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A Finite Time Convergent Least-Squares Modification of the Dynamic Regressor Extension and Mixing Algorithm

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Abstract: The recently proposed Dynamic Regressor Extension and Mixing (DREM) algorithm can be used to estimate the parameters of structured uncertainties contained in the mathematical model of a plant. In order to provide an adaptation that is less sensitive to the unavoidable mismatch between a plant and its model a least-squares based modification of the DREM estimator is proposed in this paper. The modified estimator yields significantly better estimation results as illustrated by the conducted real-world experiment and its parameter estimates also converge within finite time.

Keywords: Nonlinear observers and filter design, parameter identification, finite time estimation, parameter estimation, nonlinear regressor

1. INTRODUCTION

Although the Dynamic Regressor Extension and Mixing (DREM) algorithm was proposed relatively recently in Aranovskiy et al. (2017), it already has been implemented in several real-world applications, e.g. Yi et al. (2018), Borisov et al. (2017), Bazylev et al. (2018) and Schiffer et al. (2018). In terms of the estimation of a vector of constant parameters two interesting properties of the DREM are:

- The absolute value of each element of the parameter estimation error vector is monotonically decreasing as shown in detail in Belov et al. (2018).
- There is a condition for global asymptotic stability of the parameter estimation errors at zero which does not require persistent excitation.

However, the DREM is derived under the assumption that there is no mismatch between the plant and its mathematical model. As such a mismatch is unavoidable considering real-world plants, a modification of the DREM algorithm is proposed in this paper which is optimal with respect to a cost function similar to the "Least-Squares With Exponential Forgetting" described in Slotine and Li (1991). While maintaining the aforementioned advantages of the DREM in the case of ideal conditions, the modified algorithm yields significantly better estimation results in the presence of a mismatch between plant and model as illustrated by the conducted real-world experiment.

When the mismatch between plant and model is included in the originally proposed estimator dynamics, the monotonicity of the elements of the parameter estimation error vector can no longer be guaranteed and the condition for global asymptotic stability no longer applies (both with and without the least-squares modification). An estimator dynamics, similar to the discontinuous gradient algorithm from Rueda-Escobedo and Moreno (2016), is proposed in this paper. Combining the proposed estimator dynamics with the aforementioned least-squares modification yields two conditions for each element of the parameter estimation error vector:

- A condition for its absolute value to be monotonically decreasing.
- A condition such that it becomes zero within finite time.

Both conditions do not require persistent excitation and are, from a practical point of view, straight forward to realize. However, the parameter estimation error has to be redefined as the difference between the estimated parameter vector and the optimal solution (which is not generally equal to the actual parameter vector).

2. THE DREM PARAMETER ESTIMATION METHOD

2.1 A brief review of the DREM method

As an illustrative example, consider the system

$$\dot{x}(t) = f(x(t)) + g(x(t))\left(u(t) + \mathbf{m}^{\mathrm{T}}(x(t), u(t))\mathbf{\Theta}\right), \quad (1)$$

where $x(t)$ is the scalar state variable which depends on time t, $u(t)$ is the scalar actuating signal $f(x)$, $g(x)$, and $\mathbf{m}^{\mathrm{T}}(x, u)$ are given functions and
$\mathbf{\Theta} = [\Theta_1 \, \dots \, \Theta_q]^{\mathrm{T}}$ is a vector of q unknown constants.

Assuming that $g(x(t)) \neq 0 \; \forall t \geq 0$, i.e., the actuating signal u and the structured uncertainty $\mathbf{m}^{\mathrm{T}}\mathbf{\Theta}$ have an impact on the dynamics of the state variable $\forall t$, equation (1) can be written as

$$y(t) = \frac{\dot{x}(t) - f(x)}{g(x)} - u(t) = \mathbf{m}^{\mathrm{T}}(x,u)\mathbf{\Theta}, \qquad (2)$$

where, in a standard parameter estimation framework, the introduced function $y(t)$ is assumed to be known/measured. Based on the representation given in (2) and assuming sufficient excitation, the Dynamic Regressor Extension and Mixing (DREM) algorithm proposed in Aranovskiy et al. (2017) can be used to estimate $\mathbf{\Theta}$. This is done by applying q stable linear filters to both y and $\mathbf{m}^{\mathrm{T}}$. Denoting the outputs of the i^{th} filter as $y_{fi}(t)$ and $\mathbf{m}_{fi}^{\mathrm{T}}(t)$, respectively, the system of equations

$$\underbrace{[y_{f1} \; \dots \; y_{fq}]^{\mathrm{T}}}_{\mathbf{Y}_e} = \underbrace{[\mathbf{m}_{f1} \; \dots \; \mathbf{m}_{fq}]^{\mathrm{T}}}_{\mathbf{M}_e} \mathbf{\Theta} \qquad (3)$$

