# Optimal Design of Adaptive Structures
# vs. Optimal Adaption of Structural Design

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Abstract

Taking advantage of adaptivity in the field of civil engineering is an ongoing research topic. Integration of adaptive elements in the load-bearing structure is already well established in many other engineering fields. First investigations promise large material saving potentials also in the field of civil engineering, especially when it comes to high-rise buildings or wide spanned structures like roofs or bridges. In times of emission problems and shortage of materials, the potentials of adaptive civil structures open various new possibilities.

In the design and optimization process of adaptive civil structures, we address the differences between classical approaches for passive systems and new practices considering adaptivity. By using a suitable actuator placement, it is possible to manipulate the displacements of the structure as well as the force distribution within the structure. Both material and energy savings can be accomplished with an integrated design of the adaptive structure taking into account the actuation, suitable combination of structural design and actuator placement. For demonstration of the differences in the design process and in the resulting optimized structure, we use a small case study on a truss structure, which is inspired by a high-rise building, and consider static loads.

*Keywords:* Civil engineering, adaptive structures, structural optimization, smart structures, integrative design

## 1. INTRODUCTION

In times of a growing world population and increasing pollution of the environment, there is the need for an imminent change in the building sector. As stated in the United Nations Environment Programme (2011) the building sector today contributes to the world's condition with a share of about one third of global carbon emissions and about 40% by volume of global solid waste production. Incorporating adaptivity into load bearing structures is one promising way to face the upcoming challenges.

The first ideas to include active elements into civil structures to control occurring vibrations came up in the 60s and many articles were published on this topic. The first idea to use adaptivity to control also static deformations and forces in the structure were introduced by Zuk and Clark (1970). A good overview can be found in Soong (1990) and Utku (1998). The optimization of the structure of an adaptive system considering the actuation was introduced by Kirsch in the 1970s, an overview can be found in Kirsch (1991). The focus of many available papers lies

on the design of the control system, whereas we will focus on the design of the structure itself. Including adaptivity to generate the lightest structures possible was proceeded by Sobek et al. (2000). The subsequent work of Teuffel (2004), amongst others, at the University of Stuttgart was an important step towards a new design methodology that was recently used by Senatore et al. (2019) to design adaptive structures. The promising results lead to the initiation of the Collaborative Research Center SFB 1244 in 2017 with the title "Adaptive Skins and Structures for the Built Environment of Tomorrow". The aim of the Research Center is not only to include adaptivity in the field of civil engineering, which is strongly affected by safety and durability requirements of buildings, but also the design of actuators and the layout of the control system are important aspects, that are addressed.

Design of engineering structures has to change in order to cope with the modified requirements. In the past, properties like stiffness, strength, stability and sensitivity but also durability could only be affected by structural design until the construction was finished. The possibility to implement active elements offers new options to affect those properties not only in the design phase with a suitable structural design but also during the operating phase, when the structure is already built and in service. However, there are new challenges like resilience of the control system, fail-safe and safe-life designs. To find a suitable structural design, also the objectives change towards designing structures that are well adaptable. One

Figure 1. *Stuttgarter Träger* with single vehicle in passive (top) and active (bottom) state [Teuffel (2004)]

approach towards optimizing the adaptability of structures using reduced order models is presented by Fröhlich et al. (2019). The main questions the authors are addressing in the SFB 1244 are: is an optimal passive structure also an optimal adaptive structure and how can a good adaptive structure be characterized and designed? This paper shows a proof of concept for our theories towards the answers to these questions.

To get an idea of our theories, a short thought experiment is considered: the first automobiles were, to put it in simple terms, basically horse-drawn carriages, where the horses in the front were replaced by an engine in the back. Only later it was figured out how to build a real "motor car" for which it is not enough to simply replace one drive technology with another. For structures, the mass (material) should be replaced by energy (adaptive elements), it is therefore necessary to think in entirely new categories. For example, the adaptability of structures is a feature that has not yet been systematically investigated. For active displacement manipulation in mechanical structures there was already a lot of research going on also in other fields of engineering. There are systems in which the internal forces are not controllable by actuators at all, others allow substantial manipulations. The systematic analysis of these and other characteristics is not only a new topic in the field of structural analysis but also the prerequisite for the target-oriented design of adaptive structures. We are looking at structures in view of accessibility to adaptation and control from a structural mechanics point of view. We focus neither on specific methods of structural control nor on their optimization.

