Distributed Active Faults Diagnosis for Systems with Conditionally Dependent Faults ⋆

Ondřej Straka * **Ivo Punčochář** *

* *Department of Cybernetics, European Centre of Excellence NTIS, Faculty of Applied Sciences, University of West Bohemia,*
Address: Univerzitní 8, 306 14 Pilsen, Czech Republic
(e-mail: {straka30,ivop}@kky.zcu.cz).

Abstract: The paper deals with active fault diagnosis of stochastic large scale systems for the cases when the fault in one subsystem may change probability of occurrence of faults in other subsystems. The multiple model framework is considered and each subsystem is represented by a set of models describing fault-free and faulty behavior. The transitions between them are characterized probabilistically. The paper proposes two active fault diagnosis algorithms, a decentralized one and a distributed one. Their performance is compared in a numerical example.

Keywords: fault diagnosis, large scale systems, multiple models, interacting multiple models

1. INTRODUCTION

Complexity and degree of integration of large scale systems (LSSs) increase their liability to faults, which are undesirable changes in a monitored system caused by external or internal incidents. Since the faults may cause failures of the monitored system with catastrophic consequences, it is essential to detect them reliably and quickly by a fault diagnosis (FD) method.

The literature recognizes two fundamental FD approaches, which differ in the interaction with the monitored system. In the *passive* approach, the decisions generated by a FD algorithm are based on passive observations of monitored system quantities (Isermann, 2011; Blanke et al., 2016). The *active* approach, in addition to processing the measurable quantities, generates an input signal to excite the monitored system. The excitation purpose is to obtain more information, which helps to detect faults that may be challenging to detect using the passive FD.

In the last decade, the active FD (AFD) approach has gained in popularity (Ashari et al., 2012; Punčochář et al., 2015; Boem et al., 2019). For the AFD for stochastic systems, the multiple-model framework is used almost exclusively to describe fault-free and faulty models of the system (Blackmore et al., 2008).

Limited communication bandwidth and computational power are two main reasons for developing special algorithms for LSSs. In Punčochář and Straka (2019), a new AFD framework for stochastic LSSs was introduced and three architectures – centralized, decentralized, and distributed were proposed. The paper Straka and Punčochář (2019) addressed the problem of AFD for LSSs where only noisy indirect measurements were available to the AFD nodes. The papers assumed independent behavior of the faults of LSS subsystems, i.e., the subsystem behavior (faulty or fault-free) does not affect behavior of other subsystems. This may not be always true. For example, a fault in a transport network channel increases workload of other channels, which are thus more susceptible to faults.

The aim of this paper is to remove such an assumption and to design an AFD algorithm for LSSs where the faults are conditionally dependent. In particular, two architectures, the decentralized and the distributed will be considered.

The paper is structured as follows: Section 2 provides the LSS description and the AFD problem formulation. The decentralized and the distributed solutions are proposed in sections 3 and 4, respectively. Section 5 presents a numerical illustration.

2. PROBLEM FORMULATION

This section specifies the LSS and formulates the AFD problem. The block diagram for the AFD system is depicted in Fig. 1.

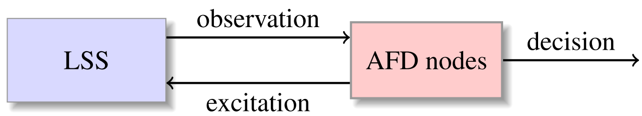

Fig. 1. AFD block diagram.

2.1 System Specification

In principle, the AFD for an LSS can be designed in a centralized manner, which is not, however, computationally feasible. A decomposition of the LSS and subsequent decentralized or distributed design of the AFD (Punčochář and Straka, 2019) represent a feasible concept. In this paper, the LSS is assumed to be decomposed into weakly coupled subsystems with separate control inputs.

The LSS Σ consists of N weakly coupled subsystems [1] ${}^{n}\Sigma,\ n \in \mathcal{N} = \{1, 2, \ldots, N\}$.

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[1] The subsystem ${}^{n}\Sigma$ coupled due to appearance of $\mathbf{x}_k$ in (1a) affects other subsystems to a lesser extent than it is affected by the dynamics of ${}^{n}\mathbf{x}_k$.

Each subsystem is described at a time step $k \in \mathcal{T} = \{0, 1, 2, \ldots\}$ by a time-invariant discrete-time stochastic multiple-model expressed in terms of a stochastic difference equation describing the local state dynamics and a stochastic algebraic equation describing relations between the local state and observation [2]

$$^{n}\Sigma: \ ^{n}\mathbf{x}_{k+1} = {}^{n}\mathbf{f}\left(\mathbf{x}_k, {}^{n}\mu_k, {}^{n}\mathbf{u}_k\right) + {}^{n}\mathbf{F}({}^{n}\mu_k)\,{}^{n}\mathbf{w}_k, \quad (1a)$$

$$^{n}\mathbf{y}_k = {}^{n}\mathbf{h}\left({}^{n}\mathbf{x}_k, {}^{n}\mu_k\right) + {}^{n}\mathbf{H}({}^{n}\mu_k)\,{}^{n}\mathbf{v}_k, \quad (1b)$$

where ${}^{n}\mathbf{x}_k \in \mathbb{R}^{({}^{n}D_x)}$ is a continuous part of the local state related to ${}^{n}\Sigma$, ${}^{n}\mu_k \in {}^{n}\mathcal{M} = \{1, 2, \ldots, {}^{n}M\}$ is a discrete part of the local state representing an index into a set of possible models of subsystem ${}^{n}\Sigma$ behavior [3], ${}^{n}\mathbf{u}_k \in {}^{n}\mathcal{U} \subset \mathbb{R}^{({}^{n}D_u)}$ is an input of ${}^{n}\Sigma$, ${}^{n}\mathbf{w}_k \in \mathbb{R}^{({}^{n}D_x)}$ is a state noise of ${}^{n}\Sigma$ described by the probability density function (PDF) $p_{{}^{n}\mathbf{w}_k}$, ${}^{n}\mathbf{y}_k \in \mathbb{R}^{({}^{n}D_y)}$ is the observation of both ${}^{n}\mathbf{x}_k$ and ${}^{n}\mu_k$ and ${}^{n}\mathbf{v}_k \in \mathbb{R}^{({}^{n}D_y)}$ is an observation noise of ${}^{n}\Sigma$ described by the PDF $p_{{}^{n}\mathbf{v}_k}$. The set of possible models includes a model for the fault-free behavior, ${}^{n}\mu_k = 1$, and also several models for possible faulty behavior, ${}^{n}\mu_k \in \{2, \ldots, {}^{n}M\}$.

