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Event-Triggered Control for Switched Systems in Network Environments ⋆

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Abstract: This paper is concerned with event-triggered control for a switched system in network environments. Firstly, a novel event-triggering communication scheme with switching features is proposed. The switching features are taken into full consideration to guarantee the current sampled data to be transmitted if a switch occurs between the last sampling instant and the current sampling instant. The newly proposed event-triggering scheme is advantageous in dealing with switched networked control systems. Secondly, under the event-triggering scheme, an asynchronously switched time-delay system model is established by taking into account effects of network-induced delays. Finally, a mode-dependent state feedback controller gain and event generator parameters co-design method is proposed for the asynchronously switched time-delay system. System performance analysis demonstrates the effectiveness of the proposed methods.

Keywords: Asynchronously switched system; event-triggering; network-induced delays.

1. INTRODUCTION

Switched systems are found wide-ranging potential applications in mechanical systems, intelligent transportation systems, and constrained robot systems, to name a few. In general, a switched system, which is conducive to modeling systems subject to sudden change of system parameters and/or structures, consists of a group of subsystems and a time-dependent switching law. In recent years, great interests have been aroused to analyze stability (Park and Park (2019); Xiang et al. (2018)), optimal control (Hara and Konishi (2019)), controllability and observability (Küsters and Trenn (2018)) of switched systems.

In modern control applications, system components including samplers, controllers and actuators are usually connected through communication networks (Zhang et al. (2017)). The introduction of communication networks provides numerous advantages in satisfying actual control demands, for instance, low installation and maintenance costs, high availability and scalability, and reduced volume of wiring (Ge et al. (2017)). Due to these benefits of networked control systems and widespread applications of switched systems, many researchers pay their attention to switched systems in network environments (Deaecto et al. (2015); Li and Chen (2019); Ren et al. (2018)). However, the introduction of communication networks gives rise to network-induced delays, which may bring some new difficulties to performance analysis and controller design for switched networked control systems (SNCSs). For SNCSs, how to take network-induced delays into consideration and to establish network-based switched system models are of paramount significance and still unresolved. Addressing these issues is the first motivation of this paper.

Traditionally, the sampled data are transmitted in a time-triggered way (Wang et al. (2018); Yue et al. (2013); Zhang et al. (2017)). By this means, however, all the sampled data are sent to the controller and the actuator, which unavoidably leads to a waste of communication resources since some superfluous sampled data have virtually no influence over the control performance (Peng and Yang (2013); Yue et al. (2013)). Thus, it is of great interest to devise more efficient transmission schemes to mitigate the unnecessary data transmission. Under this circumstance, on the premise of without sacrificing the control performance, the event-triggering transmission strategy is introduced to determine whether the newly sampled data packets will be transmitted or not (Peng and Yang (2013); Yue et al. (2013); Zhang et al. (2017)). By using event-triggering transmission scheme, the occupation of communication networks and computational resources can be reduced (Ding et al. (2018); Peng and Yang (2013); Yue et al. (2013)). The past few years have witnessed the ever-increasing interests in event-triggered control, see Ding et al. (2018); Zhang et al. (2017). Some interesting results about event-triggered control for switched systems have also been achieved (Ma et al. (2016); Qi et al. (2017); Ren et al. (2018); Xiao et al. (2019)). The stability, H_∞

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control and finite-time control of switched systems were investigated under event-triggering transmission schemes in Ma et al. (2016); Qi et al. (2017); Xiao et al. (2019). For a switched system controlled through communication networks, how to design a novel event-triggering scheme to avoid the possible state fluctuation induced by the system switching is significant and has received no attention. This is the second motivation of this paper.

In view of the above analysis, this paper focuses on event-triggered control for an SNCS. The main contributions are highlighted as follows.

(i) A novel event-triggering transmission scheme taking into account switching features is proposed. The event-triggering scheme can avoid possible state fluctuation induced by system switching;

(ii) Under the event-triggering scheme, a novel asynchronously switched time-delay system model is established for the first time, in which the effects of network-induced delays are taken into full consideration. The so-called asynchronism, caused by network-induced delays and the asynchrony between sampling and switching, means that the switching of the controller modes has a lag to the switching of system modes;

(iii) A mode-dependent state feedback controller gain and event generator parameters co-design method for the SNCS is derived by analyzing the asynchronously switched system control in matched periods and mismatched periods. In matched periods, the controller mode and the system mode are synchronous, while in mismatched periods they are asynchronous.

Notation: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote n-dimensional Euclidean space and $m \times n$ real matrix space, respectively. $\mathbb{N}$ and $\mathbb{N}_+$ are, respectively, the sets of nonnegative and positive integers. A^T is the transpose of matrix A. I_n represents an $n \times n$ identity matrix. $*$ denotes the entries of a matrix implied by symmetry.

2. PROBLEM FORMULATION

Consider a switched system described by

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and the control input vector, respectively; the switching signal $\sigma(t)$ is a piecewise continuous function, and $\sigma(t) : [t_0, \infty) \to \mathcal{S} = \{1, 2, \ldots, S\}$ with S being the number of subsystems; for any $i \in \mathcal{S}$, $A_{\sigma(t)} = A_i \in \mathbb{R}^{n\times n}$, $B_{\sigma(t)} = B_i \in \mathbb{R}^{n\times m}$ are the parameters of the i-th subsystem.