The paper is structured as follows. In the Section 2, an analytical proof of concept example is presented that shows differences emanating from the used design strategies. The methodology consisting of the required structural model and the required tools are presented in Section 3. Thereafter, a case study of an exemplary two-dimensional truss structure is presented, carrying out the same approach as in the analytical example. The obtained results and a known limitation of this idea are presented in Section 4.

## 2. AN ANALYTICAL PROOF OF CONCEPT

It can be shown by an analytical example that the design and optimization process for this new type of civil
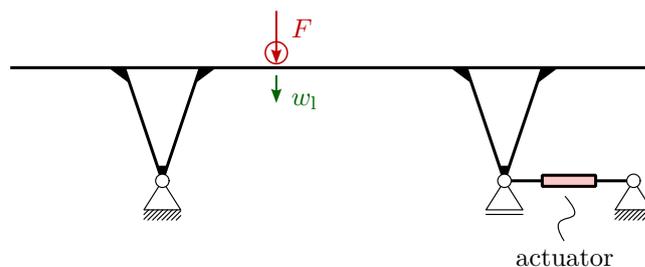


Figure 2. Simplified model of the *Stuttgarter Träger* in side view

structures has to differ from well established processes. The *Stuttgarter Träger* is an important built example by Teuffel (2004), see Figure 1 for an image of the *Stuttgarter Träger* in passive and active state. It is an abstract model of an adaptive bridge. There is a single vehicle moving from one end to the other. Figure 2 shows the side view of the model. The right support can be moved horizontally by a linear actuator and thereby introduce a rotation into the beam thus manipulating the vertical deflection in the span. By this actuation, the deflection of the bridge below the moving load can be canceled.

Our analytical example is inspired by the functionality of the *Stuttgarter Träger*, but, to keep it simple, it is different in dimensions and not considering nonlinear and dynamic effects. The aim is to show the difference in performance of the resulting structure when a classical design approach is used to minimize displacements with adding adaptivity afterwards to a design approach considering the adaptivity already in the design and structural optimization.

The objective function is to minimize the required action of the actuator and as the only design variable the distance of the supports $a$ is used. Figure 3 shows a simplified mechanical model that allows an analytical solution. We only consider the horizontal beam, model the effect of the moving load as a single force $F$ and model the actuation with a single moment $M_A$ at the active support. We are using the differential equation for the Euler-Bernoulli beam model. Assuming constant Young's modulus and constant cross section ($EI =$const.), the governing equation is

$$\frac{\mathrm{d}^4 w(x)}{\mathrm{d}x^4} = \frac{q(x)}{EI}. \tag{1}$$

The deflection at position $x$ is represented by $w(x)$ and the load is specified by $q(x)$. For the moving load at position $x_l$ we use a Dirac type loading function $q(x) = F\delta(x - x_l)$. The resultant is calculated as $\int_0^L q(x)\,\mathrm{d}x = F$. We can solve the equation for the three parts of the beam sepa-
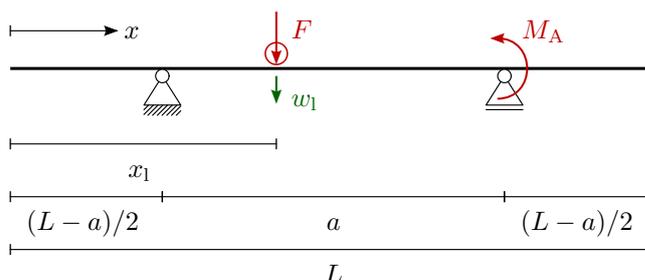


Figure 3. Further abstraction of the model of the *Stuttgarter Träger* in side view
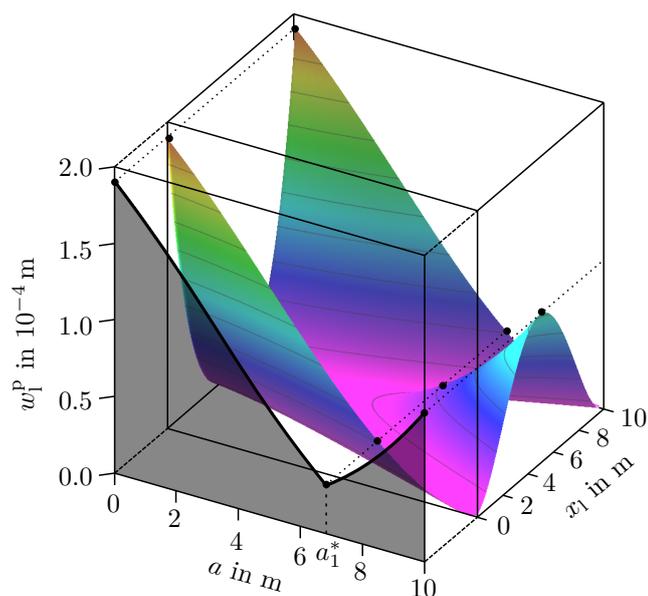
Figure 4. Deflection $w_1^p$ plotted versus support distance $a$ and load position $x_1$