The functions ${}^{n}\mathbf{f} : \mathbb{R}^{(D_x)} \times {}^{n}\mathcal{M} \times {}^{n}\mathcal{U} \mapsto \mathbb{R}^{{}^{n}D_x}$, ${}^{n}\mathbf{h} : \mathbb{R}^{({}^{n}D_x)} \times {}^{n}\mathcal{M} \mapsto \mathbb{R}^{{}^{n}D_y}$, ${}^{n}\mathbf{F} : {}^{n}\mathcal{M} \mapsto \mathbb{R}^{({}^{n}D_x \times {}^{n}D_x)}$, ${}^{n}\mathbf{H} : {}^{n}\mathcal{M} \mapsto \mathbb{R}^{({}^{n}D_y \times {}^{n}D_y)}$, the PDFs $p_{{}^{n}\mathbf{w}_k}$ and $p_{{}^{n}\mathbf{v}_k}$, and the transition probability (2) are known. The initial condition $\mathbf{x}_0$ is described by a known PDF $p_{\mathbf{x}_0}$.

The indices $\boldsymbol{\mu}_k \triangleq [{}^{1}\mu_k, \ldots, {}^{N}\mu_k] \in \mathcal{M} = {}^{1}\mathcal{M} \times \ldots \times {}^{N}\mathcal{M}$ form a discrete part of the state of Σ, which is assumed to follow a Markov process with transition probability

$$Pr(\boldsymbol{\mu}_{k+1}|\boldsymbol{\mu}_k) \quad (2)$$

with $\boldsymbol{\mu}_0$ given by a known prior probability $Pr(\boldsymbol{\mu}_0)$.

The composition $\mathbf{x}_k \triangleq [{}^{1}\mathbf{x}_k^{\mathrm{T}}, \ldots, {}^{N}\mathbf{x}_k^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{D_x}$ is a continuous part of the state of Σ with $D_x = \sum_{n=1}^{N} {}^{n}D_x$, $\mathbf{u}_k \triangleq [{}^{1}\mathbf{u}_k^{\mathrm{T}}, \ldots, {}^{N}\mathbf{u}_k^{\mathrm{T}}]^{\mathrm{T}} \in \mathcal{U} = {}^{1}\mathcal{U} \times \ldots \times {}^{N}\mathcal{U}$ is an input of the LSS and the composition $\mathbf{y}_k \triangleq [{}^{1}\mathbf{y}_k^{\mathrm{T}}, \ldots, {}^{N}\mathbf{y}_k^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{D_y}$ is an observation of Σ with $D_y = \sum_{n=1}^{N} {}^{n}D_y$.

The variables $\mathbf{x}_k$ and $\boldsymbol{\mu}_k$ constituting the state of Σ are unknown and are indirectly observable through $\mathbf{y}_k$.

The decomposition of Σ into interconnected subsystems ${}^{n}\Sigma$ is assumed to satisfy the following conditions:

(i) The noises of the subsystems are white and mutually independent,

$$p({}^{1}\mathbf{w}_k, \ldots, {}^{N}\mathbf{w}_k, {}^{1}\mathbf{v}_k, \ldots, {}^{N}\mathbf{v}_k) = \prod_{n=1}^{N} p_{{}^{n}\mathbf{w}_k}({}^{n}\mathbf{w}_k)\, p_{{}^{n}\mathbf{v}_k}({}^{n}\mathbf{v}_k),$$

(ii) The initial states ${}^{n}\mathbf{x}_0$ and the initial model indices ${}^{n}\mu_0$ are also independent and mutually independent, i.e., $p(\mathbf{x}_0, \boldsymbol{\mu}_0) = \prod_{n=1}^{N} p_{{}^{n}\mathbf{x}_0}({}^{n}\mathbf{x}_0) Pr({}^{n}\mu_0)$.

The assumption (i) expresses conditional independence between ${}^{i}\mathbf{x}_{k+1}$ and ${}^{j}\mathbf{x}_{k+1}$ given $\mathbf{x}_k$, $\mathbf{u}_k$, and $\boldsymbol{\mu}_k$, i.e. given the current state and the control input of Σ, the subsystems ${}^{i}\Sigma$ and ${}^{j}\Sigma$ behave independently at the next time step. In Punčochář and Straka (2019) and Straka and Punčochář (2019) also the following assumption was considered $Pr(\boldsymbol{\mu}_{k+1}|\boldsymbol{\mu}_k) = \prod_{n=1}^{N} Pr({}^{n}\mu_{k+1}|\,{}^{n}\mu_k)$, which corresponds to the case when occurrence of a fault in a subsystem does not influence probabilities of occurrences of faults in other subsystems. The faults do not spread from one subsystem to another. In this paper, however, such assumption is abandoned and the model indices are assumed conditionally dependent in the sense that

$$Pr(\boldsymbol{\mu}_{k+1}|\boldsymbol{\mu}_k) = \prod_{n=1}^{N} Pr({}^{n}\mu_{k+1}|\boldsymbol{\mu}_k), \quad (3)$$

where $Pr({}^{n}\mu_{k+1}|\boldsymbol{\mu}_k) \neq Pr({}^{n}\mu_{k+1}|\,{}^{n}\mu_k)$.

2.2 *AFD problem specification*

The AFD problem is formulated as designing a function transforming the complete available information to a decision about the faults (subsystem models) and to an input excitation signal. The role of the signal is to excite the system to improve the detection quality [4]. The AFD system can be described as

$$\Delta: \quad \begin{bmatrix}\mathbf{d}_k \\ \mathbf{u}_k\end{bmatrix} = \boldsymbol{\rho}_k\left(\mathbf{I}_0^k\right) = \begin{bmatrix}\boldsymbol{\sigma}_k\left(\mathbf{I}_0^k\right) \\ \gamma_k\left(\mathbf{I}_0^k\right)\end{bmatrix}, \quad (4)$$

where $\mathbf{I}_0^k \triangleq [(\mathbf{y}_0^k)^{\mathrm{T}}, (\mathbf{u}_0^{k-1})^{\mathrm{T}}]^{\mathrm{T}} \in \mathcal{I}^k$ denotes global data observed up to $k \in \mathcal{T}$ with $\mathcal{I}^k \triangleq \mathbb{R}^{(k+1)D_y} \times \mathcal{U}^k$, $\mathcal{U}^k \triangleq \mathcal{U} \times \ldots \times \mathcal{U}$. The vector $\mathbf{d}_k \triangleq [{}^{1}d_k, {}^{2}d_k, \ldots, {}^{N}d_k]^{\mathrm{T}} \in \mathcal{M}$ consists of the decisions ${}^{n}d_k \in {}^{n}\mathcal{M}$ about the model indices ${}^{n}\mu_k$, $\boldsymbol{\sigma}_k : \mathcal{I}^k \mapsto \mathcal{M}$ represents the fault detector at k, and $\gamma_k : \mathcal{I}^k \mapsto \mathcal{U}$ describes the input signal generator.