can be generated, where, for the purpose of readability, the time argument is skipped. This can be multiplied with the adjoint matrix of $\mathbf{M}_e$, i.e., $\mathrm{adj}\,(\mathbf{M}_e)$, which, furthermore yields q scalar decoupled equations

$$Y_i = \phi\Theta_i, \qquad i = 1, ..., q, \qquad (4)$$

where

$$[Y_1 \; \dots \; Y_q]^{\mathrm{T}} = \mathrm{adj}\,(\mathbf{M}_e)\,\mathbf{Y}_e \qquad (5)$$

$$\phi = \mathrm{adj}\,(\mathbf{M}_e)\,\mathbf{M}_e = \det(\mathbf{M}_e) \;\in \mathbb{R}. \qquad (6)$$

Following the estimator dynamics proposed in Aranovskiy et al. (2017), the estimates $\hat{\Theta}_i$ for each unknown parameter Θ_i are governed by

$$\dot{\hat{\Theta}}_i = \gamma_i\phi\left(Y_i - \phi\hat{\Theta}_i\right), \qquad (7)$$

with the tuning parameter $\gamma_i > 0$. As $\dot{\mathbf{\Theta}} = \mathbf{0}$, the dynamics of the estimation errors $\tilde{\Theta}_i = \hat{\Theta}_i - \Theta_i$ are given by

$$\dot{\tilde{\Theta}}_i = \dot{\hat{\Theta}}_i = -\gamma_i\phi^2\tilde{\Theta}_i, \qquad i = 1, ..., q. \qquad (8)$$

A detailed analysis of those dynamics as well as a discussion about the characteristics of ϕ for ensuring estimation error convergence is given in Aranovskiy et al. (2017).

2.2 A modification of the originally proposed dynamics

Since the evolution of ϕ can not be expected to be similar for different experiments, its impact on the estimator dynamics is not desired. In this paper the modified parameter estimation dynamics

$$\dot{\hat{\Theta}}_i = \gamma_i \frac{\phi}{\max\{\phi^2, \; \phi_{\max}^2\}}\left(Y_i - \phi\hat{\Theta}_i\right), \qquad (9)$$

where $\phi_{\max}$ also is a positive constant, is proposed. This yields

$$\dot{\tilde{\Theta}}_i = \dot{\hat{\Theta}}_i = \begin{cases} -\gamma_i\tilde{\Theta}_i & |\phi| \geq \phi_{\max} \\ -\gamma_i\left(\dfrac{\phi}{\phi_{\max}}\right)^2\tilde{\Theta}_i & \text{else} \end{cases} \qquad (10)$$

with the additional tuning parameter $\phi_{\max} > 0$. Hence, the function ϕ only influences the estimator dynamics when $|\phi| < \phi_{\max}$.

3. DREM EMBEDDED LEAST SQUARES APPROACH

A mathematical model always represents an approximation of the considered system and, in addition, measured variables are corrupted by noise. This mismatch can be taken into account by extending (2) to

$$y(t) = \mathbf{m}^{\mathrm{T}}(x(t), u(t))\mathbf{\Theta} + w(t), \qquad (11)$$

where $w(t)$ represents the mentioned mismatch between system and model as well as a noisy measurement of y. Assembling the system of equations as described in Section 2 now yields

$$\underbrace{\begin{bmatrix} y_{f1} \\ \vdots \\ y_{fq} \end{bmatrix}}_{\mathbf{Y}_e} = \underbrace{\begin{bmatrix} \mathbf{m}_{f1}^{\mathrm{T}} \\ \vdots \\ \mathbf{m}_{fq}^{\mathrm{T}} \end{bmatrix}}_{\mathbf{M}_e} \mathbf{\Theta} + \underbrace{\begin{bmatrix} w_{f1} \\ \vdots \\ w_{fq} \end{bmatrix}}_{\mathbf{W}_e} \qquad (12)$$

where w_{fi} denotes the output signal of the i^{th} filter due to the impact of w. Decoupling those equations using the adjoint matrix of $\mathbf{M}_e$ now yields

$$Y_i = \phi\Theta_i + W_i, \qquad i = 1, ..., q, \qquad (13)$$

where $[W_1 \; \dots \; W_q]^{\mathrm{T}} = \mathrm{adj}\,(\mathbf{M}_e)\,\mathbf{W}_e$. This illustrates that following the proposed DREM approach, the influence of the additional unknown terms W_i on the estimation results is difficult to be analyzed in detail. In this paper, a similar system of equations is constructed in a way that already takes the additional uncertainty term into account.

