Finsler-based Sampled-data Controller Design for Takagi-Sugeno Systems

Adriano N.D. Lopes [*,**] **Kevin Guelton** [*] **Laurent Arcese** [*]
Valter J.S. Leite [**] **Fayçal Bourahala** [***]

[*] *CReSTIC EA3804 - Université de Reims Champagne-Ardenne, Moulin de la Housse BP1039, 51100 Reims, France (e-mail: adriano.lopes,kevin.guelton,laurent.arcese@ univ-reims.fr).*
[**] *Department of Mechatronics Engineering, CEFET-MG, 400, 35503-822, Divinópolis, MG, Brazil. (e-mail: valter@ieee.org)*
[***] *LAS Laboratory - 20 August University, 21000 Skikda, Algeria (e-mail: bourahala1981@yahoo.fr)*

Abstract: This paper investigates the sampled-data control of continuous-time Takagi-Sugeno (T-S) fuzzy systems. The closed-loop dynamics is rewritten as a T-S system with input time-varying delays. In this context, asynchronous membership functions appears in the closed-loop dynamics. Thus, to reduce the conservatism of design conditions involving mismatch membership functions, a dedicated relaxation scheme is proposed. Then, from a convenient Lyapunov-Krasovskii function and the application of the Finsler's Lemma, new LMI-based conditions are proposed for the design of sampled-data Parallel-Distributed-Compensation (PDC) controllers. An example is provided to illustrate the effectiveness of the proposed design methodology in simulation, as well as to highlight their conservatism improvement regarding to previous related results from the literature.

Keywords: Sampled-data controllers, Takagi-Sugeno models, Lyapunov Krasowskii Functionals.

1. INTRODUCTION

During the last decades, sampled-data control approaches emerged as a promising research topic in control theory. It consists in the investigation of the overall closed-loop stability of continuous-time plants driven by sampled-data controllers, see e.g. (Fridman et al., 2004; Hetel et al., 2017). In this context, an elegant and powerful way to design such controllers consists is rewriting the closed-loop dynamics as a continuous-time system with input time-varying delay, also known as a time-delay approach for the stabilization of sampled-data systems (Fridman et al., 2004). If many efforts have been done for the stabilization of linear dynamical systems from sampled-data measurements, most of real applications exhibit nonlinear dynamics. Among the nonlinear control theory, Takagi-Sugeno (T-S) fuzzy models (Takagi and Sugeno, 1985) are nowadays known convenient to provide a polytopic representation of nonlinear systems as weighted sums of linear subsystems.

A vast literature is available for various T-S model-based control problems, for instance dealing with continuous-time controller design, see e.g. (Guerra et al., 2012; Cherifi et al., 2018, 2019), discrete-time ones, see e.g. (Xie et al., 2017; Lopes et al., 2020b), T-S systems with time-delays, see e.g. (Li and Liu, 2009; Bourahala et al., 2017), or also sampled-data control, see e.g. (Yoneyama, 2010; Zhang and Han, 2011; Cheng et al., 2017). Indeed, thanks to their convex polytopic structures, stability conditions and controller design conditions for T-S systems are usually studied via Lyapunov approaches and solved in the Linear Matrix Inequality (LMI) framework. Nevertheless, these LMI-based results provide only sufficient conditions and so suffer from conservatism, which reduction is an important and common challenge for the T-S community, see e.g. (Sala, 2009) and references therein.

When dealing with sampled-data control, a convenient way to check the conservatism of the design conditions is to search for the maximal allowable sampling period $\bar{\eta}$, with which the closed-loop dynamics is stabilized. In this context, successive conservatism improvements have been obtained. For instance, a Lyapunov-Krasovskii function (LKF) and relaxation techniques based on the Leibniz-Newton formula and free-weighting matrix has been considered in (Yoneyama, 2010). Then, since the delayed membership functions involved in the controller part are mismatching the ones involved in the continuous-time plant to be controlled, the upper bounds of the asynchronous errors of the membership functions has been introduced in the design conditions (Zhang and Han, 2011). In (Zhu et al., 2012), an enlargement scheme has been introduced in the stabilization criteria. Furthermore, the variation ranges of membership functions within variable sampling intervals has been considered in (Zhu et al., 2013). More recently, a structured vertex separator has been used to reduce the number of LMIs constraints (Cheng et al., 2017).

This paper follow the same objective as the above mentioned ones, i.e. conservatism improvement for the design of sampled-data controllers for T-S systems. In this context, new LMI-based conditions are proposed from the choice of a convenient augmented LKF candidate, extensions of Jensen's inequalities, and by applying the Finsler's Lemma. The effectiveness of the proposed result will be illustrated and compared to related previous studies through the benchmark of the inverted pendulum on a cart.

Notations. Stars * in symmetric matrices denote block transpose quantities. We denote the set of integers $\mathcal{I}_r = \{1, ..., r\}$. For any square matrix M, $\mathcal{H}(M) = M + M^T$. I is an identity matrix with appropriate dimension. For vectors v_1, v_2,..., v_n, $col\{v_1, v_2, ..., v_n\} = \left[\, v_1^T \;\, v_2^T \;\, \ldots \;\, v_n^T \,\right]^T$.

2. PRELIMINARIES

Let us consider a continuous-time T-S system given by:

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(z(t)) \left(A_i x(t) + B_i u(t)\right) \qquad (1)$$

where $z(t) = [\, z_1(t) \ldots z_p(t) \,] \in \mathbb{R}^p$ is a known vector of premise variables which only depends (for control purpose) on the entries of the state vector $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ is the control input vector, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ are known constant matrices describing the dynamics of each polytope and $\alpha_i(z(t)) \geq 0$ are the membership functions satisfying the convex properties $\sum_{i=1}^{r} \alpha_i(z(t)) = 1$.

In this paper, we consider the stabilization of T-S systems (1) from the following sampled-data PDC control law:

$$u(t) = \sum_{i=1}^{r} \alpha_i(z(t_k)) K_i X^{-1} x(t_k) \qquad (2)$$

where $K_i \in \mathbb{R}^{m \times n}$ and $X^{-1} \in \mathbb{R}^{n \times n}$, for $i \in \mathcal{I}_r$, are the controller gain matrices to be designed, a zero holder is employed $\forall t \in [t_k, t_{k+1})$ to maintain $x(t_k)$ from the aperiodic sampling instants $t_k \geq 0$ such that:

$$t_{k+1} - t_k \leq \eta_k \leq \bar{\eta} \qquad (3)$$

where the inner sampling period $\eta_k > 0$ can be non uniform over samples with a maximal allowable sampling period $\bar{\eta}$ to be estimated.

