

# Distributed Finite-time Bipartite Consensus of Multi-agent Systems via Event-triggered Control <sup>\*</sup>

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**Abstract:** This paper investigates a distributed finite-time event-triggered bipartite consensus control for multi-agent systems. Under scenarios of energy limitation, an event-triggered strategy coupled with a nonlinear distributed control protocol is proposed only relying on local information, where the controller only updates at triggered instants. We proved that when the antagonistic network contains a spanning tree, the event-triggered controller can drive all agents to reach consensus value with an identical magnitude but opposite signs. Moreover, both the convergence time depending on the initial state and the positive lower bound of inter-event times are achieved. Simulation results show that the proposed controller has better disturbance rejection properties and can achieve bipartite consensus faster compared to an asymptotic controller.

*Keywords:* Finite-time, Multi-agent Systems, Signed Graph, Bipartite Consensus, Event-triggered Control

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## 1. INTRODUCTION

In the past decade, many researchers have focused on studies of coordination control of multi-agent systems (MASs) Xu et al. (2020); Li et al. (2008); Xu et al. (2019), including problems of consensus Li et al. (2008); Ning et al. (2019), flocking Olfati-Saber (2006), formation control Li et al. (2019), etc. Notably, the consensus has been considered as one of the fundamental coordination problems on networks, which indicates that the agreement of all agents is on a typical quantity. One of the characteristic features of the existing solutions is that the relationships between agents are always modeled by a graph associated with a non-negative neighbor weight matrix on their communication graph. However, in practice, networks with antagonistic interactions are conventional in social network theory De Meo et al. (2014); Bao et al. (2016); Wang et al. (2016). To be specific, the topology of the network is referred to as a signed network. Since signed graphs with cooperative and antagonistic interactions can be described as positive and negative neighbor weights, it is difficult for MASs to reach consensus. Recently, a notion of bipartite consensus was proposed with the assistance of the signed network

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<sup>\*</sup> The work was supported by the National Natural Science Foundation of China (Grant Nos. 51879022, 91648120, 61633002, 51575005, 61503008), the Beijing Natural Science Foundation (No. 4192026), the Fundamental Research Funds for the Central Universities (Grants Nos. 3132019037, 3132019197) and the Academy of Finland (Grant No. 315660). Jin Tao and Mingyi Xu are corresponding authors.

theory Altafini (2012), where all agents converge to values with the same magnitude but opposite signs.

In general, bipartite consensus of MASs is assumed to have continuous measurement and/or control signals, i.e., the system continuously monitors the state of each agent and the control protocol updates all the time Altafini (2012); Valcher and Misra (2014). This is unrealistic in practical applications. Therefore, it is extremely significant to design a reasonable information transmission and sharing mechanism. Further, sampled-data control was applied in MASs Wen et al. (2017); Liu et al. (2017); Ma et al. (2018), where the measurements are taken periodically according to a constant period and the control protocol updates synchronously. However, such a sampled-data control also leads to excessive consumption of both communication and computation resources. In order to solve the problem of resource utilization, an event-triggered consensus control method for MAS cite ma2018, Li20192, ren2019e is proposed, where an event-triggered condition is firstly constructed, and then the current system is sampled and transmitted when the conditions are satisfied.

On the other hand, the convergence time is a significant performance indicator of MASs. In most existing works, protocols only achieve state consensus in an infinite time interval, that is, the consensus is only achieved asymptotically. However, the stability of MASs in a finite time interval needs to be considered in many cases. The finite-time stability focuses on the behavior of system responses

over a finite time interval Wang et al. (2014); Huang et al. (2012). Therefore, it is valuable to conduct research on the finite-time stability of MASs. Moreover, the multi-agent finite-time stability analysis has also elicited the attention of many researchers Wang and Xiao (2010); Feng et al. (2016).