The AFD seeks sequences of functions $\boldsymbol{\sigma}_0^{\infty}$ and γ_0^{∞} that minimize the following additive discounted criterion

$$J(\boldsymbol{\sigma}_0^{\infty}, \gamma_0^{\infty}) = \lim_{F \to \infty} \mathrm{E}\left\{\sum_{k=0}^{F} \eta^k L^{\mathrm{d}}(\boldsymbol{\mu}_k, \mathbf{d}_k)\right\}, \quad (5)$$

where $\mathrm{E}\{\cdot\}$ is the expectation over all involved random variables, $\eta \in (0, 1)$ is a chosen discount factor, and $L^{\mathrm{d}} : \mathcal{M} \times \mathcal{M} \mapsto \mathbb{R}^{+}$ is a detection cost function that allows different costs to be assigned for selecting the vector of decisions $\mathbf{d}_k$ when the vector of model indices $\boldsymbol{\mu}_k$ is actually in effect. If the costs are not related across the subsystems, it is reasonable to consider

$$L^{\mathrm{d}}(\boldsymbol{\mu}_k, \mathbf{d}_k) = \sum_{n=1}^{N} {}^{n}L^{\mathrm{d}}\left({}^{n}\mu_k, {}^{n}d_k\right), \quad (6)$$

where ${}^{n}L^{\mathrm{d}} : {}^{n}\mathcal{M} \times {}^{n}\mathcal{M} \mapsto \mathbb{R}^{+}$ penalizes discrepancy between the model index ${}^{n}\mu_k$ (${}^{n}\Sigma$ true behavior) and the decision ${}^{n}d_k$.

3. DESIGN OF DECENTRALIZED AFD

In the decentralized AFD architecture depicted in Fig. 2, each subsystem ${}^{n}\Sigma$ is monitored by an AFD node ${}^{n}\Delta$, which can access only the subsystem observations and knows only the model describing the subsystem behavior. Since the AFD nodes do not communicate, they can be described by

$${}^{n}\Delta: \begin{bmatrix}{}^{n}d_k \\ {}^{n}\mathbf{u}_k\end{bmatrix} = \begin{bmatrix}{}^{n}\sigma_k^{\mathrm{dec}}\left({}^{n}\mathbf{I}_0^k\right) \\ {}^{n}\gamma_k^{\mathrm{dec}}\left({}^{n}\mathbf{I}_0^k\right)\end{bmatrix}, \quad (7)$$

where ${}^{n}\mathbf{I}_0^k \triangleq [({}^{n}\mathbf{y}_0^k)^{\mathrm{T}}, ({}^{n}\mathbf{u}_0^{k-1})^{\mathrm{T}}]^{\mathrm{T}} \in {}^{n}\mathcal{I}^k$ denotes local data observed up to $k \in \mathcal{T}$ by the n-th AFD node with ${}^{n}\mathcal{I}^k \triangleq$

[2] Note that the following notation is used throughout the text. A variable or a function with left superscript pertains to the corresponding subsystem, whereas a variable or a function without the left superscript relates to the whole LSS. The variable with the right subscript and superscript $\mathbf{x}_i^j \triangleq [\mathbf{x}_i^{\mathrm{T}}, \mathbf{x}_{i+1}^{\mathrm{T}}, \ldots, \mathbf{x}_j^{\mathrm{T}}]^{\mathrm{T}}$ with $j > i$ stands for the whole sequence of variables stacked into a column vector.

[3] That is, ${}^{n}\mu_k$ is the index of the unknown model that is active at time k on ${}^{n}\Sigma$.

[4] Note that the input signal may also possess a control role, where a compromise between the roles is established by the criterion.

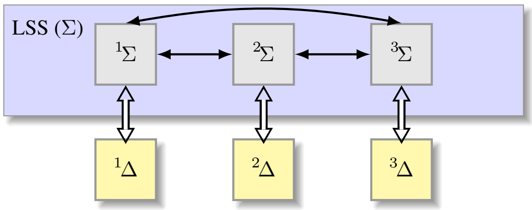

Fig. 2. Decentralized AFD system structure

$\mathbb{R}^{(k+1)(^nD_y)} \times {}^n\mathcal{U}^k$, ${}^n\mathcal{U}^k \triangleq {}^n\mathcal{U} \times \ldots \times {}^n\mathcal{U}$. The function ${}^n\sigma_k^{\text{dec}} : {}^n\mathcal{I}^k \mapsto {}^n\mathcal{M}$ is the fault detector at the time step k, and ${}^n\gamma_k^{\text{dec}} : {}^n\mathcal{I}^k \mapsto {}^n\mathcal{U}$ is the input signal generator at k.

To avoid the computationally complex centralized design, the weak couplings among subsystems are neglected and the following approximate subsystem model is considered

$$^n\hat{\Sigma} : {}^n\mathbf{x}_{k+1} = {}^n\hat{\mathbf{f}}({}^n\mathbf{x}_k, {}^n\mu_k, {}^n\mathbf{u}_k) + {}^n\mathbf{F}({}^n\mu_k)\, {}^n\mathbf{w}_k, \tag{8a}$$
$$^n\mathbf{y}_k = {}^n\mathbf{h}({}^n\mathbf{x}_k, {}^n\mu_k) + {}^n\mathbf{H}({}^n\mu_k)\, {}^n\mathbf{v}_k,\ n \in \mathcal{N}, \tag{8b}$$

where ${}^n\hat{\mathbf{f}}$ is an approximation of ${}^n\mathbf{f}$ neglecting the coupling.

The elimination of the coupling in the transition probabilities (3) requires specification of the transition probabilities

$$Pr({}^n\mu_{k+1}|\, {}^n\mu_k) \tag{9}$$

which are given by

$$Pr({}^n\mu_{k+1}|\, {}^n\mu_k) = \sum_{\bar{n}\mu_k,\ \bar{n}\in\bar{\mathcal{N}}} Pr({}^n\mu_{k+1}|\boldsymbol{\mu}_k) \frac{Pr(\boldsymbol{\mu}_k)}{Pr({}^n\mu_k)}, \tag{10}$$

where $\bar{\mathcal{N}} \triangleq \mathcal{N} \setminus \{n\}$. As the infinite horizon is considered, the term $\frac{Pr(\boldsymbol{\mu}_k)}{Pr({}^n\mu_k)}$ can be approximated as

$$\frac{Pr(\boldsymbol{\mu}_k)}{Pr({}^n\mu_k)} \approx \lim_{k\to\infty} \frac{Pr(\boldsymbol{\mu}_k)}{Pr({}^n\mu_k)} \tag{11}$$

provided that the stationary probabilities exist. For such approximate model (8) and (9), the AFD nodes (7) can be designed independently for each subsystem.