As $w(t)$ is unknown, it can not be guaranteed that an estimation of $\mathbf{\Theta}$ will converge towards the true value of $\mathbf{\Theta}$. However, an estimation $\hat{\mathbf{\Theta}}$ which converges towards an optimal approximation $\hat{\mathbf{\Theta}}_{\mathrm{opt}}(t)$ of $\mathbf{\Theta}$ is proposed in this section. This approximation is optimal in the sense that it minimizes the cost function

$$J(\hat{\mathbf{\Theta}}(t)) = \int_{\tau=0}^{t} \left(y(\tau) - \mathbf{m}^{\mathrm{T}}(\tau)\hat{\mathbf{\Theta}}(t)\right)^2 h(t-\tau)d\tau, \qquad (14a)$$

$$\hat{\mathbf{\Theta}}_{\mathrm{opt}}(t) = \underset{\hat{\mathbf{\Theta}}(t)}{\arg\min}\; J(\hat{\mathbf{\Theta}}(t)). \qquad (14b)$$

A similar cost function is used in the "Least-Squares With Exponential Forgetting" algorithm described in Slotine and Li (1991). The cost function is the integral over the quadratic error weighted by the function $h(t-\tau)$ which needs to satisfy the following properties:

- $h(t) \geq 0 \;\forall t$ so the weighting of the error never becomes negative.
- $h(t)$ is the impulse response of a BIBO stable filter (this filter will be implemented).

For example the function $h(t-\tau) = e^{\frac{\tau - t}{T}}$ with $T > 0$ can be used: the weighting of previous errors exponentially decays and $h(t) = e^{-\frac{t}{T}}$ is the impulse response of a PT_1 filter.

The gradient of the cost function with respect to $\hat{\mathbf{\Theta}}(t)$ is zero at the optimum, i.e.,

$$\left[\nabla_{\hat{\mathbf{\Theta}}(t)} J(\hat{\mathbf{\Theta}}(t))\right]\Big|_{\hat{\mathbf{\Theta}}_{\mathrm{opt}}(t)} = \mathbf{0}, \qquad (15)$$

which yields an equation system of the structure $\mathbf{Y}_e = \mathbf{M}_e\hat{\mathbf{\Theta}}_{\mathrm{opt}}(t)$ and is given by

$$\underbrace{\int_{\tau=0}^{t} \mathbf{m}(\tau) y(\tau) h(t-\tau) d\tau}_{=(\mathbf{m}y*h)(t)=:\mathbf{Y}_e} = \underbrace{\int_{\tau=0}^{t} \mathbf{m}(\tau)\mathbf{m}^{\mathrm{T}}(\tau) h(t-\tau) d\tau}_{=(\mathbf{m}\mathbf{m}^{\mathrm{T}}*h)(t)=:\mathbf{M}_e} \hat{\mathbf{\Theta}}_{\mathrm{opt}}(t) \tag{16}$$

where $\mathbf{M}_e$ and $\mathbf{Y}_e$ are both known functions filtered by a BIBO stable filter which has the impulse response $h(t)$. This equation system can be decoupled by multiplying with $\mathrm{adj}(\mathbf{M}_e)$ like in Section 2 yielding the q scalar equations

$$Y_i = \phi \hat{\Theta}_{\mathrm{opt},i}, \qquad i = 1, ..., q. \tag{17}$$

The dynamics of the q estimates $\hat{\Theta}_i$ for Θ_i can again be chosen as given in (7) or as proposed in (9).

It is interesting to note that in contrast to the approach based on (3), a generalization to the higher dimensional case $\mathbf{y}(t) = \mathbf{M}(t)\mathbf{\Theta}$ with $\mathbf{y} \in \mathbb{R}^n$ and $\mathbf{M} \in \mathbb{R}^{n\times q}$ is straight forward: applying the same steps like in the scalar case also yields an equation system $\mathbf{Y}_e = \mathbf{M}_e \hat{\mathbf{\Theta}}_{\mathrm{opt}}(t)$ with $\mathbf{Y}_e \in \mathbb{R}^q$ and $\mathbf{M}_e \in \mathbb{R}^{q\times q}$.

A different approach to make the DREM algorithm robust with respect to noise is presented in Wang et al. (2019). Instead of obtaining the equation system via optimization, it is assumed that (13) applies and then a dynamics for $\hat{\Theta}_i$ is generated that is less susceptible to the noise term W_i than the originally proposed dynamics. No such noise terms remain when using (17) since the noise is considered by replacing Θ_i with $\hat{\Theta}_{\mathrm{opt},i}$ instead.

