mode control objective is reducing the number of infected individuals to zero through the desired tracking scheme of a reference function. The Lyapunov stability theorem and Barbalat's lemma are used to examine the success of the tracking scheme. Lack of apriori knowledge related to the boundedness of the parametric uncertainties is settled using an adaptive method by updating the switching gain of sliding mode control so that the strategy is called the adaptive sliding mode control. Chattering problem that often appears in the application of sliding mode control can be reduced. Numerical simulations show that the adaptive sliding mode control satisfies the controller objectives and able to handle parametric uncertainties.

***Keywords**:* adaptive gain, chattering, cholera, epidemiological model, parametric uncertainties, sliding mode control.

1. INTRODUCTION

Cholera is an acute diarrhoeal infection due to the consumption of food or drink that is contaminated by bacteria *Vibrio cholerae*. Cholera can kill within 24 hours if left untreated. Cholera transmission mostly caused by poor sanitation and lack of access to clean water. Researchers have estimated that every year, there are 1.3 to 4 million cases with 21,000 to 143,000 deaths worldwide (Ali, Nelson, Lopez, & Sack, 2015). Cholera is still endemic in some parts of Africa, South Asia, and Latin America with the latest case occurring in Yaman, in June 2019 with 85,681 cases in a month (World Health Organization, 2019).

Since 1979, several mathematical models related to the spread of cholera have been proposed. Capasso and Fontana built a simple mathematical model consisting of infected human populations and bacteria *V. cholerae* based on the epidemic phenomenon of cholera in the European Mediterranean in the summer of 1973 (Capasso & Fontana, 1979). Then, Codeco developed the Cappaso and Fontana models by adding vulnerable human populations along with the assumption that there was contact between susceptible individuals and water sources contaminated with bacteria *V. cholerae* (Codeco, 2001). Cui et al. proposed a treatment strategy in the form of vaccination so that individuals who are vaccinated have temporary immunity against cholera (Cui, Wu, & Zhou, 2014). According to the World Health Organization (WHO) in the Weekly Epidemiological Record (World Health Organization, 2017), for the first six months after vaccination, vaccination provide about 85% protection, which decreases to 50% during the first year and after two years the level of protection decreases to less than 50%. Therefore, WHO recommends vaccination only as a companion strategy in controlling cholera. Lemos-P. et al. proposed a cholera epidemic model with optimal control using a quarantine strategy by isolating and treating infected individuals until they recover (Lemos-P., Silva, & Torres, 2017).

The dynamics of disease transmission also have the potential to be accompanied by factors of uncertainty and inaccuracy. Therefore, the uncertainty factor also needs to be considered while forming the control strategy. A control strategy known as robust control developed to accommodate these uncertainty factors. However, we cannot expect one particular robust control method to apply to all nonlinear systems.

Sliding mode control is one of the nonlinear control methods that successfully overcome the parameter uncertainty factor. The sliding mode control can bring the system output to the desired reference function through the tracking scheme and overcome the parameter uncertainty in the model. The information on the bound or range of the parameter values is assumed to be known for designing the sliding mode control. Adaptive rules are applied in the sliding mode control design to eliminate the need for information related to these boundaries. As a result, the strategy is called the adaptive sliding mode control. Although it successfully applied in various engineering fields, the adaptive sliding mode control is still rarely applied to the epidemic models. Several studies related to the application of adaptive sliding mode control in the field of epidemiology include the general mathematical

model in the form of SEIR (Susceptible-Exposed-Infected-Resistant) with vaccination control by Ibeas et al. (Ibeas, de la. Sen, & Quesada, 2014) as well as in the influenza epidemic model with vaccinations and treatment controls by Sharifi and Moradi (Sharifi & Moradi, 2017). Therefore, for the first time in this paper, an adaptive sliding mode control strategy will be applied to the cholera epidemic model.

The paper consists of five sections. The first section is an introduction that includes background, a brief description of the problem, and the applied method. The cholera epidemic model described in the second section. The design of the adaptive sliding mode control to deal with disease transmission and parameter uncertainty of cholera epidemic models described in the third section. The fourth section discusses the numerical simulation of applying the methods outlined in the third section. The fifth section is the conclusion as a summary of the results obtained from the research written in this paper.

2. CHOLERA EPIDEMIC MODEL

The SIQRB epidemic model of cholera described by the following equations (Lemos-P., Silva, & Torres, 2017):

$$\begin{aligned}
\dot{S}(t) &= \Lambda - \frac{\beta B(t)}{\kappa + B(t)} S(t) + \omega R(t) - \mu S(t), \\
\dot{I}(t) &= \frac{\beta B(t)}{\kappa + B(t)} S(t) - \delta u(t) I(t) - (\alpha_1 + \mu) I(t), \\
\dot{Q}(t) &= \delta u(t) I(t) - (\varepsilon + \alpha_2 + \mu) Q(t), \\
\dot{R}(t) &= \varepsilon Q(t) - (\omega + \mu) R(t), \\
\dot{B}(t) &= \eta I(t) - d B(t),
\end{aligned} \tag{1}$$

where control function $u(t)$ represents the fraction of infected individuals $I(t)$ that are submitted to treatment in quarantine until complete recovery. The control $u(t)$ takes the value in the closed set [0,1] so $u = 0$ means no control measure and $u = 1$ means all infected individuals are put under quarantine. δ is the rate of infected individually to be in quarantine.