Different from the above mentioned results, this paper mainly focus on finite-time distributed event-triggered bipartite consensus control for MASs. The main contributions of this paper can be summarized as follows: Firstly, a new finite-time bipartite consensus protocol based on the event-triggered control strategy for MASs is presented, and the system stability is proved. Secondly, the lower bound of the inter-event time is reached to guarantee that there is no Zeno behavior. At last, the upper bound of convergence time which depends only on initial-state properties is obtained.

The remainder of this paper is organized as follows. Preliminary definitions and the problem formulation are presented in Section 2. The main results are presented in Sections 3. Section 4 discusses the simulation examples before we conclude in Section 5.

## 2. PROMBLEM FORMULATION

In this section, we first collect some notions and basic concepts from algebraic graph theory, which will be used throughout this paper. Then, the concerned system model and bipartite consensus problem are formulated.

### 2.1 Preliminaries

$\mathbb{R}$  represents the set of real numbers.  $\mathbb{R}^{n \times n}$  denotes a  $n \times n$  real matrix.  $\mathbf{1}_N$  and  $\mathbf{0}_N$  stand for the  $N$  dimension column vectors with all entries 1 and 0, respectively.  $|S|$  is the number of elements of the set  $S$ . The matrix  $\text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$  denotes a diagonal matrix with diagonal entries  $\lambda_1, \lambda_2, \dots, \lambda_N$ .

The interaction network among agents is described by a undirected signed graph  $\mathcal{G}$ ,  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ , where  $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$  denotes a set of nodes,  $\mathcal{E} = \mathcal{V} \times \mathcal{V}$  denotes a set of edges, and  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$  is a matrix of the signed weights of  $\mathcal{G}$ . Here,  $a_{ij} \neq 0$  if and only if  $(v_j, v_i) \in \mathcal{E}$ , otherwise  $a_{ij} = 0$ . Beside, graphs with self-loops  $a_{ii} = 0, (i = 1, 2, \dots, n)$  is not taken into consideration. For a signed graph  $\mathcal{G}$ , the edge set  $\mathcal{E} = \mathcal{E}^+ \cup \mathcal{E}^-$ , where  $\mathcal{E}^+ = \{(j, i) | a_{ij} > 0\}$  and  $\mathcal{E}^- = \{(j, i) | a_{ij} < 0\}$ . Moreover, agent  $j$  is called a neighbor of agent  $i$ .  $N_i = \{(j, i) \in \mathcal{E}, j \neq i\}$  is used to represent the neighbor set of agent  $i$ . A path from node  $v_i$  to node  $v_j$  is a finite sequence of edges in the form of  $(v_i, v_{i_{k_1}}), (v_{i_{k_1}}, v_{i_{k_2}}), \dots, (v_{i_{k_l}}, v_j), i \neq j$ .

Define the Laplacian matrix  $L$  of a signed graph as

$$L = \text{diag} \left( \sum_{k \in N_1} |a_{1k}|, \sum_{k \in N_2} |a_{2k}|, \dots, \sum_{k \in N_n} |a_{nk}| \right) - \mathcal{A}$$

Then, the eigenvalues of  $L$  can be indicated by a decreasing order

$$\lambda_n(L) \geq \dots \geq \lambda_2(L) > \lambda_1(L) = 0$$

Given any signed graph  $\mathcal{G}$ , if there exists a bipartition with  $\mathcal{V}_1$  and  $\mathcal{V}_2$ , that satisfies  $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ ,  $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ . When  $a_{ij} \geq 0$  for  $\forall v_i, v_j \in \mathcal{V}_q (q \in \{1, 2\})$  and  $a_{ij} \leq 0$  for  $\forall v_i \in \mathcal{V}_q, v_j \in \mathcal{V}_r, q \neq r (r \in \{1, 2\})$ ,  $\mathcal{G}$  can be regarded to be structurally balanced, otherwise  $\mathcal{G}$  is structurally unbalanced.

The following three lemmas are introduced to facilitate subsequent proofs and analysis.