The solution to the problem of the optimal AFD uses the Bellman functional equation (Punčochář et al., 2015) for the optimal input generation. If the state of the subsystem defined as a composition of ${}^n\mu_k$ and ${}^n\mathbf{x}_k$

$$^n\mathbf{s}_k \triangleq [({}^n\mathbf{x}_k)^{\mathrm{T}}, {}^n\mu_k]^{\mathrm{T}} \in {}^n\mathcal{S} = \mathbb{R}^{^nD_x} \times {}^n\mathcal{M}, \tag{12}$$

is known, it would be the Bellman function argument.

However, the subsystem state ${}^n\mathbf{s}_k$ is unknown and thus the searching for (7) is referred to as the problem with imperfect state information. The subsystem state will be replaced in the Bellman function by an information state ${}^n\boldsymbol{\xi}_k$ (defined later) that is obtained from the observed data by an estimation algorithm. In this way, the problem with imperfect state information is reformulated to the problem with perfect state information. That is, instead of the propagation of the unknown state ${}^n\mathbf{s}_k$, known information about it, given by ${}^n\boldsymbol{\xi}_k$ obtained by the estimation algorithm should be propagated.

The information state consists of the information about ${}^n\mathbf{s}_k$ given ${}^n\mathbf{I}_0^k$. Its dynamics is obtained by coupling the subsystem behavior (8) and a state estimation algorithm and can be expressed as

$$p({}^n\mathbf{s}_{k+1}|\, {}^n\mathbf{I}_0^{k+1}) = {}^n\boldsymbol{\varphi}\left(p({}^n\mathbf{s}_k|\, {}^n\mathbf{I}_0^k), {}^n\mathbf{u}_k, {}^n\mathbf{y}_{k+1}\right), \tag{13}$$

where $p({}^n\mathbf{s}_k|\, {}^n\mathbf{I}_0^k)$ is the conditional PDF provided by the estimation algorithm based on the Bayesian recursive relations (BRRs) (Bar-Shalom et al., 2001) and ${}^n\boldsymbol{\varphi} : \mathcal{L} \times {}^n\mathcal{U} \times \mathbb{R}^{^nD_y} \mapsto \mathcal{L}$ is the mapping that describes the evolution of the conditional PDF of the state and $\mathcal{L}$ is a set of all possible PDFs. The model (13) is denoted as the perfect state information model. Note that the future output ${}^n\mathbf{y}_{k+1}$ in (13) is considered to be a random variable with the conditional PDF $p({}^n\mathbf{y}_{k+1}|\, {}^n\mathbf{I}_0^k, {}^n\mathbf{u}_k)$.

Note that the estimation algorithm generates only an approximation of $p({}^n\mathbf{s}_k|\, {}^n\mathbf{I}_0^k)$ in the form of a mixture PDF with a fixed number of terms as an exact solution to the BRRs involves exponential increase of the number of mixture terms, which is not feasible. For the purpose of keeping the number of terms fixed, the paper utilizes the Generalized Pseudo-Bayes algorithm of the 2nd order (GPB2) (Straka and Punčochář, 2019).

The information stored in $p({}^n\mathbf{s}_k|\, {}^n\mathbf{I}_0^k)$ consists of ${}^nM$ terms $p({}^n\mathbf{x}_k|\, {}^n\mathbf{I}_0^k, {}^n\mu_k)$, ${}^n\mu_k \in {}^n\mathcal{M}$ and corresponding probabilities $Pr({}^n\mu_k|\, {}^n\mathbf{I}_0^k)$ and has to be transformed to the information state ${}^n\boldsymbol{\xi}_k$. Since the model index ${}^n\mu_k$ is a discrete random variable, the conditional probability $Pr({}^n\mu_k|\, {}^n\mathbf{I}_0^k)$ can be represented by a column vector ${}^n\boldsymbol{\pi}_k({}^n\mathbf{I}_0^k) \in {}^n\mathcal{P}$, where ${}^n\mathcal{P}$ is a set of ${}^nM$-dimensional vectors with non-negative elements that add up to one. The unknown continuous state ${}^n\mathbf{x}_k$ is described by the PDF $p({}^n\mathbf{x}_k|\, {}^n\mathbf{I}_0^k, {}^n\mu_k)$, which can be represented exactly or approximately by a finite number of statistics. The sufficient statistics can be the mean and the covariance matrix, if the (extended) Kalman filter is used for estimating the continuous part of the state (Punčochář et al., 2015) or particles and their weights if the particle filter is used (Škach et al., 2017). Hence, the information state ${}^n\boldsymbol{\xi}_k$ of the perfect state information model can be defined as the composition of the probabilities ${}^n\boldsymbol{\pi}_k({}^n\mathbf{I}_0^k)$ and ${}^nM$ statistics $p({}^n\mathbf{x}_k|\, {}^n\mathbf{I}_0^k, {}^n\mu_k)$, ${}^n\mu_k \in {}^n\mathcal{M}$.

Then, at any time step $k \in \mathcal{T}$, the perfect state information model for the information state ${}^n\boldsymbol{\xi}_k$ is given as

$$^n\boldsymbol{\xi}_{k+1} = {}^n\boldsymbol{\phi}\left({}^n\boldsymbol{\xi}_k, {}^n\mathbf{u}_k, {}^n\mathbf{y}_{k+1}\right), \tag{14}$$

where ${}^n\boldsymbol{\phi} : {}^n\mathcal{G} \times {}^n\mathcal{U} \times \mathbb{R}^{^nD_y} \mapsto {}^n\mathcal{G}$ is a mapping representing the composition of the subsystem (1) and the estimation algorithm.