For actual $t \in [t_k, t_{k+1})$, let $\tau(t) = t - t_k \in [0, \eta_k)$ with $\dot{\tau}(t) = 1$, the control law (2) can be rewritten as:

$$u(t) = \sum_{i=1}^{r} \alpha_i(z(t - \tau(t))) K_i X^{-1} x(t - \tau(t)) \qquad (4)$$

In the sequel, for fuzzy summations of matrices we denote $M_\alpha = \sum_{i=1}^{r} \alpha_i(z(t)) M_i$, $M_{\bar{\alpha}} = \sum_{i=1}^{r} \alpha_i(z(t - \tau(t))) M_i$ and $M_{\alpha\bar{\alpha}} = \sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i(t) \alpha_j(z(t - \tau(t))) M_{ij}$. Substituting (4) in (1) gives the closed-loop dynamics as:

$$\dot{x}(t) = A_\alpha x(t) + B_\alpha K_{\bar{\alpha}} X^{-1} x(t - \tau(t)) \qquad (5)$$

Problem statement. Provide relaxed LMI-based conditions for the design of the gain matrices K_i and X such that the sampled-data closed-loop dynamics (5) is asymptotically stable.

The following lemmas will be considered for the proofs of the main results proposed in the next section.

Lemma 1. (Xie, 1996): Let X and Y be matrices of appropriate dimensions. For any matrix $T > 0$, the following inequality is true:

$$X^T Y + Y^T X \leq X^T T X + Y^T T^{-1} Y \qquad (6)$$

Lemma 2. (Tuan et al., 2001): For $(i, j) \in \mathcal{I}_r^2$, Let Γ_{ij} be matrices of appropriate dimensions. The inequality $\Gamma_{\alpha\alpha} < 0$ is satisfied if the following conditions hold:

$$\forall i \in \mathcal{I}_r : \Gamma_{ii} < 0 \qquad (7)$$

$$\forall (i, j) \in \mathcal{I}_r^2, i \neq j : \frac{2}{r-1} \Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0 \qquad (8)$$

Lemma 3. (Zhang and Han, 2013): For any constant matrix $R \in \mathbb{R}^{n \times n}$, $R = R^T > 0$, a scalar function $\tau(t)$ with $0 < \tau(t) \leq \tau_M$ and a vector function $\dot{x} : [-\tau_M, 0] \to \mathbb{R}^n$ such that the integration concerned is well defined, let

$$\int_{t-\tau(t)}^{t} \dot{x}(s) ds = E \psi(t) \qquad (9)$$

where $E \in \mathbb{R}^{n \times k}$ and $\psi(t) \in \mathbb{R}^k$. Then the following inequality holds for any matrix $M \in \mathbb{R}^{n \times k}$

$$-\int_{t-\tau(t)}^{t} \dot{x}^T(s) R \dot{x}(s) ds \leq \psi^T(t) \Upsilon_1 \psi(t) \qquad (10)$$

where $\Upsilon_1 = -E^T M - M^T E + \tau(t) M^T R^{-1} M$.

Lemma 4. (Fridman, 2014) For any matrix $P = P^T > 0$ with appropriate dimensions, $\tau(t) \in [0, \eta_k)$ and $\dot{\tau}(t) = 1$, the following inequality holds:

$$\int_{t-\tau(t)}^{t} x^T(s) P x(s) ds \geq \eta_k^{-1} \int_{t-\tau(t)}^{t} x^T(s) P \int_{t-\tau(t)}^{t} x(s) ds \qquad (11)$$

Lemma 5. (Skelton et al., 1998) Let $\xi \in \mathbb{R}^n$, $G \in \mathbb{R}^{m \times n}$ and $Q = Q^T \in \mathbb{R}^{n \times n}$ such that $rank(G) < n$. The following statements are equivalent.

$$\xi^T Q \xi < 0, \quad \forall \xi \in \{\xi \in \mathbb{R}^n : \xi \neq 0, G\xi = 0\} \qquad (12)$$

$$\exists R \in \mathbb{R}^{n \times m} : Q + RG + G^T R^T < 0 \qquad (13)$$

3. MAIN RESULTS

Let us recall that the relaxation scheme expressed in Lemma 2 cannot be directly employed in the context of sampled-data control since the closed-loop dynamics (5) involves a double fuzzy sum structure with asynchronous membership functions ($\alpha\bar{\alpha}$). Therefore, before deriving LMI-based conditions for the design of sampled-data PDC controllers (2) dedicated to stabilize continuous-time T-S fuzzy models (1), we will first propose a generic relaxation scheme to cope with this drawback.

3.1 Asynchronous double fuzzy sums relaxation

To deal with parameterized matrix inequalities involving double fuzzy sum structures with asynchronous membership functions ($\alpha\bar{\alpha}$), we propose the following theorem.

Theorem 1. For $(i, j) \in \mathcal{I}_r^2$, let Λ_{ij} be matrices of appropriate dimensions and assume, $\forall t$, $|\dot{\alpha}_i(t)| \leq \phi_i$. The inequality $\Lambda_{\alpha\bar{\alpha}} < 0$ is satisfied if there exists diagonal matrices $T_{ij} > 0$ such that conditions (7) and (8) hold with:

$$\Gamma_{ij} = \begin{bmatrix} \Lambda_{ij} + \frac{r-1}{16} T_{ij} & (*) & \ldots & (*) \\ \sigma_1 \bar{\Lambda}_{ij1} & -T_{ij} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sigma_{r-1} \bar{\Lambda}_{ijr-1} & 0 & 0 & -T_{ij} \end{bmatrix} \qquad (14)$$

where, $\forall q \in \mathcal{I}_{r-1}$, $\bar{\Lambda}_{ijq} = \Lambda_{iq} + \Lambda_{jq} - \Lambda_{ir} - \Lambda_{jr}$ and $\sigma_q = \min\{1, \phi_q \bar{\eta}\}$.

Proof. Using the short hand notation for memberships functions $\alpha_i = \alpha_i(z(t))$ and $\bar{\alpha}_i = \alpha_i(z(t-\tau(t)))$ we have:

$$\begin{aligned}\Lambda_{\alpha\bar{\alpha}} &= \sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i\bar{\alpha}_j\Lambda_{ij} = \sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i\alpha_j\left(\Lambda_{ij} + \sum_{q=1}^{r}(\bar{\alpha}_q-\alpha_q)\Lambda_{iq}\right)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i\alpha_j\left(\Lambda_{ij} + \sum_{q=1}^{r}\frac{\bar{\alpha}_q-\alpha_q}{2}(\Lambda_{iq}+\Lambda_{jq})\right)\end{aligned} \tag{15}$$

Since $\sum_{q=1}^{r}(\bar{\alpha}_q-\alpha_q) = 0 \Leftrightarrow (\bar{\alpha}_r-\alpha_r) = -\sum_{q=1}^{r-1}(\bar{\alpha}_q-\alpha_q)$, $\forall(i,j)$ we can write:

$$\sum_{q=1}^{r}\frac{\bar{\alpha}_q-\alpha_q}{2}(\Lambda_{iq}+\Lambda_{jq}) = \sum_{q=1}^{r-1}\frac{\bar{\alpha}_q-\alpha_q}{2}\bar{\Lambda}_{ijq} \tag{16}$$

with $\bar{\Lambda}_{ijq} = \Lambda_{iq}+\Lambda_{jq}-\Lambda_{ir}-\Lambda_{jr}$.

