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Development of Three-degree-of-freedom Zero-compliance Mechanism for Micro Force Measurement with a Cantilever

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Abstract: A multi-degree-of-freedom zero-compliance mechanism is developed for measuring micro force with a cantilever. The force acting on a tip of the cantilever is usually estimated from the displacement of the tip caused by the force. However, the position and attitude of the tip vary after force acts on the tip from the original ones. To keep them invariant, a three-degree-of-freedom zero compliance mechanism is designed, fabricated and installed into a force measurement system. In the developed measurement system, the force is estimated from the displacement/attitude of a detection point of the zero-compliance mechanism. It is confirmed analytically and experimentally that both vertical displacement and attitude of the detection point are proportional to the force applied to the tip.

Keywords: Measuring transducers, Force, Mechanisms, Cascade, Position control, Attitude control, Kinematics, Modelling.

1. INTRODUCTION

Force is one of the most fundamental physical quantities. The precise measurement of force is important in many fields of science and engineering. There are various methods of measuring force (Stefanescu, 2011). In detecting microforces, for example, in atomic force microscopes, a cantilever is often used to detect small force with high sensitivity (Binnig and Quate, 1986; Okamoto et al., 2002; Sekiguchi et al., 2011). Force acting on a tip installed at the end of the cantilever causes a defection of the cantilever. From this deflection, the amplitude of force is estimated. To achieve high-sensitivity detection, therefore, the stiffness of the cantilever should be sufficiently low.

In conventional force measurement with a cantilever, therefore, the position and the attitude of the tip varies from the original ones that are defined when the force does not act on the tip. As a result, several conditions may change during the measurement. For example, the force in measuring may be greater than the force acting on the tip at the original position because the distance between the tip and the force source decreases due to the deflection of the cantilever. It can cause some error or at least uncertainty in the measurement.

Mizuno et al. (2012) have proposed force measurement with zero-compliance mechanism. Conceptually, this mechanism is a series connection of two springs with positive stiffness and with negative stiffness with the same amplitude of stiffness. Force to be measured is assumed to act on the end of the connected springs; this end is defined as the point of action. The most important characteristic is that this point does not displace; the deflection of the positive-stiffness spring is cancelled by the deflection of the negative-stiffness spring. Instead, the amplitude of force is estimated from the deflection of the connection point; this point is defined as the detection point. It displaces in proportion to the applied force (Mizuno et al., 2015). One of the advantages of this method is the invariance of the conditions during measurement; the distance between the point of action and the force source is kept constant (Mizuno et al., 2017).

In this work, the concept of force measurement with zero-compliance mechanism is applied to force measurement with a cantilever. To make the position and the attitude of the tip invariant during measurement, a three-degree-of-freedom zero-compliance mechanism is proposed and fabricated. The basic characteristics of the fabricated measurement system are studied analytically. The efficacy of the proposed measurement system with a cantilever is demonstrated experimentally.

2. PRINCIPLES OF MEASUREMENT

2.1 Zero-compliance mechanism

Figure 1 shows a zero-compliance mechanism for force measurement. The point of action A is suspended by a series-connected suspension (Suspension I and Suspension II); the connection point becomes the detection point D. The stiffness of the connected suspensions, denoted by k_c, is given by