Given the information state ${}^n\boldsymbol{\xi}_k$, it suffices to consider the active fault detector as a time-invariant system that is described as

$$^n\Delta : \quad \begin{bmatrix} {}^nd_k \\ {}^n\mathbf{u}_k \end{bmatrix} = \begin{bmatrix} {}^n\bar{\sigma}^{\text{dec}}({}^n\boldsymbol{\xi}_k) \\ {}^n\bar{\gamma}^{\text{dec}}({}^n\boldsymbol{\xi}_k) \end{bmatrix}, \tag{15}$$

where ${}^n\bar{\sigma}^{\text{dec}} : {}^n\mathcal{G} \mapsto {}^n\mathcal{M}$ and ${}^n\bar{\gamma}^{\text{dec}} : {}^n\mathcal{G} \mapsto {}^n\mathcal{U}$ are unknown. The cost function for the perfect state information model equivalent to ${}^nL^{\text{d}}$ in (6) can be shown to satisfy

$$^n\bar{L}^{\text{d}}({}^n\boldsymbol{\xi}_k, {}^nd_k) = \sum_{^n\mu_k} {}^nL^{\text{d}}({}^n\mu_k, {}^nd_k) Pr({}^n\mu_k|\, {}^n\mathbf{I}_0^k). \tag{16}$$

The reformulated problem given by (14–16) can be solved by the Bellman functional equation (Vrabie et al., 2013).

The optimal active fault detector is determined by finding a function ${}^nV : {}^n\mathcal{G} \mapsto \mathbb{R}$ solving the Bellman functional equation

$$^nV({}^n\boldsymbol{\xi}_k) = \min_{^nd' \in {}^n\mathcal{M}} {}^n\bar{L}^{\text{d}}({}^n\boldsymbol{\xi}_k, {}^nd') + \eta \min_{^n\mathbf{u}' \in {}^n\mathcal{U}} \mathrm{E}\{{}^nV({}^n\boldsymbol{\xi}_{k+1})|\, {}^n\boldsymbol{\xi}_k, {}^n\mathbf{u}_k = {}^n\mathbf{u}'\}, \tag{17}$$

The Bellman function ${}^nV$ can be computed off-line as it depends only on the known PDF $p({}^n\mathbf{x}_{k+1}|\, {}^n\mathbf{x}_k, {}^n\mu_k, {}^n\mathbf{u}_k)$, transition probabilities $Pr({}^n\mu_{k+1}|\, {}^n\mu_k)$, measurement PDF $p({}^n\mathbf{y}_k|\, {}^n\mathbf{x}_k, {}^n\mu_k)$, cost function ${}^nL^{\text{d}}$, the discount factor η, and the estimation algorithm. The optimal decisions and optimal inputs can be determined on-line by solving much simpler optimization problems. The Bellman function V is not required to determine the optimal decision

$$^{n}d_k = {}^{n}\bar{\sigma}^{\mathrm{dec}*}({}^{n}\boldsymbol{\xi}_k) = \arg\min_{^{n}d' \in {}^{n}\mathcal{M}} {}^{n}\bar{L}^{\mathrm{d}}({}^{n}\boldsymbol{\xi}_k, {}^{n}d'). \quad (18)$$

On the other hand, the optimal input signal generator uses the Bellman function V to generate inputs as

$$^{n}\mathbf{u}_k = {}^{n}\bar{\gamma}^{\mathrm{dec}*}({}^{n}\boldsymbol{\xi}_k) = \arg\min_{^{n}\mathbf{u}' \in {}^{n}\mathcal{U}} \mathrm{E}\{{}^{n}V({}^{n}\boldsymbol{\xi}_{k+1})|{}^{n}\boldsymbol{\xi}_k, {}^{n}\mathbf{u}_k = {}^{n}\mathbf{u}'\}.$$

The computational costs for the decentralized AFD are much lower than they would be for the centralized case as the dimension of the information state for the centralized case would be significantly higher leading to high dimension of the statistics and larger number of modes. Moreover, the Bellman functional equations for the decentralized case can be solved in parallel.

4. DESIGN OF DISTRIBUTED AFD

The scheme of the distributed AFD is depicted in Fig. 3. The

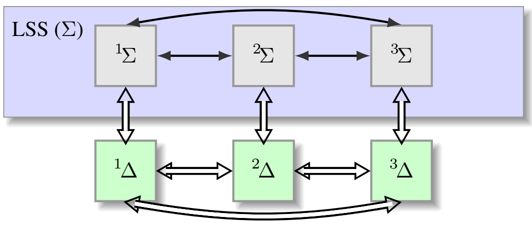

Fig. 3. Distributed AFD system structure

design of the active fault detector in the distributed architecture will utilize the decentralized AFD (Punčochář and Straka, 2019). More specifically, the input generator for the distributed AFD architecture will be designed in the form of the input signal generator ${}^{n}\gamma_k^{\mathrm{dec}}$ for the decentralized AFD architecture and will use the information state ${}^{n}\boldsymbol{\xi}_k$ [5]. Note that designing the input signal generator for the full model (1a) would lead to the impractical centralized AFD (Punčochář and Straka, 2019).

On the other hand, the optimal decision generator design proceeds from the full model (1a). The state estimation being the core of the optimal decision generator requires a communication among the AFD nodes to calculate the estimate of the complete continuous part of the state $\mathbf{x}_k$ and the probabilities of the discrete part of the state $\boldsymbol{\mu}_k$. As a consequence, the decentralized and the distributed AFDs differ in the estimation algorithm.

Now, the distributed AFD algorithm steps are specified:

Filtering: [6] Suppose, the prediction PDF of the state ${}^{n}\mathbf{s}_k$

$$p({}^{n}\mathbf{s}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}) = p({}^{n}\mathbf{x}_k, {}^{n}\mu_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}) \quad (19)$$

is given by

- ${}^{n}M$ densities $p({}^{n}\mathbf{x}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}, {}^{n}\mu_{k-1})$,
- $({}^{n}M)^2$ probabilities $Pr({}^{n}\mu_k, {}^{n}\mu_{k-1}|\mathbf{I}_0^{k-1})$

Then, the filtering PDF $p({}^{n}\mathbf{s}_k|\,{}^{n}\breve{\mathbf{I}}_0^k)$ can be factorized as

$$p({}^{n}\mathbf{s}_k|\,{}^{n}\breve{\mathbf{I}}_0^k) = \sum_{^{n}\mu_{k-1}} p({}^{n}\mathbf{x}_k, {}^{n}\mu_k, {}^{n}\mu_{k-1}|\,{}^{n}\breve{\mathbf{I}}_0^k)$$
$$= \sum_{^{n}\mu_{k-1}} p({}^{n}\mathbf{x}_k|\,{}^{n}\breve{\mathbf{I}}_0^k, {}^{n}\mu_k, {}^{n}\mu_{k-1}) Pr({}^{n}\mu_k, {}^{n}\mu_{k-1}|\,{}^{n}\breve{\mathbf{I}}_0^k), \quad (20)$$

where the symbol ${}^{n}\breve{\mathbf{I}}_0^k$ denotes a composition of the *past* global data $\mathbf{I}_0^{k-1}$ related to the LSS Σ and *present* data ${}^{n}\mathbf{y}_k$, and ${}^{n}\mathbf{u}_{k-1}$ related to the subsystem ${}^{n}\Sigma$, i.e., ${}^{n}\breve{\mathbf{I}}_0^k \triangleq [(\mathbf{I}_0^{k-1})^{\mathsf{T}}, ({}^{n}\mathbf{y}_k)^{\mathsf{T}}, ({}^{n}\mathbf{u}_{k-1})^{\mathsf{T}}]^{\mathsf{T}}$. Individual terms in (20) are given by