The variables and parameters of the model are described in Table 1 and Table 2 as follows:

Table 1. Variable for SIQRB model (1), (Lemos-P., Silva, & Torres, 2017).

Variable	Description	Unit
$S(t)$	Susceptible individuals at time t	person
$I(t)$	Infected individuals at time t	person
$Q(t)$	Quarantined individuals at time t	person
$R(t)$	Recovered individuals at time t	person
$B(t)$	Bacterial concentration at time t	cell/ml

Table 2. Parameter value for SIQRB model (1), (Lemos-P., Silva, & Torres, 2017).

Description	Parameter	Value
Recruitment rate	Λ	24.4N(0)/365000 (day^{-1})
Natural death rate	μ	2.2493×10^{-5} (day^{-1})
Ingestion rate	β	0.8 (day^{-1})
Half saturation constant	κ	10^6 (cell/ml)
Immunity waning rate	ω	0.4/365 (day^{-1})
Recovery rate	ε	0.2 (day^{-1})
Death rate (infected)	α_1	0.015 (day^{-1})
Death rate (quarantined)	α_2	0.0001 (day^{-1})
Shedding rate (infected)	η	10 (cell/ml $day^{-1}person^{-1}$)
Bacteria death rate	d	0.33 (day^{-1})

3. ADAPTIVE SLIDING MODE CONTROL FOR CHOLERA EPIDEMIC MODEL

Sliding mode control is a nonlinear control technique designed to bring the state of the system to a sliding surface and keep it on the sliding surface. Therefore, Liu and Wang stated that there are two steps in the sliding mode control design. The first step is designing a sliding surface so that the plant restricted to the sliding surface has the desired system response. The second step is constructing a switched feedback gains necessary to drive the plant's state trajectory to the sliding surface (Liu & Wang, 2012). The Lyapunov stability theorem and Barbalat's lemma guarantee the success of the sliding mode control scheme.

3.1 Sliding Mode Control Design

There are two advantages of the sliding mode control (Liu & Wang, 2012). First is the ability to adjust system behavior based on a particular sliding surface selection. Second, the response of the closed loop system can become insensitive to uncertainty including the uncertainty of model parameters and the system interference. The sliding mode control built in two phases. The first phase is the reaching phase when the state trajectory is driven towards the sliding surface while the second phase is the sliding phase when the state trajectory is moving towards the origin along the sliding surface (Shtessel, Edwards, Fridman, & Levant, 2010).

In this subsection, a sliding mode control designed for the cholera epidemic model (1) to asymptotically reduce the number of infected individuals over a certain period while handling parameter uncertainty. It assumed that the parameter uncertainty factor lies in the parameters of the ingestion rate (β) and the rate of disease-related death of infected individuals (α_1). Therefore, the tracking error defined as:

$$e_r(t) = I(t) - I_{ref}(t), \tag{2}$$

where $I(t)$ represents the number of infected individuals at time t while $I_{ref}(t)$ represents the reference function that satisfies:

$$I_{ref}(0) = I(0), \tag{3}$$

$$I_{ref}(t) \to 0, \; t \to \infty. \tag{4}$$

Equation (3) states that the initial tracking error is zero ($e_r = 0$). Equation (4) states that the reference function asymptotically goes to zero. Reference functions can be formed as desired as long as they meet (3) and (4). Therefore, for simulation purposes, the following reference function is used:

$$I_{ref}(t) = (I(0) - I(t_f))e^{-\xi t} + I(t_f), \tag{5}$$

with the parameter $\xi > 0$ set the rate of convergence of the reference function, $I(0)$ represents the initial number of infected individuals, and $I(t_f)$ represents the number of infected individuals at the end of the simulation. The sliding mode controller consists of the equivalent control (u_{eq}) and the switching control (u_{sw}). The equivalent control maintains the system in the sliding surface while the switching control forces the system towards the sliding surface. A sliding variable $\sigma(t)$ which satisfies the control objective defined as:

$$\sigma(t) = e_r(t). \tag{6}$$

The sliding surface which guarantees the achievement of control objective given by:

$$\sigma(t) = e_r(t) = 0. \tag{7}$$

To maintain the position of state on sliding surface, it necessary to apply $\sigma(t) = \dot{\sigma}(t) = 0$. Time-derivative of the sliding variable gives

$$\begin{aligned} \dot{\sigma}(t) &= \dot{e}_r \\ &= \dot{I}(t) + \xi(I(0) - I(t_f))e^{-\xi t}, \end{aligned} \tag{8}$$

so by substituting $\dot{\sigma}(t) = 0$ in (8), we get the control value $u(t)$ which is then called equivalent control u_{eq} in the form

$$\begin{aligned} u_{eq} = \frac{1}{\delta I(t)}\Bigg(&\frac{\beta B(t)}{\kappa + B(t)} S(t) - (\alpha_1 + \mu) I(t) \\ &+ \xi\Big(I(0) - I(t_f)\Big)e^{-\xi t}\Bigg), \end{aligned} \tag{9}$$

