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Sampled-data output feedback controllers for nonlinear systems with time-varying measurement and control delays ⋆

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Abstract: In this paper, we propose sampled-data output feedback controllers for nonlinear systems with time-varying measurement and input delays. A state prediction is generated by chains of saturated high-gain observers with switching error-correction terms and the state prediction is used to stabilize the system with saturated controls. The observers reconstruct the unmeasurable states at different delayed time-instants, which partition the maximal variation interval of the time-varying delays. The density of these delayed time instant depend both on the magnitude of the delays and the growth rate of the nonlinearities. Our sampled-data feedback controllers are obtained as zero-order discretizations of continuous time controllers.

Keywords: Delay systems, time-varying measurement and input delays, dynamic state predictors, continuous-time and sampled-data output feedback controllers.

1. INTRODUCTION

The problem of reconstructing the unmeasurable state variables by using the delayed output measurements is long-standing. The literature is vast under this regard and we will not refer this here. The reconstructed unmeasurable state variables can be used for stabilization in the presence of delayed controls. A solution is to set to zero the input delay and then searching for upper bounds on the input delays that the closed-loop system can tolerate while still realizing the desired goal. This often involves Lyapunov-Krasovskii functionals (as discussed in Fridman et al. (2008) and Mazenc et al. (2008), which often lead to satisfactory results when the delay is small. In Mazenc et al. (2008) a prediction based approach is used to construct globally asymptotically stabilizing control laws for time-varying systems using state-feedback. This approach differs from the classical reduction model approach or the prediction based approaches introduced by Krstic (as in Bresch-Pietri et al. (2014)) which also involve distributed terms. Several dynamic extensions are used, making it possible to obtain a prediction of the state variable without using distributed terms. Many contributions, including Ahmed-Ali et al. (2014), use several dynamic extensions to carry out state prediction, but to the best of our knowledge, they do not apply to the problem we consider here and they use distributed terms.

In Zhou (2012) linear time-invariant systems are considered under additional eigenvalue conditions and controllability conditions or bounds on the delays, without robustness to uncertainty; on the other hand, Zhou (2014) covers time-varying linear systems and give sufficient conditions for stabilizability under pseudo-predictor feedback using an integral delay system.

Most of the above papers are focused on the state-feedback problem with globally Lipschitz or linear dynamics. Only Cacace et al. (2014b) and Ahmed-Ali et al. (2014) cover the output-feedback case with large delays but restricting to globally Lipschitz dynamics and only Lei et al. (2016) covers feedback linearizable systems but restricting to small delays. In this paper, we remove globally Lipschitz assumptions or linearity assumptions on the system by introducing techniques based on incremental homogeneity properties (Battilotti (2014)) and propose sampled-data output-feedback stabilizers for nonlinear systems with time-varying measurement and input delays. Following the idea of chains of linear observers (Cacace et al. (2014b), we generate a state prediction by chains of nonlinear (high-gain) observers that reconstruct the unmeasurable state at different delayed time-instants, which partition the maximal variation interval of the time-varying delays. The number of observers is in general larger as the maximum delay is larger. Our remarkable improvement of this idea relies in the fact that the number of observers, in the presence of strong nonlinearities, should depend also on the growth rate of the nonlinearities. Stronger nonlinearities require a larger number of observers. The state prediction is used by a nonlinear controller to stabilize the system through the delayed control input. The novelty of our stabilizer is the use of a nonlinear (saturated) controller processing the state estimates given by a chain of nonlinear observers with saturated estimates and switching error-correction terms. Saturations (or alternatively rate limiters) take care of the strong nonlinearities of the system and avoid the peaking phenomenon (well-known for systems with no delays). Switching error-correction terms take care of the time-varying delays. First, we design continuous-time output-feedback stabilizers and, from these, we obtain sampled-data output-feedback stabilizers as zero-order hold discretization of the proposed continuous-time stabilizers. Sampled-data predictors and controllers were studied in

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Karafyllis et al. (2012a) under assumptions on the system very similar to the ones used for continuous-time controllers in the literature and most contributions show only *practical* asymptotic stability (see Shim et al. (2003) for instance).

2. NOTATION

(**N1**) $\mathbb{R}^n$ (resp. $\mathbb{R}^{n\times s}$) is the set of n-dimensional real column vectors (resp. $n\times s$ matrices). $\mathbb{R}_\geqslant$ (resp. $\mathbb{R}^n_\geqslant$, $\mathbb{R}^{n\times s}_\geqslant$) denotes the set of non-negative real numbers (resp. vectors in $\mathbb{R}^n$, matrices in $\mathbb{R}^{n\times s}$, with non-negative real elements). $\mathbb{R}_>$ (resp. $\mathbb{R}^n_>$) denotes the set of positive real numbers (resp. vectors in $\mathbb{R}^n$ with real positive entries). $(\mathbb{R}^n)^*$ is the dual space of $\mathbb{R}^n$ (space of row vectors).

(**N2**) For any matrix $A \in \mathbb{R}^{p\times n}$ we denote by $A_{i,j}$ the (i,j)-th element of A and for any vector $v \in \mathbb{R}^n$ (or $v \in (\mathbb{R}^n)^*$) we denote by v_i the i-th element of v. Also, we may write vectors $v \in \mathbb{R}^n$ as $(v_1,\ldots,v_n)^T$, vectors $w \in (\mathbb{R}^n)^*$ as $(w_1,\ldots,w_n)$ and matrices $A \in \mathbb{R}^{s\times n}$ either as $A = [v_1,\ldots,v_n]$ (i.e. by columns) or $A = [w_1^T,\ldots,w_s^T]^T$ (i.e. by rows). I_n is the $n\times n$ identity matrix. We retain a similar notation for functions.