Note that, $\forall q \in \mathcal{I}_r$ we have:

$$-1 \le \alpha_q - \bar{\alpha}_q \le 1 \tag{17}$$

Moreover, by assuming $\forall t$, $|\dot{\alpha}_q(t)| \le \phi_q$ and since $\tau(t) \in [0,\eta_k)$ with $\eta_k \le \bar{\eta}$, we also have:

$$-\phi_q\bar{\eta} \le \alpha_q - \bar{\alpha}_q = \int_{t-\tau(t)}^{t}\dot{\alpha}_q(s)ds \le \phi_q\bar{\eta} \tag{18}$$

Thus, from (17) and (18), we can assert that:

$$-1 \le \frac{\bar{\alpha}_q-\alpha_q}{\sigma_q} \le 1 \text{ with } \sigma_q = \min\{1, \phi_q\bar{\eta}\} \tag{19}$$

Let us now rewrite (16) as:

$$\sum_{q=1}^{r-1}\frac{\bar{\alpha}_q-\alpha_q}{2}\bar{\Lambda}_{ijq} = \mathcal{H}_e\left(\frac{1}{4}\underbrace{\begin{bmatrix} I & \dots & I\end{bmatrix}}_{r-1 \text{ times } I}\Delta_{\alpha\bar{\alpha}}\nabla_{ij}\right) \tag{20}$$

where:

$$\Delta_{\alpha\bar{\alpha}} = \begin{bmatrix} \frac{\bar{\alpha}_1-\alpha_1}{\sigma_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\bar{\alpha}_{r-1}-\alpha_{r-1}}{\sigma_{r-1}}\end{bmatrix} \text{ and } \nabla_{ij} = \begin{bmatrix}\sigma_1\bar{\Lambda}_{ij1}\\ \vdots \\ \sigma_{r-1}\bar{\Lambda}_{ijr-1}\end{bmatrix}$$

From Lemma 1, for any matrices $T_{ij} > 0$, it yields:

$$\sum_{q=1}^{r-1}\frac{\bar{\alpha}_q-\alpha_q}{2}\bar{\Lambda}_{ijq} \le \frac{r-1}{16}T_{ij} + \nabla_{ij}^T\Delta_{\alpha\bar{\alpha}}T_{ij}^{-1}\Delta_{\alpha\bar{\alpha}}\nabla_{ij} \tag{21}$$

Now, let $T_{ij} > 0$ be diagonal matrices, since $\Delta_{\alpha\bar{\alpha}}$ is also diagonal and $\Delta_{\alpha\bar{\alpha}}\Delta_{\alpha\bar{\alpha}} \le I$, $\Delta_{\alpha\bar{\alpha}}T_{ij}^{-1}\Delta_{\alpha\bar{\alpha}} = \Delta_{\alpha\bar{\alpha}}\Delta_{\alpha\bar{\alpha}}T_{ij}^{-1} \le T_{ij}^{-1}$. Thus, considering (15), (16) and (21) and applying Lemma 2, then applying the Schur complement, we obtain the conditions expressed in theorem 1. □

3.2 LMI-based sampled-data controller design

The following theorem summarizes the proposed relaxed LMI-based sampled-data controller (2) design conditions for T-S systems (1).

Theorem 2. : Let $(i,j) \in \mathcal{I}_r^2$ and assume that there exists the scalars $\phi_i > 0$ such that $\forall t, |\dot{\alpha}_i(t)| \le \phi_i$. For aperiodic sampling periods $\eta_k \le \bar{\eta}$ ($\bar{\eta}$ to be maximized), the T-S fuzzy model (1) is stabilized by the sampled-data PDC controller (2) if there exists the matrices $0 < \bar{L} = \bar{L}^T \in \mathbb{R}^{n\times n}$, $\bar{M}_j = \bar{M}_j^T \in \mathbb{R}^{4n\times 4n}$, $\bar{N}_j = \bar{N}_j^T \in \mathbb{R}^{2n\times 2n}$, $\bar{P}_{11ij} = \bar{P}_{11ij}^T \in \mathbb{R}^{n\times n}$, $\bar{P}_{22ij} = \bar{P}_{22ij}^T \in \mathbb{R}^{n\times n}$, $\bar{P}_{12ij} \in \mathbb{R}^{n\times n}$, $X \in \mathbb{R}^{n\times n}$, $K_j \in \mathbb{R}^{m\times n}$, $\bar{Y}_{ij} \in \mathbb{R}^{4n\times n}$, $\bar{U}_{ij} = \bar{U}_{ij}^T \in \mathbb{R}^{3n\times 3n}$ and the scalars ε_1, ε_2 and ε_3, such that the conditions of Theorem 1 are satisfied with:

$$\Lambda_{ij} = \begin{bmatrix} \Lambda_{ij}^{11} & * & * & * & * & * \\ 0 & \Lambda_{ij}^{22} & * & * & * & \\ 0 & \bar{\eta}\bar{Y}_{ij} & -\bar{\eta}\bar{P}_{22ij} & * & * & * \\ 0 & \bar{\eta}\tilde{W}_j & 0 & -\bar{U}_{ij} & * & * \\ 0 & 0 & 0 & 0 & \bar{M}_j^0 - \bar{U}_{ij} & * \\ 0 & 0 & 0 & 0 & 0 & -\bar{P}_{11ij}\end{bmatrix} < 0 \tag{22}$$

with:

$$\Lambda_{ij}^{11} = \bar{\Phi}_{\Sigma ij}^0 + \mathbb{I}_\varepsilon\bar{G}_{ij} + \bar{G}_{ij}^T\mathbb{I}_\varepsilon^T,$$

$$\Lambda_{ij}^{22} = \eta_k^2\tilde{M}_j^0 + \eta_k(\bar{\Phi}_{\Sigma j}^1 - \tilde{P}_{ij}) + \bar{\Phi}_{\Sigma ij}^0 + \mathbb{I}_\varepsilon\bar{G}_{ij} + \bar{G}_{ij}^T\mathbb{I}_\varepsilon^T,$$

$$\mathbb{I}_\varepsilon = \begin{bmatrix} I & \varepsilon_1 I & \varepsilon_2 I & \varepsilon_3 I\end{bmatrix}^T, \bar{G}_{ij} = \begin{bmatrix} A_iX & B_iK_j & 0 & -X\end{bmatrix}, \tilde{W}_j = \begin{bmatrix} 0 & W_j\end{bmatrix},$$

$$\bar{\Phi}_{\Sigma j}^1 = \mathcal{H}\left(\bar{\eta}\mathbb{E}_1^T\bar{M}_j\mathbb{E}_2 - \mathbb{E}_1^T\bar{M}_j\mathbb{E}_1\right) - \mathcal{H}\left(\mathbb{E}_4^T\bar{N}_j\mathbb{E}_5\right),$$

$$\begin{aligned}\bar{\Phi}_{\Sigma ij}^0 = {}& \bar{\eta}\mathbb{E}_1^T\bar{M}_j\mathbb{E}_1 + \mathcal{H}\left(\bar{\eta}\mathbb{E}_4^T\bar{N}_j\mathbb{E}_5\right) - \mathcal{H}\left(E^T\bar{Y}_{ij}\right)\\ &+ \begin{bmatrix} \bar{\eta}\bar{P}_{11ij} - \bar{P}_{12j} & 0 & 0 & \bar{L} + \bar{\eta}\bar{P}_{12j} \\ 0 & \bar{P}_{12j} & 0 & 0 \\ 0 & 0 & -\eta_k^{-1}\bar{P}_{11ij} & 0 \\ * & 0 & 0 & \bar{\eta}\bar{P}_{22ij}\end{bmatrix},\end{aligned}$$