Definition 1. (Hespanha and Morse (1999)) For any constants $t_2 > t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ be the times of switches during the time interval (t_1, t_2). T_{ad} is called average dwell time (ADT) if

$$N_\sigma(t_1, t_2) \leq N_0 + \frac{t_2 - t_1}{T_{ad}} \tag{2}$$

holds for all $t_2 > t_1 \geq 0$ and a scalar $N_0 \geq 1$.

2.1 A Novel Event-Triggering Scheme

In this subsection, we introduce a novel event-triggering communication scheme which takes into account switching features.

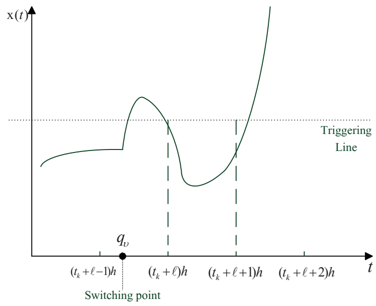

Fig. 1. State fluctuation induced by system switching.

We consider the above switched system in network environments. The sensor is assumed to be clock driven with h being its sampling interval. In this case, 0, h, $2h$,$\cdots$,kh,$\cdots$ ($k \in \mathbb{N}$) are the sampling instants. An event generator to be designed will decide whether the sampled state $x(kh)$ is transmitted or not. Denote the event-triggering instants as $t_0h, t_1h, t_2h, \cdots, t_kh, \cdots$. Subsequently, the corresponding state data $x(t_0h)$, $x(t_1h)$, $x(t_2h)$, $\cdots$, $x(t_kh)$, $\cdots$ will be transmitted to the controller.

Without loss of generality, this paper considers event-triggering scheme in the sensor. The results can be extended to deal with the SNCS considering the event-triggered sensor and controller. The event-triggering scheme considering switching features is described as

$$t_{k+1}h = t_kh + \min_{\ell} \Big\{\ell h : \text{i) or ii) is violated}\Big\}, \tag{3}$$

where

$$\begin{aligned} \text{i)} \quad & e^T((t_k+\ell)h)\Phi e((t_k+\ell)h) \\ & < \delta x^T((t_k+\ell)h)\Phi x((t_k+\ell)h), \\ \text{ii)} \quad & \sigma((t_k+\ell)h) = \sigma(t_kh), \end{aligned}$$

with $0 < \delta \leq 1$, $e((t_k + \ell)h) = x((t_k + \ell)h) - x(t_kh)$, and Φ being a matrix to be designed.

Remark 1. Fig. 1 describes the state fluctuation that may occur when using traditional event-triggering schemes. As observed from Fig. 1, if the event generator (3) is used, $t_{k+\ell}h$ will be a triggering instant since q_ν is a switching instant between $(t_k + \ell - 1)h$ and $(t_k + \ell)h$. The event generator (3), which takes switching features into full consideration, responds the switching signal in a timely manner, and further reduces possible state fluctuation induced by system switching. In addition, it should be mentioned that Φ and δ can be system mode dependent.

2.2 Asynchronously Switched System Modelling

In this subsection, an asynchronously switched time-delay system model is established under the event-triggering scheme (3).

Due to the introduction of communication networks between the sensor, the controller and the actuator, network-induced delays are unavoidable and have received considerable attention (Peng and Yang (2013); Wang and Han (2018); Yue et al. (2013)). τ_k denotes the transmission time from the instant t_kh to the instant when the actuator receives the corresponding data, where $\tau_k \in [\underline{\tau},\ \overline{\tau}]$, $0 < \underline{\tau} < \overline{\tau}$. Thus, the control input based on released state $x(t_kh)$ will be received by the actuator at the time $t_kh+\tau_k$.

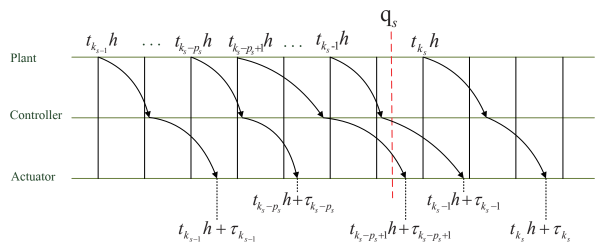

Fig. 2. Signal transmission around q_s.

Suppose that data packets are transmitted and received sequentially. Then one has that $t_k h + \tau_k < t_{k+1}h + \tau_{k+1}$.

The mode-dependent controller is designed as

$$u(t) = K_{\sigma(t_k h)} x(t_k h),\ t \in [t_k h + \tau_k,\ t_{k+1}h + \tau_{k+1}), \quad (4)$$

where $K_{\sigma(t_k h)}$ is the controller gain to be designed.

Accordingly, the system (1) is rewritten as

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}K_{\sigma(t_k h)}x(t_k h), \\ \qquad\qquad t \in [t_k h + \tau_k,\ t_{k+1}h + \tau_{k+1}). \end{cases} \quad (5)$$

We are now in a position to describe the matched periods and mismatched periods for the switched system (5).

