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Finite-time Frequency Estimator for Harmonic Signal

Alexey A. Bobtsov [*,**,***] **Anastasiia O. Vediakova** [****]
Nikolay A. Nikolaev [*] **Olga V. Slita** [*] **Anton A. Pyrkin** [*]
Alexey A. Vedyakov [*]

[*] *Department of Control Systems and Robotics, ITMO University, Saint Petersburg, Russia (e-mail: bobtsov@mail.ru, nanikolaev@itmo.ru, ovslita@itmo.ru, a.pyrkin@gmail.com, vediakov@gmail.com)*
[**] *Laboratory "Control of Complex Systems", Institute of Problems of Mechanical Engineering, V.O., Bolshoj pr., 61, St. Petersburg, 199178, Russia*
[***] *Center for Technologies in Robotics and Mechatronics Components, Innopolis University, Innopolis, Russia*
[****] *Department of Computer Applications and Systems, Saint Petersburg State University, Saint Petersburg, Russia (e-mail: vediakova@gmail.com)*

Abstract: This paper is devoted to a frequency estimation of a pure sinusoidal signal in finite-time. The parameterization is based on applying delay operators to a measurable signal. The result is the first-order linear regression model with one parameter, which depends on the signal frequency. The proposed method of finite-time estimation consists of two steps. On the first step, the standard gradient descent method is used to estimate the regression model parameter. On the next step using algebraic equations, finite-time frequency estimate is found. The described method does not require measuring or calculating derivatives of the input signal and uses one integrator for the gradient method and another one for the finite-time estimation. The efficiency of the proposed approach is demonstrated through the set of numerical simulations.

Keywords: parameter estimation, observers, frequency estimation, finite-time estimation

1. INTRODUCTION

The parameter estimation of sinusoidal signals is an important and classical problem which has received much attention from control system theory, see for instance Stoica (1993). From a control systems perspective, an online estimation is the main field of interest. We are interested here in the continuous-time estimation of the frequency ω of a pure sinusoidal signal $y(t) = A\sin(\omega t + \phi)$, where $y(t)$ is measurable. There are a lot of parameter estimation methods of sinusoidal signals such as Fourier analysis, least-squares method, Kalman's filter, adaptive notch filter, state-variable filtering techniques etc., see for instance some recent works Pin et al. (2017), Fedele et al. (2016), Chen et al. (2017), Na et al. (2015) Yang and Zhao (2011).

An online frequency estimation is widely used in many practical applications, for example, as an essential part of active noise and vibration control, and disturbance rejection systems Pyrkin and Bobtsov (2016), in a hard disk drive servo system in order to increase data density Goh et al. (2001), in precise positioning systems for nanotechnology Aphale et al. (2008), in dynamic positioning systems for vessels in the presence of waves, winds, and current Takahashi et al. (2007), in the processing of measurement data in gravimetry Koshaev et al. (2019). In power systems, a frequency estimation is used for load balancing, fault detection, enhance power quality Xia et al. (2012); Phan et al. (2016). The sensorless speed estimation approach based on real-time frequency estimation is proposed by Roque et al. (2014).

During the last two decades, a global convergent online frequency estimation is extensively studied. Some approaches are not restricted to the case of a single sinusoid, in particular, a signal with bias is considered in Bobtsov and Romasheva (2007); Bobtsov (2008); Fedele et al. (2010); Chen et al. (2015); Pin et al. (2017), and the multisinusoidal case is presented in Pyrkin et al. (2015); Fedele et al. (2016); Pin et al. (2015); Chen et al. (2014); Hou (2012); Carnevale and Galeani (2011); Sharma and Kar (2008); Carnevale and Astolfi (2008); Hou (2007); Marino and Tomei (2002); Obregon-Pulido et al. (2002); Bobtsov et al. (2002); Bobtsov and Lyamin (2000). Estimator with the smallest possible dimension, $3n-1$ ($3n$ for biased signal), is constructed in Carnevale and Astolfi (2008); Pyrkin et al. (2015). Estimation problem of the rest multiharmonic signal parameters is also solved, in particular, by Pyrkin et al. (2015); Na et al. (2015); Chen et al. (2015); Fedele

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(2012). Some researchers extend methods to the case when measured signal is corrupted by noise Aranovskii et al. (2007); Pyrkin et al. (2011); Fedele (2012); Chen et al. (2014, 2015); Na et al. (2015); Fedele et al. (2016); Pin et al. (2017).