Figure 5. Absolute value of actuation moment $M_A$ plotted versus support distance $a$ and load position $x_1$

rately and use suitable transition and boundary conditions in order to connect the beams and to model supports and actuation. The differential equation (1) for this simplified model can be solved analytically for the deflection $w(x, x_1, M_A, a)$ by integration and the integration constants can be determined using boundary and transition conditions. By substituting $x = x_1$, the deflection at the position of the load $w_1(x_1, M_A, a)$ depending on the position of the load $x_1$, the actuation moment $M_A$ and on the distance of the supports $a$ is calculated. To pursue the defined goal to minimize the required action of the actuator means to minimize the maximum absolute value of the required actuation moment $M_A$ in the end of the design process.

Following values are assumed:

$$EI = 218750\,\mathrm{kNm}^2$$
$$L = 10.0\,\mathrm{m}$$
$$F = 1.0\,\mathrm{kN}.$$

An apparently self-evident design approach, which is denoted as Approach 1 in the sequel, would be to design a structure with the lowest possible maximum deflection in the passive state ($M_A = 0$) and actuate afterwards such that the deflection vanishes. The deflection in the passive state $w_1^p(x_1, a)$ is shown in Figure 4 as a two-dimensional surface plotted versus the two remaining variables load position $x_1$ and support distance $a$. In the projection in $w_1$-$a$-plane, the upper edge of the surface shows the maximum deflection values for all load positions (see front of Figure 4). It can be seen that in this design the optimal support distance is $a_1^* = 6.89\,\mathrm{m}$ and the maximum appearing deflection is $3.1 \cdot 10^{-5}\,\mathrm{m}$. The required actuation moment $M_{A1}$, which is calculated by solving the equation $w_1(x_1, M_A, a_1^*) = 0$ for $M_A$ is shown in Figure 6 as dashed line.

The integrative design approach for adaptive structures (Approach 2) is fundamentally different: it it considers
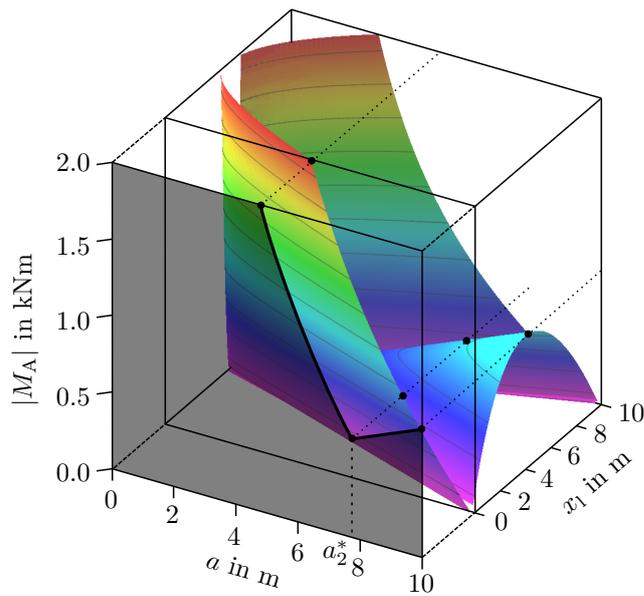
the adaptivity directly from the beginning and optimizes the absolute value of the actuation moment $|M_A|$ using the zero displacement condition $w_1(x_1, M_A, a) = 0$ as a constraint. The equation is solved for $M_A$ and the absolute value is computed. The obtained result can be seen in Figure 5 again depending on the two variables $x_1$ and $a$. Using the same perspective as before it can be seen, that the support distance, that results in the minimum value for the actuation moment over all loading positions, is $a_2^* = 7.70\,\mathrm{m}$. The resulting actuation moment is shown in Figure 6 as dotted line.
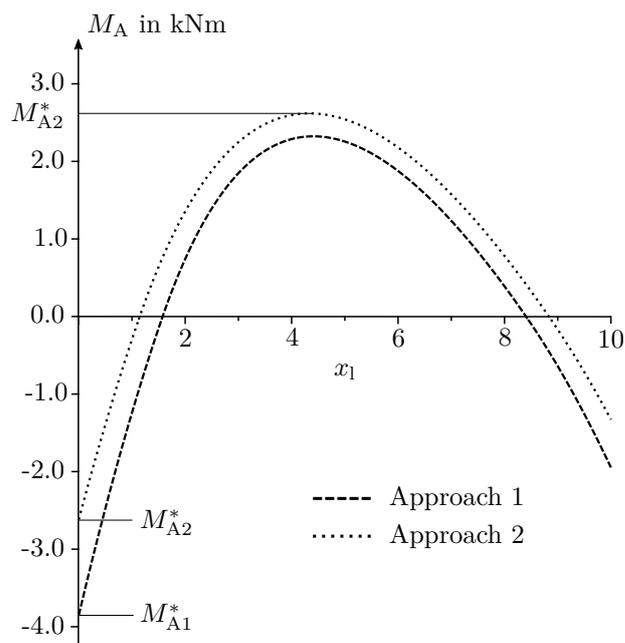


