Heat Balancing in Cooling Systems using Distributed Pumping

Carsten Skovmose Kallesøe^{*,**} Brian Kongsgaard Nielsen^{*} Agisilaos Tsouvalas^{*}

 * Grundfos Holding A/S, Poul Due Jensens vej 7, DK-8850 Bjerringbro, Denmark (e-mail: {ckallesoe,bknielsen,atsouvalas}@grundfos.com).
 ** Aalborg University, Fredrik Bajers vej 7, DK-9220 Aalborg, Denmark (e-mail: csk@es.aau.dk)

Abstract: Hydronic cooling systems are often used in large buildings. In these systems, chilled water is transported from the chillers to Air Handling Units (AHUs) via a water distribution system. The temperature of the exhaust air from the AHU is controlled by the flow of the chilled water in AHUs. Installation costs are very important for the building sector, which has lead to a new structure of the hydronic system using distributed pumps. This paper derives a control scheme for this new structure that ensures flow requirements at the chillers and controls the air temperature at the local AHUs. Saturation problems are well known in hydronic systems and can lead to poor control performance in parts of the building. To mitigate this problem the hydronic system needs to be hydraulically balanced. With the proposed control this balancing procedure is handled automatically. The balancing algorithm is verified by a Modelica model of an office building.

Keywords: Cooling Systems, Distributed Optimization.

1. INTRODUCTION

Due to the structure of the building sector, installation cost, including cost for Heating, Ventilation, and Air Condition (HVAC) systems, is often an important factor for decision makers, even though, compared to the operational cost the installation cost is often insignificant. The energy consumption in the building sector is substantial, with just below 12.000 GW in 2016 used for cooling alone IEA (2016). Half of this is used in commercial buildings. Commissioning of HVAC systems, including hydraulic balancing of cooling systems, is very important to ensure energy efficient operation of buildings Reid Hart et al. (2017). This is supported by Kolb et al. (2017), showing several case studies on heating systems of one family houses revealing an energy saving potential between 8%and 23%. In Kohl (2001) it is argued that commissioning is extremely important for the building to be operated properly, however it is often not done due to primary cost. In other cases the hydraulic balance obtained at initial commissioning is destroyed by changes made to the HVAC system at a later time.

Commissioning of the hydronic system is a standard procedure, which is well described in several hand books, see for example Petitjean (2000). Especially, performance is degraded in a subset of the zones during re-cooling after system switch-off and under high load conditions when the hydronic networks are not well balanced. In Taylor and Stein (2002) different strategies are evaluated with respect to the impact on the system performance and the cost, leading to the conclusion that only for large buildings the system performance improvement may justify the extra cost associated with balancing. However, an adaptive balancing scheme is proposed in Kallesøe et al. (2019) eliminating the cost associated with balancing.

Our starting point is a new type of hydronic system for carrying the cooling load from the chillers to the Air Handling Units (AHU). The idea is to design the system to include only the necessary elements leading to reduced installation costs. In the proposed system, flows trough the AHUs and chillers are controlled by distributed pumps placed at the AHUs. Therefore, control valves at the AHUs and dedicated chiller pumps are not present in the system. The removal of the valves and thereby the pressure losses over these will lead to energy savings on the pump operation. However, these savings depend on the specific efficiencies of the installed pumps and are therefore hard to quantify in general. The mechanical structure of the new system is more simple than for a conventional system but puts extra tasks on the control.

An algorithm for controlling the new type of hydronic system is presented. The algorithm will ensure that the chiller flow requirements are fulfilled and the cooling load is balanced between the AHUs especially in overload situations. The algorithm will be distributed between the pumps and optimal operation is ensured by communication between the pumps. The distributed implementation is expected to make installation easier as extra control units are eliminated.

The distributed balancing controller is derived by considering a dual optimization problem. This balancing setup is similar to the setup proposed in Kallesøe et al. (2019), where a conventional hydronic cooling system is considered. Dual optimization has been used for deriving distributed optimization and control in for example Rantzer (2009), Chang and Nedic (2014), Liang et al. (2018), and Wang and Elia (2011). In our work we distribute the calculations of the optimization problem between the AHU controllers, and communicate necessary Lagrange multipliers between the units. As proposed in Wang and Elia (2011) the optimization is implemented as a control system leading to an "easy to install" balancing system for the new type of hydronic system.