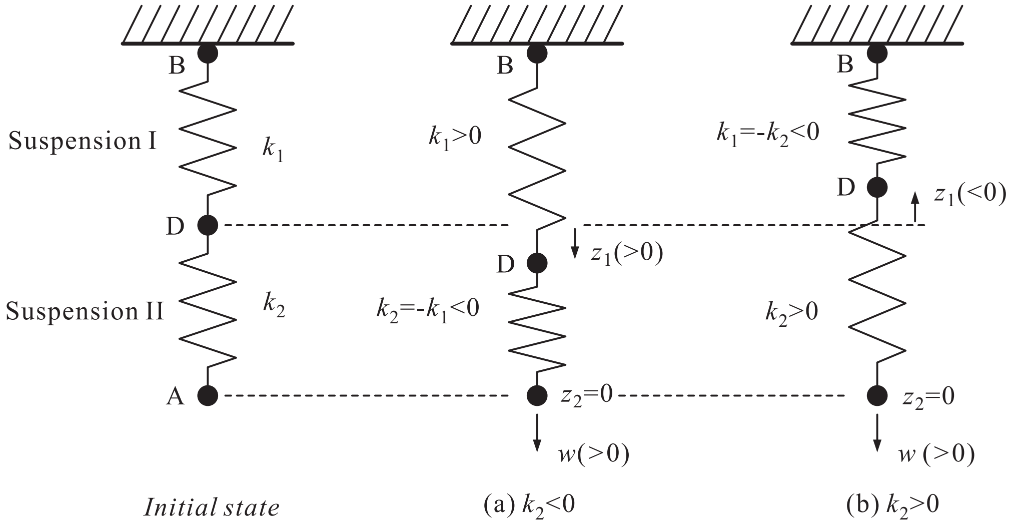


Fig.1 Force measurement using zero-compliance mechanism

$$k_c = \frac{k_1 k_2}{k_1 + k_2}, \tag{1}$$

where k_i: stiffness of each suspension. This equation shows that the total stiffness becomes lower than that of each suspension when normal springs are connected. However, if one of the suspensions has negative stiffness that satisfies

$$k_1 = -k_2, \tag{2}$$

the resultant stiffness becomes infinite, that is

$$|k_c| = \infty. \tag{3}$$

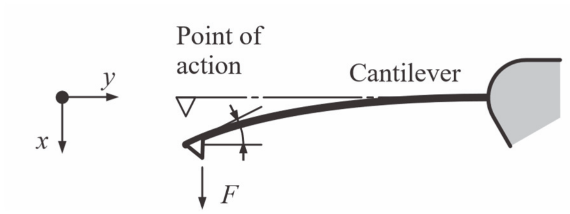
It indicates that the point of action does not displace even if force acts on this point as if measured by the *null method*. In contrast, the detection point displaces proportionally to the force acting on the body as

$$z_1 = \frac{w}{k_1} = -\frac{w}{k_2}. \tag{4}$$

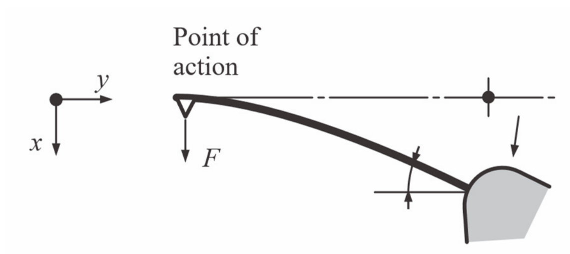
Therefore, the force can be estimated from the displacement of the detection point as if measured by the *deflection method*. It is noted that high resolution is expected when low-stiffness suspensions are used.

2.2 Force detection with a cantilever

In atomic force microscopes, a cantilever is used to detect small force with high sensitivity (Binnig and Quate, 1986). The amplitude of force is determined from the deflection of the cantilever caused by the force. Figure 2(a) shows the expected behaviour of a cantilever when a vertical force acts on a tip at the end of the cantilever. The tip displaces in the horizon direction, which is denoted by the y-axis in the figure, in addition to vertical direction that is denoted by the z-axis. Moreover, the angular displacement around the x-axis also



(a) Behaviour of cantilever when force is applied



(b) Desired states

Fig.2 Behaviour of cantilever

appears. It indicates that the position and the attitude of the end of the cantilever vary from the original ones that is defined when no force acts on the tip. It can cause some error or at least uncertainty in the measurement mainly because the measurement conditions change. Therefore, the states shown by Fig.2(b) is targeted in this work.

2.3 Zero-compliance mechanism for force measurement with a cantilever

Figure 2 indicates the necessity of a three-degree-of-freedom (3-DoF) mechanism to maintain the position and attitude of the point of action when a cantilever is used for force detection. The proposed mechanism is shown by Fig.3. A three-link hinge is used as the 3-DoF mechanism. It has three variable-length (active) links and two fixed-length (passive) links. The former links, which are named as Link 1, Link 2 and Link 3, comprise a triangle whose angles vary freely. One of the variable-length links (Link 1) and the latter links comprise another triangle whose angles also vary freely. One of the fixed-length links is fixed on the base. The two triangles share Link 1 as a common side. A cantilever is fixed at the vertex opposite to the common side as an extension of Link 2. This vertex becomes the detection point. The position and the attitude of the detection point can be adjusted by varying the lengths of the former links.

When a vertical force acts on the point of action (tip at the end of the cantilever), the point of action displaces in both vertical and horizontal direction, and the attitude also varies as shown by Fig.2a. However, the position and the attitude of the point force can be maintained at the original ones by varying the link length as shown by Fig.3b. It indicates that force can be measured in the invariant conditions with the proposed mechanism.