(**N3**) We denote by $\mathbf{C}^0(\mathscr{X},\mathscr{Y})$, $\mathscr{X} \subset \mathbb{R}^n$ and $\mathscr{Y} \subset \mathbb{R}^s$, the set of continuous functions $\alpha : \mathscr{X} \to \mathscr{Y}$. Moreover, $\mathcal{K}$ denotes the set of strictly increasing functions $\alpha \in \mathbf{C}^0(\mathbb{R}_\geqslant,\mathbb{R}_\geqslant)$ such that $\alpha(0) = 0$, $\mathcal{K}_\infty$ denotes the set of functions $\alpha \in \mathcal{K}$ such that $\alpha(s) \to +\infty$ as $s \to +\infty$. Also, $\mathcal{L}$ denotes the set of strictly decreasing functions $\alpha \in \mathbf{C}^0(\mathbb{R}_\geqslant,\mathbb{R}_>)$ such that $\alpha(s) \to 0$ as $s \to +\infty$ and by $\mathcal{KL}$ denotes the set of functions $\alpha \in \mathbf{C}^0(\mathbb{R}_\geqslant \times \mathbb{R}_\geqslant,\mathbb{R}_\geqslant)$ such that $\alpha(s,\cdot) \in \mathcal{L}$ and $\alpha(\cdot,s) \in \mathcal{K}$ for each $s \in \mathbb{R}_\geqslant$.

(**N4**) For $\epsilon \in \mathbb{R}_>$, the group of dilations $\mathcal{G} = (\epsilon^{\mathfrak{r}},\diamond)$ is the set of elements $\epsilon^{\mathfrak{r}} := (\epsilon^{\mathfrak{r}_1},\ldots,\epsilon^{\mathfrak{r}_n})^T \in \mathbb{R}^n$, $\mathfrak{r} \in \mathbb{R}^n$, with group operation $\epsilon^{\mathfrak{r}'}\diamond\epsilon^{\mathfrak{r}''} = \epsilon^{\mathfrak{r}'+\mathfrak{r}''}$ and identity element $\epsilon^{\mathbf{0}_n} := \mathbf{1}_n := (1,\ldots,1)^T$ where $\mathbf{0}_n := (0,\ldots,0)^T$.

Also, we define the $\epsilon^{\mathfrak{r}}-$*dilation of* $v \in \mathbb{R}^n$ as the left group action $\diamond$ on $\mathbb{R}^n$ defined as $\epsilon^{\mathfrak{r}} \diamond v := (\epsilon^{\mathfrak{r}_1}v_1,\cdots,\epsilon^{\mathfrak{r}_n}v_n)^T$. Similarly, we define the $\epsilon^{\mathfrak{r}}-$*dilation of* $w \in (\mathbb{R}^n)^*$ as the right group action $\diamond$ on $(\mathbb{R}^n)^*$ defined as $w \diamond \epsilon^{\mathfrak{r}} := (\epsilon^{\mathfrak{r}_1}w_1,\cdots,\epsilon^{\mathfrak{r}_n}w_n)$.

By extension, we can define the *left* $\epsilon^{\mathfrak{r}}-$*dilation of* $A := [w_1^T,\ldots,w_s^T]^T \in \mathbb{R}^{n\times s}$ as the left group action $\diamond$ on $\mathbb{R}^{n\times s}$ defined as $\epsilon^{\mathfrak{r}} \diamond A := [\epsilon^{\mathfrak{r}_1}w_1^T,\ldots,\epsilon^{\mathfrak{r}_n}w_n^T]^T$ and the *right* $\epsilon^{\mathfrak{r}}-$*dilation of* $A := [v_1,\ldots,v_n] \in \mathbb{R}^{s\times n}$ as the right group action $\diamond$ on $\mathbb{R}^{s\times n}$ defined as $A \diamond \epsilon^{\mathfrak{r}} := [\epsilon^{\mathfrak{r}_1}v_1,\cdots,\epsilon^{\mathfrak{r}_n}v_1]$. The dilation's properties used in this paper are given in the appendix.

(**N5**) A *saturation function* σ_l *with saturation levels* $l \in \mathbb{R}^n_>$ is a function $\sigma_l(x) := (\sigma_{l_1}(x_1),\ldots,\sigma_{l_n}(x_n))^T$, $x \in \mathbb{R}^n$, such that for each $i = 1,\ldots,n$ and $x_i \in \mathbb{R}$:

$$\sigma_{l_i}(x_i) = \begin{cases} x_i & |x_i| \leqslant l_i \\ \mathrm{sign}(x_i)l_i & \text{otherwise.} \end{cases} \tag{1}$$

3. THE CLASS OF SYSTEMS AND PROBLEM STATEMENT

We consider continuous-time nonlinear systems with delayed measurement $\mathbf{y}$ and input $\mathbf{u}$:

$$\dot{\mathbf{x}}_t = A\mathbf{x}_t + B\mathbf{u}_{t-\mathbf{c}_t} + \phi(\mathbf{x}_t),\ t \geqslant -c_\infty - 2d_\infty, \tag{2}$$

$$\mathbf{y}_t = C\mathbf{x}_{t-\mathbf{d}_t} + \psi(\mathbf{x}_{t-\mathbf{d}_t}),\ t \geqslant 0 \tag{3}$$

with state $\mathbf{x}_t \in \mathbb{R}^n$, measurements $\mathbf{y}_t \in \mathbb{R}$, continuous measurement delay $\mathbf{d}_t \in \mathbb{R}_\geqslant$, known up to time t and bounded by a known constant d_∞, input $\mathbf{u}_t \in \mathbb{R}$ and continuous input delay $\mathbf{c}_t \in \mathbb{R}_\geqslant$, known up to time t and bounded by a known constant c_∞ (full extension to multiple-input multiple-output systems and robustness issues are contained in Battilotti (2019)). The assumption that the delays are known is realistic in many applications. The input $\mathbf{u}_t$ is set to zero for $t \leqslant c_\infty$. We assume that ϕ and ψ are locally Lipschitz. The matrices A, B are in Brunowski form with $C = (1,0,\ldots,0)$. The problem we want to solve in this paper is to design I) continuous-time stabilizers of (2) using the output information $\mathbf{y}_t$ and II) obtain from these stabilizers sampled-data stabilizers using the sampled output information $\mathbf{y}_{t_h}$, $t_h := hT$ ($h \in \mathbb{N}$ and $T \in \mathbb{R}_>$ the sampling period).