The paper starts by presenting the new hydronic system and the underlying model in Section 2. The model is used in Section 3 to setup an optimization problem that solves the system balance and ensures minimum chiller flow with minimum disturbances on the temperature control. The solver for the optimization problem is distributed between the control units in Section 4. The developed algorithm is tested on a Dymola model of a building in Section 5, exemplifying the usability and benefits of the algorithm. The paper ends with some concluding remarks.

1.1 Nomenclature

We use i, j, k as index elements in vectors, such that x_i is the *i*th element of x. For recursive algorithms the step index is $x^{(t)}$ for the *t*th step or time instant. For signal saturation we use $[x]_+$, such that $[x]_+ = \max\{0, x\}$.

2. COOLING SYSTEMS

A sketch of the considered new hydronic system is shown in Fig. 1. The system contains two chillers producing the cooling water. The chillers control the outlet supply temperature θ_c to a fixed reference. Dependent on the load, one or both chillers are activated by opening the valves v_1 and v_2 . Dependent on the number of active chillers, a minimum chiller flow \underline{q}_c must always be obtained to avoid icing inside evaporators. In conventional hydronic systems this flow is ensured by separating the chiller flows from the AHU flows using a bypass placed between the chillers and the distribution system.



Fig. 1. Sketch of the cooling system under consideration.

The distribution part of the cooling system is composed of n Air Handling Units (AHUs) supplied with cooling water via the distributed pumps. As in conventional Variable Air Volume (VAV) systems, the flow trough each of the AHUs is controlled such that the exhaust air temperature T_i is kept at its reference value. This temperature control is handled by a local PI controller measuring the exhaust air temperature and controlling the pump speed. In a conventional system these PI controllers would have actuated valves instead.

The heat exchangers shown in Fig. 1 are well described by finite volume approximations. Here, the heat exchanger is modeled using one water volume V_w and one air volume V_a . This leads to the following model of the heat exchanger dynamics

$$C_w V_w \dot{\theta}_i = C_w q_i (\theta_c - \theta_i) - B(\theta_i - T_i)$$
(1a)

$$C_a V_a \dot{T}_i = C_a Q_i (T_a - T_i) + B(\theta_i - T_i) , \qquad (1b)$$

where the temperatures of the water and air are θ_i and T_i respectively, and B is the heat transfer coefficient that describes the energy transfer between the water and the air. The temperature T_a is the outside air temperature feeding the AHU and Q_i is the air flow trough the heat exchanger. θ_c is the supply water temperature, and finally, C_w and C_a are the heat capacities of the water and air respectively.

For cooling systems the following assumptions about the operating conditions of the heat exchanger always hold.

Assumption 1. The supply water temperature θ_c is always lower than the outside air temperature T_a .

In our control setup the exhaust air temperature T_i from the heat exchanger is controlled by a local controller and is kept at its reference value T_i^* . Hence, the balancing controller focuses on controlling the steady state conditions of the system. Assuming steady state conditions of the heat exchanger model (1) ($\dot{\theta}_i = \dot{T}_i = 0$), the following relation between the setpoint for the steady state exhaust air temperature T_i^* and the steady state flow q_i^* is found

$$T_{i}^{*} = \frac{C_{w}(C_{a}Q_{i}T_{a} + B\theta_{c})q_{i}^{*} + BC_{a}Q_{i}T_{a}}{C_{w}(C_{a}Q_{i} + B)q_{i}^{*} + BC_{a}Q_{i}} .$$
 (2)

We assume positive water and air flows $q_i^* > 0$ and $Q_i > 0$, and assume the supply water temperature $\theta_c > 0$ and the air temperature $T_a > 0$. Then, from (2) the exhaust air temperature T_i^* is also positive. In fact it is easy to see that $T_i^* = T_a$ for $q_i^* = 0$ and for $q_i^* \to \infty$ the exhaust air temperature $T_i^* = (C_a Q_i T_a + B \theta_c)/(C_a Q_i + B)$.