Figure 6. Required actuation moments over all load positions for the different design approaches
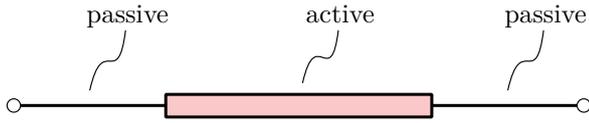
Figure 7. Linear actuator element

The maximum actuation moment for the classical approach is $M_{A1}^* = -3.81\,\mathrm{kNm}$ and for the adaptive optimized structure $M_{A2}^* = \pm2.64\,\mathrm{kNm}$. As the result of this investigation, the maximum needed actuation moment can be reduced by about 30%. The important conclusion is, that by using design Approach 2, the resulting structure will have a different design and will perform better.

## 3. METHODOLOGY

In order to bring this insight to a more realistic setup, we consider a discrete truss structure undergoing only small deformations and neglecting dynamic effects. There are established methods to manipulate the dynamic behavior of civil structures to a certain extent, but in statics there is very little. The equation of motion for this linear system is given by

$$\mathbf{Kd} = \mathbf{f}, \tag{2}$$

consisting of the stiffness matrix $\mathbf{K} \in \mathbb{R}^{n \times n}$ with $n$ representing the number of degrees of freedom of the model. The displacement vector $\mathbf{d} \in \mathbb{R}^n$ and the vector of loads $\mathbf{f}_{\mathrm{ext}} + \mathbf{f}_{\mathrm{act}} = \mathbf{f} \in \mathbb{R}^n$ complete Eqn. (2). The load vector consists of two parts, the external loads $\mathbf{f}_{\mathrm{ext}} \in \mathbb{R}^n$ and the actuation $\mathbf{f}_{\mathrm{act}} = \mathbf{Bu}$ with the input matrix $\mathbf{B} \in \mathbb{R}^{n \times m}$ and the input $\mathbf{u} \in \mathbb{R}^m$. The parameter $m$ represents the number of actuators that are implemented into the structure. The problem of actuator placement in order to obtain $m$ and $\mathbf{B}$ is not addressed here, methods from Wagner et al. (2018) are applied. To keep it as simple as possible, the selected input for the following study is the stroke of the actuator. The stiffness matrix $\mathbf{K}$ is assembled from the element stiffnesses, which depend on the cross sectional area of the truss elements. Thus, the stiffness matrix can be regarded a function of the vector $\mathbf{a}$, which collects the cross sectional areas of all elements, $\mathbf{K} = \mathbf{K}(\mathbf{a})$. The input of the actuators $\mathbf{u}$ is also an unknown variable, such that the load vector can be regarded as a function of the actuator action $\mathbf{f} = \mathbf{f}(\mathbf{u})$.

To simulate the actuation properly, an active truss finite element is introduced, which can directly apply a prescribed stroke $u$ with no further pre- and postprocessing steps. The element is a straight two-dimensional element with both ends hinged. The extension of the formulation to three dimensions is straight forward. In Figure 7 the used element is shown. The position and the length of the active part are arbitrary, here a symmetric assembly is visualized. Material and cross section in passive and active parts may vary. External loads are allowed only to act on the end points of the element.