Suppose that q_s is the s-th switching instant ($s = 1,\ 2,\ 3,\ \cdots$), and $q_s \in (t_{k_s-1}h,\ t_{k_s}h]$, where $t_{k_s}h$ is the s-th event-triggering instant activated by the event-triggering condition ii) in (3), $t_{k_s-1}h$ is the latest event-triggering instant prior to $t_{k_s}h$. Obviously, $\bigcup\limits_{s=0}^{\infty}\{t_{k_s}h\} \subset \bigcup\limits_{k=0}^{\infty}\{t_k h\}$, where $k_0 = 0$.

It is easy to see that $t_{k_s}h - q_s < h$. Moreover, due to the existence of network-induced delays, there exists a positive integer p_s such that $t_{k_s-p_s}h + \tau_{k_s-p_s} \le q_s < t_{k_s-p_s+1}h + \tau_{k_s-p_s+1}$, where $t_{k_s-p_s}h$ and $t_{k_s-p_s+1}h$ are also event-triggering instants.

In what follows, we will prove that the matched period and mismatched period are, respectively, $[t_{k_{s-1}}h + \tau_{k_{s-1}},\ q_s)$ and $[q_s,\ t_{k_s}h + \tau_{k_s})$, for $s = 1,\ 2,\ 3,\ \cdots$. Without loss of generality, suppose that the time intervals between any two switches are *larger than or equal to* $h + \overline{\tau}$.

On one hand, since $t_{k_s}h + \tau_{k_s} - q_s = (t_{k_s}h - q_s) + \tau_{k_s} < h + \overline{\tau}$, it is easy to see that the times of switching is 1 from q_s to $t_{k_s}h + \tau_{k_s}$, which implies that $q_{s+1} > t_{k_s}h + \tau_{k_s}$.

On the other hand, since $t_{k_s-p_s}h + \tau_{k_s-p_s} \le q_s < t_{k_s-p_s+1}h + \tau_{k_s-p_s+1}$ and the times of switching is 1 from q_{s-1} to $t_{k_{s-1}}h + \tau_{k_{s-1}}$, one has that $q_{s-1} < t_{k_{s-1}}h + \tau_{k_{s-1}} \le t_{k_s-p_s}h + \tau_{k_s-p_s}$.

Obviously, one has that for $s = 1,\ 2,\ 3,\ \cdots$,

$$\begin{aligned} &\text{matched period}: &&[t_{k_{s-1}}h + \tau_{k_{s-1}},\ q_s), \\ &\text{mismatched period}: &&[q_s,\ t_{k_s}h + \tau_{k_s}). \end{aligned}$$

Fig. 2 depicts the signal transmission for the switched system (5) around the s-th switching instant q_s. This figure demonstrates the matched period $[t_{k_{s-1}}h + \tau_{k_{s-1}},\ q_s)$ and the mismatched period $[q_s,\ t_{k_s}h + \tau_{k_s})$ clearly.

Next, for $s = 1,\ 2,\ 3,\ \cdots$, we discuss an asynchronously switched system modelling in the time interval $[t_{k_{s-1}}h + \tau_{k_{s-1}},\ t_{k_s}h + \tau_{k_s})$. It is easy to see that $\bigcup\limits_{s=1}^{\infty}[t_{k_{s-1}}h + \tau_{k_{s-1}},\ t_{k_s}h + \tau_{k_s}) = \bigcup\limits_{k=0}^{\infty}[t_k h + \tau_k,\ t_{k+1}h + \tau_{k+1})$.

Motivated by Yue et al. (2013), we introduce an artificial time-varying delay $\tau(t)$ and a new vector function $e_{k_s}(t)$ for $t \in [t_{k_{s-1}}h + \tau_{k_{s-1}},\ t_{k_s}h + \tau_{k_s})$, where k_{s-1} can be $k_s - 1,\ k_s - 2,\ k_s - 3,\ \cdots$. Taking $k_{s-1} = k_s - 2$ for example, one can construct $\tau(t)$ and $e_{k_s}(t)$ as follows.