4. MODIFIED ESTIMATOR DYNAMICS: FINITE TIME CONVERGENCE

Instead of using the estimator dynamics given in (7) or (9) the dynamics of $\hat{\Theta}$ now is assumed to be captured by

$$\dot{\hat{\Theta}}_i = \tilde{\gamma}_i \operatorname{sign}\left[\phi\left(Y_i - \phi\hat{\Theta}_i\right)\right], \tag{18}$$

which is similar to the discontinuous gradient algorithm presented in Rueda-Escobedo and Moreno (2016). Of course, this can be applied whether Y_i and ϕ are obtained via the least-squares modification or as originally proposed. In (18)

$$\tilde{\gamma}_i = \begin{cases} \gamma_i & |\phi| \geq \phi_{\max} \\ \gamma_i \left(\dfrac{\phi}{\phi_{\max}}\right)^2 & \text{else} \end{cases}, \tag{19}$$

where γ_i and $\phi_{\max}$ are positive tuning parameters. Under ideal conditions, i.e., $Y_i = \phi\Theta_i$, the dynamics of the estimation errors $\tilde{\Theta}_i = \hat{\Theta}_i - \Theta_i$ are given by

$$\dot{\tilde{\Theta}}_i = -\tilde{\gamma}_i \operatorname{sign}\left(\phi^2\right) \operatorname{sign}\left(\tilde{\Theta}_i\right), \qquad i = 1, ..., q \tag{20}$$

where it is important to remember that Θ_i is constant. If $|\phi| > 0$ then $\operatorname{sign}\left(\phi^2\right) = 1$ and $\tilde{\gamma}_i > 0$ so $|\tilde{\Theta}_i|$ decreases. Note that $|\phi| > 0$ is equivalent to "the solution of (14b) is unique", i.e., "there is a unique least-squares solution" when using the least-squares modification.

The convergence properties of the different DREM based estimation algorithms are investigated in the next section of this paper since $|\phi| > 0$ is not sufficient for finite time convergence and, additionally, the dynamics of $\tilde{\Theta}_i$ are no longer given by (20) when the mismatch between plant and model is considered.

5. STABILITY ANALYSIS

5.1 Unperturbed case

The dynamics of the estimation errors of the originally proposed DREM under ideal conditions are given by (8). As shown in Aranovskiy et al. (2017) it is characterized by the estimation error property

$$\lim_{t\to\infty} \tilde{\Theta}_i(t) = 0 \quad \Leftrightarrow \quad \lim_{t\to\infty} \int_{\tau=0}^{t} \phi^2(\tau) d\tau = \infty. \tag{21}$$

Also under ideal conditions, the dynamics of the estimation errors of the DREM using the dynamics for finite time convergence are given by (20). Similar to Barabanov and Ortega (2017), this is investigated for $\tilde{\Theta}_i(t) \neq 0$ and yields the dynamics

$$\frac{d}{dt}|\tilde{\Theta}_i| = \operatorname{sign}\left(\tilde{\Theta}_i\right)\dot{\tilde{\Theta}}_i = -\gamma_i \frac{\phi^2}{\max\left\{\phi^2,\ \phi_{\max}^2\right\}} \tag{22}$$

which has the solution

$$|\tilde{\Theta}_i(t)| = |\tilde{\Theta}_i(0)| - \gamma_i \int_{\tau=0}^{t} \frac{\phi^2(\tau)}{\max\left\{\phi^2(\tau),\ \phi_{\max}^2\right\}} d\tau. \tag{23}$$

Therefore $\tilde{\Theta}_i(t)$ becomes zero if there exists T_i s.t.

$$\int_{\tau=0}^{T_i} \frac{\phi^2(\tau)}{\max\left\{\phi^2(\tau),\ \phi_{\max}^2\right\}} d\tau = \frac{|\tilde{\Theta}_i(0)|}{\gamma_i} \tag{24}$$

where $|\tilde{\Theta}_i(0)|$ is not known but finite. If $\tilde{\Theta}_i(T_i) = 0$ then $\tilde{\Theta}_i(t) = 0\ \forall t \geq T_i$ since, as of (20), also $\dot{\tilde{\Theta}}_i(t) = 0\ \forall t \geq T_i$. Therefore (24) is a necessary and sufficient condition for the estimation error $\tilde{\Theta}_i$ to converge to 0 within finite time under ideal conditions.

5.2 Perturbed case

While above results illustrate the advantages of the dynamics for finite time convergence in (18), they no longer apply when the mismatch between plant and model is considered. As already mentioned in Section 3, this is not investigated further for the originally proposed DREM.