$$\tilde{M}_j^0 = \begin{bmatrix} \bar{U}_{ij} - \bar{M}_j^0 & 0 \\ 0 & 0\end{bmatrix}, \bar{M}_j^0 = \begin{bmatrix} \mathcal{H}(\bar{M}_{13j} + \bar{M}_{34j}) & \bar{M}_{23j}^T - \bar{M}_{34j} & \bar{M}_{33j}^T \\ * & 0 & 0 \\ * & 0 & 0\end{bmatrix},$$

$$\bar{M}_j = \begin{bmatrix} \bar{M}_{11j} & \bar{M}_{12j} & \bar{M}_{13j} & \bar{M}_{14j} \\ * & \bar{M}_{22j} & \bar{M}_{23j} & \bar{M}_{24j} \\ * & * & \bar{M}_{33j} & \bar{M}_{34j} \\ * & * & * & \bar{M}_{44j}\end{bmatrix}, \tilde{P}_{ij} = \begin{bmatrix} \bar{P}_{11ij} & 0 & 0 & \bar{P}_{12j} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ * & 0 & 0 & \bar{P}_{22ij}\end{bmatrix},$$

$$\bar{W}_j = \begin{bmatrix} \mathcal{H}(\bar{M}_{14j}) + \bar{M}_{11j} + \bar{M}_{44j} \\ \bar{M}_{24j} - \bar{M}_{44j} + \bar{M}_{12j}^T - \bar{M}_{14j}^T \\ \bar{M}_{34j} + \bar{M}_{13j}^T\end{bmatrix}, \mathbb{E}_1 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ I & -I & 0 & 0\end{bmatrix},$$

$$\mathbb{E}_2 = \begin{bmatrix} 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & I\end{bmatrix}, \mathbb{E}_4 = \begin{bmatrix} 0 & 0 & I & 0 \\ I & -I & 0 & 0\end{bmatrix}, \mathbb{E}_5 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I\end{bmatrix}.$$

Proof. Let us consider the following LKF candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \tag{23}$$

where:

$$V_1(t) = x(t)^T Lx(t) \tag{24}$$

$$V_2(t) = (\eta_k\tau(t) - \tau^2(t))\zeta^T(t)M_{\bar{\alpha}}\zeta(t) \tag{25}$$

$$V_3(t) = (\eta_k - \tau(t))\rho^T(t)N_{\bar{\alpha}}\rho(t) \tag{26}$$

$$V_4(t) = (\eta_k - \tau(t))\int_{t-\tau(t)}^{t}\chi^T(s)P_{\alpha\bar{\alpha}}\chi(s)ds \tag{27}$$

with $\chi(t) = col\{x(t), \dot{x}(t)\}$ and:

$$\zeta(t) = col\left\{x(t), x(t-\tau(t)), \int_{t-\tau(t)}^{t}x(s)ds, \int_{t-\tau(t)}^{t}\dot{x}(s)ds\right\},$$

$$\rho(t) = col\left\{\int_{t-\tau(t)}^{t}x(s)ds, \int_{t-\tau(t)}^{t}\dot{x}(s)ds\right\}.$$

Assuming $L = L^T > 0$, the whole LKF (23) is continuous and positive at each sample time t_k since we have $V_1(t_k) > 0$ and $V_\ell(t_k^-) = V_\ell(t_k) = 0$, for $\ell = 2, ..., 4$. Hence, since the LKF $V(t)$ is continuous $\forall t \in [t_k, t_{k+1})$, if it can be proven to be monotonously decreasing during this interval, then it is positive $\forall t \in [0, +\infty)$ and the closed-loop dynamics (5) is stable. That is to say if, $\forall t \in [t_k, t_{k+1})$:

$$\dot{V}(t) = \dot{V}_1(t)) + \dot{V}_2(t)) + \dot{V}_3(t)) + \dot{V}_4(t)) < 0 \tag{28}$$

To make up the stability conditions, let us consider the following extended state vector:

$$\xi(t) = col\left\{x(t), x(t-\tau(t)), \int_{t-\tau(t)}^{t} x(s)ds, \dot{x}(t)\right\} \tag{29}$$

The derivative of $V_1(t)$ is:

$$\dot{V}_1(t) = 2x^T(t)L\dot{x}(t) = \xi^T(t)\Phi_1^0\xi(t), \Phi_1^0 = \begin{bmatrix} 0&0&0&L\\ 0&0&0&0\\ 0&0&0&0\\ L&0&0&0 \end{bmatrix} \tag{30}$$

Then, for $V_2(t)$ we have:

$$\begin{aligned}\dot{V}_2(t) =&(\eta_k - 2\tau(t))\zeta^T(t)M_{\bar{\alpha}}\zeta(t)\\ &+ 2(\eta_k\tau(t) - \tau^2(t))\zeta^T(t)M_{\bar{\alpha}}\dot{\zeta}(t)\end{aligned} \tag{31}$$

i.e. since $\zeta(t) = \mathbb{E}_1\xi(t)$ and $\dot{\zeta}(t) = \mathbb{E}_2\xi(t)$:

$$\begin{aligned}\dot{V}_2(t) =&\tau^2(t)\xi^T(t)\Phi_{2\bar{\alpha}}^2\xi(t)\\ &+ \tau(t)\xi^T(t)\Phi_{2\bar{\alpha}}^1\xi(t) + \xi^T(t)\Phi_{2\bar{\alpha}}^0\xi(t)\end{aligned} \tag{32}$$

with $\Phi_{2\bar{\alpha}}^2 = -\mathcal{H}\left(\mathbb{E}_1^T M_{\bar{\alpha}}\mathbb{E}_2\right)$,
$\Phi_{2\bar{\alpha}}^1 = \mathcal{H}\left(\eta_k\mathbb{E}_1^T M_{\bar{\alpha}}\mathbb{E}_2 - \mathbb{E}_1^T M_{\bar{\alpha}}\mathbb{E}_1\right)$ and $\Phi_{2\bar{\alpha}}^0 = \eta_k\mathbb{E}_1^T M_{\bar{\alpha}}\mathbb{E}_1$.
Now the derivative of $V_3(t)$ is:

$$\dot{V}_3(t) = -\rho^T(t)N_{\bar{\alpha}}\rho(t) + 2(\eta_k - \tau(t))\rho^T(t)N_{\bar{\alpha}}\dot{\rho}(t) \tag{33}$$