4. CONTINUOUS-TIME STABILIZERS

The continuous-time stabilizer we propose consists of a controller together with a certain number of chained observers. These observers are chained in the sense that each observer in the chain computes the estimate of the state of the controlled process, delayed by a sufficiently small relative amount, and hands over a certain amount of information (like its own estimate) to the next one in the chain. The approach of using chained sub-predictors for coping with large delays is not new (Germani et al. (2002)). The novelty here is to consider the measurement and control delays $\mathbf{d}_t$ and $\mathbf{c}_t$ forming together a large delay $\mathbf{d}_t + \mathbf{c}_t$ (from the last received measurement to the first applied control action) and the partition of the delay interval $[-c_\infty, d_\infty]$ into an increasing sequence of points $\{p^{(j)}\}_{j=1,\ldots,\nu}$, which determines the number ν of sub-predictors. Another important novelty is that ν depends not only on how large is the delay but also on the growth rate of the nonlinearities of the controlled process (tunable chain length ν). According to this partition, each observer of the chain computes an estimate of the delayed state $\mathbf{x}_t^{(j)} := \mathbf{x}_{t-p^{(j-1)}}$, $j = 2,\ldots,\nu+1$, denoted by $\hat{\mathbf{x}}_t^{(j)}$. The first element of the observer chain is an observer which computes the estimate $\hat{\mathbf{x}}_t^{(\nu+1)}$ of the (maximally) delayed state $\mathbf{x}_t^{(\nu+1)} := \mathbf{x}_{t-d_\infty}$ and the last element of the observer chain is an observer which computes the estimate $\hat{\mathbf{x}}_t^{(2)}$ of the state $\mathbf{x}_t^{(2)} := \mathbf{x}_{t+c_\infty}$ (i.e. a c_∞-step prediction). The control action $\mathbf{u}_t$ is defined by processing the estimate $\hat{\mathbf{x}}_t^{(j+1)}$ such that $-\mathbf{c}_t \in [p^{(j)}, p^{(j+1)}]$ in so that, when delayed by $\mathbf{c}_t$ at the input $\mathbf{u}_{t-\mathbf{c}_t}$ of the system, it is "close" to the estimate of $\mathbf{x}_t$. The partition of the interval $[-c_\infty, d_\infty]$ into a sequence of points $\{p^{(j)}\}_{j=1,\ldots,\nu}$ is made precise as follows.

A real sequence $\{p^{(j)}\}_{j=1,\ldots,\nu}$ *is a* δ*-fine partition of an interval* $[a,b] \subset \mathbb{R}$, $\delta \in \mathbb{R}_>$, *if* $\nu = \lceil\frac{b-a}{\delta}\rceil + 1$, $p^{(j)} := a + (j-1)\delta$ *for* $j = 1,\ldots,\nu-1$ *and* $p^{(\nu)} := b$.

Notice that the number N depends on the refinement δ of the partition and $p^{(\nu)} - p^{(\nu-1)} \leqslant \delta$ with $p^{(\nu)} - p^{(\nu-1)} = \delta$ if and only if $\frac{b-a}{\delta}$ is integer. In what follows, we consider δ-fine partitions $\{p^{(j)}\}_{j=1,\ldots,\nu}$ of the interval

$[-c, d_\infty]$ including the point 0 and an auxiliary extra point $p^{(\nu+1)} > d_\infty$ such that $p^{(\nu+1)} - p^{(\nu)} \leqslant \delta$ and we assume that $p^{(\nu_0)} = 0$ for some $\nu_0 \in \{1, \ldots, \nu\}$ (we will say that *the partition is extended and centered at* 0).

4.1 The observer chain with tunable length

Each observer of the chain, say the j-th observer of the chain, manipulates a certain amount of information, according to the relative values of the delay $\mathbf{d}_t$ with respect to the partition of $[-c_\infty, d_\infty]$: typically, when $\mathbf{d}_t$ is large the observer will process the estimate $\hat{\mathbf{x}}_t^{(j+1)}$ handed over by the preceding observer in the chain, while for small values of $\mathbf{d}_t$ the observer will use the available outputs $\mathbf{y}_t$ and, if necessary, past outputs $\mathbf{y}_s$, $s \leqslant t$. Different data processing of the above type determine different innovations for each observer to guarantee convergence of the estimate to the delayed state. As already stated, we assume that $\mathbf{d}_t$ is bounded by d_∞ and continuous. When the delay $\mathbf{d}_t$ is continuous, past measurements are available for processing continuously in time up to t.

Let's get into the technical structure of each observer in the chain. Let $\mathfrak{f}^{(o)} \in \mathbb{R}^n$, $\mathfrak{r} \in \mathbb{R}^n_>, \epsilon, l^{(o)} \in \mathbb{R}_>$ and diagonal positive definite $\Gamma^{(o)} \in \mathbb{R}^{n\times n}$ be design parameters. Moreover, in accordance with the notation $\mathbf{x}_t^{(j+1)} := \mathbf{x}_{t-p^{(j)}}$ set $\mathbf{u}_t^{(j+1)} := \mathbf{u}_{t-p^{(j)}}$. The observer chain is described by

$$\begin{aligned}\dot{\hat{\mathbf{x}}}_t^{(j)} &= A\hat{\mathbf{x}}_t^{(j)} + B\mathbf{u}_{t-\mathbf{c}_t}^{(j)} \\ &\quad + \phi\left(\sigma_{\lambda^{(o)}(\epsilon)}\left(\hat{\mathbf{x}}_t^{(j)}\right)\right) + {P^{(o)}}^{-1}C^T R^{(o)}\mathbf{z}_t^{(j)}, \\ &\quad j = 2, \ldots, \nu+1,\ t \geqslant 0, \end{aligned} \tag{4}$$

with saturation function $\sigma_{\lambda^{(o)}(\epsilon)}$ and saturation levels $\lambda^{(o)}(\epsilon) := l^{(o)}\epsilon^{\mathfrak{r}}$, matrices

$$\begin{aligned} &P^{(o)} = (I_n - G^{(o)}A^T)^T \diamond \epsilon^{-2\mathfrak{r}} \diamond (I_n - G^{(o)}A^T), \\ &R^{(o)} = C(\epsilon^{-\mathfrak{r}} \diamond G^{(o)} \diamond \epsilon^{-\mathfrak{r}})C^T,\ G^{(o)} = \epsilon^{\mathfrak{f}^{(o)}} \diamond \Gamma^{(o)} \diamond \epsilon^{\mathfrak{f}^{(o)}}, \end{aligned} \tag{5}$$

and innovations $\mathbf{z}_t^{(j)}$ defined as follows:

▫ for $j = \nu_0 + 1, \ldots, \nu + 1$

$$\mathbf{z}_t^{(j)} := \begin{cases} \mathbf{y}_{t^{(j)}} - C\hat{\mathbf{x}}_{t-\mathbf{s}_t^{(j)}}^{(j)} - \psi\left(\sigma_{\lambda^{(o)}(\epsilon)}\left(\hat{\mathbf{x}}_{t-\mathbf{s}_t^{(j)}}^{(j)}\right)\right) \\ \qquad\qquad \text{if } \mathbf{d}_t \in [0, p^{(j-1)}), \\ \mathbf{y}_t - C\hat{\mathbf{x}}_{t-\mathbf{s}_t^{(j)}}^{(j)} - \psi\left(\sigma_{\lambda^{(o)}(\epsilon)}\left(\hat{\mathbf{x}}_{t-\mathbf{s}_t^{(j)}}^{(j)}\right)\right) \\ \qquad\qquad \text{if } \mathbf{d}_t \in [p^{(j-1)}, p^{(j)}], \\ C(\hat{\mathbf{x}}_t^{(j+1)} - \hat{\mathbf{x}}_{t-\mathbf{s}_t^{(j)}}^{(j)}) \quad \text{if } \mathbf{d}_t \in (p^{(j)}, p^{(\nu+1)}] \end{cases} \tag{6}$$

(where $\mathbf{y}_{t^{(j)}}$ is the past output at $t^{(j)} \in [0, t]$ such that $t^{(j)} - \mathbf{d}_{t^{(j)}} = t - p^{(j-1)}$: $t^{(j)}$ does exist by continuity of the measurement delay) with delay

$$\mathbf{s}_t^{(j)} := \begin{cases} 0 & \text{if } \mathbf{d}_t \in [0, p^{(j-1)}), \\ \mathbf{d}_t - p^{(j-1)} & \text{if } \mathbf{d}_t \in [p^{(j-1)}, p^{(j)}], \\ p^{(j)} - p^{(j-1)} & \text{if } \mathbf{d}_t \in (p^{(j)}, p^{(\nu+1)}], \end{cases} \tag{7}$$

▫ for $j = 2, \ldots, \nu_0$

$$\mathbf{z}_t^{(j)} := C(\hat{\mathbf{x}}_t^{(j+1)} - \hat{\mathbf{x}}_{t-\mathbf{s}_t^{(j)}}^{(j)}) \tag{8}$$

$$\mathbf{s}_t^{(j)} := p^{(j)} - p^{(j-1)}. \tag{9}$$

Each observer is initialized as follows:

$$\hat{\mathbf{x}}_\theta^{(j)} := 0, \forall \theta \in [-c_\infty - 2d_\infty, 0] \tag{10}$$

(this particular initialization is motivated by sake of simplicity).The length ν of the chain depends not only on the magnitude of the delays but also on the nonlinearities of the system and it is a critical parameter in our design. Notice that when $\mathbf{d}_t \in [0, p^{(j-1)})$, $j = \nu_0 + 1, \ldots, \nu + 1$, the past outputs $\mathbf{y}_{t^{(j)}}$ ($t^{(j)} < t$, where $t^{(j)} = t + \mathbf{d}_{t^{(j)}} - p^{(j-1)}$) is processed for the innovation $\mathbf{z}_t^{(j)}$. The estimate $\mathbf{x}_t^{(j)}$ is not delayed ($\mathbf{s}_t^{(j)} = 0$). Notice that for the implementation of this step we need the past outputs $\mathbf{y}_{t^{(j)}}$ ($t^{(j)} < t$) and this requires the continuity of $\mathbf{d}_t$. This is the only point for which the continuity of $\mathbf{d}_t$ is needed. If $\mathbf{d}_t$ is not continuous the output $\mathbf{y}_{t^{(j)}}$ may be not available for processing. In this case we may think to reconstruct the value $\mathbf{y}_{t^{(j)}}$ from the past outputs (exactly or approximately using for instance sinc-functions). As it appears from (6), (8), the chained structure is given by the estimate $\hat{\mathbf{x}}_t^{(j+1)}$ of $\mathbf{x}_t^{(j+1)}$ computed by the $(j+1)$-th observer in the chain and handed over to the j-th observer only either when $\mathbf{d}_t \in (p^{(j)}, p^{(\nu+1)}]$ ((6)) and for the observers which compute state interpolations (i.e. past values of the state: (8)) or for the observers which compute state predictions (i.e. future values of the state: (8)). Notice that each observer (4) is a copy of the system (2), delayed by the amount $p^{(j-1)}$, with saturated estimates $\sigma_{\lambda^{(o)}(\epsilon)}(\hat{\mathbf{x}}_t^{(j)})$ and updated by the innovation process $\mathbf{z}_t^{(j)}$, weighted by the gain matrix ${P^{(o)}}^{-1}C^T R^{(o)}$. The gain matrix is defined as a suitable dilated transformation with parameter ϵ, which follows very naturally from the incremental homogeneity assumptions on the process nonlinearities f which will be introduced in the section 4.3. The importance of saturating the estimates when trying to reconstruct the state of a nonlinear system with delay-free measurements has been pointed out in various works since the late 90's.