The derivation of the elemental matrices for one active truss element is briefly described in the following. Starting point is the total potential energy $\Pi^{\mathrm{tot}}$ of an element given by

$$\Pi^{\mathrm{tot}}[d(x)] = \Pi^{\mathrm{int}}[d(x)] + \Pi^{\mathrm{ext}}[d(x)]. \tag{3}$$
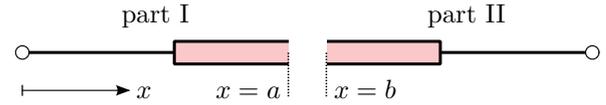


Figure 8. Partitioning of the linear actuator element

The total potential energy consists of both the internal and the external energy and is a functional depending on the displacement field in the element $d(x)$. To find the displacement field such that the structure is in equilibrium, the first variation $\delta\Pi^{\mathrm{tot}}[d(x)]$ must vanish, because the total potential energy has a minimum for this particular displacement field. The stroke load case can be interpreted as a discontinuity in the displacement field. To model this discontiuity, the element is separated into two parts. The connection between the parts in the points $a$ and $b$ in the center of the active part of the element, shown in Figure 8, can be described by the equation

$$d(b) - d(a) = u. \tag{4}$$

Adding this constraint equation to the minimization problem using the Lagrangian multiplier $\lambda$ yields the Lagrangian functional

$$\begin{aligned}\mathcal{L}[d, \lambda] = {} & \Pi^{\mathrm{int}}[d(x)] + \Pi^{\mathrm{ext}}[d(x)] \\ & + \lambda\left(d(b) - d(a) - u\right).\end{aligned} \tag{5}$$

For this derivation we assume geometrically and materially linear behaviour. Fully nonlinear formulation of this element is straightforward using nonlinear stress and strain measures. After computing the first variation, the approximation of the displacement field by a finite element approach using suitable shape functions in conjunction with nodal degrees of freedom is inserted. Afterwards, the application of the fundamental lemma of variational calculus yields the linear system of equations for a single element:

$$\tilde{\mathbf{k}}\tilde{\mathbf{d}} = \tilde{\mathbf{f}}, \tag{6}$$

in detail:

$$\begin{bmatrix} \mathbf{k}_{\mathrm{I}} & \mathbf{0} & \mathbf{g}_{\mathrm{I}}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{k}_{\mathrm{II}} & \mathbf{g}_{\mathrm{II}}^{\mathrm{T}} \\ \mathbf{g}_{\mathrm{I}} & \mathbf{g}_{\mathrm{II}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{d}_{\mathrm{I}} \\ \mathbf{d}_{\mathrm{II}} \\ f_{\mathrm{A}} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{\mathrm{I}} \\ \mathbf{f}_{\mathrm{II}} \\ u \end{bmatrix}. \tag{7}$$

All quantities for part I are labeled with index I and quantities of part II with index II, respectively. The matrices $\mathbf{k}_{\mathrm{I}}$ and $\mathbf{k}_{\mathrm{II}}$ describe the particular elastic stiffness matrices. The geometrical coupling between the separated parts is established by the coupling matrices $\mathbf{g}_{\mathrm{I}}$ and $\mathbf{g}_{\mathrm{II}}$. Load and displacement vectors are separated in displacements of the two parts and the additional parameter $f_{\mathrm{A}}$ and $u$. These parameters describe the discretized Lagrangian multiplier, representing the actuator force $f_{\mathrm{A}}$, which is therefore directly computed when solving the linear system of equations, and the stroke in the actuator $u$, respectively. Using linear shape functions, globally oriented degrees of freedom at the endpoints, and axially oriented degrees of freedom at the transition between passive and active parts and for the geometrical coupling, the vectors and matrices have the following dimensions:

$$\mathbf{k}_{\mathrm{I}}, \ \mathbf{k}_{\mathrm{II}} \in \mathbb{R}^{4 \times 4}$$

$$\mathbf{g}_{\mathrm{I}}, \ \mathbf{g}_{\mathrm{II}} \in \mathbb{R}^{1 \times 4}$$

$$\mathbf{d}_{\mathrm{I}}, \ \mathbf{d}_{\mathrm{II}}, \ \mathbf{f}_{\mathrm{I}}, \ \mathbf{f}_{\mathrm{II}} \in \mathbb{R}^4.$$

Assembly of the global stiffness matrix and the global load vector are straightforward. The element can be used also
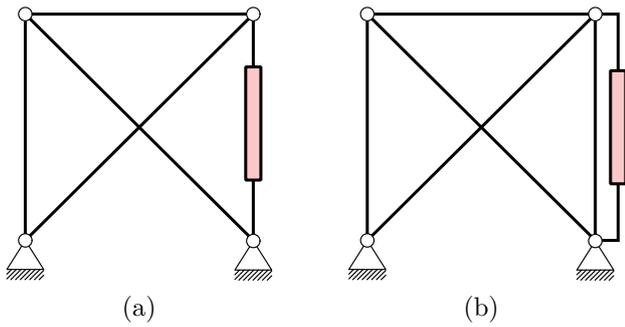
Figure 9. Actuation principles: serial (a) and parallel (b) actuation

with force input instead of stroke input, but this is not considered here. Static condensation on element level can be applied to reduce the number of degrees of freedom without further simplification.