Since $\rho(t) = \mathbb{E}_4\xi(t)$ and $\dot{\rho}(t) = \mathbb{E}_5\xi(t)$ we can write:

$$\dot{V}_3(t) = \tau(t)\xi^T(t)\Phi_{3\bar{\alpha}}^1\xi(t) + \xi^T(t)\Phi_{3\bar{\alpha}}^0\xi(t) \tag{34}$$

with $\Phi_{3\bar{\alpha}}^0 = \mathcal{H}\left(\eta_k\mathbb{E}_4^T N_{\bar{\alpha}}\mathbb{E}_5\right) - \mathbb{E}_4^T N_{\bar{\alpha}}\mathbb{E}_4$ and $\Phi_{3\bar{\alpha}}^1 = -\mathcal{H}\left(\mathbb{E}_4^T N_{\bar{\alpha}}\mathbb{E}_5\right)$.
Taking the derivative of $V_4(t)$ we get:

$$\dot{V}_4(t) = (\eta_k - \tau(t))\chi^T(t)P_{\alpha\bar{\alpha}}\chi(t) - \int_{t-\tau(t)}^{t}\chi^T(s)P_{\alpha\bar{\alpha}}\chi(s)ds \tag{35}$$

Assuming $P_{\alpha\bar{\alpha}} = \begin{bmatrix} P_{11\alpha\bar{\alpha}} & P_{12\bar{\alpha}}\\ * & P_{22\alpha\bar{\alpha}}\end{bmatrix}$ leads to:

$$\begin{aligned}\dot{V}_4(t) =& (\eta_k - \tau(t))\chi^T(t)P_{\alpha\bar{\alpha}}\chi(t) - \int_{t-\tau(t)}^{t} x^T(s)P_{11\alpha\bar{\alpha}}x(s)ds\\ &- \int_{t-\tau(t)}^{t}\dot{x}^T(s)P_{22\alpha\bar{\alpha}}\dot{x}(s)ds - 2\int_{t-\tau(t)}^{t} x^T(s)P_{12\bar{\alpha}}\dot{x}(s)ds\end{aligned} \tag{36}$$

That is to say:

$$\begin{aligned}\dot{V}_4(t) =& (\eta_k - \tau(t))\chi^T(t)P_{\alpha\bar{\alpha}}\chi(t) - \int_{t-\tau(t)}^{t} x^T(s)P_{11\alpha\bar{\alpha}}x(s)ds\\ &- \int_{t-\tau(t)}^{t}\dot{x}^T(s)P_{22\alpha\bar{\alpha}}\dot{x}(s)ds - x^T(t)P_{12\bar{\alpha}}x(t)\\ &+ x^T(t-\tau(t))P_{12\bar{\alpha}}x(t-\tau(t))\end{aligned} \tag{37}$$

Assuming $P_{11\alpha\bar{\alpha}} > 0$ and applying Lemma 4 on the first intergal term, we have:

$$\begin{aligned}\dot{V}_4(t) \leq& (\eta_k - \tau(t))\chi^T(t)P_{\alpha\bar{\alpha}}\chi(t)\\ &- \eta_k^{-1}\int_{t-\tau(t)}^{t} x^T(s)P_{11\alpha\bar{\alpha}}\int_{t-\tau(t)}^{t} x(s)ds\\ &- \int_{t-\tau(t)}^{t}\dot{x}^T(s)P_{22\alpha\bar{\alpha}}\dot{x}(s)ds - x^T(t)P_{12\bar{\alpha}}x(t)\\ &+ x^T(t-\tau(t))P_{12\bar{\alpha}}x(t-\tau(t))\end{aligned} \tag{38}$$

Assuming $P_{22\alpha\bar{\alpha}} > 0$, for the second integral term, note that:

$$\int_{t-\tau(t)}^{t}\dot{x}^T(s)ds = \underbrace{\begin{bmatrix} I & -I & 0 & 0\end{bmatrix}}_{E}\xi(t) \tag{39}$$

Hence, applying Lemma 3, for any matrix $Y_{\alpha\bar{\alpha}}$ we have:

$$\begin{aligned}\dot{V}_4(t) \leq& (\eta_k - \tau(t))\chi^T(t)P_{\alpha\bar{\alpha}}\chi(t)\\ &- \eta_k^{-1}\int_{t-\tau(t)}^{t} x^T(s)P_{11\alpha\bar{\alpha}}\int_{t-\tau(t)}^{t} x(s)ds\\ &+ \xi^T(t)\left(-E^T Y_{\alpha\bar{\alpha}} - Y_{\alpha\bar{\alpha}}^T E + \tau(t)Y_{\alpha\bar{\alpha}}^T P_{22\alpha\bar{\alpha}}^{-1}Y_{\alpha\bar{\alpha}}\right)\xi(t)\\ &- x^T(t)P_{12\bar{\alpha}}x(t) + x^T(t-\tau(t))P_{12\bar{\alpha}}x(t-\tau(t))\end{aligned} \tag{40}$$

Or, equivalently:

$$\dot{V}_4(t) \leq \tau(t)\xi^T(t)\Phi_{4\alpha\bar{\alpha}}^1\xi(t) + \xi^T(t)\Phi_{4\alpha\bar{\alpha}}^0\xi(t) \tag{41}$$

with $\Phi_{4\alpha\bar{\alpha}}^1 = Y_{\alpha\bar{\alpha}}^T P_{22\alpha\bar{\alpha}}^{-1}Y_{\alpha\bar{\alpha}} - \tilde{P}_{\alpha\bar{\alpha}}$, $\tilde{P}_{\alpha\bar{\alpha}} = \begin{bmatrix} P_{11\alpha\bar{\alpha}} & 0 & 0 & P_{12\bar{\alpha}}\\ 0&0&0&0\\ 0&0&0&0\\ * & 0 & 0 & P_{22\alpha\bar{\alpha}}\end{bmatrix}$,

$$\Phi_{4\alpha\bar{\alpha}}^0 = -\mathcal{H}\left(E^T Y_{\alpha\bar{\alpha}}\right) + \begin{bmatrix} \eta_k P_{11\alpha\bar{\alpha}} - P_{12\bar{\alpha}} & 0 & 0 & \eta_k P_{12\bar{\alpha}}\\ 0 & P_{12\bar{\alpha}} & 0 & 0\\ 0 & 0 & -\eta_k^{-1}P_{11\alpha\bar{\alpha}} & 0\\ * & 0 & 0 & \eta_k P_{22\alpha\bar{\alpha}}\end{bmatrix}.$$

So, from (30), (32), (34) and (41), the inequality (28) is satisfied if:

$$\begin{aligned}\mathcal{P}(\tau(t)) =& \tau^2(t)\xi^T(t)\Phi_{2\bar{\alpha}}^2\xi(t)\\ &+ \tau(t)\xi^T(t)(\Phi_{2\bar{\alpha}}^1 + \Phi_{3\bar{\alpha}}^1 + \Phi_{4\alpha\bar{\alpha}}^1)\xi(t)\\ &+ \xi^T(t)(\Phi_1^0 + \Phi_{2\bar{\alpha}}^0 + \Phi_{3\bar{\alpha}}^0 + \Phi_{4\alpha\bar{\alpha}}^0)\xi(t) < 0\end{aligned} \tag{42}$$

Note that, $\forall\xi(t)$ the polynomial $\mathcal{P}(\tau(t)) = 0$ is convex if:

$$\xi^T(t)\Phi_{2\bar{\alpha}}^2\xi(t) > 0 \quad\Leftrightarrow\quad \Phi_{2\bar{\alpha}}^2 > 0 \tag{43}$$