- Case 1: If
$$t_{k_s-2}h + h + \overline{\tau} \ge t_{k_s-1}h + \tau_{k_s-1},$$
$$t_{k_s-1}h + h + \overline{\tau} \ge t_{k_s}h + \tau_{k_s},$$
define $\tau(t)$ as
$$\tau(t) = \begin{cases} t - t_{k_s-2}h, & t \in \mathcal{X}_1, \\ t - t_{k_s-1}h, & t \in \mathcal{X}_2, \end{cases}$$
where
$$\mathcal{X}_1 = [t_{k_s-2}h + \tau_{k_s-2},\ t_{k_s-1}h + \tau_{k_s-1}),$$
$$\mathcal{X}_2 = [t_{k_s-1}h + \tau_{k_s-1},\ t_{k_s}h + \tau_{k_s}).$$
Define $e_{k_s}(t)$ as
$$e_{k_s}(t) = 0.$$
- Case 2: If
$$t_{k_s-2}h + h + \overline{\tau} < t_{k_s-1}h + \tau_{k_s-1},$$
$$t_{k_s-1}h + h + \overline{\tau} \ge t_{k_s}h + \tau_{k_s},$$
define $\tau(t)$ as
$$\tau(t) = \begin{cases} t - t_{k_s-2}h, & t \in \mathcal{Y}_0, \\ t - t_{k_s-2}h - ah, & t \in \mathcal{Y}_a, \\ t - t_{k_s-2}h - \bar{a}h, & t \in \mathcal{Y}_{\bar{a}}, \\ t - t_{k_s-1}h, & t \in \mathcal{X}_2, \end{cases}$$
where $a = 1,\ 2,\ \cdots,\ \bar{a} - 1$, and
$$\mathcal{Y}_0 = [t_{k_s-2}h + \tau_{k_s-2},\ t_{k_s-2}h + h + \overline{\tau}),$$
$$\mathcal{Y}_a = [t_{k_s-2}h + ah + \overline{\tau},\ t_{k_s-2}h + ah + h + \overline{\tau}),$$
$$\mathcal{Y}_{\bar{a}} = [t_{k_s-2}h + \bar{a}h + \overline{\tau},\ t_{k_s-1}h + \tau_{k_s-1}).$$
Define $e_{k_s}(t)$ as
$$e_{k_s}(t) = \begin{cases} 0, & t \in \mathcal{Y}_0 \bigcup \mathcal{X}_2, \\ x(t_{k_s-2}h) - x(t_{k_s-2}h + ah), & t \in \mathcal{Y}_a, \\ x(t_{k_s-2}h) - x(t_{k_s-2}h + \bar{a}h), & t \in \mathcal{Y}_{\bar{a}}. \end{cases}$$
- Case 3: If
$$t_{k_s-2}h + h + \overline{\tau} \ge t_{k_s-1}h + \tau_{k_s-1},$$
$$t_{k_s-1}h + h + \overline{\tau} < t_{k_s}h + \tau_{k_s},$$
define $\tau(t)$ as
$$\tau(t) = \begin{cases} t - t_{k_s-2}h, & t \in \mathcal{X}_1, \\ t - t_{k_s-1}h, & t \in \mathcal{Z}_0, \\ t - t_{k_s-1}h - bh, & t \in \mathcal{Z}_b, \\ t - t_{k_s-1}h - \bar{b}h, & t \in \mathcal{Z}_{\bar{b}}, \end{cases}$$
where $b = 1,\ 2,\ \cdots,\ \bar{b} - 1$, and
$$\mathcal{Z}_0 = [t_{k_s-1}h + \tau_{k_s-1},\ t_{k_s-1}h + h + \overline{\tau}),$$
$$\mathcal{Z}_b = [t_{k_s-1}h + bh + \overline{\tau},\ t_{k_s-1}h + bh + h + \overline{\tau}),$$
$$\mathcal{Z}_{\bar{b}} = [t_{k_s-1}h + \bar{b}h + \overline{\tau},\ t_{k_s}h + \tau_{k_s}).$$
Define $e_{k_s}(t)$ as
$$e_{k_s}(t) = \begin{cases} 0, & t \in \mathcal{X}_1 \bigcup \mathcal{Z}_0, \\ x(t_{k_s-1}h) - x(t_{k_s-1}h + bh), & t \in \mathcal{Z}_b, \\ x(t_{k_s-1}h) - x(t_{k_s-1}h + \bar{b}h), & t \in \mathcal{Z}_{\bar{b}}. \end{cases}$$

- Case 4: If
$$t_{k_s-2}h + h + \overline{\tau} < t_{k_s-1}h + \tau_{k_s-1},$$
$$t_{k_s-1}h + h + \overline{\tau} < t_{k_s}h + \tau_{k_s},$$
define $\tau(t)$ as
$$\tau(t) = \begin{cases} t - t_{k_s-2}h, & t \in \mathcal{Y}_0, \\ t - t_{k_s-2}h - ah, & t \in \mathcal{Y}_a, \\ t - t_{k_s-2}h - \bar{a}h, & t \in \mathcal{Y}_{\bar{a}}, \\ t - t_{k_s-1}h, & t \in \mathcal{Z}_0, \\ t - t_{k_s-1}h - bh, & t \in \mathcal{Z}_b, \\ t - t_{k_s-1}h - \bar{b}h, & t \in \mathcal{Z}_{\bar{b}}. \end{cases}$$
Define $e_{k_s}(t)$ as
$$e_{k_s}(t) = \begin{cases} 0, & t \in \mathcal{Y}_0 \bigcup \mathcal{Z}_0, \\ x(t_{k_s-2}h) - x(t_{k_s-2}h + ah), & t \in \mathcal{Y}_a, \\ x(t_{k_s-2}h) - x(t_{k_s-2}h + \bar{a}h), & t \in \mathcal{Y}_{\bar{a}}, \\ x(t_{k_s-1}h) - x(t_{k_s-1}h + bh), & t \in \mathcal{Z}_b, \\ x(t_{k_s-1}h) - x(t_{k_s-1}h + \bar{b}h), & t \in \mathcal{Z}_{\bar{b}}. \end{cases}$$

From these four cases, it is easy to obtain that
$$\underline{\tau} \le \tau(t) \le h + \overline{\tau}.$$

Remark 2. From the event-triggering condition i) and the definitions of $\tau(t)$ and $e_{k_s}(t)$, it is easy to derive that for $t \in [t_{k_{s-1}}h + \tau_{k_{s-1}},\ t_{k_s}h + \tau_{k_s})$,
$$e^T_{k_s}(t)\Phi e_{k_s}(t) < \delta x^T(t - \tau(t))\Phi x(t - \tau(t)). \tag{6}$$