To integrate these linear actuators into a truss structure, two different actuation concepts are possible. Considering a serial setup, see Figure 9(a), the passive truss element is replaced by an actuator or an actuator is added, where no truss was before. The parallel setup in Figure 9(b) results, if the truss element is not replaced by the actuator, but the actuator is added in parallel to the truss, see Wagner et al. (2018). Both can be modeled using the presented active truss element. In cases with large dead load to be transferred trough the active trusses, it may be helpful to consider using a parallel actuator setup, to control e.g. only the dynamic effects in the structure, but we will not address this topic here. In the following case study, we assume serial actuation of the structure.

To find the optimized designs for arbitrary truss structures, an analytical solution and a graphical representation of the objective function are not possible. Therefore, structural optimization is carried out using a gradient based optimization algorithm in MATLAB. Whenever possible, the gradients of the objective functions and of the constraint functions are calculated using the complex step derivative approximation to overcome the difficult determination of the suitable step sizes for numerical differentiation. For further reading on this topic, see Squire and Trapp (1998).

On the one hand it is possible to remove the passive element from the model and add the actuation force as an external force, but this results very often in non-regular stiffness matrices. On the other hand, serial actuation can be considered in the simulation by using a parallel actuator setup in combination with a suitable output equation using the feed through matrix. It can be understood as using the sum of the forces in the serial actuator and in an imaginary parallel passive truss. The output respectively the normal forces in the structure are calculated as

## 4. CASE STUDY AND RESULTS

The process used in the case study considering the two-dimensional truss structure shown in Figure 10 is adopted from the proof of concept example in Section 2. The small example showed significant differences in the performance of an adaptive structure, depending on the design and optimization process. The equivalent investigation for truss structures can be stated as follows. Again pursuing the de-
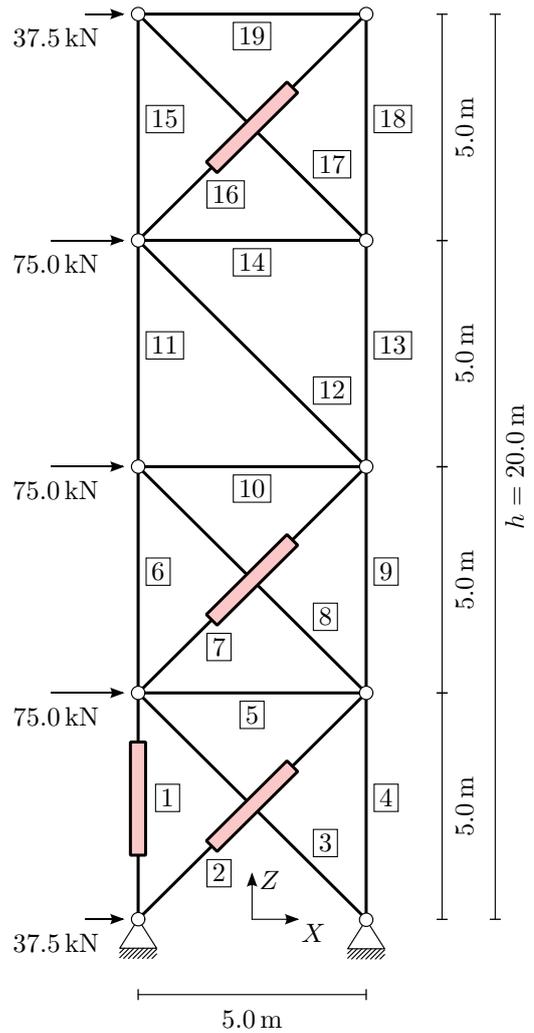


Figure 10. Exemplary truss structure

fined goal to minimize the required action of the actuators means in this case to minimize the required mechanical actuation work $W_A$. To ensure comparability we fix the mass of the structure $m$ as the sum of all element masses to 1000 kg and keep this value constant for all following approaches. The task is to distribute the cross sections in the structure such that the resulting structure needs minimal mechanical actuation work. Fixing the mass is obviously is not very realistic, but it helps to illustrate the significant differences between making a optimized passive structure adaptive and optimizing an adaptive structure.