Using the equations (17) obtained via the least-squares modification and the dynamics (18), the dynamics of the estimation errors $\tilde{\Theta}_i = \hat{\Theta}_i - \hat{\Theta}_{\mathrm{opt},i}$ are now given by

$$\dot{\tilde{\Theta}}_i = \dot{\hat{\Theta}}_i - \dot{\hat{\Theta}}_{\mathrm{opt},i} = -\tilde{\gamma}_i \operatorname{sign}\left(\phi^2\right) \operatorname{sign}\left(\tilde{\Theta}_i\right) - \dot{\hat{\Theta}}_{\mathrm{opt},i},$$
$$i = 1, ..., q. \tag{25}$$

Investigating this for $\tilde{\Theta}_i(t) \neq 0$ yields the dynamics

$$\begin{aligned} \frac{d}{dt}|\tilde{\Theta}_i| &= \operatorname{sign}\left(\tilde{\Theta}_i\right)\dot{\tilde{\Theta}}_i \\ &= -\gamma_i \frac{\phi^2}{\max\left\{\phi^2,\ \phi_{\max}^2\right\}} - \operatorname{sign}\left(\tilde{\Theta}_i\right)\dot{\hat{\Theta}}_{\mathrm{opt},i} \end{aligned} \tag{26}$$

which has the solution

$$|\tilde{\Theta}_i(t)| = |\tilde{\Theta}_i(0)| - \int\limits_{\tau=0}^{t} \left[\gamma_i \frac{\phi^2(\tau)}{\max\{\phi^2(\tau),\ \phi_{\max}^2\}} + \mathrm{sign}\left(\tilde{\Theta}_i(\tau)\right) \dot{\hat{\Theta}}_{\mathrm{opt},i}(\tau) \right] d\tau. \tag{27}$$

Therefore $\tilde{\Theta}_i(t)$ becomes 0 within finite time if there exists T_i s.t.

$$\int\limits_{\tau=0}^{T_i} \left[\gamma_i \frac{\phi^2(\tau)}{\max\{\phi^2(\tau),\ \phi_{\max}^2\}} - |\dot{\hat{\Theta}}_{\mathrm{opt},i}(\tau)| \right] d\tau = |\tilde{\Theta}_i(0)|. \tag{28}$$

Additionally, it is obvious from (26) that $|\tilde{\Theta}_i(t)|$ is strictly monotonically decreasing on $[t_1,\ t_2]$ if

$$\frac{\phi^2(t)}{\max\{\phi^2(t),\ \phi_{\max}^2\}} > \frac{|\dot{\hat{\Theta}}_{\mathrm{opt},i}(t)|}{\gamma_i} \quad \forall t \in [t_1,\ t_2] \tag{29}$$

and $\tilde{\Theta}_i(t) \neq 0$ on $[t_1,\ t_2]$. Both above conditions are sufficient but not necessary.

6. REAL-WORLD EXPERIMENT

Different DREM based estimators are applied to an electric circuit in the laboratory, its schematic is shown in Fig. 1. The circuit consists of the resistors R_1, R_2 and R_3 and of the capacitor C. The input voltage is denoted by u, the voltage drop at the capacitor is labeled as u_C.

6.1 Mathematical model

The dynamics of the electric circuit are given by

$$i_C = C\frac{du_C}{dt} = i - i_3 = \frac{u - u_C}{R_1 + R_2} - \frac{u_C}{R_3}, \tag{30}$$

where i_C, i and i_3 denote the current through the capacitor, through the resistors R_1 and R_2 and through the resistor R_3 respectively. Using the definition $x = u_C$ the above given dynamics of the electric circuit is captured by

$$\dot{x} = f(x) + g(x)\left(u + \mathbf{m}^{\mathrm{T}}(x,u)\mathbf{\Theta}\right) \tag{31}$$

as specified in Section 2, where

$$f(x) = -\frac{x}{CR_1}, \qquad g(x) = \frac{1}{CR_1},$$
$$\mathbf{m}^{\mathrm{T}}\mathbf{\Theta} = \underbrace{(-u)}_{m_1}\underbrace{\frac{R_2}{R_1+R_2}}_{\Theta_1} + \underbrace{x}_{m_2}\underbrace{\left(\frac{R_2}{R_1+R_2} - \frac{R_1}{R_3}\right)}_{\Theta_2}. \tag{32}$$

The input voltage u is used as actuating signal, the voltage $u_C = x$ is measured. The parameters of this plant were identified using offline identification procedures and are given by $R_1 = 8.2k\Omega$, $R_2 = 1.8k\Omega$, $R_3 = 840k\Omega$ and $C = 52\mu F$. This yields $\Theta_1 = 0.18$ and $\Theta_2 = 0.17$.