- $({}^{n}M)^2$ densities $p({}^{n}\mathbf{x}_k|\,{}^{n}\breve{\mathbf{I}}_0^{k-1}, {}^{n}\mu_k, {}^{n}\mu_{k-1})$,
- $({}^{n}M)^2$ probabilities $Pr({}^{n}\mu_k, {}^{n}\mu_{k-1}|\,{}^{n}\breve{\mathbf{I}}_0^{k-1})$,

computed by

$$p({}^{n}\mathbf{x}_k|\,{}^{n}\breve{\mathbf{I}}_0^k, {}^{n}\mu_k, {}^{n}\mu_{k-1}) = \frac{p\left({}^{n}\mathbf{y}_k|\,{}^{n}\mathbf{x}_k, {}^{n}\mu_k\right) p\left({}^{n}\mathbf{x}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}, {}^{n}\mu_{k-1}\right)}{p({}^{n}\mathbf{y}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}, {}^{n}\mu_k, {}^{n}\mu_{k-1})}, \quad (21)$$

and

$$Pr({}^{n}\mu_k, {}^{n}\mu_{k-1}|\,{}^{n}\breve{\mathbf{I}}_0^k) = \quad (22)$$
$$\frac{p({}^{n}\mathbf{y}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}, {}^{n}\mu_k, {}^{n}\mu_{k-1}) Pr({}^{n}\mu_k, {}^{n}\mu_{k-1}|\mathbf{I}_0^{k-1})}{\sum_{^{n}\mu_k, {}^{n}\mu_{k-1}} p({}^{n}\mathbf{y}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}, {}^{n}\mu_k, {}^{n}\mu_{k-1}) Pr({}^{n}\mu_k, {}^{n}\mu_{k-1}|\mathbf{I}_0^{k-1})}.$$

The measurement prediction PDF in (21) and (22) is

$$p({}^{n}\mathbf{y}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}, {}^{n}\mu_k, {}^{n}\mu_{k-1}) =$$
$$\int p\left({}^{n}\mathbf{y}_k|\,{}^{n}\mathbf{x}_k, {}^{n}\mu_k\right) p\left({}^{n}\mathbf{x}_k|\mathbf{I}_0^{k-1}, {}^{n}\mathbf{u}_{k-1}, {}^{n}\mu_{k-1}\right) \mathrm{d}\,{}^{n}\mathbf{x}_k \quad (23)$$

Note that $p\left({}^{n}\mathbf{y}_k|\,{}^{n}\mathbf{x}_k, {}^{n}\mu_k\right)$ is obtained from (1b).

GPB2: The GPB2 algorithm is used to prevent the increase of the number of terms of the state estimate PDF $p({}^{n}\mathbf{s}_k|\,{}^{n}\breve{\mathbf{I}}_0^k)$ caused by considering multiple models. Essentially, the sum in the filtering pdf (20) is merged to a single term for each ${}^{n}\mu_k$.

Then, the filtering PDF is approximated by

$$p({}^{n}\mathbf{s}_k|\,{}^{n}\breve{\mathbf{I}}_0^k) = p({}^{n}\mathbf{x}_k, {}^{n}\mu_k|\,{}^{n}\breve{\mathbf{I}}_0^k) \approx$$
$$p({}^{n}\mathbf{x}_k|\,{}^{n}\breve{\mathbf{I}}_0^k, {}^{n}\mu_k) Pr({}^{n}\mu_k|\,{}^{n}\breve{\mathbf{I}}_0^k), \quad (24)$$

given by

- ${}^{n}M$ densities $p({}^{n}\mathbf{x}_k|\,{}^{n}\breve{\mathbf{I}}_0^k, {}^{n}\mu_k)$,
- ${}^{n}M$ probabilities $Pr({}^{n}\mu_k|\,{}^{n}\breve{\mathbf{I}}_0^k)$,

Decision generation: The optimal decision generator is

$$^{n}d_k = {}^{n}\bar{\sigma}^{\mathrm{dis}*}({}^{n}\boldsymbol{\xi}_k) = \arg\min_{^{n}d' \in {}^{n}\mathcal{M}} {}^{n}\bar{L}^{\mathrm{d}}({}^{n}\boldsymbol{\xi}_k, {}^{n}d'), \quad (25)$$

where the information state ${}^{n}\boldsymbol{\xi}_k$ is constructed from the estimates $p({}^{n}\mathbf{x}_k|\,{}^{n}\breve{\mathbf{I}}_0^k, {}^{n}\mu_k)$ and $Pr({}^{n}\mu_k|\,{}^{n}\breve{\mathbf{I}}_0^k)$.

Input generation: The input signal generator is equal to the signal generator of the decentralized AFD

$$^{n}\mathbf{u}_k = {}^{n}\bar{\gamma}^{\mathrm{dis}*}(\boldsymbol{\xi}_k) = \arg\min_{^{n}\mathbf{u}' \in {}^{n}\mathcal{U}} \mathrm{E}\left\{{}^{n}V({}^{n}\boldsymbol{\xi}_{k+1})\,|\,{}^{n}\boldsymbol{\xi}_k, {}^{n}\mathbf{u}_k = {}^{n}\mathbf{u}'\right\},$$

where ${}^{n}\boldsymbol{\xi}_{k+1}$ is calculated by the perfect information model for the distributed architecture.

Fusion: To calculate the prediction PDF $p({}^{n}\mathbf{s}_{k+1}|\mathbf{I}_0^k, \mathbf{u}_k)$ using the model dynamics (1a) and (2), the AFD nodes must

[5] Hence, the information state ${}^{n}\boldsymbol{\xi}_k$ dynamics for the Bellman function calculation is the same as in the decentralized AFD.

[6] Note that the filtering step is simpler and the GPB2 step is unnecessary at the first time step.

communicate their filtering estimates. For this reason, the AFD node ${}^{n}\Delta$ sends its filtering estimate, i.e., the PDF $p({}^{n}\mathbf{s}_k|\,{}^{n}\breve{\mathbf{I}}_0^k)$ to all other AFD nodes. The estimates received by the AFD node ${}^{n}\Delta$ from all other nodes [7] are fused to obtain $p(\mathbf{s}_k|\mathbf{I}_0^k)$. Now, the fusions of the PDFs and the probabilities will be discussed separately.