The uncertainty factor causes the parameters of the system exactly not known. Therefore for practical purposes, nominal parameters are used by defining:

$$\begin{aligned} p &= \frac{\beta B(t)}{\kappa + B(t)} S(t), \quad & \hat{p} &= \frac{\hat{\beta} B(t)}{\kappa + B(t)} S(t), \\ q &= \alpha_1 + \mu, & \hat{q} &= \hat{\alpha}_1 + \mu, \end{aligned} \tag{10}$$

where $\hat{\beta}$ and $\hat{\alpha}_1$ are the nominal parameters of β and α_1. Therefore, equivalent control (9) becomes

$$\hat{u}_{eq} = \frac{1}{\delta I(t)}\Big(\hat{p} - \hat{q} I(t) + \xi\Big(I(0) - I(t_f)\Big)e^{-\xi t}\Big). \tag{11}$$

The application of the equivalent control (11) to the system (1) may not fulfill the control objective since the equivalent control works with the assumption that the sliding surface achieved. Hence, the equivalent control extended by selecting a switching control in the form of a constant rate of achievement rule (Liu & Wang, 2012) as follows:

$$u_{sw} = \frac{g(x,t)}{\delta I(t)} \operatorname{sgn}(\sigma(t)), \tag{12}$$

where sign function (sgn) defined as:

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ -1 & \text{if } x < 0. \end{cases} \tag{13}$$

As a result, the control law becomes

$$u(t) = \hat{u}_{eq} + u_{sw}, \tag{14}$$

with $\hat{u}_{eq}$ in (11) and u_{sw} in (12). The switching gain $g(x,t)$ will be determined to handle parameter uncertainty so that the control rules in (14) can guarantee the convergence of the tracking error (2). Switching gain $g(x,t)$ is defined based on the following assumptions (Ibeas, de la. Sen, & Quesada, 2014):

Assumption 1 There is a state-dependent function $b(x,t)$ such that the following upper-bounding holds:

$$\left|p - \hat{p} - (q - \hat{q})I(t)\right| \le b(x,t),\ t \ge 0. \tag{15}$$

Assumption 2 The upper-bounding function $b(x,t)$ is known.

Assumption 3 The switching gain $g(x,t)$ is selected as

$$g(x,t) = b(x,t) + k,\ k > 0. \tag{16}$$

Assumption 1 is related to the upper bound of uncertainty in parameters. Note that the function $b(x,t)$ exists because the model parameterized by certain parameters despite being unknown. To present it clearly, the inequality (15) can be rewritten as

$$\begin{aligned} \left|p - \hat{p} - (q - \hat{q})I(t)\right| &= \left|\delta I(t) u_{eq,true}(t) - \delta I(t) u_{eq}(t)\right|, \\ &= \delta I(t)\left|u_{eq,true}(t) - u_{eq}(t)\right|, \\ &\le b(x,t), \end{aligned} \tag{17}$$

where $u_{eq,true}$ is the control function in terms of actual value, while u_{eq} is the control function in terms of nominal value. Thus, based on (17), Assumption 1 states that the difference between the application of quarantine with actual parameters and nominal parameters is bounded by a certain upper bounded function. This assumption is directly related to the error that occurs in quarantine treatment due to parameter uncertainty. Assumption 2 states that the upper-bounding $b(x,t)$ is known. This is a general assumption in a sliding mode control system (Slotine & Li, 1991). Then, Assumption 3 defines how the switching gain $g(x,t)$ must be chosen.

Furthermore, Assumption 2 will be simplified in the next subsection, where the switching gain $g(x,t)$ is adapted online

eliminating the required information related to the parameter bound values. Now, the following Theorem 1 (modified from (Ibeas, de la. Sen, & Quesada, 2014)) can be proven.