In that case, the inequality (42) is satisfied if:

$$\mathcal{P}(0) < 0 \text{ and } \mathcal{P}(\eta_k) < 0 \tag{44}$$

Focus first on the inequality (43) and assume:

$$M_{\bar{\alpha}} = M_{\bar{\alpha}}^T = \begin{bmatrix} M_{11\bar{\alpha}} & M_{12\bar{\alpha}} & M_{13\bar{\alpha}} & M_{14\bar{\alpha}}\\ * & M_{22\bar{\alpha}} & M_{23\bar{\alpha}} & M_{24\bar{\alpha}}\\ * & * & M_{33\bar{\alpha}} & M_{34\bar{\alpha}}\\ * & * & * & M_{44\bar{\alpha}}\end{bmatrix} \tag{45}$$

Thus, from (32) we have:

$$\Phi_{2\bar{\alpha}}^2 = -\begin{bmatrix} M_{\bar{\alpha}}^0 & W_{\bar{\alpha}}\\ * & 0\end{bmatrix} \tag{46}$$

with $M_{\bar{\alpha}}^0 = \begin{bmatrix} \mathcal{H}(M_{13\bar{\alpha}} + M_{34\bar{\alpha}}) & M_{23\bar{\alpha}}^T - M_{34\bar{\alpha}} & M_{33\bar{\alpha}}^T\\ * & 0 & 0\\ * & 0 & 0\end{bmatrix}$,

and $W_{\bar{\alpha}} = \begin{bmatrix} \mathcal{H}(M_{14\bar{\alpha}}) + M_{11\bar{\alpha}} + M_{44\bar{\alpha}}\\ M_{24\bar{\alpha}} - M_{44\bar{\alpha}} + M_{12\bar{\alpha}}^T - M_{14\bar{\alpha}}^T\\ M_{34\bar{\alpha}} + M_{13\bar{\alpha}}^T\end{bmatrix}$.

Let $U_{\alpha\bar{\alpha}} \in \mathbb{R}^{3n\times 3n}$ regular and consider the null terms $W_{\bar{\alpha}}^T U_{\alpha\bar{\alpha}}^{-1}W_{\bar{\alpha}} - W_{\bar{\alpha}}^T U_{\alpha\bar{\alpha}}^{-1}W_{\bar{\alpha}} = 0$ and $M_{\bar{\alpha}}^0 - M_{\bar{\alpha}}^0 = 0$. Applying the Schur complement we can write:

$$\begin{bmatrix} U_{\alpha\bar{\alpha}} - M_{\bar{\alpha}}^0 + M_{\bar{\alpha}}^0 & W_{\bar{\alpha}}\\ W_{\bar{\alpha}}^T & W_{\bar{\alpha}}^T U_{\alpha\bar{\alpha}}^{-1}W_{\bar{\alpha}}\end{bmatrix} = 0 \tag{47}$$

Thus, we can also write:

$$\Phi_{2\alpha\bar{\alpha}}^2 = -\begin{bmatrix} M_{\bar{\alpha}}^0 & W_{\bar{\alpha}}\\ W_{\bar{\alpha}}^T & 0\end{bmatrix} = \begin{bmatrix} U_{\alpha\bar{\alpha}} - M_{\bar{\alpha}}^0 & 0\\ 0 & W_{\bar{\alpha}}^T U_{\alpha\bar{\alpha}}^{-1}W_{\bar{\alpha}}\end{bmatrix} \tag{48}$$

Hence, considering now $\Phi_{2\alpha\bar{\alpha}}^2$ in (42) as the right-hand matrix of (48), the inequality (43) holds if:

$$U_{\alpha\bar{\alpha}} > 0 \text{ and } U_{\alpha\bar{\alpha}} - M_{\bar{\alpha}}^0 > 0 \tag{49}$$

Now, before dealing with (44), we will introduce the closed-loop dynamics into the stability conditions. To do so, note that (5) is equivalent, with $G_{\alpha\bar{\alpha}} = \begin{bmatrix} A_\alpha & B_\alpha K_{\bar{\alpha}}X^{-1} & 0 & -I\end{bmatrix}$, to $G_{\alpha\bar{\alpha}}\xi(t) = 0$. Moreover, (42) can be rewritten as:

$$\xi^T(t)\left(\tau^2(t)\Phi_{2\alpha\bar{\alpha}}^2 + \tau(t)(\Phi_{\Sigma\bar{\alpha}}^1 + \Phi_{4\alpha\bar{\alpha}}^1) + \Phi_{\Sigma\alpha\bar{\alpha}}^0\right)\xi(t) < 0 \tag{50}$$

with $\Phi^1_{\Sigma\bar{\alpha}} = \Phi^1_{2\bar{\alpha}} + \Phi^1_{3\bar{\alpha}}$ and $\Phi^0_{\Sigma\alpha\bar{\alpha}} = \sum_{q=1}^{3} \Phi^0_{q\bar{\alpha}} + \Phi^0_{4\alpha\bar{\alpha}}$. So, we can apply Lemma 5 and the inequality (50) is satisfied if there exists $R \in \mathbb{R}^{4n\times n}$ such that:

$$\tau^2(t)\Phi^2_{2\alpha\bar{\alpha}} + \tau(t)(\Phi^1_{\Sigma\bar{\alpha}} + \Phi^1_{4\alpha\bar{\alpha}}) + \Phi^0_{\Sigma\alpha\bar{\alpha}} + RG_{\alpha\bar{\alpha}} + G^T_{\alpha\bar{\alpha}}R^T < 0 \quad (51)$$

Hence, (44) is satisfied if the following inequalities hold:

$$\Phi^0_{\Sigma\alpha\bar{\alpha}} + RG_{\alpha\bar{\alpha}} + G^T_{\alpha\bar{\alpha}}R^T < 0 \quad (52)$$

$$\eta_k^2\Phi^2_{2\alpha\bar{\alpha}} + \eta_k(\Phi^1_{\Sigma\bar{\alpha}} + \Phi^1_{4\alpha\bar{\alpha}}) + \Phi^0_{\Sigma\alpha\bar{\alpha}} + RG_{\alpha\bar{\alpha}} + G^T_{\alpha\bar{\alpha}}R^T < 0 \quad (53)$$

Let X regular and $R = \left[\, X^{-1} \ \varepsilon_1 X^{-1} \ \varepsilon_2 X^{-1} \ \varepsilon_3 X^{-1} \,\right]^T$. To deal with (52), pre- and post-multiplying it respectively by $diag\,[\, X \ X \ X \ X \,]^T$ and its transpose, we obtain:

$$\Lambda^{11}_{\alpha\bar{\alpha}} = \bar{\Phi}^0_{\Sigma\alpha\bar{\alpha}} + \mathbb{I}_\varepsilon \bar{G}_{\alpha\bar{\alpha}} + \bar{G}^T_{\alpha\bar{\alpha}}\mathbb{I}^T_\varepsilon < 0 \quad (54)$$