The fusion must respect the fact, that the local states ${}^{n}\mathbf{x}_k$ for $n \in \mathcal{N}$ and $k > 0$ given by the PDF $p({}^{n}\mathbf{x}_k|\,{}^{n}\breve{\mathbf{I}}_0^k, {}^{n}\mu_k)$ are conditionally dependent but the degree of dependence is unknown. If the estimates are represented by the means and covariance matrices, the unknown dependency issue can be solved by the covariance intersection technique (Julier and Uhlmann, 1997). Such fusion then leads to an estimate of $\mathbf{x}_k$. As a result of the fusion, each node contains the estimate of the full continuous state $p(\mathbf{x}_k|\mathbf{I}_0^k, \boldsymbol{\mu}_k)$.

Calculation of the probability of the system model indices $Pr(\boldsymbol{\mu}_k|\mathbf{I}_0^k)$ is ambiguous as the dependency of the model indices is unknown for $k > 0$. This paper will approximate the conditional probability $Pr(\boldsymbol{\mu}_k|\mathbf{I}_0^k)$ by

$$Pr(\boldsymbol{\mu}_k|\mathbf{I}_0^k) \approx \prod_{n\in\mathcal{N}} Pr({}^{n}\mu_k|\,{}^{n}\breve{\mathbf{I}}_0^k). \quad (26)$$

Note that such approximation maximizes the Shannon entropy of $Pr(\boldsymbol{\mu}_k|\mathbf{I}_0^k)$.

Prediction: The prediction PDFs $p({}^{n}\mathbf{x}_{k+1}|\mathbf{I}_0^k, {}^{n}\mathbf{u}_k, \boldsymbol{\mu}_k)$ and probabilities $Pr({}^{n}\mu_{k+1}, \boldsymbol{\mu}_k|\mathbf{I}_0^k)$ are calculated from

$$p({}^{n}\mathbf{x}_{k+1}|\mathbf{I}_0^k, {}^{n}\mathbf{u}_k, \boldsymbol{\mu}_k) = \int p({}^{n}\mathbf{x}_{k+1}|\mathbf{x}_k, {}^{n}\mathbf{u}_k, {}^{n}\mu_k) p(\mathbf{x}_k|\mathbf{I}_0^k, \boldsymbol{\mu}_k) \mathrm{d}\mathbf{x}_k \quad (27)$$

and

$$Pr({}^{n}\mu_{k+1}, \boldsymbol{\mu}_k|\mathbf{I}_0^k) = Pr({}^{n}\mu_{k+1}|\boldsymbol{\mu}_k) Pr(\boldsymbol{\mu}_k|\mathbf{I}_0^k). \quad (28)$$

Note that $p({}^{n}\mathbf{x}_{k+1}|\mathbf{x}_k, \mathbf{u}_k, {}^{n}\mu_k)$ is obtained from (1a).

Now, the terms in (27) and (28) related to the remaining nodes ${}^{\bar{n}}\Sigma$ can be eliminated by marginalization and subsequent merging to conserve computational costs similarly to the GPB2 step as

$$p({}^{n}\mathbf{x}_{k+1}|\mathbf{I}_0^k, {}^{n}\mathbf{u}_k, {}^{n}\mu_k) \approx \sum_{\substack{{}^{\bar{n}}\mu_k \\ \bar{n}\in\bar{\mathcal{N}}}} p({}^{n}\mathbf{x}_{k+1}|\mathbf{I}_0^k, {}^{n}\mathbf{u}_k, \boldsymbol{\mu}_k) Pr(\boldsymbol{\mu}_k|\mathbf{I}_0^k)$$

and

$$Pr({}^{n}\mu_{k+1}, {}^{n}\mu_k|\mathbf{I}_0^k) \approx \sum_{\substack{{}^{\bar{n}}\mu_k \\ \bar{n}\in\bar{\mathcal{N}}}} Pr({}^{n}\mu_{k+1}, \boldsymbol{\mu}_k|\mathbf{I}_0^k).$$

5. NUMERICAL ILLUSTRATION

To conserve space, the decentralized and distributed architectures are compared by means of a simple numerical example only [8]. Let us consider the system Σ that consists of two weakly coupled multiple-model linear subsystems

$$
\begin{aligned}
{}^{1}\Sigma: {}^{1}\mathbf{x}_{k+1} &= {}^{1}\mathbf{A}({}^{1}\mu_k)\mathbf{x}_k + {}^{1}B({}^{1}\mu_k)\,{}^{1}u_k + {}^{1}G({}^{1}\mu_k)\,{}^{1}w_k,\\
{}^{1}y_k &= {}^{1}C({}^{1}\mu_k)\,{}^{1}\mathbf{x}_k + {}^{1}H({}^{1}\mu_k)\,{}^{1}v_k\\
{}^{2}\Sigma: {}^{2}\mathbf{x}_{k+1} &= {}^{2}\mathbf{A}({}^{2}\mu_k)\mathbf{x}_k + {}^{2}B({}^{2}\mu_k)\,{}^{2}u_k + {}^{1}G({}^{2}\mu_k)\,{}^{2}w_k,\\
{}^{2}y_k &= {}^{2}C({}^{2}\mu_k)\,{}^{2}\mathbf{x}_k + {}^{2}H({}^{2}\mu_k)\,{}^{2}v_k
\end{aligned}
$$

where both subsystems have two modes with

$$
\begin{aligned}
{}^{1}\mathbf{A}(1) &= [0.98\ 0.01], {}^{1}B(1) = 0.01, \quad {}^{1}G(1) = \sqrt{0.003},\\
{}^{1}\mathbf{A}(2) &= [0.92\ 0.03], {}^{1}B(2) = 0.08, \quad {}^{1}G(2) = \sqrt{0.003},\\
{}^{2}\mathbf{A}(1) &= [0.02\ 0.93], {}^{2}B(1) = 0.07, \quad {}^{2}G(1) = \sqrt{0.002},\\
{}^{2}\mathbf{A}(2) &= [0.01\ 0.93], {}^{2}B(2) = 0.09, \quad {}^{2}G(2) = \sqrt{0.002}.
\end{aligned}
$$

Both modes of each subsystem have the same observation models, ${}^{1}C(1) = {}^{1}C(2) = 1$, ${}^{1}H(1) = {}^{1}H(2) = 0.01$, ${}^{2}C(1) = {}^{2}C(2) = 1$, ${}^{2}H(1) = {}^{2}H(2) = 0.01$. The transition probabilities of the modes for each subsystem are given in Table 1. Note that occurrence of a faults at a subsystem corresponds to increased probability of fault occurring in the other subsystem.