4.2 The controller

Let $\mathfrak{f}^{(s)} \in \mathbb{R}^n$, $l^{(s)} \in \mathbb{R}_>$ and diagonal positive definite $\Gamma^{(s)} \in \mathbb{R}^{n\times n}$ be design parameters. The controller is defined as

$$\begin{aligned}\mathbf{u}_t &:= -R^{(s)}B^T P^{(s)}(I_n - A^T G^{(s)}) \times \\ &\quad \times \sigma_{\lambda^{(s)}(\epsilon)}\Big((I_n - A^T G^{(s)})^{-1}\hat{\mathbf{x}}_t^{(j+1)}\Big) \\ &\quad \text{if } -\mathbf{c}_t \in [p^{(j)}, p^{(j+1)}). \end{aligned} \tag{11}$$

with saturation function $\sigma_{\lambda^{(s)}(\epsilon)}$ and saturation levels $\lambda^{(s)}(\epsilon) := l^{(s)}\epsilon^{\mathfrak{r}}$ (in general $\neq \lambda^{(o)}(\epsilon)$) and

$$\begin{aligned} &P^{(s)} = (I_n - A^T G^{(s)})^{-T} \diamond \epsilon^{-2\mathfrak{r}} \diamond (I_n - A^T G^{(s)})^{-1}, \\ &R^{(s)} = B^T(\epsilon^{\mathfrak{r}} \diamond G^{(s)} \diamond \epsilon^{\mathfrak{r}})B,\ G^{(s)} = \epsilon^{\mathfrak{f}^{(s)}} \diamond \Gamma^{(s)} \diamond \epsilon^{\mathfrak{f}^{(s)}}. \end{aligned} \tag{12}$$

Notice that the control $\mathbf{u}_t$ changes according to the relative position of $-\mathbf{c}_t$ with respect to the partition of $[-c_\infty, 0]$. Also, notice that the controller (11) comes out from the composition of a linear controller with the saturation $\sigma_{\lambda^{(s)}(\epsilon)}(\cdot)$. The linear controller is characterized by a gain matrix $R^{(s)}B^T P^{(s)}$ defined as a suitable dilated transformation with parameter ϵ, which follows very naturally from the incremental homogeneity assumptions on the process nonlinearities f which will be introduced in the section 4.3. The importance of saturating the control when trying to asymptotically stabilize a delay-free nonlinear system by output feedback has been pointed out since the late 90's. Here, we prove the important fact that also in the presence of delays it is important to saturate the (delayed) control action.

4.3 Main assumptions and results

Our assumptions on the system (2) are the following (see a review of incremental homogeneity in appendix A).

(H0) (*forward completeness*): *the trajectories* $\mathbf{x}_t$ *of (2) satisfy the following inequality: there exist* $\mu \in \mathbb{R}_>$ *and continuously differentiable and proper* $U : \mathbb{R}^n \to \mathbb{R}_\geqslant$ *and* $\kappa \in \mathcal{K}_\infty$ *such that for all* $t \geqslant -c_\infty - 2d_\infty$

$$\dot{U}(\mathbf{x}_t)|_{(2)} \leqslant \mu U(\mathbf{x}_t) + \kappa(\|\mathbf{u}_{t-\mathbf{c}_t}\|), \tag{13}$$

(H1) (*state feedback design*): *for some degrees* $\mathfrak{f}^{(s)} \in \mathbb{R}^n$ *and weights* $\mathfrak{r} \in \mathbb{R}^n_>$ *such that*

$$\begin{aligned} &\mathfrak{f}^{(s)}_{j-1} \leqslant \hat{\mathfrak{f}}^{(s)}_j \leqslant \mathfrak{f}^{(s)}_j,\ j = 2, \ldots, n, \\ &\hat{\mathfrak{f}}^{(s)}_1 := \mathfrak{f}^{(s)}_1,\ \hat{\mathfrak{f}}^{(s)}_j := \mathfrak{r}_j - \mathfrak{r}_{j-1} - \mathfrak{f}^{(s)}_{j-1},\ j = 2, \ldots, n, \end{aligned} \tag{14}$$

f is homogeneous in the upper bound with quadruples $(\mathfrak{r}, \mathfrak{r} + \mathfrak{f}^{(s)}, \hat{\mathfrak{f}}^{(s)}, \Phi^{(s)}(x))$ *and lower triangular* $\Phi^{(s)}(0)$*,*

(H2) (*observer design*): *for some degrees* $\mathfrak{f}^{(o)} \in \mathbb{R}^n$ *and weights* $\mathfrak{r} \in \mathbb{R}^n_>$ *such that*

$$\begin{aligned} &2\mathfrak{f}^{(o)}_j - \mathfrak{f}^{(o)}_{j-1} \leqslant \hat{\mathfrak{f}}^{(o)}_{j-1} \leqslant \mathfrak{f}^{(o)}_{j-1},\ j = 2, \ldots, n, \\ &\hat{\mathfrak{f}}^{(o)}_j := \mathfrak{r}_{j+1} - \mathfrak{r}_j - \mathfrak{f}^{(o)}_{j+1},\ j = 1, \ldots, n-1,\ \hat{\mathfrak{f}}^{(o)}_n := \mathfrak{f}^{(o)}_n, \end{aligned} \tag{15}$$

ϕ is incrementally homogeneous in the upper bound with quadruples $(\mathfrak{r}, \mathfrak{r} + \hat{\mathfrak{f}}^{(o)}, \mathfrak{f}^{(o)}, \Phi^{(o)}(x', x''))$ *and lower triangular* $\Phi^{(o)}(0, 0)$*.*

(H3) *(state feedback performances recovery):* $\mathfrak{f}^{(o)}_n > \mathfrak{f}^{(s)}_n$.