A desired property of the algorithm is global convergence of the estimation error to zero. It allows proving the stability of the closed-loop system with such estimator. The first method with global convergence is proposed by Hsu et al. (1999). In this paper, the authors modify the continuous-time version of the adaptive notch filter Regalia (1991) by scaling and normalizing procedure.

Usually, researchers use Lyapunov methods to analyze convergence of the estimates. A different approach is described in Sharma and Kar (2008), where asymptotic convergence is proved using contraction theory.

Recently the problem of finite-time estimation has become very popular, see for instance Pin et al. (2017); Na et al. (2015); Pyrkin et al. (2011); Gerasimov et al. (2018); Ortega et al. (2019b).

In this paper, we propose a novel finite-time frequency estimation of a pure sinusoidal signal. The first-order linear regression model is constructed using delay operators and some trigonometric identities. The standard gradient descent algorithm estimates the regression model parameter. With this estimate, the frequency value is reconstructed at finite-time.

This paper is organized as follows. The problem statement is described in Section 2. The main result is presented in Section 3, where linear regression model and finite-time estimator are constructed. In Section 4 the computer simulation results of the proposed algorithm are included confirming the efficiency of the approach.

2. PROBLEM FORMULATION

Consider the measured signal

$$y(t) = A\sin(\omega t + \phi), \tag{1}$$

where $A \in \mathbb{R}_+$ is the amplitude, $\omega \in \mathbb{R}_+$ is the frequency and $\phi \in \mathbb{R}$ is the phase. A, ω and ϕ are unknown constant parameters..

The goal is to construct an estimate $\hat{\omega}(t)$ of the frequency ω with finite-time convergence of the error $\tilde{\omega}(t) = \omega - \hat{\omega}(t)$ to zero.

Assumption 1. The upper bound on the signal frequency ω is known and equal to $\bar{\omega}$.

3. MAIN RESULT

3.1 Preliminaries

In Aranovskiy et al. (2010) authors proposed a simple frequency identification algorithm of a biased harmonic signal. Dynamical order of this algorithm is equal to three that is less than similar algorithms have:

$$\begin{cases} \dot{\chi}(t) &= k_1\dot{\zeta}(t)\left[-2\alpha\ddot{\zeta}(t) - \alpha^2\dot{\zeta}(t)\right] \\ & \quad -k_1\dot{\zeta}^2(t)\hat{\theta}(t) - k_1\ddot{\zeta}(t)y(t), \\ \hat{\theta}_1(t) &= \chi(t) + k_1\dot{\zeta}_1(t)y(t), \\ \dot{\zeta}_1(t) &= \zeta_2(t), \\ \dot{\zeta}_2(t) &= -2\alpha\zeta_2(t) - \alpha^2\zeta_1(t) + y(t), \\ \zeta(t) &= \zeta_1(t), \end{cases} \tag{2}$$

where α and k_1 are positive constant parameters, the frequency estimation is equal to $\hat{\omega}_1(t) = \sqrt{\left|\hat{\theta}_1(t)\right|}$.

In Gromov et al. (2017) a first order estimator for a pure sinusoidal signal is proposed. In that paper authors proposed a new parameterization of the sinusoidal signal which is based on the delay operators. The parameterization leads to a linear regression model and then the standard gradient descent method can be applied to estimate the parameter

$$\begin{cases} \hat{\omega}_2(t) &= \arccos(\hat{\beta}(t))/\tau, \\ \dot{\hat{\beta}}(t) &= k_2 y_1(t)\left[\psi(t) - 2\hat{\beta}(t)y_1(t)\right], \\ \psi(t) &= y(t) + y_2(t), \\ y_1(t) &= y(t-\tau), \\ y_2(t) &= y(t-2\tau), \end{cases} \tag{3}$$

where k_2 is positive coefficient, τ is constant delay.