Suppose that the system mode shifts from i to j at the instant q_s, where $i, j \in \mathcal{S}, i \neq j$. Then, for $t \in [t_{k_{s-1}}h + \tau_{k_{s-1}},\ t_{k_s}h + \tau_{k_s})$, the system (5) can be rewritten as
$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i K_i x(t-\tau(t)) + B_i K_i e_{k_s}(t), \\ \qquad t \in [t_{k_{s-1}}h + \tau_{k_{s-1}},\ q_s), \\ \dot{x}(t) = A_j x(t) + B_j K_i x(t-\tau(t)) + B_j K_i e_{k_s}(t), \\ \qquad t \in [q_s,\ t_{k_s}h + \tau_{k_s}). \end{cases} \tag{7}$$

The initial condition of the state $x(t)$ is given as
$$x(l) = \varphi(l),\ l \in [-h - \overline{\tau},\ 0],$$
where $\varphi(l)$ is a continuous function in $[-h - \overline{\tau},\ 0]$.

3. CONTROLLER AND EVENT GENERATOR CO-DESIGN

In this section, the stabilizing controller gain and event generator co-design method for the asynchronously switched time-delay system (7) is proposed to optimize the system performance.

For this purpose, define a switching signal $\sigma'(t)$ representing the switching pattern of the controller mode. Then from the above analysis and (7), one has that
$$\begin{cases} N_{\sigma'}(t_0, t) = N_\sigma(t_0, t), & t \in [t_{k_{s-1}}h + \tau_{k_{s-1}},\ q_s), \\ N_{\sigma'}(t_0, t) = N_\sigma(t_0, t) - 1, & t \in [q_s,\ t_{k_s}h + \tau_{k_s}). \end{cases} \tag{8}$$

For briefness, define $\tilde{\tau}_1 = \underline{\tau},\ \tilde{\tau}_2 = h + \overline{\tau},\ \tilde{\tau} = \max\{\underline{\tau},\ h + \overline{\tau} - \underline{\tau}\}$. Then, we state and establish the following result.

Theorem 1. Given positive scalars ρ_1, ρ_2, and $\mu \ge 1$, for any $i,\ j \in \mathcal{S},\ i \neq j$, the system (7) is exponentially stable with the average dwell time T_{ad} satisfying
$$T_{ad} > T^*_{ad} = \frac{\ln\mu + (\tilde{\tau}_2 + \tilde{\tau})(\rho_1 + \rho_2)}{\rho_1}, \tag{9}$$
if there exist an adjustable parameter η_i, matrices $\tilde{P}_i > 0$, $\tilde{Q}_{1i} > 0$, $\tilde{Q}_{2i} > 0$, $\tilde{R}_{1i} > 0$, $\tilde{R}_{2i} > 0$, and Y with appropriate dimensions, such that
$$\begin{aligned} \tilde{P}_i \le \mu\tilde{P}_j,\ \tilde{Q}_{1i} \le \mu\tilde{Q}_{1j},\ \tilde{Q}_{2i} \le \mu\tilde{Q}_{2j}, \\ \tilde{R}_{1i} \le \mu\tilde{R}_{1j},\ \tilde{R}_{2i} \le \mu\tilde{R}_{2j}, \end{aligned} \tag{10}$$
$$\tilde{\Lambda}_{1i} = \begin{bmatrix} \tilde{\Gamma}_{i11} & e^{-\rho_1\tilde{\tau}_1}\tilde{R}_{1i} & 0 & \tilde{\Gamma}_{i14} & B_i\tilde{K}_i & B_i\tilde{K}_i \\ * & \tilde{\Gamma}_{i22} & 0 & 0 & e^{-\rho_1\tilde{\tau}_2}\tilde{R}_{2i} & 0 \\ * & * & \tilde{\Gamma}_{i33} & 0 & e^{-\rho_1\tilde{\tau}_2}\tilde{R}_{2i} & 0 \\ * & * & * & \tilde{\Gamma}_{i44} & \eta_i B_i\tilde{K}_i & \eta_i B_i\tilde{K}_i \\ * & * & * & * & \tilde{\Gamma}_{i55} & 0 \\ * & * & * & * & * & -\tilde{\Phi} \end{bmatrix} < 0, \tag{11}$$
$$\tilde{\Lambda}_{2i} = \begin{bmatrix} \tilde{\Upsilon}_{i11} & \tilde{R}_{1i} & 0 & \tilde{\Upsilon}_{i14} & B_j\tilde{K}_i & B_j\tilde{K}_i \\ * & \tilde{\Upsilon}_{i22} & 0 & 0 & e^{\rho_2\tilde{\tau}_1}\tilde{R}_{2i} & 0 \\ * & * & \tilde{\Upsilon}_{i33} & 0 & e^{\rho_2\tilde{\tau}_1}\tilde{R}_{2i} & 0 \\ * & * & * & \tilde{\Upsilon}_{i44} & \eta_i B_j\tilde{K}_i & \eta_i B_j\tilde{K}_i \\ * & * & * & * & \tilde{\Upsilon}_{i55} & 0 \\ * & * & * & * & * & -\tilde{\Phi} \end{bmatrix} < 0, \tag{12}$$
where
$$\begin{aligned}
\tilde{\Gamma}_{i11} &= \rho_1\tilde{P}_i + \tilde{Q}_{1i} - e^{-\rho_1\tilde{\tau}_1}\tilde{R}_{1i} + Y^T A_i^T + A_i Y, \\
\tilde{\Gamma}_{i14} &= \tilde{P}_i - Y + \eta_i Y^T A_i^T, \\
\tilde{\Gamma}_{i22} &= -e^{-\rho_1\tilde{\tau}_1}\tilde{Q}_{1i} + e^{-\rho_1\tilde{\tau}_1}\tilde{Q}_{2i} - e^{-\rho_1\tilde{\tau}_1}\tilde{R}_{1i} - e^{-\rho_1\tilde{\tau}_2}\tilde{R}_{2i}, \\
\tilde{\Gamma}_{i33} &= -e^{-\rho_1\tilde{\tau}_2}\tilde{Q}_{2i} - e^{-\rho_1\tilde{\tau}_2}\tilde{R}_{2i}, \\
\tilde{\Gamma}_{i44} &= \tilde{\tau}_1^2\tilde{R}_{1i} + (\tilde{\tau}_2 - \tilde{\tau}_1)^2\tilde{R}_{2i} - \eta_i Y - \eta_i Y^T, \\
\tilde{\Gamma}_{i55} &= \delta\tilde{\Phi} - 2e^{-\rho_1\tilde{\tau}_2}\tilde{R}_{2i}, \\
\tilde{\Upsilon}_{i11} &= -\rho_2\tilde{P}_i + \tilde{Q}_{1i} - \tilde{R}_{1i} + Y^T A_j^T + A_j Y, \\
\tilde{\Upsilon}_{i14} &= \tilde{P}_i - Y + \eta_i Y^T A_j^T, \\
\tilde{\Upsilon}_{i22} &= -e^{\rho_2\tilde{\tau}_1}\tilde{Q}_{1i} + e^{\rho_2\tilde{\tau}_1}\tilde{Q}_{2i} - \tilde{R}_{1i} - e^{\rho_2\tilde{\tau}_1}\tilde{R}_{2i}, \\
\tilde{\Upsilon}_{i33} &= -e^{\rho_2\tilde{\tau}_2}\tilde{Q}_{2i} - e^{\rho_2\tilde{\tau}_1}\tilde{R}_{2i}, \\
\tilde{\Upsilon}_{i44} &= \tilde{\tau}_1^2\tilde{R}_{1i} + (\tilde{\tau}_2 - \tilde{\tau}_1)^2\tilde{R}_{2i} - \eta_i Y - \eta_i Y^T, \\
\tilde{\Upsilon}_{i55} &= \delta\tilde{\Phi} - 2e^{\rho_2\tilde{\tau}_1}\tilde{R}_{2i}.
\end{aligned}$$
Moreover, the controller gain matrix $K_i = \tilde{K}_i Y^{-1}$, and the event generator parameter matrix $\Phi = Y^{-T}\tilde{\Phi}Y^{-1}$.