In the derivation of the balancing control algorithm the first derivative of the reference temperature T_i^* with respect to the supply flow q_i^* is needed. This derivative is given by

$$\frac{dT_{i}^{*}}{dq_{i}^{*}} = -\frac{B^{2}C_{w}C_{a}Q_{i}\left(T_{a}-\theta_{c}\right)}{\left(C_{w}\left(C_{a}Q_{i}+B\right)q_{i}^{*}+BC_{a}Q_{i}\right)^{2}} < 0, \quad (3)$$

where the last inequality is due to Assumption 1, meaning that the nominator of (3) is positive, and therefore the first order derivative is negative always.

The AHUs and the chillers are connected via a pipe network that transports the cooling water from the chillers to the AHUs. This means that the flow of one AHU affects the pressure conditions of the other AHUs. These hydraulic relations are expressed by setting up the pressure loop equations for each of the n loops associated with the AHUs. The pressure loop for the *i*th AHU is given by

$$0 = \Delta p_i - R_c \left(\sum_{j=1}^n q_j\right)^2 - \sum_{k=1}^i 2R_i \left(\sum_{j=k}^n q_j\right)^2 - r_i q_i^2,$$
(4)

where Δp_i is the pressure provided by the pump of the *i*th loop and q_i is the flow through the *i*th loop. The

hydraulic resistance of the chillers is denoted R_c . R_i is the pipe resistance in the supply and return pipe, see Fig. 1. Finally r_i is the pipe resistance of the *i*th branch.

The pump pressures Δp_i are provided by centrifugal pumps, which are well modeled by the following polynomial equation

$$\Delta p_i = -a_i q_i^2 + b_i \omega_i^2 , \qquad (5)$$

where $a_i > 0$ and $b_i > 0$ are parameters describing the pump, and ω_i is the pump speed of the *i*th pump. Due to physical constraints in the pump and the pump speed controller, the pump speed ω_i is restricted to be within some limits, such that

$$0 \le \omega_i \le \overline{\omega}_i . \tag{6}$$

3. OPTIMAL PUMP OPERATION

In this work the local AHU exhaust air temperature T_i is controlled by a local PI controller, without integrator anti-windup. This controller ensures that the exhaust air temperature equals its reference T_i^* , whenever the pump is not saturated. When the pump saturates upwards the AHU is not able to deliver the cooling power needed. This phenomenon is called starvation, which leads to reduced comfort in the connected rooms. Starvation must therefore be reduced to a minimum. Beside minimizing the effect of pump saturation, a minimum flow through the chillers \underline{q}_c must always be obtained to avoid icing in the evaporator.

The proposed algorithm, minimizes the effect of pump saturation and low chiller flow conditions during steady state operation by solving a constrained optimization problem. The objective is to minimize the steady state control error of the PI-controllers. The control error for the *i*th AHU is given by

$$\Delta T_i^* = T_i^* - \bar{T}_i^* , \qquad (7)$$

where \overline{T}_i^* is the reference temperature. By minimizing the square sum of the steady state control errors ΔT_i^* subject to constraints, the effect of pump saturation and low chiller flow is minimized. The AHU control, based on this optimization problem, leads to a minimal average increase in the exhaust air temperature, and therefore minimum average reduction in room comfort.

Introducing the pump model (5) in the pipe loop model (4) and solving for the pump speed leads to the following expression for the squared pump speed ω_i

$$\omega_i^2 = f_i(q) \equiv \frac{a_i + r_i}{b_i} q_i^2 + \frac{R_c}{b_i} \left(\sum_{j=1}^n q_j\right)^2 + \sum_{k=1}^i 2\frac{R_i}{b_i} \left(\sum_{j=k}^n q_j\right)^2 , \quad (8)$$

where $q = [q_1, \dots, q_n]^T$ is a vector of all the branch flows. The flows of the individual branches are forced to be positive by non-return valves, e.g. see Fig. 1. Thus, the constraint $q \ge 0$ is imposed on the system, and (8) is true for $q \ge 0$ only. The speed of the *i*th pump ω_i must fulfill the requirements (6), leading to upper constraints on the pumps speed. When the speed of the *i*th pump is below $\overline{\omega}_i$, the exhaust air temperature of the associated AHU can be controlled to its reference value. This means that pump saturation is not occurring at the given branch. Beside ensuring the flow needed to obtain the reference temperature at each AHU, the flow through the chillers q_c must always be larger than the required minimum chiller flow \underline{q}_c . This leads to the following constraint on the chiller flow