Theorem 1. *Consider the SIQRB epidemic model (1) with the control law (14). Thus if Assumptions 1, 2, and 3 hold, then tracking error* $(e_r(t))$ *vanishes asymptotically.*

Proof. Using the Lyapunov stability theorem, consider the Lyapunov candidate function:

$$V(t) = \frac{1}{2}\sigma^2(t). \tag{18}$$

Time-derivative of (18) is calculated as

$$\begin{aligned}\dot{V}(t) &= \sigma(t)\dot{\sigma}(t)\\ &= \sigma(t)\left(\frac{\beta B(t)}{\kappa + B(t)}S(t) - \delta u(t)I(t) - (\alpha_1 + \mu)I(t)\right.\\ &\qquad \left. + \xi(I(0) - I(t_f))e^{-\xi t}\right)\\ &= \sigma(t)\left[(p-\hat{p}) - (q-\hat{q})I(t) - g(x,t)\,\mathrm{sgn}(\sigma(t))\right]\\ &\le \sigma(t)\left[b(x,t)\,\mathrm{sgn}(\sigma(t)) - g(x,t)\,\mathrm{sgn}(\sigma(t))\right]\\ &= \sigma(t)\left[-k\,\mathrm{sgn}(\sigma(t))\right]\\ &= -k\,|\,\sigma(t)\,| \le 0.\end{aligned}$$

Pay attention to the use of Assumptions 1, 2, and 3 which indicate that the time-derivative of $V(t)$ for $V(t) \neq 0$ is always negative definite. Thus, according to the Lyapunov stability theorem (Khalil, 2002), the origin $(\sigma(t) = 0)$ is asymptotically stable, so that $\sigma(t) \to 0$ as $t \to \infty$. This means that the sliding mode control brings the tracking error $(e_r(t))$ to zero asymptotically. ■

In this way, $I(t) \to I_{ref}(t)$ as $t \to \infty$ so that the number of infected individuals also goes to zero. Thus, this theorem guarantees that sliding mode control can meet the control objective, which is reducing the cholera transmission with the presence of uncertainty parameters in the epidemic model.

However, there is a drawback to this approach, given that the upper bound $(b(x,t))$ must be known (Assumptions 1 and 2) for defining gain $(g(x,t))$ that depends on that value (Assumption 3). In some cases, this information may not be known in advance and complicates the application of the proposed control. Therefore, adaptation over time to the value of switching gain needs to be done to avoid the required a priori knowledge related to the upper bound $(b(x,t))$.

3.2 Adapting Switching Gain of Sliding Mode Control

In this subsection, the switching gain $(g(x,t))$ in (12) is adapted online to eliminate the required information regarding the upper-bound $(b(x,t))$ under Assumption 2. However, Assumption 1 regarding the upper-bound still applied even though it is not used explicitly in the controller design process. Besides, the following Assumption 4 is an extension of Assumption 1 and will be used to prove the stability of the control scheme.

Assumption 4 (Ibeas, de la. Sen, & Quesada, 2014) There exist a finite, potentially unknown, positive constant $\bar{b}$ such that

$$\bar{b} \ge b(x,t). \tag{19}$$

Note that Assumption 1 states that the upper-bound is a state-depending function, while Assumption 4 states that $b(x,t)$ is upper-bounded by a constant. From a biological perspective, this assumption states that the absolute error between nominal and actual parameters in the model bounded by a constant. However, it should be noted that the upper-bound may not be known explicitly in the control design and only used in the proof of stability. Furthermore, the switching gain adaptation process starts with a value of zero and then increases it until the sliding condition met. Therefore, the switching control (16) changed into

$$\hat{u}_{sw} = \frac{\hat{g}(t)}{\delta I(t)}\,\mathrm{sgn}(\sigma(t)). \tag{20}$$

As a result, the control law u on (14) now becomes

$$\begin{aligned}u = &\frac{1}{\delta I(t)}\left(\hat{p} - \hat{q}I(t) + \xi\left(I(0) - I(t_f)\right)e^{-\xi t}\right)\\ &+ \frac{\hat{g}(t)}{\delta I(t)}\,\mathrm{sgn}(\sigma(t)),\end{aligned} \tag{21}$$

where $\hat{g}(t)$ denotes the time-varying switching gain that updated by the following adaptive rule:

$$\frac{d\hat{g}(t)}{dt} = \Gamma\,\sigma(t)\mathrm{sgn}(\sigma(t)) = \Gamma\,|\,\sigma(t)\,|,\ \hat{g}(0) = 0, \tag{22}$$

where Γ is a positive constant.

The switching gain $\hat{g}(t)$ may be divergent given its derivative in (22) is definite non-negative all the time. The key that guarantees the success of tracking error towards zero is the positive constant $\bar{b}$ in Assumption 4 exists despite being unknown. Thus, the switching gain value rises until an upper-bound (19) reached. Then, the system converges to the sliding surface $(\sigma(t) = 0)$ stopping the increase of the switching gain $\hat{g}(t)$. This scheme makes the control objective fulfilled as how the following Proposition 1 applies.