Proof: For $t \in [t_{k_{s-1}}h + \tau_{k_{s-1}}, q_s)$, construct the Lyapunov functional candidate as
$$\begin{aligned} V_{1i}(x_t) =& x^T(t)P_i x(t) + \int_{t-\tilde{\tau}_1}^{t} e^{\rho_1(s-t)}x^T(s)Q_{1i}x(s)ds \\ &+ \int_{t-\tilde{\tau}_2}^{t-\tilde{\tau}_1} e^{\rho_1(s-t)}x^T(s)Q_{2i}x(s)ds \\ &+ \tilde{\tau}_1\int_{-\tilde{\tau}_1}^{0}\int_{t+\theta}^{t} e^{\rho_1(s-t)}\dot{x}^T(s)R_{1i}\dot{x}(s)dsd\theta \\ &+ (\tilde{\tau}_2 - \tilde{\tau}_1)\int_{-\tilde{\tau}_2}^{-\tilde{\tau}_1}\int_{t+\theta}^{t} e^{\rho_1(s-t)}\dot{x}^T(s)R_{2i}\dot{x}(s)dsd\theta. \end{aligned}$$
From (7), for any positive scalar η_i and a real matrix X, one has
$$\begin{aligned} 2(x^T(t)X^T + \eta_i\dot{x}^T(t)X^T)\Big(A_i x(t) + B_i K_i x(t - \tau(t)) \\ + B_i K_i e_{k_s}(t) - \dot{x}(t)\Big) = 0. \end{aligned} \tag{13}$$
Combining (6), (13) and Jensen inequality together, one has

$$
\begin{aligned}
&\dot{V}_{1i}(x_t) + \rho_1 V_{1i}(x_t) \\
\leq &\dot{V}_{1i}(x_t) + \rho_1 V_{1i}(x_t) \\
&+ \delta x^T(t-\tau(t))\Phi x(t-\tau(t)) - e^T_{k_s}(t)\Phi e_{k_s}(t) \\
\leq &\zeta^T(t)\Lambda_{1i}\zeta(t),
\end{aligned}
$$

where

$$
\Lambda_{1i} = \begin{bmatrix} \Gamma_{i11} & e^{-\rho_1\tilde{\tau}_1}R_{1i} & 0 & \Gamma_{i14} & X^T B_i K_i & X^T B_i K_i \\ * & \Gamma_{i22} & 0 & 0 & e^{-\rho_1\tilde{\tau}_2}R_{2i} & 0 \\ * & * & \Gamma_{i33} & 0 & e^{-\rho_1\tilde{\tau}_2}R_{2i} & 0 \\ * & * & * & \Gamma_{i44} & \eta_i X^T B_i K_i & \eta_i X^T B_i K_i \\ * & * & * & * & \Gamma_{i55} & 0 \\ * & * & * & * & * & -\Phi \end{bmatrix},
$$