It is shown that the described method provides exponential semi-global convergence of the frequency estimation error to zero.

In this paper the results obtained in Gromov et al. (2017) are extended. Initial equation is parameterized to receive a regression model. Then an algorithm which provides convergence of estimate to a real value within a finite time is proposed.

3.2 Parameterization

Let us construct a linear regression model with measurable variables and a vector of constant parameters that depends on the unknown frequency ω.

Consider two auxiliary transport delay operators with the following outputs

$$y_1(t) = y(t-\tau), \tag{4}$$
$$y_2(t) = y(t-2\tau), \tag{5}$$

where $\tau \in \mathbb{R}_+$ is a known constant. The signals (4) and (5) can be rewritten as

$$\begin{aligned} y_1(t) &= y(t-\tau) = A\sin(\omega t - \omega\tau + \phi) \\ &= A\sin(\omega t + \varphi)\cos(\omega\tau) - A\cos(\omega t + \phi)\sin(\omega\tau) \\ &= y(t)\cos(\omega\tau) - A\cos(\omega t + \phi)\sin(\omega\tau), \end{aligned} \tag{6}$$

$$\begin{aligned} y_2(t) &= y(t-2\tau) = A\sin(\omega t - 2\omega\tau + \phi) \\ &= A\sin(\omega t + \phi)\cos(2\omega\tau) - A\cos(\omega t + \phi)\sin(2\omega\tau) \\ &= y(t)\cos(2\omega\tau) - A\cos(\omega t + \phi)\sin(2\omega\tau). \end{aligned} \tag{7}$$

Consider a new variable

$$\nu(t) = y_1(t)\sin(2\omega\tau) - y_2(t)\sin(\omega\tau). \tag{8}$$

Let us transform the model (8):

$$\begin{aligned}\nu(t) &= 2y_1(t)\sin(\omega\tau)\cos(\omega\tau) - y_2(t)\sin(\omega\tau)\\ &= [y(t)\cos(\omega\tau) - A\cos(\omega t+\phi)\sin(\omega\tau)]\sin(2\omega\tau)\\ &\quad - [y(t)\cos(2\omega\tau) - A\cos(\omega t+\phi)\sin(2\omega\tau)]\sin(\omega\tau)\\ &= y(t)\left[\cos(\omega\tau)\sin(2\omega\tau) - \cos(2\omega\tau)\sin(\omega\tau)\right]\\ &= y(t)\left[2\cos^2(\omega\tau)\sin(\omega\tau) - \sin(\omega\tau)(\cos^2(\omega\tau)\right.\\ &\quad \left. - \sin^2(\omega\tau))\right] = y(t)\sin(\omega\tau).\end{aligned}$$

So we can write

$$2y_1(t)\sin(\omega\tau)\cos(\omega\tau) - y_2(t)\sin(\omega\tau) = y(t)\sin(\omega\tau). \quad (9)$$

After dividing both sides of the last equation by $\sin(\omega\tau)$ we have

$$\begin{aligned}2y_1(t)\cos(\omega\tau) - y_2(t) &= y(t),\\ \cos(\omega\tau)y_1(t) &= \frac{1}{2}\left[y(t) + y_2(t)\right]. \quad (10)\end{aligned}$$

Remark 1. To guarantee that $\sin(\omega\tau) \neq 0$ the delay τ should be chosen such that

$$\tau < \frac{\pi}{\bar{\omega}}. \quad (11)$$

From equation (10) a regression model can be written as

$$z(t) = \theta_3\phi(t), \quad (12)$$

where

$$z(t) = \frac{1}{2}\left[y(t) + y_2(t)\right], \quad (13)$$

$\phi(t) = y_1(t)$ and $\theta_3 = \cos(\omega\tau)$.