Then, to deal with (53), apply first the Schur complement on $\Phi^1_{4\alpha\bar{\alpha}}$ and $\Phi^2_{2\alpha\bar{\alpha}}$ written as the right-hand matrix of (48), then pre- and post-multiplying it respectively by $diag\,[\, X \ X \ X \ X \ X \,]^T$ and its transpose, we obtain:

$$\begin{bmatrix} \Lambda^{22}_{\alpha\bar{\alpha}} & * & * \\ \eta_k Y_{\alpha\bar{\alpha}} & -\eta_k P_{22\alpha\bar{\alpha}} & * \\ \eta_k \tilde{W}_{\bar{\alpha}} & 0 & -U_{\alpha\bar{\alpha}} \end{bmatrix} < 0 \quad (55)$$

with $\Lambda^{22}_{\alpha\bar{\alpha}} = \eta_k^2 \tilde{M}^0_{\bar{\alpha}} + \eta_k(\bar{\Phi}^1_{\Sigma\bar{\alpha}} - \tilde{P}_{\alpha\bar{\alpha}}) + \bar{\Phi}^0_{\Sigma\alpha\bar{\alpha}} + \mathbb{I}_\varepsilon \bar{G}_{\alpha\bar{\alpha}} + \bar{G}^T_{\alpha\bar{\alpha}}\mathbb{I}^T_\varepsilon$, $\tilde{M}^0_{\bar{\alpha}} = \begin{bmatrix} U_{\alpha\bar{\alpha}} - M^0_{\bar{\alpha}} & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{W}_{\bar{\alpha}} = [\, 0 \, W_{\bar{\alpha}} \,]$ and where, in (54) and (55), $\mathbb{I}_\varepsilon = [\, I \ \varepsilon_1 I \ \varepsilon_2 I \ \varepsilon_3 I \,]^T$, $\bar{G}_{\alpha\bar{\alpha}} = [\, A_\alpha X \ B_\alpha K_{\bar{\alpha}} \ 0 \ -X \,]$ and all decision matrices inside $\bar{\Phi}^2_{2\alpha\bar{\alpha}}$, $\bar{\Phi}^1_{\Sigma\bar{\alpha}}$, $\bar{\Phi}^1_{4\alpha\bar{\alpha}}$ and $\bar{\Phi}^0_{\Sigma\alpha\bar{\alpha}}$ belong to the bijective change of variables $\bar{D} = X^T D X$, $D = \{L, M_{11\bar{\alpha}}, \ldots, M_{44\bar{\alpha}}, N_{11\bar{\alpha}}, \ldots, N_{22\bar{\alpha}}, P_{11\alpha\bar{\alpha}}, P_{12\bar{\alpha}}, P_{22\alpha\bar{\alpha}}\}$. Finally, concatenating (49), (54), (55), $P_{11\alpha\bar{\alpha}} > 0$ and $P_{22\alpha\bar{\alpha}} > 0$ into the same parameterized LMI $\Lambda_{\alpha\bar{\alpha}} < 0$, then applying Theorem 1, we obtain the conditions expressed in theorem 2. □

Remark 1. The conditions expressed in Theorem 2 are not strictly LMI because of the parameters ε_1, ε_2 and ε_3. However, as state in many previous works applying the Finsler's Lemma, see e.g. (Oliveira et al., 2011; Bourahala et al., 2017; Cherifi et al., 2018, 2019), these parameters are usually tuned offline by grid search.

4. ILLUSTRATIVE EXAMPLE

To illustrate the effectiveness of the proposed method, we consider the benchmark of the inverted pendulum on a cart that has been used for comparative purposes in many previous related T-S model-based sampled-data controller design studies (see the references shown in Table 1). A T-S fuzzy model (1) with $r = 2$ is proposed in (Wang et al., 1996) for such inverted pendulum. This model, valid for $|x_1(t)| < \pi/2$ and $|x_2(t)| \le \pi$, is given by the polytopes:

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ \frac{-a}{4l/3 - aml} \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ \frac{-a\beta}{4l/3 - aml\beta^2} \end{bmatrix}, \ \beta = cos(88°),$$

and the membership functions:

$$\alpha_1(x_1(t)) = \begin{cases} 1 - \frac{2}{\pi}x_1(t), & \text{if } 0 \le x_1(t) < \frac{\pi}{2}, \\ 1 + \frac{2}{\pi}x_1(t), & \text{if } -\frac{\pi}{2} < x_1(t) < 0, \end{cases}$$

$$\alpha_2(x_1(t)) = 1 - \alpha_1(x_1(t))$$

where $x_1(t)$ denotes the angle (rad) of the pendulum from the erect position and $x_2(t)$ is the angular velocity (rad/s), $g = 9.8m/s^2$ is the acceleration of the gravity, $m = 2kg$ is the mass of the pendulum, $M = 8kg$ is the mass of the cart, $l = 0.5m$ is the half length of the pendulum, $a = 1/(m + M)$ and the input $u(t)$ corresponds to the actuator force applied to the cart (in N).

The conditions of Theorem 2 has been solved with MATLAB using YALMIP and SeDuMi (Lofberg, 2004). The maximal allowable upper bound $\bar{\eta} = 51ms$ has been found with $\varepsilon_1 = 5.5$, $\varepsilon_2 = 3$, $\varepsilon_3 = 0.31$ and assuming $\sigma_1 = \sigma_2 = 2\bar{\eta}$. The obtained controller gains are given by:

$$K_1 = \begin{bmatrix} 0.2231 & -0.0321 \end{bmatrix} \quad X = \begin{bmatrix} 0.0065 & -0.0186 \\ -0.0174 & 0.0708 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 1.8690 & 9.5775 \end{bmatrix}$$

As shown in Table 1, the maximal allowable upper bound $\bar{\eta}$ obtained with the present approach outperform previous related results by at least 21.43%. This shows the significant convervatism improvement raised by Theorem 2.

Table 1. Comparison of maximal $\bar{\eta}$ obtained with related previous studies.

Method	$\bar{\eta}$ (ms)
(Yoneyama, 2010)	9
(Zhu and Wang, 2011)	13
(Zhang and Han, 2011)	16
(Zhu et al., 2012)	19
(Zhu et al., 2013)	24
(Cheng et al., 2017)	42
Theorem 2	51

Applying the designed sampled-data controller (4), Fig. 1(a) shows the closed-loop state trajectories of the inverted pendulum from the initial condition $x(0) = [\pi/4 \ 0]^T$ with a fixed sampling period $\eta_k = \bar{\eta} = 51ms$. In addition, Fig. 1(b) shows the same simulation but with random aperiodic sampling periods $\eta_k \in [0, 51ms]$. This demonstrate the effectiveness of the proposed sampled-data controller design methodology for T-S fuzzy systems.

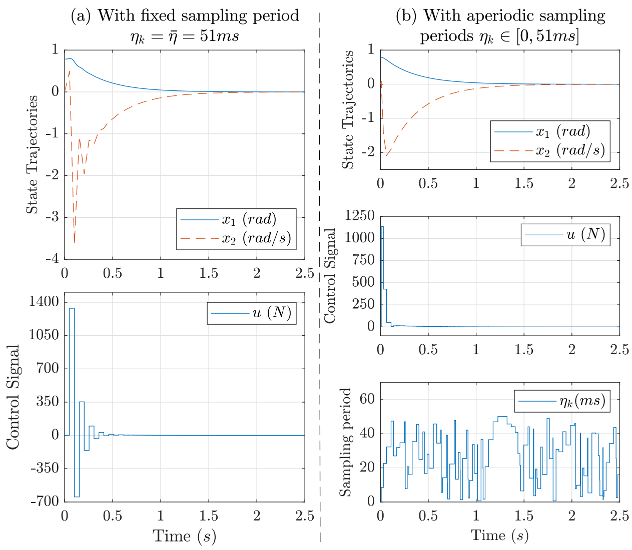

Fig. 1. Closed-loop simulations of the considered inverted pendulum with sampled-data controller.