Table 1. Transition probabilities of the modes.

		$[{}^{1}\mu_k, {}^{2}\mu_k]$			
		[1,1]	[1,2]	[2,1]	[2,2]
${}^{1}\mu_{k+1}$	1	0.99	0.9	0.1	0.02
	2	0.01	0.1	0.9	0.98
${}^{2}\mu_{k+1}$	1	0.99	0.05	0.9	0.01
	2	0.01	0.95	0.1	0.99

The state noises ${}^{1}\mathbf{w}_k$ and ${}^{2}\mathbf{w}_k$ have both standard Gaussian PDF $p_{{}^{1}\mathbf{w}_k} = p_{{}^{2}\mathbf{w}_k} = \mathcal{N}\{0, 1\}$. The initial condition $\mathbf{x}_0$ has Gaussian PDF $\mathcal{N}\{\mathbf{0}, 0.01 \cdot \mathbf{I}\}$ and initial $\boldsymbol{\mu}_0$ has probability $P(\boldsymbol{\mu}_0 = [1\ 1]^{\mathrm{T}}) = 1$, which means that each subsystem starts from mode 1 (i.e., fault-free behavior). The admissible inputs of subsystems are ${}^{1}\mathcal{U} = {}^{2}\mathcal{U} = \{-1, 0, 1\}$. The detection cost function ${}^{n}L^{\mathrm{d}}$ is the zero-one function ${}^{n}L^{\mathrm{d}}({}^{n}\mu_k, {}^{n}d_k) = 1 - \delta_{{}^{n}\mu_k, {}^{n}d_k}$ where $\delta_{i,j}$ is the Kronecker delta. The discount factor is $\eta = 0.9$.

The system faults were detected within both decentralized and distributed architectures by a passive FD (PFD) approach and the proposed AFD approach. The continuous part of the state ${}^{n}\mathbf{x}_k$ was estimated in all cases using the Kalman filter (KF). Hence, the statistics in the information state were given by the mean and covariance matrix.

The PFD used the excitation input generated by the function $\sin(0.2k)$ quantized to values $\{-1, 0, 1\}$, while the excitation input for the AFD was calculated using the Bellman function obtained off-line by the value iteration algorithm.

The decentralized architecture ignored the coupling between the subsystems in the dynamic matrices. Also, the conditional dependency of the model indices between the subsystems was ignored using (10) with (11).

Ignoring the coupling degraded the accuracy of conditional mode probabilities, nevertheless it made the Bellman function calculation for the AFD feasible. Two independent input signal generators were designed and thus the information state was only five-dimensional for each input signal generator, i.e. ${}^{n}\boldsymbol{\xi}_k \in \mathbb{R}^5$, $n = 1, 2$. It consisted of the scalar mean $\mathrm{E}[{}^{n}\mathbf{x}_k|\mathbf{I}_0^k]$ and variance $\mathrm{var}[{}^{n}\mathbf{x}_k|\mathbf{I}_0^k]$ provided by the KF for both models and

[7] If the information from all other nodes is not available to a node (e.g. for the reason of missing communication), the node must utilize the approximate subsystem models used in the decentralized AFD design.

[8] Note that the system is linear for simplicity even though the AFD algorithm was proposed for a general nonlinear system.

the probability of the first model. Hence, the value iteration algorithm was performed over the grids given as $[-1 : 0.2 : 1] \times [-1 : 0.2 : 1] \times [0 : 2 \cdot 10^{-5} : 2 \cdot 10^{-4}] \times [0 : 2 \cdot 10^{-5} : 2 \cdot 10^{-4}] \times [0 : 0.02 : 1]$ with 732 050 discrete states. The decentralized architecture ignored the coupling during both the off-line Bellman function calculation and the on-line detection generation.

Within the AFD distributed architecture, the on-line detection generation considered the full model and the algorithm operated as described in Section 4. As mentioned in Section 4, the input signal generator was the same as the input signal generator of the decentralized architecture. Hence, the value iteration algorithm was performed over the same grid with 732 050 discrete states. Note that the online memory requirements for storing the Bellman function, which was computed in the off-line part of the AFD algorithm, were 12 [MB] for both architectures. If the input signal generator were computed for the distributed architecture without neglecting the subsystem coupling, it would lead to a 22-dimensional information state. This would make the Bellman function calculation extremely difficult. Note that such computation would correspond to the AFD with centralized architecture.

The performance of the decentralized and distributed AFD was evaluated using the 10^5 Monte Carlo (MC) simulations where each MC simulation was run over the finite time horizon $F = 400$. The estimate $\hat{J}$ of the criterion obtained by the MC simulations and time requirements $T_{\text{on-line}}$ of a single run of the algorithm (i.e. the on-line decision generation) [9] are given in Table 2.

Table 2. Performance of decentralized and distributed PFD and AFD architectures.

	architecture	$\hat{J}$	$T_{\text{on-line}}$
PFD	decentralized	1.46	0.27
	distributed	1.29	0.42
AFD	decentralized	1.41	0.36
	distributed	1.22	0.51

From the table, it follows that the AFD achieves lower criterion value than the PFD for both architectures. The price paid is slightly higher computational costs of the AFD in comparison with the PFD, which is caused by selecting the optimum excitation input from the table representing the Bellman function by the AFD whereas the PFD only calculates the quantized sine function value.

When comparing the AFD architectures, the distributed architecture achieves significantly lower criterion value than the decentralized architecture. The reason is the fact that the decision generation of the distributed architecture does not neglect the coupling between the subsystems and its individual nodes have more information due to their communication. Naturally, this leads to higher computational costs due to the fusion step, which is not present in the decentralized architecture and the prediction step, which involves full coupling between the subsystems.

[9] All the numerical simulations in the paper were performed using the R2019a version of Matlab® software running on the PC equipped with Intel® CoreTM i7–4790 CPU (3.60 [GHz]).

6. CONCLUSION

The paper dealt with active fault diagnosis of large-scale stochastic systems within the multiple-model framework. The aim was to respect possible influence of the faults among the subsystems. For this purpose, the paper considered conditionally dependent faults. First, the decentralized architecture was considered, which neglects the coupling among the subsystems. For this architecture, the design of the optimal active detector consisting of the input signal generator and the fault detector was illustrated. Then, the AFD within the distributed architecture was proposed and its fault detector algorithm was described in detail. The simple numerical example illustrated better performance of the distributed architecture in comparison with the decentralized architecture.

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