3.3 Gradient estimator

Parameter θ_3 can be estimated from equation (12) using the standard gradient descent method:

$$\dot{\hat{\theta}}_3(t) = -k_3\phi^2(t)\hat{\theta}_3(t) + k_3\phi(t)z(t), \quad (14)$$

where k_3 is a positive coefficient.

Then frequency can be found from (14):

$$\hat{\omega}_3(t) = \frac{1}{\tau}\arccos\left(\hat{\theta}_3(t)\right). \quad (15)$$

3.4 Finite-time estimator

In the previous subsection parameterization of model (1) is carried out and the gradient estimator is used to reconstruct the frequency ω. Now we can obtain a finite-time observer, see for instance Gerasimov et al. (2018), Ortega et al. (2019b), Ortega et al. (2019a).

Proposition 1. Consider regression model (12) and the parameter estimator (14).

Then for

$$t \geq t_1 : \int_0^{t_1} \phi^2(\tau)\mathrm{d}\tau > 0, \quad (16)$$

we can find a finite-time parameter estimator of θ

$$\hat{\theta}_F(t) = \frac{\hat{\theta}_3(t) - \hat{\theta}_3(0)w_c(t)}{1 - w_c(t)}, \quad (17)$$

where

$$w_c(t) = \begin{cases}\mu & \text{if } w(t) \geq \mu,\\ w(t) & \text{otherwise,}\end{cases} \quad (18)$$

with $\mu \in (0, 1)$,

$$\dot{w}(t) = -k_3\phi^2(t)w(t), \quad w(0) = 1. \quad (19)$$

Proof. Consider the error function $\tilde{\theta}_3(t) = \hat{\theta}_3(t) - \theta_3$, then for derivative of $\tilde{\theta}_3(t)$ we have

$$\dot{\tilde{\theta}}_3(t) = \dot{\hat{\theta}}_3(t) - \dot{\theta}_3 = -k_3\phi^2(t)\tilde{\theta}_3(t). \quad (20)$$

Solution of the previous equation can be found in the following form

$$\tilde{\theta}_3(t) = \tilde{\theta}_3(0)\mathrm{e}^{-\int_0^t k_3\phi^2(\tau)\mathrm{d}\tau} = \tilde{\theta}_3(0)w(t). \quad (21)$$

Then for (21) we have

$$\begin{cases}\tilde{\theta}_3(t) = & \tilde{\theta}_3(0)w(t) = \left[\hat{\theta}_3(0) - \theta_3\right]w(t),\\ \tilde{\theta}_3(t) = & \hat{\theta}_3(t) - \theta_3.\end{cases} \quad (22)$$

From (22) we obtain

$$\theta_3 = \hat{\theta}_F(t) = \frac{\hat{\theta}_3(t) - \hat{\theta}_3(0)w(t)}{1 - w(t)}. \quad (23)$$

Using (23) to find $\hat{\omega}_F(t)$ from $\hat{\omega}_F(t) = \frac{1}{\tau}\arccos\left(\hat{\theta}_F(t)\right)$ completes the proof.

Increasing a value of μ decreases estimation duration. However, if the value is close to one, then $1 - w(t)$ is close to zero and sensitivity to noise is high.

4. SIMULATION RESULTS

In this section, we present the simulation results that illustrate the efficiency of the proposed estimation algorithm. All simulations have been performed in Mathworks Simulink.

Let us compare the simulation results of algorithm described in this paper and algorithms proposed in Aranovskiy et al. (2010) and Gromov et al. (2017). The following parameters we use for simulations:

- $\omega = 1$, $A = 1$, $\phi = 0$ in (1);
- $\alpha = 1$, $k_1 = 10$ and initial conditions $\chi(0) = 0$, $\zeta_1(0) = 0$ and $\zeta_2(0) = 0$ in algorithm (2).
- $k_2 = 10$, $\tau = 0.1$ and $\hat{\beta}(0) = 1$ in algorithm (3);
- $k_3 = 10$, $\tau = 0.1$, $\mu = 0.99$, $t_1 = 2\tau$ and $\hat{\theta}_3(0) = 1$ for finite-time method (17)–(19).