Assumptions (H1) and (H2) are enough general for coping with large classes of nonlinear systems: the nonlinearities must satisfy some incremental homogeneity conditions, one for state-feedback design (H1) and one for observer design (H2). The additional condition (H3) is a fast recovery condition (through state reconstruction) of the closed-loop performances achieved by state-feedback and couples the state-feedback design with the observer design. Output feedback controllers are obtained from the state-feedback controllers by processing the state estimates instead of the true (unknown) values of the state. Notice that $\Phi^{(s)}(0)$ (resp. $\Phi^{(o)}(0, 0)$) is required to be lower triangular, which implies that f, when at least once differentiable, has a lower triangular linearization at 0. This implies that the linearization of (2) at 0 is controllable. Assumptions based on incremental homogeneity similar to (H1)-(H3) have been considered in Battilotti (2014) for designing controllers for systems with no delays. In this paper, we consider more general control and observer structures than the ones introduced in Battilotti (2014) with *ad hoc* techniques for the choice of the gain matrices and saturation levels as well as for the closed-loop stability analysis. It is not difficult to check out assumptions (H1) and (H2). In general, this kind of assumptions amount to solve a set of algebraic inequalities in the unknowns $\mathfrak{r} \in \mathbb{R}^n_>$ and $\mathfrak{f}^{(\cdot)} \in \mathbb{R}^n$. For example the system

$$\begin{aligned} \dot{\mathbf{x}}_{1,t} &= \mathbf{x}_{2,t} + \mathbf{x}_{1,t} \\ \dot{\mathbf{x}}_{2,t} &= -\mathbf{x}_{1,t} + (1 - \mathbf{x}^2_{1,t})\mathbf{x}_{2,t} + \mathbf{u}_{t-\mathbf{c}_t} \end{aligned} \tag{16}$$

satisfies all the assumptions (H1)-(H3) with $\phi(\mathbf{x}) = (\mathbf{x}_1, -\mathbf{x}_1 + (1 - \mathbf{x}^2_1)\mathbf{x}_2)^T$, $\mathfrak{r} = (1, 3)^T$, $\mathfrak{f}^{(s)} = (1, 1)^T$, $\mathfrak{f}^{(o)} = (4, 2)^T$ and suitable $\Phi^{(s)}(x)$ and $\Phi^{(o)}(x', x'')$ (which we leave to the reader) with lower triangular $\Phi^{(s)}(0)$ and $\Phi^{(o)}(0, 0)$.

Assumption (H0) is a standard assumption for forward completeness and it can be relaxed by requiring that the trajectories of (2) satisfy (13) only up to time $t = c_\infty$ (i.e. forward completeness for the open-loop system). For instance, assumption (H0) holds for (16) with $U(x) = \|x\|^2$, $\mu = 3$ and $\kappa = s^2$.

The important stabilization result of this paper is the following (the proof is quite long and omitted for lack of space: see Battilotti (2019) for technical details).

Theorem 1. Let $\mathcal{C} \subset \mathbb{R}^n$ *be a given compact set. Under assumptions* (H0)-(H3) *there exist diagonal positive definite* $\Gamma^{(l)} \in \mathbb{R}^{n\times n}$, $l^{(l)} \in \mathbb{R}_>$, $l \in \{s, o\}$, $\epsilon \in \mathbb{R}_>$, $\delta \in \mathbb{R}_>$ *and a δ-fine partition* $\{p^{(j)}\}_{j=1,\ldots,\nu}$ *of* $[-c_\infty, d_\infty]$*, extended and centered at* 0*, such that the solutions* $(\mathbf{x}_t, \hat{\mathbf{x}}^{(j)}_t)$*,* $j = 2, \ldots, \nu + 1$*, of (2), (3), (4), (11), with* $\mathbf{x}_{-c_\infty-2d_\infty} \in \mathcal{C}$*, are bounded for all* $t \geqslant -c_\infty - 2d_\infty$ *and* $\lim_{t\to+\infty} \mathbf{x}_t = 0$*.*

The continuous-time controller (4), (11) guarantees asymptotic stability of (2) for all initial conditions $\mathbf{x}_{-c_\infty-2d_\infty} \in \mathcal{C}$, where $\mathcal{C}$ is an *a priori* given compact set. In this sense our controller (4), (11) *semi-globally* asymptotically stabilizes (2). Boundedness and convergence are uniform (in the sense of $\mathcal{KL}$ functions).

4.4 Example and simulations

For testing our stabilizer we consider the system

$$\begin{aligned} \dot{\mathbf{x}}_{1,t} &= \mathbf{x}_{2,t} \\ \dot{\mathbf{x}}_{2,t} &= -\mathbf{x}_{1,t} + (1 - \mathbf{x}^2_{1,t})\mathbf{x}_{2,t} + \mathbf{u}_{t-1},\ \mathbf{y}_t = \mathbf{x}_{1,t-\mathbf{d}_t} \end{aligned} \tag{17}$$

The control delay is constantly $= c$ while the measurements are taken over intervals of the form $[1.1h, 1.1h + 1]$ for $h = 0, 1, \ldots$ and are supplied at a high rate during the subsequent time interval $[1.1h + 1, 1.1(h + 1)]$. Correspondingly, the measurement delay profile is $\mathbf{d}_t$ as follows: $\mathbf{d}_t = t - 1.1h$ if $t \in [1.1h, 1.1h + 1]$ and $\mathbf{d}_t = 1 - 10(t - 1.1h - 1)$ if $t \in [1.1h + 1, 1.1(h + 1)]$, $h = 0, 1, \ldots$, and it is bounded by $d_\infty = 1$. Moreover, the input delay is $c = 1$. System (17) satisfies assumptions (H0)-(H3) of theorem 1 with $\mathfrak{r}_1 = 1/8$, $\mathfrak{r}_2 = 3/8$, $\mathfrak{f}^{(s)}_1 = \mathfrak{f}^{(s)}_2 = 1/8$, $\mathfrak{f}^{(o)}_1 = 1/2$ and $\mathfrak{f}^{(o)}_2 = 1/4$.

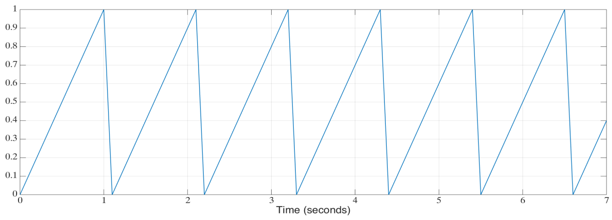

Fig. 1. Measurement delay δ_t.