The considered truss structure consists of 19 elements and is inspired by high-rise buildings. We consider a wind load case with a constant value of 15.0 kN/m in $X$-direction across the height of the structure imposed as nodal loads of 75.0 kN and 37.5 kN, respectively. The dead load of the members is also considered in the calculation. Elements 1, 2, 7 and 16 are serial actuators. In the computation of $W_A$, we do not consider the recovery of the mechanical energy if negative work is performed. The work is calculated as the sum over all actuators multiplying the absolute values of actuator force and required stroke. We assume that the active elements consist of the same material and have the same constant cross section along the element as a passive element. For the material, a structural

steel S235 with the following properties is assumed:

$$E = 2.10 \cdot 10^8 \, \text{kN/m}^2$$
$$f_\text{y} = 2.35 \cdot 10^5 \, \text{kN/m}^2$$
$$\rho = 78.5 \, \text{kN/m}^3$$

For an element $e$ in compression, the maximum allowable stress is calculated as the minimum of the yielding strength of the material $f_\text{y}$ and Euler's critical buckling stress for this element $\sigma_{\text{b},e}$. The assumed cross section for the members is a square hollow box with outer dimensions of $0.18\,\text{m} \times 0.18\,\text{m}$. The admissible values for the cross sectional areas are bounded to be greater than zero and smaller than or equal to the fully filled square box.

In the design of high-rise structures, a reasonable constraint is to limit the horizontal deformations for a building with height $h$ to a value of $d_\text{H} \leq h/500$ aside of other limitations, for instance, on inter-storey drift or on maximum accelerations.

In Approach 1 the first step means to determine the distribution of cross sectional areas in an optimal way to minimize the maximum appearing displacement at any horizontal degree of freedom $\mathbf{d}_\text{H}$, cf. minimizing the maximum deflection of the beam in Approach 1 in Section 2. The resulting minimization problem is formulated as:

$$\min_{\mathbf{a}} \quad \|\mathbf{d}_\text{H}(\mathbf{a})\|_\infty$$

s.t.    mass constraint:
$$m(\mathbf{a}) = 1000\,\text{kg}$$
stress constraint:
$$-\min\{f_\text{y}, \sigma_{\text{b},e}\} \leq \sigma_e(\mathbf{a}) \leq f_\text{y} \ \forall \ e = 1..19.$$

Whenever possible, the gradients of the objective functions and of the constraint functions are calculated using the complex step derivative approximation to overcome the difficult determination of the suitable step sizes for numerical differentiation. For further reading on this topic, see Squire and Trapp (1998).

The minimum is found at $\|\mathbf{d}_\text{H}\|_\infty = 6.0 \cdot 10^{-2}\,\text{m}$, which is larger than the admissible horizontal deformation of $h/500 = 4.0 \cdot 10^{-2}\,\text{m}$. The resulting distribution of cross sectional areas $\mathbf{a}_1^*$ is shown in Figure 11(left). The second step is to calculate the necessary mechanical actuation work for Approach 1 $W_{\text{A}1}$, which is required to prevent displacements exceeding $h/500$ at any horizontal degree of freedom. The cross sectional areas are given by $\mathbf{a}_1^*$ and the input for the actuators $\mathbf{u}$ has to be determined. This is formulated as a second optimization problem:

$$\min_{\mathbf{u}} \quad W_{\text{A}1}(\mathbf{u})$$

s.t.    displacement constraint:
$$\|\mathbf{d}_\text{H}(\mathbf{u})\|_\infty \leq h/500$$
stress constraint:
$$-\min\{f_\text{y}, \sigma_{\text{b},e}\} \leq \sigma_e(\mathbf{u}) \leq f_\text{y} \ \forall \ e = 1..19.$$

The required mechanical work needed to fulfill all given constraints is $W_{\text{A}1}^* = 0.93\,\text{kNm}$ for this structure.