Fig. 1 demonstrates the transients of frequency estimates for algorithms from Aranovskiy et al. (2010), Gromov et al. (2017) and algorithm (17)–(19).

Fig. 2 demonstrates transients of the frequency estimates for the standard gradient decent method from (14)–(15) and for the finite-time method (time $t_1 = 0.03$s is used for simulation of finite-time observer) for sinusoidal signal $y(t) = \sin(10t)$, with parameters $\tau = 0.01$, $\mu = 0.99$ and the initial condition $\hat{\theta}_3(0) = 1$. In all cases the finite-time estimate is obtained at $t_1 = 0.03$.

Fig. 3 demonstrates the transients of the frequency estimates $\hat{\omega}_3(t)$ for the standard gradient decent method from (14)–(15) and $\hat{\omega}_F(t)$ for the proposed algorithm (17)–(19) for the sinusoidal signal $y(t) = \sin(\omega t)$ with different values of frequency ω and fixed value of parameter $k = 50$, parameters $\tau = 0.01$, $\mu = 0.99$ and the initial condition $\hat{\theta}_3(0) = 1$. In this case also the performance is independent of the frequency value.

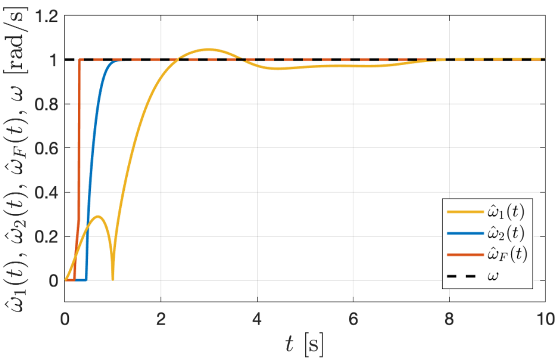

Fig. 1. The frequency ω and its estimates for algorithms: 1) $\hat{\omega}_1(t)$ for Aranovskiy et al. (2010); 2) $\hat{\omega}_2(t)$ for Gromov et al. (2017); 3) $\hat{\omega}_F(t)$ for finite-time method (17)–(19)

To demonstrate the robustness of the proposed algorithm, let us consider the noised measured signal

$$y(t) = \sin(20t) + \delta(t), \tag{24}$$

where additive noise $\delta(t)$ is simulated as a uniformly distributed process ranging within $[-0.5, 0.5]$ whose influence is shown as the blue curve plotted in Fig. 4.

The simulation result for the noise scenario with parameters $k = 3$, $\tau = 0.1$ is shown in Fig. 5. The proposed algorithm is still able to provide finite-time estimate in presence of additive disturbance.

The time behavior of $\hat{\omega}(t)$ and $\hat{\omega}_F(t)$ for measured signal $y(t)$ with an additive exponentially correlated noise $\Delta(t)$ is shown in Fig. 7. The noise is modeled by a shaping filter $W(s) = 3/(s + 50)$ with frequency-bounded input white noise of power $N = 0.1$. In this case $k = 3$, $\tau = 0.1$. The effect of the additive noise is shown as the blue curve plotted in Fig. 6. The identification algorithm demonstrates robustness to additive perturbations.

5. CONCLUSION

The finite-time frequency estimation problem for a pure sinusoidal signal without bias was considered. A linear regression model with measured variables and a vector of constant parameters that depends on the unknown frequency was obtained using transport delay operators.

Decreasing tuning gain in the gradient descent method, we decrease sensitivity to the measurement noise. Without the finite-time estimation scheme, it dramatically increases estimation duration. However, with this scheme estimate is obtained at the same predefined time, but it is less affected by noise, because performance of the proposed method is independent of the frequency value and the tuning gain of the used internally gradient descent method.

Future investigations will be devoted to extending the methodology to the case of multisinusoidal signal estimation.