Proposition 1. *Consider the SIQRB epidemic model (1) with control law (21) and (22). If Assumptions 1 and 4 hold for a finite upper bound b that may be unknown and* $\Gamma > 0$*, then tracking error* $(e_r(t))$ *asymptotically goes to zero.*

Proof. Suppose Assumption 4 applies, then there is a constant θ that satisfies $\theta \ge \bar{b} + k$ for each constant $k > 0$. Based on Lyapunov's stability theorem, a Lyapunov candidate function defined as:

$$V_1(t) = \frac{1}{2}\left(\sigma^2(t) + \frac{1}{\Gamma}(\hat{g}(t) - \theta)^2(t)\right). \tag{23}$$

Time-derivative of (23) is

$$\begin{aligned}\dot{V}_1(t) &= \sigma(t)\dot{\sigma}(t) + \frac{1}{\Gamma}(\hat{g}(t)-\theta)\dot{\hat{g}}(t) \\ &= \sigma(t)\left[(p-\hat{p}) - (q-\hat{q})I(t)\right] - \theta|\sigma(t)| \\ &\le b(x,t)|\sigma(t)| - \theta|\sigma(t)| \\ &\le \bar{b}|\sigma(t)| - \bar{b}|\sigma(t)| - k|\sigma(t)| \\ &= -k|\sigma(t)| \le 0.\end{aligned}$$

Thus, we get $V_1(t)$ positive definite and its time-derivative $\dot{V}_1(t)$ is semidefinite negative so according to the Lyapunov's stability theorem (Khalil, 2002), the origin $(\sigma(t)=0)$ is stable. The asymptotic nature cannot be shown considering that the derivative function of the Lyapunov candidate function is semidefinite negative. However, using the Barbalat's lemma (Khalil, 2002), it can be shown that the system asymptotically converges towards the sliding surface $(\sigma(t)=0)$.

Suppose $w(t)=\frac{1}{2}\sigma^2(t)$, using Barbalat's lemma, will be shown that $w(t)\to 0$ as $t\to\infty$. Time-derivative of $w(t)$ given by $w(t)=\sigma(t)\dot{\sigma}(t)$, since $\sigma(t)$ and $\dot{\sigma}(t)$ bounded, then $\dot{w}(t)$ bounded so that $w(t)$ uniformly continuous. Based on the time-derivetive of Lyapunov candidate function $(\dot{V}_1(t))$ we get

$$\dot{V}_1(t) \le -k|\sigma(t)| = -\bar{k}w(t),\ \bar{k}=2k, \qquad (24)$$

thus the integration of (24) from 0 to t as $t\to\infty$ leads to

$$V_1(0) - V_1(\infty) \ge \lim_{t\to\infty}\int_0^t \bar{k}w(\tau)d\tau. \qquad (25)$$

Since $\dot{V}_1(t)\le 0$, then $V_1(0)-V_1(\infty)\ge 0$. As a result, $\lim_{t\to\infty}\int_0^t w(\tau)d\tau$ exist and finite. Therefore, $w(t)\to 0$ as $t\to\infty$, or in other words, the convergence towards the sliding surface $(\sigma(t)=0)$ certainly achieved. ■

That is how the adaptive sliding mode control scheme succeeds in reducing the number of infected individuals and handling parameter uncertainties.

4. NUMERICAL SIMULATION

In this section, a numerical simulation is used to evaluate the performance of the proposed adaptive sliding mode control scheme for the cholera epidemic model. The initial values for each state in the model (1) takes the value of $[S(0),I(0),Q(0),R(0),B(0)]=[5750,1700,0,0,275000]$ with an observation time of 182 days, the constant $\Gamma=0.0005$, and the actual parameter values in the Table 1. The number of infected individuals is expected to decrease as the following reference function:

$$I_{ref}(t) = \left(I(0)-I(t_f)\right)\exp(-\xi t) + I(t_f), \qquad (26)$$

where $I(0)$ is the initial number of infected individuals, ξ is the exponential convergence rate of the reference function, in this case $\xi=0.05$, and $I(t_f)$ is the the expected number of infected individuals by the end of simulation, which is day 182. It is expected that at the end of the simulation no humans will be infected so $I(t_f)=0$ is selected. The results of the numerical simulation for the cholera epidemic model with adaptive sliding mode control are given in Figure 2 as follows.

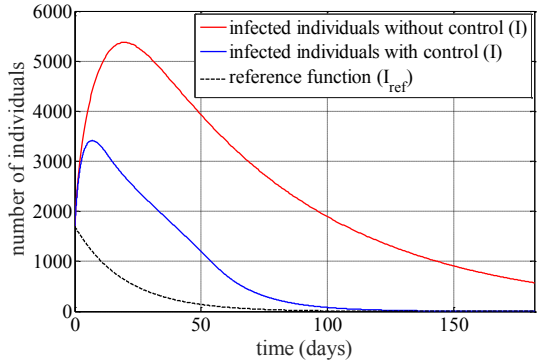

Fig 2a.Numerical simulation of the number of infected individuals.