$$\underline{q}_{c} \le q_{c} = f_{n+1}(q) \equiv \sum_{j=1}^{n} q_{j} ,$$
 (9)

where q_j is the flow through the *j*th branch. Moreover, the pump size must be chosen such that the minimum chiller flow \underline{q}_c always can be obtained. Hence, given a pipe network, the pump parameters a_i, b_i and the maximum pump speed $\overline{\omega}_i$ must at least be chosen such that the minimum chiller flow can be accommodated.

The following optimization problem leads to the optimal distribution of steady state control errors $\Delta T_1^*, \dots, \Delta T_n^*$, where *n* is the number of branches in the system.

$$\min_{q^*} \sum_{i=1}^n \frac{1}{2\kappa_i'} (\Delta T_i^*)^2 \tag{10}$$

subject to

$$f_i(q^*) - \overline{\omega}_i^2 \le 0 \tag{11a}$$

$$\underline{q}_{c} - f_{n+1}(q^{*}) \le 0$$
. (11b)

Note that $\Delta T_i^* = T_i^* - \overline{T}_i^*$ is a function of q^* , see (2). The first set of constraints are obtained by combining the physical constraints (6) and (8) and must be fulfilled for $i = 1, \dots, n$. The last constraint (11b) is obtained from (9). To solve the optimization problem we setup the Lagrangian function

$$\mathcal{L} = \sum_{i=1}^{n} \frac{1}{2\kappa_i'} (\Delta T_i^*)^2 + \sum_{j=1}^{n} \overline{\lambda}_j \left(f_j(q^*) - \overline{\omega}_j^2 \right) + \mu \left(\underline{q}_c - f_{n+1}(q^*) \right) ,$$

where $\overline{\lambda}_j, \mu \geq 0$ are the Lagrange multipliers. The functions $f_j \ j = 1 \cdots, n$ are given by (8) and the function f_{n+1} is given by (9). The temperatures ΔT_i^* are given by (2), which models the heat exchanger under steady state conditions in the individual AHUs.

The optimal set of control errors ΔT_i^* are found by calculating the derivative of \mathcal{L} with respect to the pump flows

$$0 = \sum_{i=1}^{n} \frac{1}{\kappa_i'} \Delta T_i^* \frac{dT_i^*}{dq^*} + \sum_{j=1}^{n} \overline{\lambda}_j \frac{df_j}{dq^*} - \mu \frac{df_{n+1}}{dq^*} \,.$$

From (2), $\frac{d\Delta T_i^*}{dq_j^*} = 0$ for all $i \neq j$, as ΔT_i^* is a function of q_i^* only. Moreover, (11b) shows that $\frac{f_{n+1}}{dq^*} = \mathbb{1}$. Using these relations, the expression for the optimal set of control errors ΔT_i^* is reduced to

$$0 = \Delta T_i^* \frac{d\Delta T_i^*}{dq_i^*} + \kappa_i' \left(\sum_{j=1}^n \overline{\lambda}_j \frac{df_j}{dq_i} - \mu \right) .$$
(12)

The Lagrange multipliers $\overline{\lambda}, \mu$ are found by solving the dual problem given by

$$\max_{\overline{\lambda},\mu} \mathcal{L}(\Delta T^*, \overline{\lambda}, \mu)$$

subject to $\overline{\lambda} \geq 0$, $\mu \geq 0$.

The dual problem is always concave (Boyd and Vandenberghe, 2004, p. 216) and therefore can always be solved by standard gradient methods.