3. MODELLING AND ANALYSIS

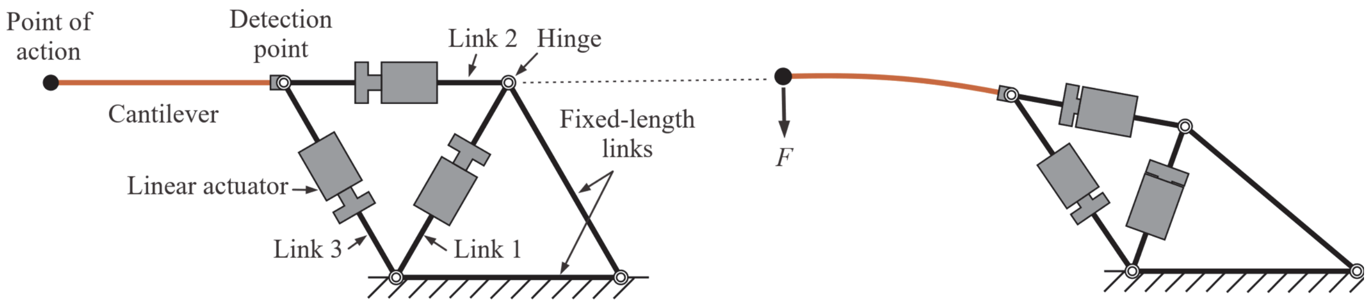
To achieve the zero-compliance states, the length of each active link must be determined appropriately when force acting on the tip. The equations necessary for the determination will be derived.

First, a reference frame Σ_0 whose origin is at the fixed point P_0, and another frame Σ_1 whose origin is at the detection point P_1 where the cantilever is fixed are considered (see Fig.4). The angle of Link 2 to the lateral direction is set to be θ_c that is equal to the attitude of the point of action when a normal force acts on the point of action. A homogeneous transformation matrix from the first coordinate system to the second coordinate system is given by (Kucuk and Bingul, 2006):

$$ {}^{0}T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_c & -\sin\theta_c & \ell_3\cos\theta_4 \\ 0 & \sin\theta_c & \cos\theta_c & \ell_3\sin\theta_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{5} $$

where θ_4 is the angle of Link 3 to the base ($=\theta_1+\theta_c$). The fourth column of ${}^{0}T_1$ corresponds to the position of the detection point

$$ P_1 = \begin{bmatrix} 0 \\ y_d \\ z_d \end{bmatrix} = \begin{bmatrix} 0 \\ \ell_3\cos\theta_4 \\ \ell_3\sin\theta_4 \end{bmatrix}. \tag{6} $$



(a) Initial state (b) Force acts on the point of action

Fig.3 Zero-compliance mechanism for force measurement with a cantilever

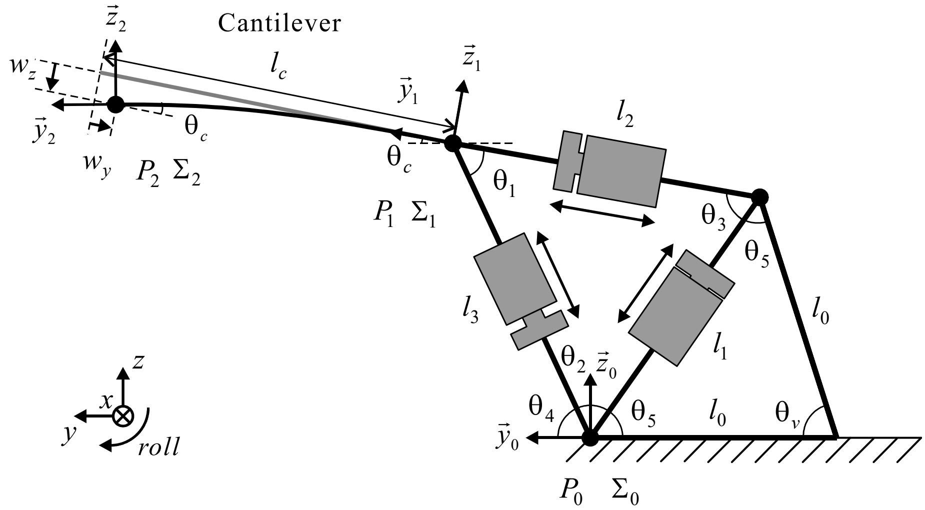


Fig.4 Model of 3-link hinge and cantilever kinematics. The parameters concerning point of action and detection point are set as shown in the figure. Derive a homogeneous transformation matrix from the origin to point of action and obtain a theoretical expression of the length of each link.