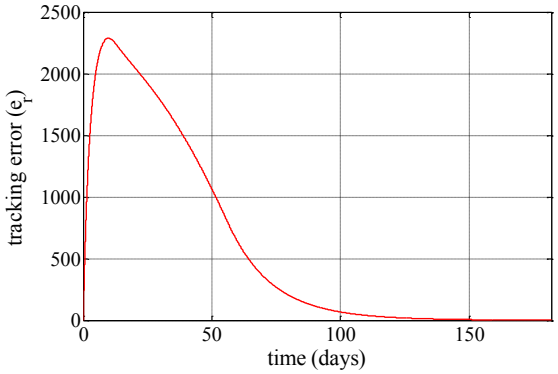

Fig 2b.Graph of tracking error.

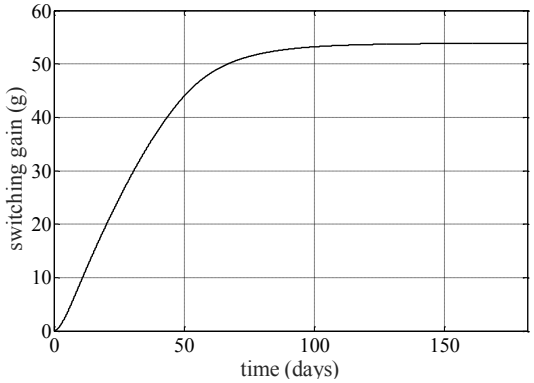

Fig 2c. Graph of adaptive switching gain $\hat{g}$.

Fig 2. Numerical simulation for cholera epidemic model with adaptive sliding mode control.

Table 1. Comparison of the average number infected individuals for 182 days.

Condition	Average number of infected human for 182 days (person)
without control	2539

with adaptive sliding mode control	778

In Figure 2a, it appears that in the end, the number of individuals infected with cholera $(I(t))$ drops towards zero following the reference function $(I_{ref}(t))$, which also drops towards zero. From Figure 2a and Table 3 it appears that the application of the proposed control scheme reduces the number of infected individuals. The use of adaptive sliding mode control reduces the average number of infected individuals by 69.4% compared to the uncontrolled treatment. The number of infected individuals with and without control treatment has the same behavior as to how they initially rise to a certain maximum value. However, using quarantine treatment, this maximum value is decreased, from 5374 on the 20th day to 3414 on the 7th day. As a result of a decrease in the number of infected individuals, there is a decrease in contact between infected individuals and the water source that is potentially contaminated by bacteria *V. cholerae*. In other words, the number of recruiting bacterial concentrations has decreased, whereas there are still mortality factors in bacterial subpopulation. Thus, the potential for cholera transmission decreased. Then in Figure 2b we see how the tracking error goes to zero so that the control objective is fulfilled, which is to reduce the number of infected humans through the tracking function of a reference function. As a result, the sliding surface $(\sigma(t)=0)$ begins to reach and the switching gain $(\hat{g}(t))$ starts to stop increasing, as shown in Figure 2c.

Next, we will examine the effect of selecting the convergence rate (ξ) of the reference function $(I_{ref}(t))$ on adaptive sliding mode control effort, tracking error, and infected individuals' dynamics $(I(t))$. Therefore, we variate the rate of convergence of the reference function as $\xi=0.01$ and $\xi=0.05$. Here is the numerical simulation of the cholera epidemic model with adaptive sliding mode control for the selected convergence rates in the reference function.

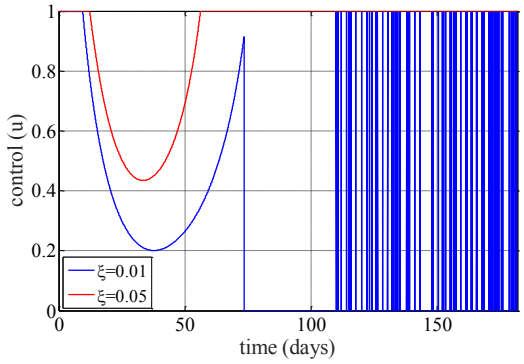

Fig 3a.Control rules with variation on the convergence rate (ξ) of the reference function (I_{ref}).

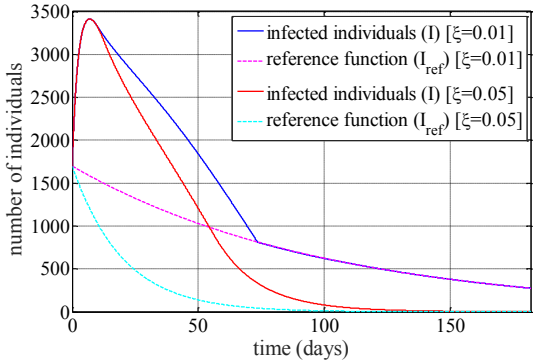

Fig 3b. Number of infected individuals with variation on the convergence rate (ξ) of the reference function (I_{ref}).