4. DISTRIBUTED PUMP OPERATION

We assume that there exists an asymptotic stable set of PI-controllers placed at the AHUs, that controls the individual exhaust air temperatures. This Means that our starting point is a set of PI-controllers at the AHUs that controls the local temperature errors ΔT_i^* to zero. Adding the correction terms given in (12) leads to a corrected controller error, which reduces the effect of pump saturation and protects the system against low chiller flow. The corrected control error is given by

$$e_i = \Delta T_i^* + \kappa_i' \left(\frac{d\Delta T_i^*}{dq_i^*}\right)^{-1} \left(\sum_{j=1}^n \overline{\lambda}_j \frac{df_j}{dq_i} - \mu\right) , \quad (13)$$

where e_i is the corrected control error for the *i*th AHU temperature controller. Note that the inverse of $\frac{d\Delta T_i^*}{dq_i^*}$ always exists due to (3). From (8) the term $\frac{df_j}{dq_i}$ in (13) is given by

$$\frac{df_j}{dq_i} = \begin{cases} 2\frac{R_c}{b_i}s_1 + \sum_{k=1}^{j} 4\frac{R_k}{b_i}s_k & , j < i\\ 2\frac{a_i + r_i}{b_i}q_i + 2\frac{R_c}{b_i}s_1 + \sum_{k=1}^{i} 4\frac{R_k}{b_i}s_k & , j = i \\ 2\frac{R_c}{b_i}s_1 + \sum_{k=1}^{i} 4\frac{R_k}{b_i}s_k & , j > i \end{cases}$$
(14)

where $s_k = \sum_{i=k}^{n} q_i$. Eq. (14) shows that $\frac{df_j}{dq_i}$ is a function of all branch flows in the system.

The optimization problem (10) and (11) is similar to the problem presented in Kallesøe et al. (2019) and we adapt a similar approach for a distributed implementation. That is, we seek an implementation, which is distributed between the local controllers at the AHUs and the only signals communicated between the local controllers are the Lagrange multipliers. To that end, we linearize the problem meaning that only sub-optimal operation can be expected.

The term $\frac{df_i}{dq_i}$ is nonlinear and is multiplied with Lagrange multipliers related to speed saturation. Speed saturation occurs when the flows of the hydronic system are at their maximum values. Maximum flow values are typically also the design flows for the system. Let \bar{q}_i^* define the design flow for the *i*th branch. The design flow depends on the load conditions imposed by the design air temperature reference \bar{T}_i^* , the design air flow \bar{Q}_i , the design ambient temperature \bar{T}_a , and the design supply water temperature $\bar{\theta}_i$. The linearized values of the $\frac{df_i}{dq_i}$ becomes

$$\alpha_{ij} = \frac{df_j}{dq_i}\Big|_{q=\overline{q}^*} \,. \tag{15}$$

The term $\frac{d\Delta T_i^*}{dq_i^*}$ is given by (3) meaning that it makes the problem non-linear. The operating conditions of the system, when the inequality constraints are active, differs from case to case. However, starvation is most likely to appear at high load condition, and is considered to be the most important. Therefore, $\frac{d\Delta T_i^*}{dq_i^*}$ is linearized around the design conditions

$$K_{i} = \frac{d\Delta T_{i}^{*}}{dq_{i}^{*}} \bigg|_{q_{i}^{*} = \overline{q}_{i}^{*}, Q_{i} = \overline{Q}_{i}, T_{a} = \overline{T}_{a}, \theta_{c} = \overline{\theta}_{c}} .$$
(16)

Introducing the linearized terms in (15) and (16) lead to a linearized version of the corrected control errors in (13)

$$e_i = \Delta T_i^* + \kappa_i \left(\sum_{j=1}^n \alpha_{ij} \overline{\lambda}_j - \mu \right) , \qquad (17)$$

where $\kappa_i = \frac{\kappa_i}{K_i}$. The update of the Lagrange multipliers is implemented as line search. Wang and Elia (2011) shows that such a line search can be implemented as a filter leading to the following discrete time filter equations

$$\overline{\lambda}_{i}^{(t+\delta t)} = \left[\overline{\lambda}_{i}^{(t)} + \kappa_{\lambda}(\overline{\omega}_{i}^{2} - f_{i}(q^{*}))|_{q_{i}^{*} = g_{i}(\Delta T_{i}^{*(t)})}\right]_{+}$$
(18)

$$\mu^{(t+\delta t)} = \left[\mu^{(t)} + \kappa_{\mu} (\underline{q}_{c} - f_{n+1}(q^{*})) |_{q_{i}^{*} = g_{i}(\Delta T_{i}^{*(t)})} \right]_{+}$$
(19)

where t is the present time and δt is the time step between two calculations.