A stabilizer has been designed according to our procedure and a simulation has been worked out with initial conditions $\mathbf{x}_{-c-2d_\infty} = (-5,-4)^T$. With such state initial conditions (a square initialization region $\mathcal{C}$ with side 10 has been guaranteed) an observer chain with $\nu = 11$ is sufficient for our aims. The interval $[-1,1]$ has been partitioned into 10 subintervals with equal length 0.2 and points $p_j = -1 + 0.2(j-1)$, $j = 1,\dots,12$ (with the extra point $p_{12} := 1.2$). The saturation levels of the estimates are set with $l^{(s)} = 0.05$ and $l^{(o)} = 0.1$, the diagonal elements of $\Gamma^{(s)}$ are respectively 1 and 10, the diagonal elements of $\Gamma^{(o)}$ are respectively 10 and 1. The closed-loop state trajectories $\mathbf{x}_t$ together with the prediction errors $\mathbf{e}_t^{(2)}$ are shown versus time in Fig. 2.

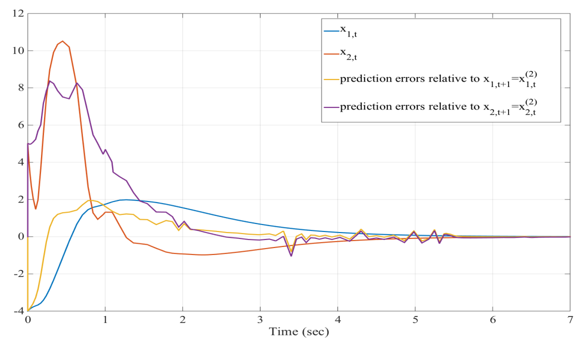

Fig. 2. Closed-loop state trajectories $\mathbf{x}_t = (\mathbf{x}_{1,t},\mathbf{x}_{2,t})^T$ and prediction errors relative to $\mathbf{x}_{t+1} = (\mathbf{x}_{1,t+1},\mathbf{x}_{2,t+1})^T$.

5. SAMPLED-DATA STABILIZERS

The design of continuous-time stabilizer for (2), (3) given in the previous section suggests naturally the way of designing a sampled-data stabilizer for (2), (3). This will consists of a sampled-data controller and a chain of sampled-data observers with sampling period T. Sampled-data stabilizers can be naturally obtained from particular classes of continuous-time stabilizers as follows. Let

$$\mathbf{u}_t = \alpha(\hat{\mathbf{x}}_{t_h}), \qquad (18)$$
$$\dot{\hat{\mathbf{x}}}_t = A\hat{\mathbf{x}}_t + \beta(\hat{\mathbf{x}}_{t_h}^{(t_0,\dots,t_k)}, \mathbf{y}_{t_h}^{(t_0,\dots,t_k)}),\ t \in [t_h, t_{h+1}),$$

$h,k \in \mathbb{N}$, $k \leqslant h$, be a continuous-time stabilizer for (2), (3) with $t_h := hT$, locally Lipschitz continuous functions α,β and $\mathbf{v}_t^{(t_0,\dots,t_k)} := (\mathbf{v}_t, \mathbf{v}_{t-t_0}, \cdots, \mathbf{v}_{t-t_k})$. A sampled-data stabilizer for (2), (3) is obtained as a zero-order hold discretization of (18) with $\mathbf{u}_t = \alpha(\hat{\mathbf{x}}_{t_h})$ for $t \in [t_h, t_{h+1})$ and

$$\hat{\mathbf{x}}_{t_{h+1}} = A_T\hat{\mathbf{x}}_{t_h} + B_T\beta(\hat{\mathbf{x}}_{t_h}^{(t_0,\dots,t_k)}, \mathbf{y}_{t_h}^{(t_0,\dots,t_k)}),\ h \in \mathbb{N}, \qquad (19)$$

where $A_T = e^{AT}$ and $B_T = \int_0^T e^{As}ds$. The stability analysis (boundedness and asymptotic convergence) of (2), (3), (19) is carried out through the stability analysis of (2), (3), (18) (therefore, following the proof of theorem 1) since the trajectories of (19) and (18) coincide at the sampling times.

With this in mind, as a first step we design a continuous-time stabilizer for (2), (3) having the form (18). From this we obtain a sampled-data stabilizer for (2), (3) according to the zero-order hold discretization procedure pointed out in (19). The δ-fine partition $\{p^{(j)}\}_{j=\dots,\nu}$ of the interval $[-c, d_\infty]$ is chosen so that each point $p^{(j)}$ (and therefore δ) is a multiple of the sampling time T. For this reason, exactly as δ in the proof of theorem 1, the sampling period T will depend on the parameter ϵ and, therefore, both on the magnitude of the delays and on the growth rate of the nonlinearities. Our continuous-time stabilizer consists of a (zero-hold in the period T) controller and a chain of ν continuous-time observers (switching after each period T). Let $P^{(j)}, R^{(j)}, G^{(j)}$, $j \in \{s,o\}$, be as in (5) and (12). The observer chain is described by:

$$\dot{\hat{\mathbf{x}}}_t^{(j)} = A\hat{\mathbf{x}}_t^{(j)} + B\mathbf{u}_{t-\mathbf{c}_t}^{(j)} \qquad (20)$$
$$+ \phi\left(\sigma_{\lambda^{(o)}(\epsilon)}\left(\hat{\mathbf{x}}_{t_h}^{(j)}\right)\right) + {P^{(o)}}^{-1}C^T R^{(o)}\mathbf{z}_{t_h}^{(j)},$$
$$j = 2,\dots,\nu+1,\ t \in [t_h, t_{h+1}),$$

with innovations $\mathbf{z}_t^{(j)}$ and delays $\mathbf{s}_t^{(j)}$ defined as in (6), (7) for $j = \nu_0+1,\dots,\nu+1$, where now $\mathbf{y}_{t^{(j)}}$ is the past output at $t^{(j)} := \max\{t_k \in [0,t]\colon t_k - \mathbf{d}_{t_k} \leqslant t - p^{(j-1)}\}$, and in (8), (9) for $j = 2,\dots,\nu_0$. Each observer is initialized as in (10). The controller is defined (compare with (11)) as

$$\mathbf{u}_t := -R^{(s)}B^T P^{(s)}(I_n - A^T G^{(s)}) \times \qquad (21)$$
$$\times\sigma_{\lambda^{(s)}(\epsilon)}\left((I_n - A^T G^{(s)})^{-1}\hat{\mathbf{x}}_{t_h}^{(j+1)}\right),\ t \in [t_h, t_{h+1}),$$

where j is such that $-\mathbf{c}_{t_h} \in [p^{(j)}, p^{(j+1)})$. It is easy to check that (20), (21) has the form (18). The following result follows from the proof of theorem 1 (see Battilotti (2019) for technical details) using the fact that the Lyapunov-Razumichin function, used in the proof, is locally quadratic around zero and the nonlinearities are locally Lipschitz.