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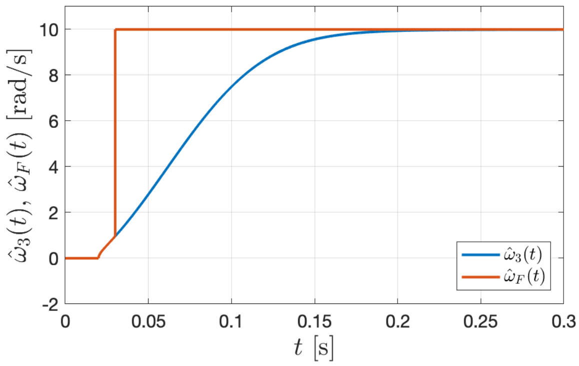
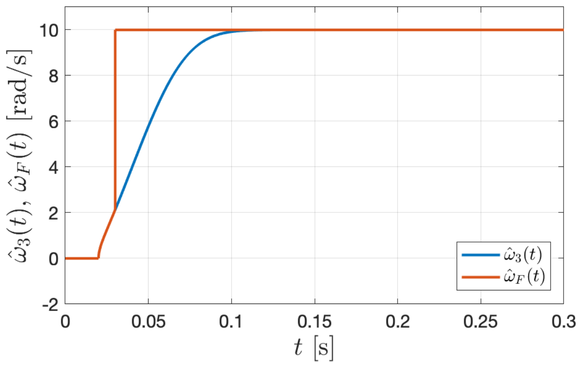
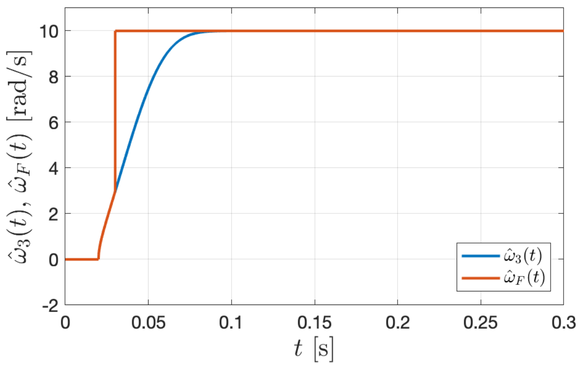


(a) $k_3 = 10$

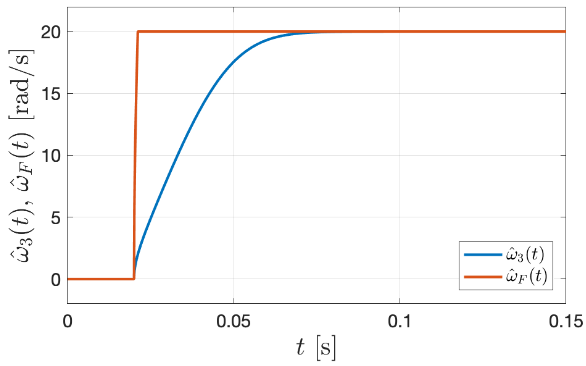


(b) $k_3 = 50$

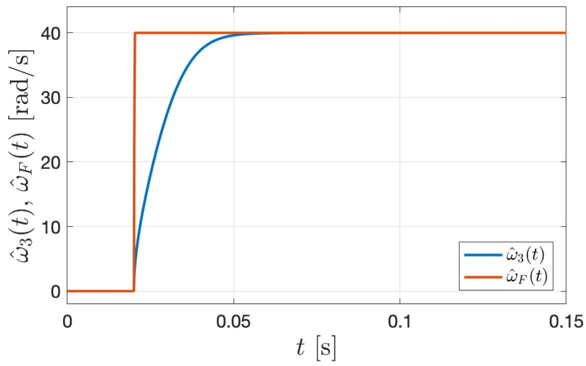


(c) $k_3 = 100$

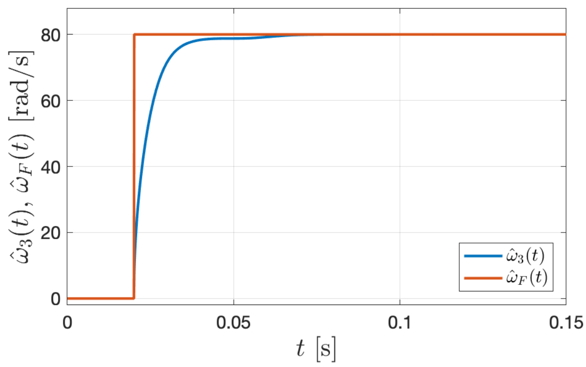
Fig. 2. The estimates $\hat{\omega}_3(t)$ and $\hat{\omega}_F(t)$ of the frequency ω for different values of parameter k_3