The constraint updates (18) and (19) are made operational by using available measurements that are proxies for the functions f_i and f_{n+1} . Firstly, consider (18). According to (8) the function $f_i(q^*)$ equals the speed needed to provide the flow q^* . The output of the PI-controllers are assumed to converge to the speeds needed to meet the temperature references and thereby the flow vector q^* . This means that the speed output from the local PI-controller can be used in (18). Defining $\Omega_i = \omega_i^2$ leads to the following local algorithm for calculating $\overline{\lambda}_i$

$$\overline{\lambda}_{i}^{(t+\delta t)} = \left[\overline{\lambda}_{i}^{(t)} + \kappa_{\lambda} (\overline{\omega}_{i}^{2} - \Omega_{i}^{(t)})\right]_{+} .$$
 (20)

Secondly, consider (19). A pressure sensor measuring the pressure across the chiller bank is used as a proxy for the chiller flow. Thereby the cost related to flow sensors is saved. The relation between the chiller flow and the pressure is given by the hydraulic conductivity of the chiller. For a hydraulic system the relations between the resistance and conductive is given by $K_c = \frac{1}{\sqrt{R_c}}$, where R_c is the chiller resistance and K_c is the conductivity. The relation between the pressure and the flow is given by

$$q_c = K_c \sqrt{\Delta p_c}$$
.

Note, that the resistance changes and thereby the conductivity when chillers are cut-in or cut-out of the chiller bank, see Fig. 1. However, as the chillers in the chiller bank are parallel connected, the pressure requirement is invariant under the number of active chillers for equal size chillers. Therefore, (19) can be reformulated to

$$\mu^{(t+\delta t)} = \left[\mu^{(t)} + \kappa_{\mu} K_c \left(\sqrt{\Delta \underline{p}_c} - \sqrt{\Delta p_c^{(t)}}\right)\right]_+, \quad (21)$$

where the low pressure constraint $\Delta \underline{p}_c$ is constant over the number of active chillers. From (21) it is evident that the dynamic of the μ filter depends on the the chiller bank conductivity K_c . However, here a constant K_c value is used and κ_{μ} is chosen such that the feedback is stable under all operating conditions. The implementation of the controllers is done locally at the AHUs, leading to the controller structure shown in Fig. 2. The controller receives the Lagrange multipliers from the other units and calculates the correct control errors (17). The errors are controlled to zero by standard PI controllers at the local AHUs. Moreover, the Lagrange multipliers related to the saturation of the local control is calculated at the local AHUs using (20), and transmitted to the other AHUs.



Fig. 2. Local heat controller and calculation unit for calculating the Lagrange multipliers handling speed saturation in the pump placed at each AHU.

The Lagrange multiplier related to the flow requirements of the chiller bank is implemented locally at the chiller using the algorithm structure shown in Fig. 3. The Lagrange multiplier related to flow saturation is updated using (21) and transmitted to the AHU controllers, shown in Fig. 2.





5. NUMERICAL STUDY

The new hydronic system architecture with distributed pumps and the proposed balancing controller, is tested on a Dymola model of the imaginary office building shown in Fig. 4. The building is divided into 2 offices zones, a laboratory area, and a canteen. Each of the zones are supplied with cooling power from 4 separate AHUs. The AHUs are supplied with cooling water via a hydronic system with distributed pumps, hence the structure shown in Fig. 1. The red line between the air handling units symbolises the forward and return pipeline of the hydronic system. The cooling is produced by a chiller bank. The minimum flow of the chiller bank is 35% of the maximum flow.