Second, another reference frame Σ_2 whose origin is at the point of action P_2 is considered. It is assumed that a vertical force F acts on P_2 and the point displaces with w_y in the horizontal direction and w_z in the vertical direction, and θ_c around the x-axis. A homogeneous transformation matrix from the second coordinate system to the third coordinate system is given by

$$ {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_c & \sin\theta_c & \ell_c - w_y \\ 0 & -\sin\theta_c & \cos\theta_c & -w_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{7} $$

From Eq.(5) and Eq.(7), a transformation matrix from the first coordinate system to the third coordinate system is obtained as

$$ {}^0T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y_{2,1} + \ell_3 \cos\theta_4 \\ 0 & 0 & 1 & z_{2,1} + \ell_3 \sin\theta_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8} $$

where

$$ y_{2,1} = (\ell_c - w_y)\cos\theta_c + w_z \sin\theta_c , \tag{9} $$

$$ z_{2,1} = (\ell_c - w_y)\sin\theta_c - w_z \cos\theta_c , \tag{10} $$

The fourth column of ${}^0T_2$ corresponds to the position of the point of action.

$$ P_2 = \begin{bmatrix} 0 \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} 0 \\ y_{2,1} + \ell_3 \cos\theta_4 \\ z_{2,1} + \ell_3 \sin\theta_4 \end{bmatrix}. \tag{11} $$

The two passive links have the same length of ℓ_0. Thereby, the second triangle is an isosceles triangle. The angle between Link 1 and each passive link is represented by $\theta_5 \ (= \theta_3 - \theta_c)$. The length of Link 1 is given by

$$ \ell_1 = 2\ell_0 \cos\theta_5 , \tag{12} $$

From (11), we get

$$ \ell_3 = \sqrt{(y_p - y_{2,1})^2 + (z_p - z_{2,1})^2} , \tag{13} $$

The position of the detection point satisfies

$$ \ell_3 \sin\theta_4 = \ell_1 \sin\theta_5 + \ell_2 \sin\theta_c \tag{14} $$

$$ \ell_3 \cos\theta_4 = -\ell_1 \cos\theta_5 + \ell_2 \cos\theta_c \tag{15} $$

From Eq.(12), Eq.(14) and Eq.(15), we get

$$ \ell_1 = \sqrt{2\ell_0(\ell_0 \cos^2\theta_c + w_{z*} \sin\theta_c - \ell_w \cos\theta_c)} , \tag{16} $$

$$ \ell_2 = \ell_0 \cos\theta_c + w_{y*} - \ell_w , \tag{17} $$

where

$$ w_{y*} = -(\ell_c - w_y)\cos\theta_c + y_p \cos\theta_c + z_p \sin\theta_c , \tag{18} $$

$$ w_{z*} = -y_p \sin\theta_c + w_z + z_p \cos\theta_c , \tag{19} $$

$$ \ell_w = \sqrt{\ell_0^2 \cos^2\theta_c + 2\ell_0 w_{z*} \sin\theta_c - w_{z*}^2} , \tag{20} $$

The length of each active link can be calculated from Eq.(13), Eq.(16) and Eq.(17).

4. EXPERIMENTAL APPARATUS

4.1 Outline of experimental setup

Figure 5 shows a picture of the fabricated measurement system. A fabricated mechanism is set at the middle of the system.

4.2 Mechanism for measurement with a cantilever

Figure 6 shows the total structure of the mechanism. An active link has a voice coil motor (VCM) as an actuator and a linear spring in parallel with the VCM. The linear spring allows a single-degree-of-freedom motion in the normal direction while it restricts the other motions. The expansion or contraction of the spring is controlled with the VCM. A rotational spring used at the connection of two links. It also allows a single-degree-of-freedom rotation between the two links with less friction.