(a) $\omega = 20$



(b) $\omega = 40$



(c) $\omega = 80$

Fig. 3. The estimates $\hat{\omega}_3(t)$ and $\hat{\omega}_F(t)$ for different values of frequency ω

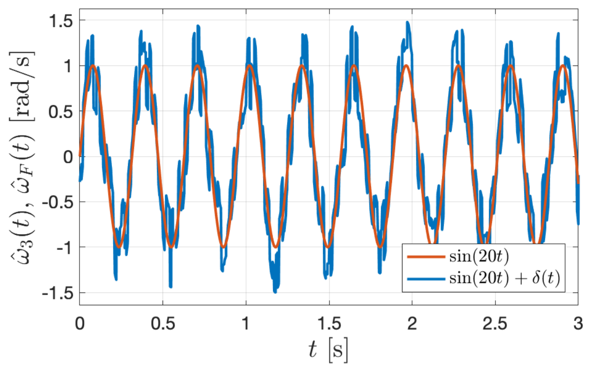


Fig. 4. The sinusoidal signal $y(t)$ with additive noise

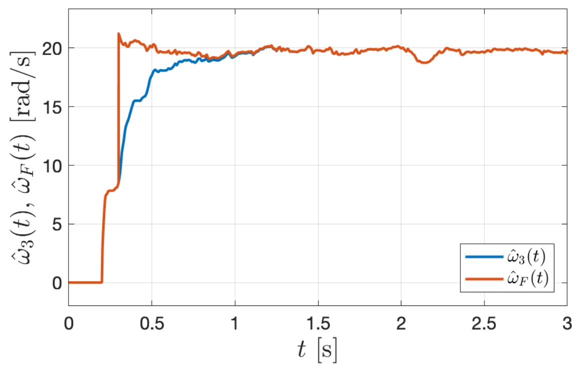


Fig. 5. The estimates $\hat{\omega}_3(t)$ and $\hat{\omega}_F(t)$ of the frequency $\omega = 20$ rad/s

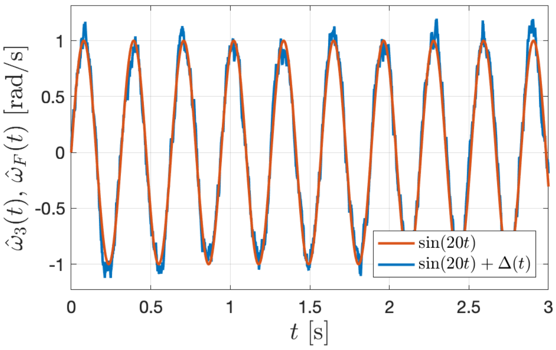


Fig. 6. The sinusoidal signal $y(t)$ with additive noise

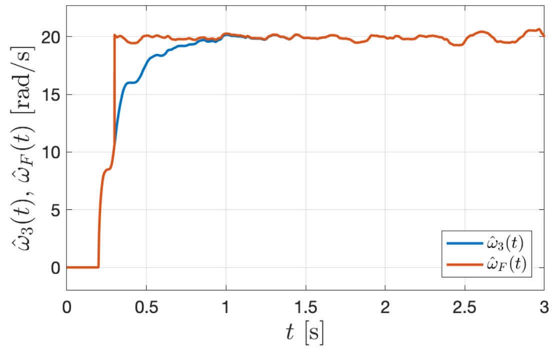


Fig. 7. The estimates $\hat{\omega}_3(t)$ and $\hat{\omega}_F(t)$ of the frequency $\omega = 20$ rad/s

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