In the experiment the load on the building is controlled by weather data and an occupancy model. The weather data are from Seville, Spain, weeks 29/30 starting at Friday 12am. The occupancy model states that the office spaces and laboratory area are occupied between 9pm and 12pm



Fig. 4. Sketch of the imaginary office building used in the numerical experiments.

and again from 13pm till 17pm during work days. The canteen is occupied from 12am till 13am also during work days. The room temperatures are controlled to $22^{\circ}C$ by air flow dampers in all four zones from 8:30pm till 22pm. During night time there is a 5°C increase of the set point. The exhaust air temperature set point for the AHUs is set to $\bar{T}_i = 18^{\circ}C$. Each of the AHUs are equipped with the control units (CTRL) shown in Fig. 2. A pressure supervisor (PS), as shown in Fig. 3, is placed at the chiller bank. The Lagrange multipliers $\bar{\lambda}_i$ $i = 1, \cdots, 4$ and μ are communicated between the units. In Fig. 4 the blue line between the CTRLs and the PS indicates the needed communication network.

The result of the numerical experiment is shown in Fig. 5. The first plot shows the temperatures in the zones and their references (dashed line), the second plot shows the air temperatures supplied to the zones as well as their references (dashed line), and the third plot shows the pressure across the chillers together with the configured minimum pressure value (dashed line). The forth and fifth plots show the Lagrange multipliers; $\overline{\lambda}_i$ for upper speed saturation of the pumps, and μ for the lower flow saturation of the chiller, respectively.

The overall goal for a cooling system is to keep the room temperatures on the reference values. In a Variable Air Volume (VAV) system the room temperatures are controlled by controlling the air flow into the rooms via dampers. From the first plot it is evident that during night time the room temperature references are not followed as the load is too low to heat up the building. In this case the dampers are closed and the energy flow in the hydronic system is almost zero.

The air flow temperatures are controlled by the AHU towards constant reference values. The second plot in Fig. 5, shows that the reference value cannot be maintained during night time. This error is caused by the minimum flow requirement of the chillers, which is measured by the PS, see Figs. 4 and 3. The PS indicates saturation by the Lagrange multiplier μ , shown in the last plot, and thereby adjusts the controller errors to maintain the necessary pressure and thereby chiller flow.

During re-cooling after night setback the pumps saturates, which leads to the Lagrange multiplies $\overline{\lambda}_i > 0$, see the forth plot of Fig. 5. As all pumps are saturated this has no impact on the operation. Whereas, during the afternoon, where the load is the highest, pump 3 saturates. The saturation of pump 3 would lead to imbalance in the



Fig. 5. Results from the numerical experiment. The first three plots show the temperatures in the zones, the exhaust air temperatures, and pressure across the chiller, respectively. The final two plots show Lagrange multipliers; $\overline{\lambda}_i$ and μ for balancing control.

system. However, as shown in the forth plot, the CTRL of at AHU 3 indicates the saturation by $\overline{\lambda}_3 > 0$, leading to a correction of all the controller errors. The error correction increases the exhaust air temperature for AHU a little (not only the saturated one), which can be accommodated by the dampers.

The cost of installing and operating this new type of hydronic system is compared to a conventional system with a shunt, a primary and a secondary pumping station, and control valves for temperature control. In Kallesøe et al. (2019) a balancing controller for a conventional system is developed and tested on a numerical simulation similar to the one used here. In the following table we have compared the cost of the active components (list prices of pumps and valve) and the cost of energy with operating the pumps for the conventional and the new hydronic layout in our imaginary building. For the cost of energy we have used the energy price of $0.24 \in /KWh$.

	CAPEX		OPEX
	Pump	Valve	Pump Energy
Conventional New	3700 € 3300 €	4800 € 0 €	$\begin{array}{c} 45 \in / \text{week} \\ 7.58 \in / \text{week} \end{array}$

It is clear that there is a huge saving potential. Besides the savings listed in the table the new hydronic system has fewer components and therefore expected lower installation costs, and a lower footprint in the mechanical room.

6. CONCLUSIONS

This paper presents a distributed control architecture that can handle pump saturation and minimum chiller flow requirements in a new hydronic system configuration. In this new setup, pumps are used for temperature control and the chiller flow is not disconnected from the AHU flows. During high load situations, saturation of the pumps can occur, leading to imbalance of the cooling distribution in the building. Moreover, during low load saturation, the minimum flow requirement of the chillers might not be fulfilled. The proposed distributed controller minimizes the effect on room comfort during these situations. Simulation studies verify that the proposed control architecture enables the use of the new hydronic system.

Future work includes stability analysis of the control system and identification of design rules for the controller constants.

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