6.2 Experiment

The voltages u and u_C are recorded using a sampling time of $T_s = 10ms$. In order to validate the mathematical model, it is simulated using the recorded input voltage u as input and a fixed step solver with step size T_s. The recorded voltages and the simulation results are shown in Fig. 2. This plot reveals that the mathematical model and its parameters capture the dynamics of the electric circuit system very accurately.

The discussed estimation schemes are also implemented using a fixed step solver with step size T_s, the recorded voltages are used as inputs.

6.3 Estimation with the originally proposed DREM

The equation system $\mathbf{Y}_e = \mathbf{M}_e\mathbf{\Theta}$ for the estimation with the originally proposed DREM is assembled as described in Section 2 using two PT_1 filters with the time constants $T_1 = 50s$ and $T_2 = 25s$, the transfer functions of those filters are

$$G_i(s) = \frac{1}{1 + sT_i}, \tag{33}$$

with $i = 1, 2$. As

$$y = \frac{\dot{x} - f(x)}{g(x)} - u = CR_1\dot{x} + x - u \tag{34}$$

can not be calculated since $\dot{x}$ is not known, y_{fi} is obtained by

$$\begin{aligned} \bar{y}_{fi}(s) &= G_i(s)\bar{y}(s) \\ &= G_i(s)CR_1 s\bar{x}(s) + G_i(s)\left(\bar{x}(s) - \bar{u}(s)\right), \end{aligned} \tag{35}$$

where s denotes the complex variable obtained by applying the Laplace transformation to involved functions of time [1]. This does not require $\dot{x}$ to be known as

$$sG_i(s) = \frac{s}{1 + sT_i} \tag{36}$$

can easily be implemented. Two different estimator dynamics are implemented. The originally proposed dynamics from (7) with $\gamma_1 = \gamma_2 = 10^7$ and the modified dynamics from (9) with $\gamma_1 = \gamma_2 = 5$ and $\phi_{\max} = 5 \cdot 10^{-4}$. In order to simulate ideal conditions, the estimator with the

[1] In this paper, corresponding Laplace transformed functions are denoted by $\bar{\bullet}(s)$, i.e., $\bar{\bullet}(s)$ represents the Laplace transformed function of $\bullet(t)$.

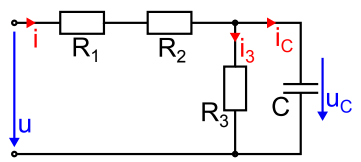

Fig. 1. Electric circuit used for the real-world experiment

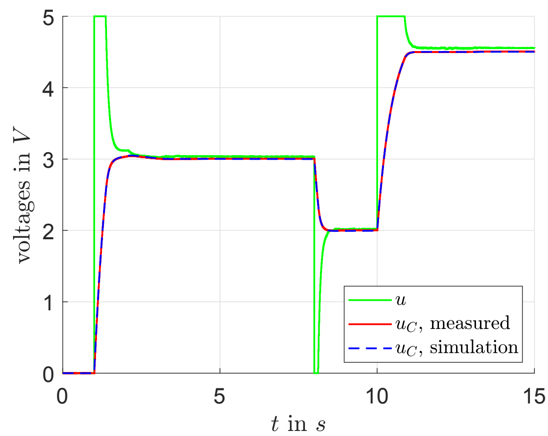

Fig. 2. Comparison of recorded and simulated voltages

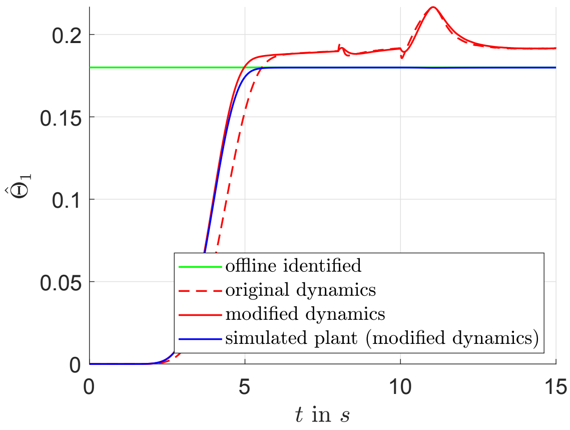

Fig. 3. Estimation of Θ_1 as originally proposed.

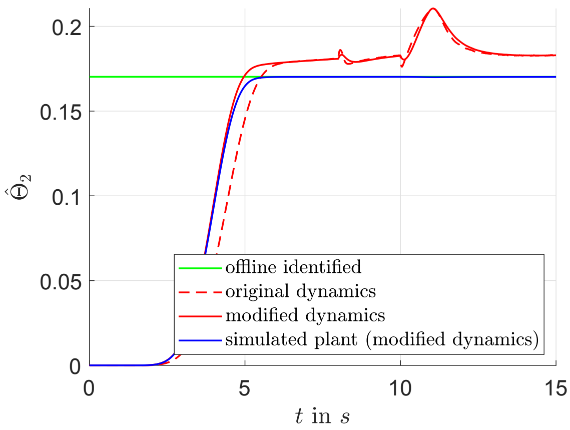

Fig. 4. Estimation of Θ_2 as originally proposed.

modified dynamics is also implemented using u_C from the simulated model instead of the recorded voltage. Note that the simulated voltage is not a piecewise constant function, since the sampling process itself would already be a mismatch between plant and model.