with

$$
\begin{aligned}
\Gamma_{i11} &= \rho_1 P_i + Q_{1i} - e^{-\rho_1\tilde{\tau}_1}R_{1i} + A_i^T X + X^T A_i, \\
\Gamma_{i14} &= P_i - X^T + \eta_i A_i^T X, \\
\Gamma_{i22} &= -e^{-\rho_1\tilde{\tau}_1}Q_{1i} + e^{-\rho_1\tilde{\tau}_1}Q_{2i} - e^{-\rho_1\tilde{\tau}_1}R_{1i} - e^{-\rho_1\tilde{\tau}_2}R_{2i}, \\
\Gamma_{i33} &= -e^{-\rho_1\tilde{\tau}_2}Q_{2i} - e^{-\rho_1\tilde{\tau}_2}R_{2i}, \\
\Gamma_{i44} &= \tilde{\tau}_1^2 R_{1i} + (\tilde{\tau}_2 - \tilde{\tau}_1)^2 R_{2i} - \eta_i X - \eta_i X^T, \\
\Gamma_{i55} &= \delta\Phi - 2e^{-\rho_1\tilde{\tau}_2}R_{2i}.
\end{aligned}
$$

If the inequality in (11) is satisfied, that is, $\tilde{\Lambda}_{1i} < 0$, one has that $\tilde{\Gamma}_{i44} < 0$. Thus, $Y + Y^T$ is a symmetric positive definite matrix, which implies that Y is nonsingular. Let $X = Y^{-1}$, $P_i = X^T\tilde{P}_i X$, $Q_{1i} = X^T\tilde{Q}_{1i}X$, $Q_{2i} = X^T\tilde{Q}_{2i}X$, $R_{1i} = X^T\tilde{R}_{1i}X$, $R_{2i} = X^T\tilde{R}_{2i}X$, $K_i = \tilde{K}_i X$, $\Phi = Y^T\tilde{\Phi}Y$. Pre- and post-multiplying $\tilde{\Lambda}_{1i} < 0$ in (11) by $diag\{X^T, X^T, X^T, X^T, X^T, X^T\}$ and its transpose, one has $\Lambda_{1i} < 0$, and

$$
\dot{V}_{1i}(x_t) + \rho_1 V_{1i}(x_t) < 0. \tag{14}
$$

Integrating both sides of (14) from $t_{k_{s-1}}h + \tau_{k_{s-1}}$ to t, one can derive that

$$
V_{1i}(x_t) \leq V_{1i}(t_{k_{s-1}}h + \tau_{k_{s-1}})e^{-\rho_1(t - t_{k_{s-1}}h - \tau_{k_{s-1}})}. \tag{15}
$$

If $t \in [q_s, t_{k_s}h + \tau_{k_s})$, construct the Lyapunov functional candidate as

$$
\begin{aligned}
V_{2i}(x_t) =& x^T(t)P_i x(t) + \int_{t-\tilde{\tau}_1}^{t} e^{\rho_2(t-s)}x^T(s)Q_{1i}x(s)ds \\
&+ \int_{t-\tilde{\tau}_2}^{t-\tilde{\tau}_1} e^{\rho_2(t-s)}x^T(s)Q_{2i}x(s)ds \\
&+ \tilde{\tau}_1\int_{-\tilde{\tau}_1}^{0}\int_{t+\theta}^{t} e^{\rho_2(t-s)}\dot{x}^T(s)R_{1i}\dot{x}(s)dsd\theta \\
&+ (\tilde{\tau}_2 - \tilde{\tau}_1)\int_{-\tilde{\tau}_2}^{-\tilde{\tau}_1}\int_{t+\theta}^{t} e^{\rho_2(t-s)}\dot{x}^T(s)R_{2i}\dot{x}(s)dsd\theta.
\end{aligned}
$$

Similarly, one has that

$$
\dot{V}_{2i}(x_t) - \rho_2 V_{2i}(x_t) < 0, \tag{16}
$$

and

$$
V_{2i}(x_t) \leq V_{2i}(q_s)e^{\rho_2(t-q_s)}. \tag{17}
$$

Obviously, one can derive that

$$
V_{2i}(q_s^+) \leq e^{(\rho_1+\rho_2)\tilde{\tau}}V_{1i}(q_s^-). \tag{18}
$$

For $t \in [t_0,\ t]$, the Lyapunov functional candidate can be chosen as

$$
V(x_t) = \begin{cases} V_{1i}(x_t), t \in [t_{k_{s-1}}h + \tau_{k_{s-1}}, q_s), \\ V_{2i}(x_t), t \in [q_s, t_{k_s}h + \tau_{k_s}), \end{cases} \quad s = 1,\ 2,\ 3, \cdots. \tag{19}
$$

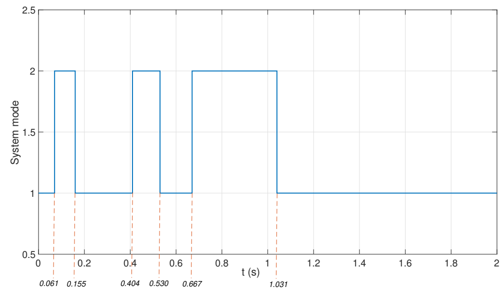

Fig. 3. Switching signal $\sigma(t)$.