4.3 Control system

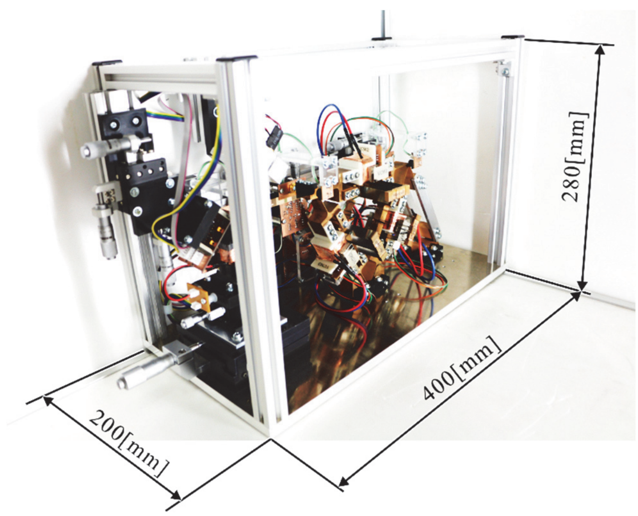


Fig.5 Picture of measurement system

Figure 7 shows a block diagram of the control system used in the experiments. To achieve the zero-compliance states, PID control is applied to the two-dimensional position and the attitude of the point of action. The outputs from the three PID controllers are transformed to the lengths of active links according to Eq.(13), Eq.(16) and Eq.(17). The VCMs are driven to achieve the commanded lengths. In each active link, a local PD control is applied to improve the transient response.

5. EXPERIMENTAL RESULT

Figure 8 shows the displacements and attitude of the point of action and those of the detection point when static force is added to the point of action in the normal direction by another VCM. The added force is 0 to 6.0 [mN]. It is clearly found from Fig.8 that the point of action keeps the position and the attitude while the detection point displaces and tilts due to the applied force. The displacement in the *z*-direction and the tilt angle are almost proportional to the applied force. It indicates that the applied force can be estimated based on these quantities. The displacement in the *y*-direction increases in proportion to the square of the applied force. When the line ℓ inclines with an angle of θ, the lateral displacement of the head is given by $\ell(1-\cos\theta)\cong\ell\theta^2/3$. It may explain this result.

6. CONCLUSIONS

The concept of force measurement with a zero-compliance mechanism was extended to measurement with a cantilever. For this purpose, a three-degree-of-freedom zero-compliance mechanism was designed and fabricated. It is characterized by a triangle with three variable-length links. It enables the mechanism to keep the position and attitude of the tip of the cantilever (point of action) at the originals even when force acts on the tip. A model of the mechanism was derived. Several kinematic analyses were conducted for this model to predict the position and attitude of the root of the cantilever (detection point). In the experiment, static force was applied to the tip of the cantilever. The experimental results demonstrate that the displacement in the normal direction and the attitude of the detection point were proportional to the applied force. It demonstrates the possibility of precise force measurement with the fabricated apparatus.