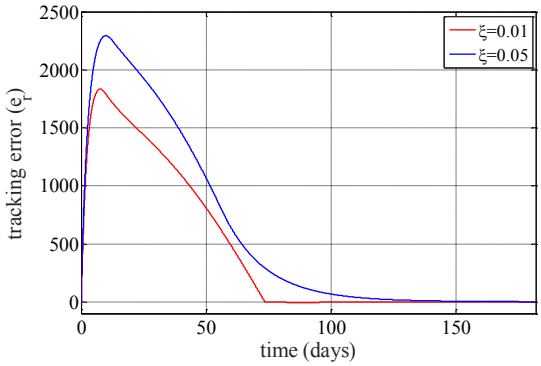

Fig 3c.Graph of tracking error with variation on the convergence rate (ξ) of the reference function (I_{ref}).

Fig 3. Numerical simulation for cholera epidemic model with variation on the convergence rate of the reference function.

In Figure 3a, the faster the reference function converges to zero, the greater the control effort given. Besides, with the rapid convergence of the reference function, the decrease in the number of infected people is faster, as shown in Figure 3b. The faster the number of infected individuals decreases does not mean that the faster tracking error goes to zero, as shown in Figure 3c.

The chattering phenomenon is a problem that often encountered in the implementation of sliding mode control (Slotine & Li, 1991) as seen in the system for the reference function with a convergence rate of $\xi=0.01$. The chattering phenomenon is seen in the switching or on/off behavior of the control (Figure 3a)) when the sliding surface begins to reach on the 74th day. In ideal sliding mode control, once the trajectory of the infected individuals touching the reference function, it should slide on the reference function, so it no longer cuts the reference function creating zig-zag motion (oscillation). However, in Figure 3b, there is an oscillation of the trajectory of the infected individuals around the reference function. This oscillation is known as chattering.

Chattering that occurs in sliding mode control is acceptable, but in practice, it would be better if chattering does not exist (Slotine & Li, 1991). This is necessary to avoid extreme control performance, such as spontaneous switching. In this

case, we can not eliminate chattering, however, we can reduce it without changing the control scheme that established before. Therefore, without loss of generality, the sign function in the switching control (u_{sw}) can be approximated using the sigmoid function, in this case, the hyperbolic tangent function used.

The following are the numerical simulation in reducing the chattering effect on the cholera epidemic model for reference function with the convergence rate of $\xi = 0.01$.

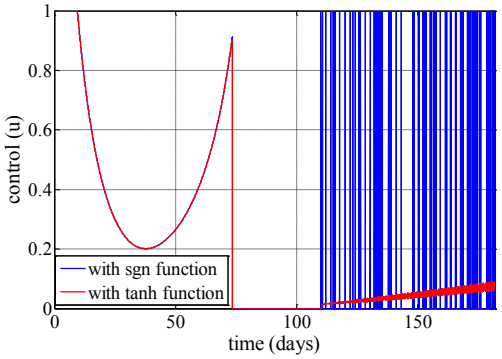

Fig 4a. Comparison on the adaptive sliding mode control with sgn and tanh function.

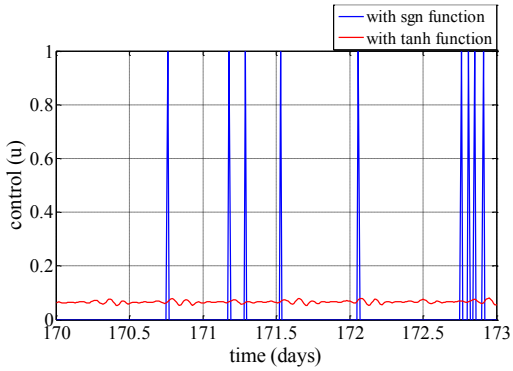

Fig 4b. Comparison on the adaptive sliding mode control to sgn and tanh function on day 170 to 173.

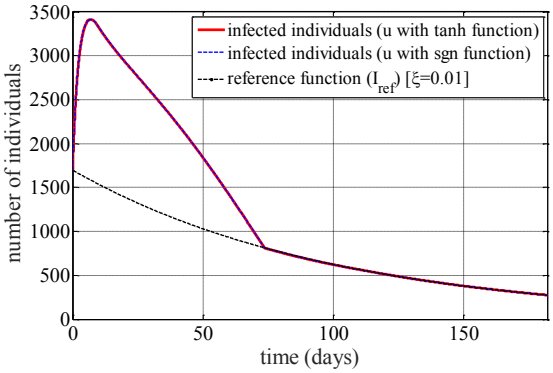

Fig 4c. Comparison on the number of infected individuals between control to sgn and tanh function.

Fig 4. Numerical simulation on the approximation of sign function using hyperbolic tangent function in reducing the effect of chattering.