The estimation results are shown in Fig. 3 and Fig. 4. Using the originally proposed and the modified dynamics yields similar estimation results, under ideal conditions the parameters are perfectly identified. While the recorded and the simulated voltages are almost the same, those results already indicate the susceptibility of this estimator to even very small mismatches between plant and model.

6.4 Comparison with the least-squares based DREM

The DREM estimator using the modified equation system is also applied to the electric circuit experiment. Again,

$$y = \frac{\dot{x} - f(x)}{g(x)} - u = CR_1\dot{x} + x - u \tag{37}$$

can not be calculated since $\dot{x}$ is not known. Similar to (35), the output y_f of a PT_1 filter with y as input is obtained. This filter is also applied to $\mathbf{m}$, which yields

$$y_f(t) = \mathbf{m}_f^{\mathrm{T}}(t)\boldsymbol{\Theta} + w_f(t). \tag{38}$$

With $h(t-\tau) = e^{\frac{\tau - t}{T}}$ where $T = 50s$, the equation system can be obtained as described in Section 3 using y_f and $\mathbf{m}_f$ instead of y and $\mathbf{m}$. Note that this causes an additional error since the solution $\hat{\boldsymbol{\Theta}}_{\mathrm{opt}}$ will not be optimal with respect to w but to w_f, which is w filtered by the PT_1 filter. Therefore, this filter uses a relatively small time constant of $T_f = 10 \cdot T_s = 0.1s$. Another possible approach would be the estimation of $\dot{x}$ via a differentiator as outlined Cruz-Zavala et al. (2011).

The least-squares based DREM is implemented using the dynamics from (9) with $\gamma_1 = \gamma_2 = 5$ and $\phi_{\max} = 5 \cdot 10^{-3}$. The estimation results are compared to the results of the estimator from Section 6.3 which also uses the dynamics from (9), i.e., the only differences between those two estimators are the equation system (once as originally proposed and once obtained via least-squares optimization) and the value of $\phi_{\max}$.

The estimation results are shown in Fig. 5 and Fig. 6. As expected, the DREM using the new least-squares based equation system yields significantly better results.

6.5 Estimation with finite-time convergent dynamics

The least-squares based DREM is implemented twice, once as explained in Section 6.4 (i.e. using the dynamics form

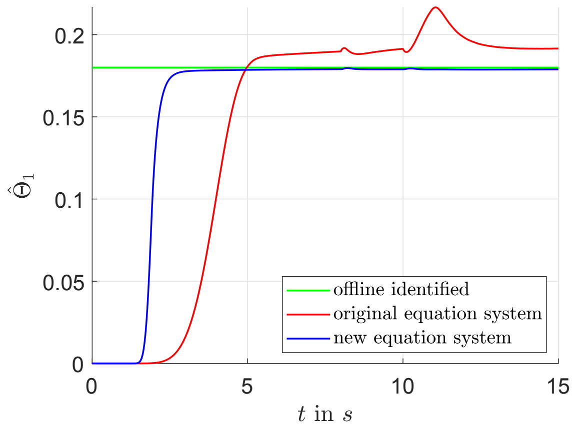

Fig. 5. Estimation of Θ_1 using the originally proposed and the new (least-squares based) equation system.

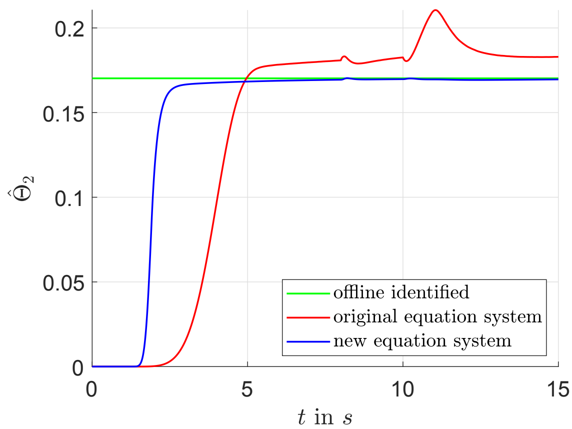

Fig. 6. Estimation of Θ_2 using the originally proposed and the new (least-squares based) equation system.