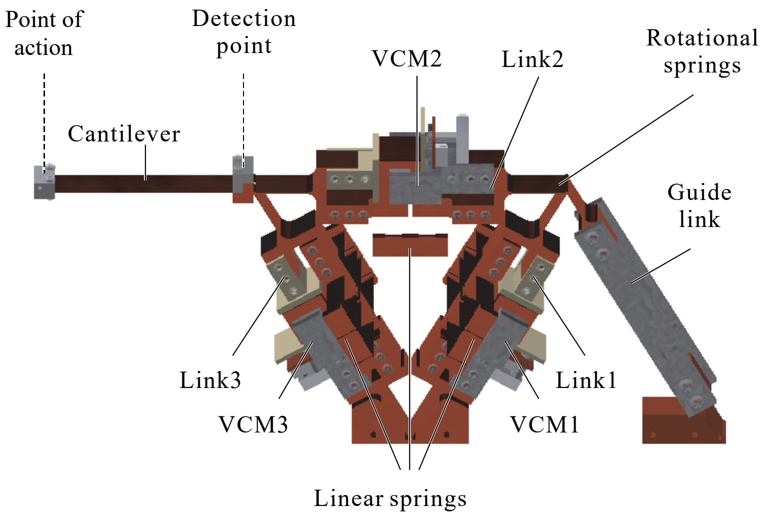


Fig.6 Three-degree-of freedom-of-motion mechanism

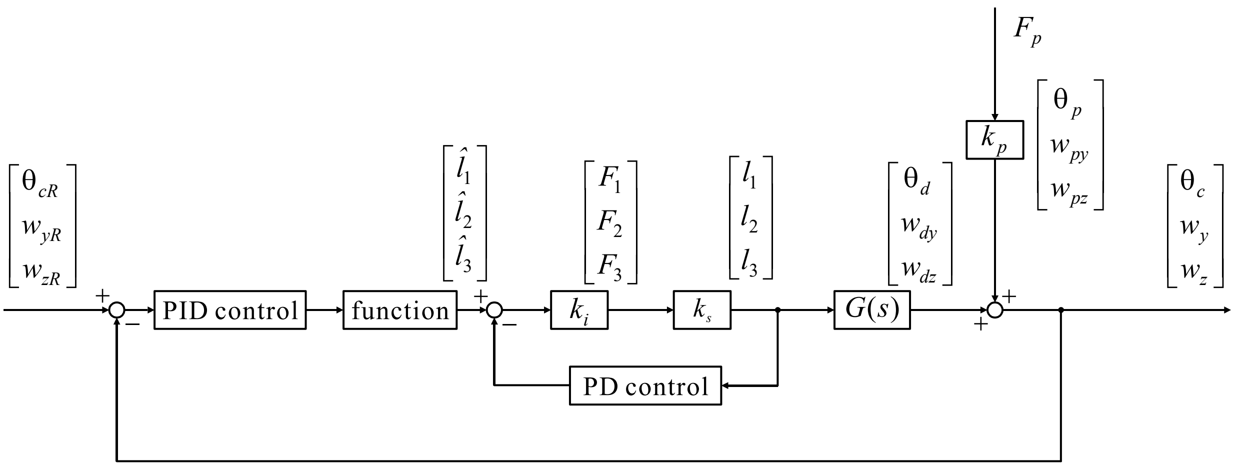
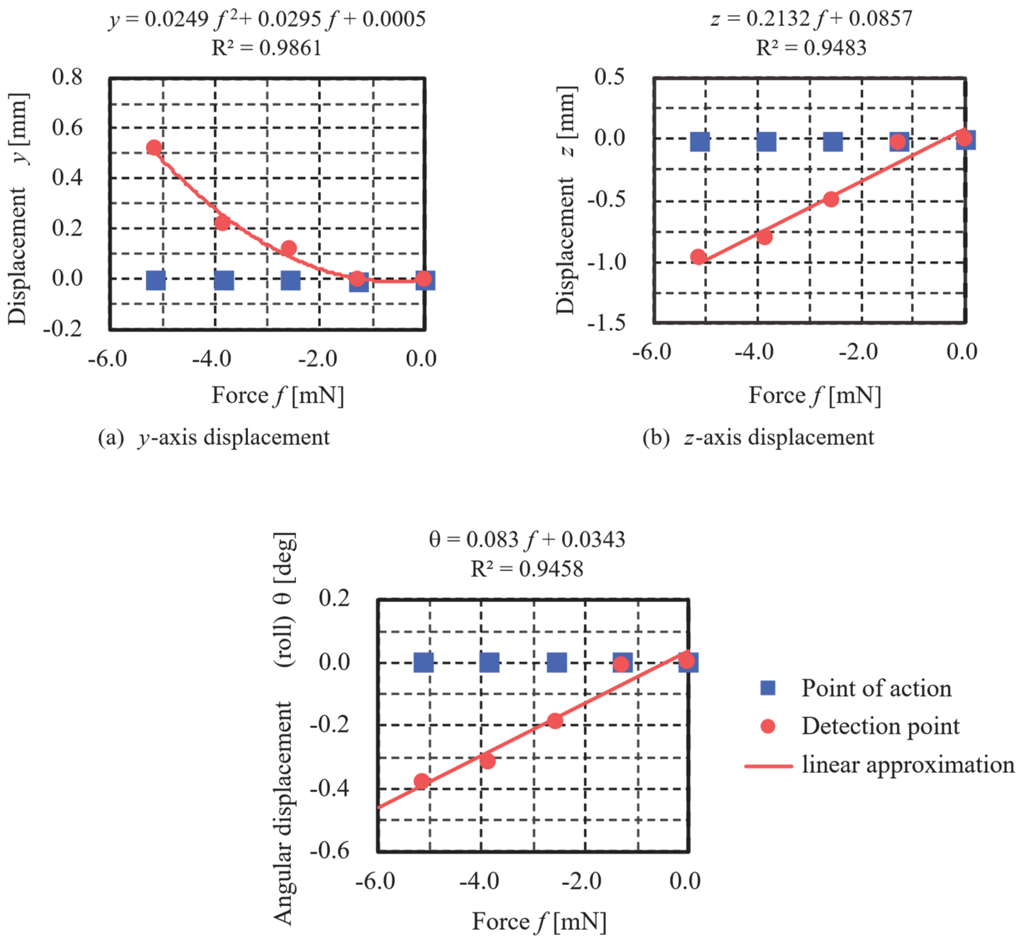


Fig.7 Block diagram of control system



(a) *y*-axis displacement (b) *z*-axis displacement

(c) roll-axis angular displacement

Fig.8 Response when force acts on the point of action

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