In Figure 4a and Figure 4b, we can see that the approximation of the sign function with the hyperbolic tangent function reduces chattering in the application of adaptive sliding mode control. As a result, changes in control values no longer occur spontaneously (extreme) in the range of [0,1] but only around [0,0.1]. Besides, the use of the hyperbolic tangent function does not affect the proposed control scheme so that the control objective still met. Furthermore, there is no significant difference between tracking the reference function for controls with the sign function or with the hyperbolic tangent function as shown in Figure 4c.

Thus, even though we cannot eliminate chattering, we can reduce it so that the control works efficiently in meeting the tracking scheme. Besides, the approximation using the hyperbolic tangent function still applies following the sliding mode control design scheme that built previously.

Furthermore, we apply some variation on the uncertainty percentage of the parameters of the cholera epidemic model to see the performance of the adaptive sliding mode control scheme in handling parameter uncertainty. Uncertainty percentage defined as the percentage of error relative to the nominal parameter against the actual parameter denoted as and formulated as:

$$\Delta = \left(\frac{\hat{\rho} - \rho}{\rho} \right) \times 100\% \tag{27}$$

with $\hat{\rho}$ represents nominal parameters, and ρ represents actual parameters with the value given in Table 1. We assume $|\Delta| \leq 1$, which means that the relative error percentage of nominal parameters and actual parameters is no more than 100%. As in the control scheme (14) that we built before, the parameter uncertainty factor lies in the parameters of the ingestion rate β and the rate of disease-related death of infected individuals α_1.

Then, for simulation purposes, the uncertainty percentages that we used are 20%, 40%, 60%, and 80%. The following are numerical simulation results for the variation of the uncertainty percentages on the application of adaptive sliding mode control for cholera epidemic model.

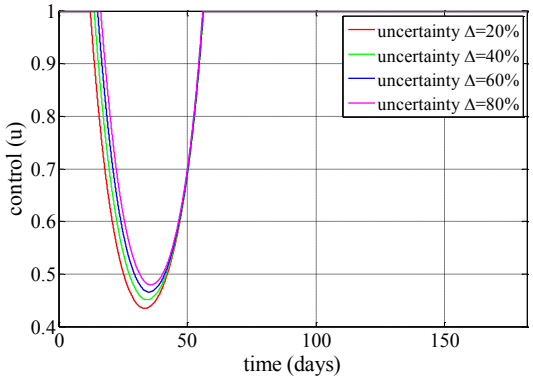

Fig 5a. Control rules for different values of parameter uncertainty.

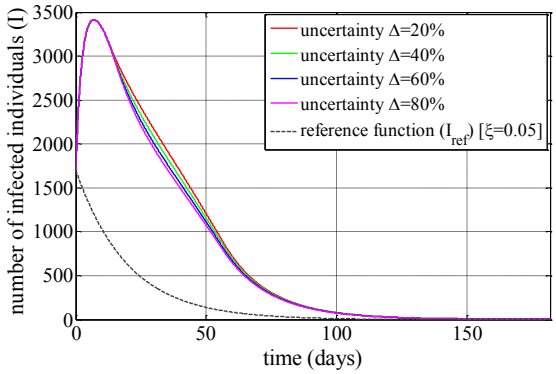

Fig 5b. Number of infected individuals for different values of parameter uncertainty.

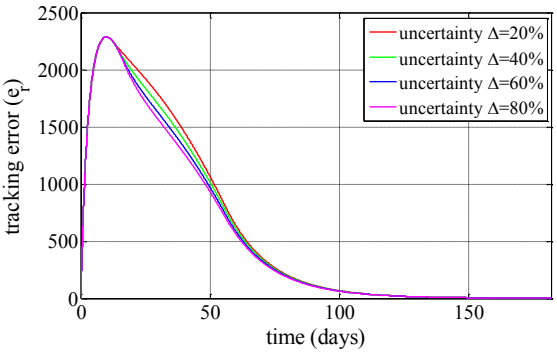

Fig 5c. Graph of tracking error for different values of parameter uncertainty.

Fig 5. Numerical simulations for cholera epidemic model with variation of parameter uncertainty.

In general, the variation of uncertainty percentage results in system dynamics that are not much different from each other. In Figure 5a, we can see the control law for the given variation of uncertainty percentage. Even though there is a parameter uncertainty factor, the control scheme in Figure 5c is still able to fulfill the control objective, which is to reduce the number of infected individuals by tracking the reference function, as shown in Figure 5b. Then, in Figure 5c, we can see how the tracking error for each variation of uncertainty percentage still goes to zero. In other words, the adaptive sliding mode control can perform well in handling the parameter uncertainty.

5. CONCLUSIONS

Based on the simulation results, it appears that the adaptive sliding mode control strategy can reduce the number of individuals infected with cholera through a reference function tracking scheme despite the existence of the parameter uncertainty. Adaptive sliding mode control with the desired reference function can suppress the average number of infected individuals and with a percentage of 69.4% compared to the uncontrolled treatment.

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