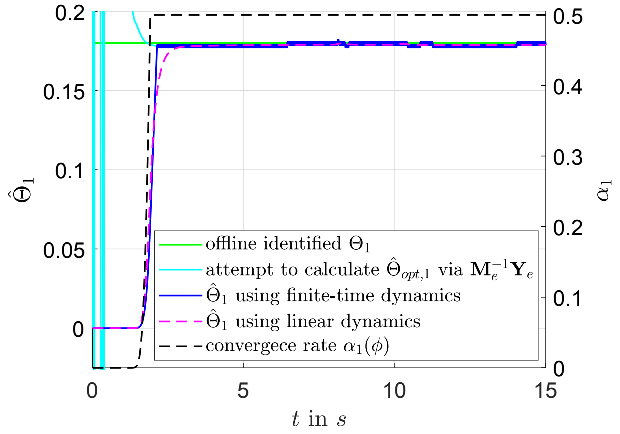

Fig. 7. Estimation of Θ_1 with finite-time convergent dynamics compared to dynamics linear in $\tilde{\Theta}_1$.

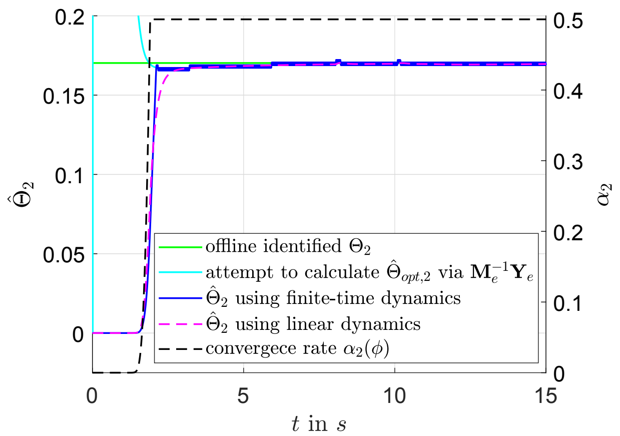

Fig. 8. Estimation of Θ_2 with finite-time convergent dynamics compared to dynamics linear in $\tilde{\Theta}_2$.

(9) with $\gamma_1 = \gamma_2 = 5$ and $\phi_{\max} = 5 \cdot 10^{-3}$) and once using the dynamics for finite-time convergence from (25) with $\gamma_1 = \gamma_2 = 0.5$ and $\phi_{\max} = 5 \cdot 10^{-3}$. All estimators are implemented in continuous-time while using a solver with fixed step size, therefore some chattering can be expected when using the finite time convergent dynamics.

The estimation results are shown in Fig. 7 and Fig. 8. Those figures also show the results of an attempt to calculate $\hat{\mathbf{\Theta}}_{\mathrm{opt}} = \mathbf{M}_e^{-1}\mathbf{Y}_e$ directly: if $\mathbf{M}_e^{-1}$ does not exist, the result from the previous simulation step for $\hat{\mathbf{\Theta}}_{\mathrm{opt}}$ is used. The rates $\alpha_i(\phi)$ at which the estimates using the finite-time convergent dynamics converge

$$\dot{\hat{\Theta}}_i = -\underbrace{\gamma_i \frac{\phi^2}{\max\{\phi^2,\ \phi_{\max}^2\}}}_{=\alpha_i(\phi)} \operatorname{sign}\left(\tilde{\Theta}_i\right) \tag{39}$$

are also shown in Fig. 7 and Fig. 8.

Since $\hat{\mathbf{\Theta}}_{\mathrm{opt}} = \mathbf{M}_e^{-1}\mathbf{Y}_e$ (if $\mathbf{M}_e^{-1}$ exists), the absolute value of $\phi = \det(\mathbf{M}_e)$ can be interpreted as a measure of the quality of the least-squares solution $\hat{\mathbf{\Theta}}_{\mathrm{opt}}$. Given this interpretation, the least-squares based DREM with the finite-time convergent dynamics has the following properties as illustrated by Fig. 7 and Fig. 8:

- If the quality of the solution of the least-squares optimization problem is considered high, the estimates converge at a fast rate (up to the specified constants γ_i) and within finite time.
- If the quality of the solution is considered low, the estimates converge at a slow rate. This is also advantageous considering the evolution of $\mathbf{M}_e^{-1}\mathbf{Y}_e$ in the respective sections of the experiment.

7. CONCLUSION

A modification of the DREM parameter estimation method by embedding it into a least squares optimization framework and enhancing it by a finite time parameter estimation error dynamics is proposed in this paper. It is demonstrated by real world experiments that the least squares based DREM yields significantly better estimation results, both with and without the finite time parameter estimation error dynamics.

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