Approach 2 uses a one step solution procedure for the given task. The cross sectional areas and the inputs for the actuators are optimized simultaneously. Also mass, displacement, and stress constraints are considered in the following problem formulation at the same time:
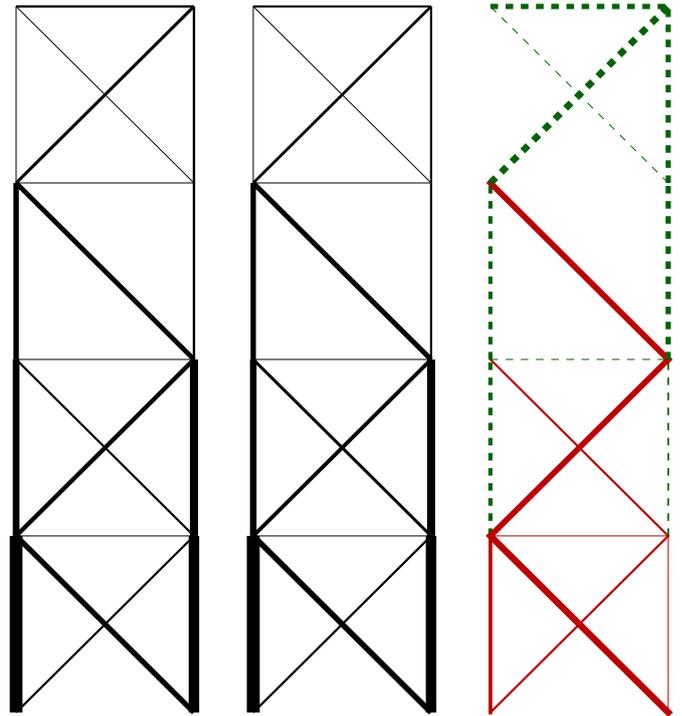


Figure 11. Distribution of cross sectional areas in Approach 1 (left), in Approach 2 (center), and the scaled absolute differences (right; with green dashed line: larger cross section in Approach 1, red line: larger cross section in Approach 2, not shown: no difference)

$$\min_{\mathbf{a},\mathbf{u}} \quad W_{\text{A}2}(\mathbf{a},\mathbf{u})$$

s.t.    mass constraint:
$$m(\mathbf{a}) = 1000\,\text{kg}$$
displacement constraint:
$$\|\mathbf{d}_\text{H}(\mathbf{a},\mathbf{u})\|_\infty \leq h/500$$
stress constraint:
$$-\min\{f_\text{y}, \sigma_{\text{b},e}\} \leq \sigma_e(\mathbf{a},\mathbf{u}) \leq f_\text{y} \ \forall \ e = 1..19.$$

The resulting distribution of cross sectional areas $\mathbf{a}_2^*$ can be found in Figure 11(center) and the resulting actuation work for the adaptive structure is $W_{\text{A}2}^* = 0.79\,\text{kNm}$. There is a decrease of actuation work by about 15%. Since the differences between the distributions of cross sectional areas are not clearly visible, Figure 11(right) shows the scaled absolute differences $\mathbf{a}_2^* - \mathbf{a}_1^*$ of the cross sections resulting from Approaches 1 and 2. The relative differences referred to $\mathbf{a}_1^*$ vary from -36% to 6%.

In mathematical terms, the dimension of the design space is richer in Approach 2 compared to Approach 1. Therefore, getting a better performing structure as result of Approach 2 is not surprising. Comparing the distribution of cross sectional areas reveals that the better adaptable truss structure has larger members and therefore higher stiffnesses in the lower part of the structure near the supports and less stiff members at the top. These necessary modifications in the structure in order to save 15% in the actuation work are not directly accessible from structural engineering point of view. This is not yet a general design guideline for adaptive structures but it shows an important aspect of the design process of an adaptive structure. The aim of the design has to change towards a structure, that

can be suitably adapted instead of designing the overall stiffest structure possible.

This comparative approach is not applicable in the design of minimum weight structures considering only a single load case. Following Kirsch (1991), there is a unique admissible distribution of cross sections that fulfills all constraints and equilibrium. In this case, both presented approaches yield the same resulting structure and no performance improvements can be achieved.

## 5. CONCLUSION AND FURTHER WORK

The paper discusses the importance of a new design procedure for adaptive structures by an analytical example and by a case study considering a two-dimensional truss structure. It can be stated that structural design has to aim in a different direction than that of passive structures if adaptivity is incorporated. The optimization problem formulations have to differ from the well-known ones, like maximizing stiffness or strength. The basic difference in performance of the different structures yielded by different design procedures and objectives were shown in the proof of concept example in Section 2. Maximizing the stiffness did not lead to a well performing adaptive structure. The approaches used in Section 2 are applied to a truss structure yielding proper results. The performance of the adaptive structure can be improved, if the adaptivity of the structure is taken into account in the design process. If an optimized passive structure is used to incorporate adaptivity, the resulting structure will exhibit worse performance.

For further work, the extension of the case study considering arbitrary load scenarios with multiple load cases and dynamic effects are important aspects. The investigation of different structural topologies is also an important topic for future work in order determine, which topology performs best when used as adaptive structure. Besides topology, attention ought to be put onto the question which role the actuator placement plays in the need of actuation energy. Also the uniqueness of the overall mass minimum considering adaptivity has to be further investigated considering multiple load cases. Structures consisting not only of truss elements but also of beam elements or other structural elements will be taken into account in future work.

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