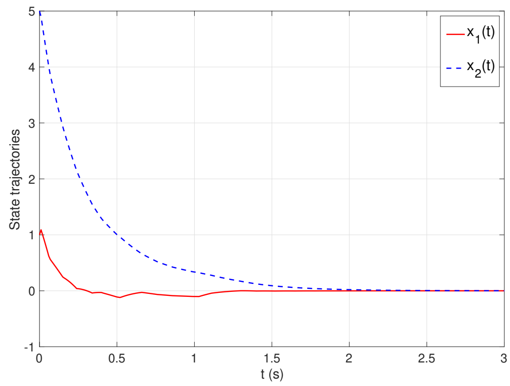

Fig. 4. State response of the switched system.

From (10), (15), and (17)-(19), one can easily obtain that

$$
V(x_t) \leq V(x_{t_0})\tilde{\rho}^{N_\sigma(t_0,t)}\mu^{N_{\sigma'}(t_0,t)}e^{\rho_2 T_{mi}(t_0,t) - \rho_1 T_{ma}(t_0,t)}, \tag{20}
$$

where $\tilde{\rho} = e^{(\rho_1+\rho_2)\tilde{\tau}}$, while $T_{mi}(t_0, t)$ and $T_{ma}(t_0, t)$ are the total mismatched periods and matched periods, respectively, for $t \in [t_0,\ t]$.

Then, motivated by the proof in Lian and Ge (2013), one can conclude that the system (7) is exponentially stable with the average dwell time satisfying (9). ∎

4. SYSTEM PERFORMANCE ANALYSIS

This section aims to demonstrate the merits and effectiveness of the asynchronously switched system modelling, and the controller and event generator co-design method.

Consider the switched system (1) with two subsystems. The system parameters are given as

$$
A_1 = \begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -2 \\ -1 \end{bmatrix},
$$
$$
A_2 = \begin{bmatrix} 1 & 0 \\ 0.5 & -1.5 \end{bmatrix}, B_2 = \begin{bmatrix} -1 \\ -1.5 \end{bmatrix}.
$$

Without loss of generality, let $N_0 = 2$, $\rho_1 = 0.2$, $\rho_2 = 0.5$, $h = 0.01s$, $\underline{\tau} = 0.02s$, $\overline{\tau} = 0.04s$, and $\mu = 1.2$. Then one has that $\tilde{\tau}_1 = \underline{\tau} = 0.02s$, $\tilde{\tau}_2 = h + \overline{\tau} = 0.05s$, $\tilde{\tau} = 0.03s$. Moreover, from (9), the average dwell time satisfies

$$
T_{ad} > T_{ad}^* = 1.1916.
$$

Select the adjustable parameters $\eta_1 = 0.1$, $\eta_2 = 0.3$. For $\delta = 0.2$, applying Theorem 1, one obtains

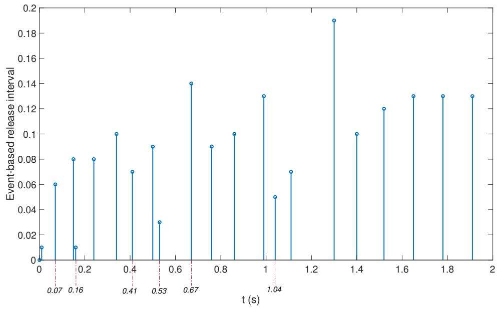

Fig. 5. Transmission instants and release intervals.

$$K_1 = \begin{bmatrix} 3.6420 \\ 0.9838 \end{bmatrix}^T, K_2 = \begin{bmatrix} 3.1008 \\ 0.6701 \end{bmatrix}^T, \Phi = \begin{bmatrix} 2.8833 & 0.8267 \\ 0.8267 & 0.4392 \end{bmatrix}.$$

The switching signal $\sigma(t)$ and switching instants of the system mode are depicted in Fig. 3. The initial condition of this system is $\varphi(l) = [1\ \ 5]^T$, $l \in [-0.05,\ 0]$. Then the state response of the switched system is given in Fig. 4, while release instants and release intervals are shown in Fig. 5. As one can see in Fig. 3, switching instants of the system mode are $0.061s$, $0.155s$, $0.404s$, $0.53s$, $0.667s$, $1.031s$. Then the associated event-triggering instants induced by switching are $0.07s$, $0.16s$, $0.41s$, $0.53s$, $0.67s$, $1.04s$, which are depicted in Fig. 5. Note that most release intervals of sampled data are larger than the length of the sampling interval h. In Fig. 5, the sensor samples and generates 200 data packets, and only 21 of them are released. The release rate is 10.5%. Thus, the burden of communication networks can be reduced greatly without sacrificing the system performance.

5. CONCLUSION

The problem of event-triggered control for a switched system in network environments has been investigated in this paper. A novel event-triggering communication scheme, which takes into account switching features for the first time, has been proposed. Then an asynchronously switched time-delay system model has been established. A mode-dependent state feedback controller gain and event generator parameters co-design method has been proposed. The proposed co-design scheme has been shown to be effective in providing good system performance and reducing the communication networks burden.

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