Remark 2. The proposed methodology for T-S systems includes linear systems as a special case. In this special case, an experimental validation of the proposed design procedure on the Quanser AERO 2 DOF Helicopter test-bed can be found in (Lopes et al., 2020a).

5. CONCLUSION

In this paper, new LMI-based conditions have been proposed for the design of sampled-data PDC controllers for continuous-time T-S fuzzy models. Conservatism improvement regarding to previous works has been achieved from the choice of the considered LKF and the Finsler's Lemma. Also, generic conditions have also been proposed to relax double fuzzy sums with asynchronous MFs involved in T-S model-based sampled-data control plants. The effectiveness of the proposed conditions, as well as their conservatism improvement regarding to previous results, have been illustrated through the well-known benchmark of an inverted pendulum on a cart.

ACKNOWLEDGEMENTS

The authors would like to thank Mr. Sam Plaid for inspiring this study. This work was partially funded by the CEFET-MG and the Brazilian agency CNPq, under the grant 311208/2019-3, and by the Region Grand-Est (France) within the CPER FFCA Project.

REFERENCES

Bourahala, F., Guelton, K., Manamanni, N., and Khaber, F. (2017). Relaxed Controller Design Conditions for Takagi–Sugeno Systems with State Time-Varying Delays. *International Journal of Fuzzy Systems*, 19(5), 1406–1416.

Cheng, W., Wang, D., Zhu, X., and Wang, Y. (2017). H_∞ stabilization for sampling fuzzy systems with asynchronous constraints on membership functions. *IEEE Access*, 5, 1524–1533.

Cherifi, A., Guelton, K., and Arcese, L. (2018). Uncertain TS model-based robust controller design with D-stability constraints—A simulation study of quadrotor attitude stabilization. *Engineering Applications of Artificial Intelligence*, 67, 419–429.

Cherifi, A., Guelton, K., Arcese, L., and Leite, V.J. (2019). Global non-quadratic d-stabilization of takagi–sugeno systems with piecewise continuous membership functions. *Applied Mathematics and Computation*, 351, 23–36.

Fridman, E. (2014). Tutorial on lyapunov-based methods for time-delay systems. *European Journal of Control*, 20(6), 271 – 283.

Fridman, E., Seuret, A., and Richard, J.P. (2004). Robust sampled-data stabilization of linear systems: an input delay approach. *Automatica*, 40(8), 1441–1446.

Guerra, T.M., Bernal, M., Guelton, K., and Labiod, S. (2012). Non-quadratic local stabilization for continuous-time Takagi-Sugeno models. *Fuzzy Sets and Systems*, 201, 40–54.

Hetel, L., Fiter, C., Omran, H., Seuret, A., Fridman, E., Richard, J.P., and Niculescu, S.I. (2017). Recent developments on the stability of systems with aperiodic sampling: An overview. *Automatica*, 76, 309–335.

Li, L. and Liu, X. (2009). New results on delay-dependent robust stability criteria of uncertain fuzzy systems with state and input delays. *Information Sciences*, 179(8), 1134–1148.

Lofberg, J. (2004). Yalmip : a toolbox for modeling and optimization in matlab. In *2004 IEEE International Conference on Robotics and Automation*, 284–289.

Lopes, A.N.D., Arcese, L., Guelton, K., and Cherifi, A. (2020a). Sampled-data controller design with application to the quanser aero 2-dof helicopter. In *2020 IEEE International Conference on Automation, Quality and Testing, Robotics (AQTR)*, 1–6.

Lopes, A.N.D., Leite, V.J.S., Silva, L.F.P., and Guelton, K. (2020b). Anti-windup ts fuzzy pi-like control for discrete-time nonlinear systems with saturated actuators. *International Journal of Fuzzy Systems*, 22, 46–61.

Oliveira, R.C.L.F., de Oliveira, M.C., and Peres, P.L.D. (2011). Robust state feedback lmi methods for continuous-time linear systems: Discussions, extensions and numerical comparisons. In *2011 IEEE International Symposium on Computer-Aided Control System Design (CACSD)*, 1038–1043.

Sala, A. (2009). On the conservativeness of fuzzy and fuzzy-polynomial control of nonlinear systems. *Ann. Rev. in Control*, 33(1), 48–58.

Skelton, R., Iwasaki, T., and Grigoriadis, K. (1998). *A unified algebraic approach to linear control design*. Taylor and Francis, London.

Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. on Systems, Man, and Cybernetics*, SMC-15(1), 116–132.

Tuan, H.D., Apkarian, P., Narikiyo, T., and Yamamoto, Y. (2001). Parameterized linear matrix inequality techniques in fuzzy control system design. *IEEE Trans. on fuzzy systems*, 9(2), 324–332.

Wang, H.O., Tanaka, K., and Griffin, M.F. (1996). An approach to fuzzy control of nonlinear systems: stability and design issues. *IEEE Trans. on Fuzzy Systems*, 4(1), 14–23.

Xie, L. (1996). Output feedback hinf control of systems with parameter uncertainty. *International Journal of control*, 63(4), 741–750.

Xie, X., Yue, D., Zhang, H., and Peng, C. (2017). Control synthesis of discrete-time ts fuzzy systems: reducing the conservatism whilst alleviating the computational burden. *IEEE Trans. on Cybernetics*, 47(9), 2480–2491.

Yoneyama, J. (2010). Robust H_∞ control of uncertain fuzzy systems under time-varying sampling. *Fuzzy Sets and Systems*, 161(6), 859 – 871.

Zhang, D. and Han, Q. (2011). H_∞ control design for network-based t-s fuzzy systems with asynchronous constraints on membership functions. In *IECON 2011 - 37th Annual Conference of the IEEE Industrial Electronics Society*, 2584–2589.

Zhang, X.M. and Han, Q.L. (2013). Novel delay-derivative-dependent stability criteria using new bounding techniques. *International Journal of Robust and Nonlinear Control*, 23(13), 1419–1432.

Zhu, X., Chen, B., Yue, D., and Wang, Y. (2012). An improved input delay approach to stabilization of fuzzy systems under variable sampling. *IEEE Trans. on Fuzzy Systems*, 20(2), 330–341.

Zhu, X.L. and Wang, Y. (2011). Stabilization for sampled-data neural network- based control systems. *IEEE Trans. Syst., Man Cybern. B, Cybern.*, 41(1), 210–221.

Zhu, X.L., Chen, B., Wang, Y., and Yue, D. (2013). H_∞ stabilization criterion with less complexity for nonuniform sampling fuzzy systems. *Fuzzy Sets and Systems*, 225, 58–73.