Theorem 2. Let $\mathcal{C} \subset \mathbb{R}^n$ be a given compact set. Under assumptions (H0)-(H3) *there exist diagonal positive definite $\Gamma^{(j)} \in \mathbb{R}^{n\times n}$, $l^{(j)} \in \mathbb{R}_>$, $j \in \{s,o\}$, $\epsilon, \delta, T \in \mathbb{R}_>$ and a δ-fine partition $\{p^{(j)}\}_{j=-1,\dots,\nu}$ of $[-c, d_\infty]$, extended and centered at 0, such that the solutions $(\mathbf{x}_t, \hat{\mathbf{x}}_t^{(j)})$, $j = 2\dots,\nu+1$, of (2), (3), (20), (21), with $\mathbf{x}_{-c_\infty-2d_\infty} \in \mathcal{C}$, are bounded for all $t \geqslant -c_\infty - 2d_\infty$ and $\lim_{t\to+\infty}\mathbf{x}_t = 0$.*

The continuous-time controller (20), (21) has the form (18) and *semi-globally* asymptotically stabilizes (2). Also in this case boundedness and convergence results are uniform (in the sense of $\mathcal{KL}$ functions). The sampled-data stabilizer, obtained from a zero-order hold discretization of the continuous-time (20), (21) as pointed out in (19), semi-globally asymptotically stabilizes (2), since the trajectories of (20), (21) and its sampled-data counterpart coincide at the sampling times.

Appendix A. INCREMENTAL GENERALIZED HOMOGENEITY

The notion of (incremental) homogeneity in a generalized sense has been introduced in Battilotti (2014) in the context of (semi-)global stabilization and observer design problems. In this appendix we shortly recall this notion in a slightly more general form. For any function ϕ let $(\Delta\phi)(x', x'') := \phi(x') - \phi(x'')$ and if ϕ is the identity function we simply write $\Delta(x', x'') := x' - x''$.

Definition 3. A parametric function $\phi(\epsilon) \in \mathbf{C}^0(\mathbb{R}^n, \mathbb{R}^l)$, $\epsilon \in \mathbb{R}_>$, *is said to be incrementally homogeneous (in the generalized sense: g.i.h.) with quadruple* $(\mathfrak{r}, \mathfrak{d}, \mathfrak{h}, \Phi(x', x''))$ *if there exist* $\mathfrak{d} \in \mathbb{R}^l, \mathfrak{h} \in \mathbb{R}^n$, $\mathfrak{r} \in \mathbb{R}^n_>$ *and* $\Phi \in \mathbf{C}^0(\mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^{l\times n})$ *such that for all* $\epsilon \in \mathbb{R}_>$ *and* $x', x'' \in \mathbb{R}^n$

$$(\Delta\phi(\epsilon))(\epsilon^{\mathfrak{r}} \diamond x', \epsilon^{\mathfrak{r}} \diamond x'') = \epsilon^{\mathfrak{d}} \diamond \left(\Phi(x', x'')\Delta(\epsilon^{\mathfrak{h}} \diamond x', \epsilon^{\mathfrak{h}} \diamond x'')\right)$$

When the variation Δ of $\phi(\epsilon)$ is computed in between the dilated points $x' := x \in \mathbb{R}^n$ and $x'' := 0$, with $\phi(\epsilon)(0) = 0$, we say $\phi(\epsilon)$ *is homogeneous (in the generalized sense: g.h.) with quadruple* $(\mathfrak{r}, \mathfrak{d}, \mathfrak{h}, \Phi'(x))$ with $\Phi'(x) := \Phi(x, 0)$.

Definition 4. A parametric function $\phi(\epsilon) \in \mathbf{C}^0(\mathbb{R}^n, \mathbb{R}^l)$, $\epsilon \in \mathbb{R}_>$, *is said to be incrementally homogeneous in the upper bound (in the generalized sense: g.i.h.u.b.) with quadruple* $(\mathfrak{r}, \mathfrak{d}, \mathfrak{h}, \Phi(x', x''))$ *if there exist* $\mathfrak{d} \in \mathbb{R}^l, \mathfrak{h} \in \mathbb{R}^n$, $\mathfrak{r} \in \mathbb{R}^n_>$, $\Phi \in \mathbf{C}^0(\mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^{l\times n}_{\geqslant})$ *such that for all* $\epsilon \in (1, +\infty)$ *and* $x', x'' \in \mathbb{R}^n$

$$\begin{aligned}&\left\langle(\Delta\phi(\epsilon))(\epsilon^{\mathfrak{r}} \diamond x', \epsilon^{\mathfrak{r}} \diamond x'')\right\rangle \\ &\qquad\le \epsilon^{\mathfrak{d}} \diamond \left(\Phi(x', x'')\left\langle\Delta(\epsilon^{\mathfrak{h}} \diamond x', \epsilon^{\mathfrak{h}} \diamond x'')\right\rangle\right)\end{aligned}$$

Notice that, in the case of g.i.h.u.b., expanding dilations (i.e. $\epsilon \in (1, +\infty)$) are considered. When the variation Δ of $\phi(\epsilon)$ is computed in between the dilated points $x' := x \in \mathbb{R}^n$ and $x'' := 0$, with $\phi(\epsilon)(0) = 0$, we say $\phi(\epsilon)$ *is homogeneous in the upper bound (in the generalized sense: g.h.u.b.) with quadruple* $(\mathfrak{r}, \mathfrak{d}, \mathfrak{h}, \Phi'(x))$ with $\Phi'(x) = \Phi(x, 0)$.

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