*Lemma 1.* (Meng et al. (2015)). For a structurally balanced signed graph  $\mathcal{G}$ , there exists a diagonal matrix  $D = \text{diag} \{\sigma_1, \sigma_2, \dots, \sigma_n\}$  such that the entries of  $DAD$  are all nonnegative, where  $\sigma_i \in \{1, -1\}, \forall i \in \mathcal{V}$ .

*Lemma 2.* (Bhat and Bernstein (2000)). Suppose that a function  $V(t) : [0, \infty) \rightarrow [0, \infty)$  is differentiable and satisfies the condition

$$\frac{dV(t)}{dt} \leq -KV(t)^\alpha$$

where  $K > 0$  and  $0 < \alpha < 1$ .

Then  $V(t)$  reaches zero at the convergence time  $T$ , and  $V(t) = 0$  for all  $t \geq T$ , where  $t = \frac{V(0)^{1-\alpha}}{K(1-\alpha)}$ .

*Lemma 3.* (Lick (2012)). Given any  $\xi_1, \xi_2, \dots, \xi_n \geq 0, 0 < p \leq 1$ , the following property is applied

$$\left( \sum_{i=1}^n \xi_i \right)^p \leq \sum_{i=1}^n \xi_i^p \leq n^{1-p} \left( \sum_{i=1}^n \xi_i \right)^p$$

### 2.2 Problem formulation

For a signed graph  $\mathcal{G}$ , consider a group of  $n$  single-integrator agents modeled as

$$\dot{x}_i(t) = u_i(t), i \in \mathcal{V} \quad (1)$$

where  $x_i(t) \in \mathbb{R}$  and  $u_i(t) \in \mathbb{R}$  are the state and the control input of agent  $i$ , respectively.

Suppose that  $n$  agents are classified into two antagonistic groups  $\mathcal{V}_1$  and  $\mathcal{V}_2$ , where  $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ ,  $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ . Obviously, it is better to describe the interaction network as a structurally balanced signed graph  $\mathcal{G}$ .

Then, the bipartite consensus for MASs is defined as follows

*Definition 1.* (Finite-time Bipartite Consensus). Given a structurally balanced signed graph  $\mathcal{G}$ , a distributed control protocol  $u_i(t, x_i(t)), i = 1, 2, \dots, N$  is designed to converge to the finite-time bipartite consensus for system (1). Namely, there exists a settling time  $T$  such that

$$\lim_{t \rightarrow T} x_i(t) = \sigma_i c, \forall i \in \mathcal{V}_1$$

and

$$\lim_{t \rightarrow T} x_j(t) = -\sigma_j c, \forall j \in \mathcal{V}_2$$

where  $\sigma_i \in \{1, -1\}$ , and  $c$  is the same absolute value of the final consensus states of all agents.

### 2.3 Event-triggered control consensus protocol

Traditional finite-time bipartite consensus control protocol is given as

$$u_i(t) = \sum_{j=1}^n a_{ij} \operatorname{sgn}(x_j(t) - \operatorname{sgn}(a_{ij})x_i(t)) |x_j(t) - \operatorname{sgn}(a_{ij})x_i(t)|^\alpha \quad (2)$$

where  $0 < \alpha < 1$ , and  $\operatorname{sgn}(\cdot)$  is the sign function, which is defined as

$$\operatorname{sgn}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

Note that the control protocol (2) needs to be updated continuously, which leads to a waste of unnecessary transmission energy and communication bandwidth. In order to solve this problem, an event-triggered strategy is applied to avoid unnecessary assumptions. Particularly, the controllers of agents only update at discrete event instants in the scenarios of continuous communication, which indicates that the controllers are regarded as zero-order holder between every two event-triggered instants.

Denote an increasing sequence  $t_0^i, t_1^i, \dots, t_k^i, \dots$  as the event instants of agent  $i$ , such that  $\hat{x}_i(t) = x_i(t_k^i)$  is the state of agent  $i$  at the  $k$ -th event instants. In the network with antagonistic interactions, an event-triggered control protocol can be designed as

$$u_i(t) = \sum_{j=1}^n a_{ij} \operatorname{sgn}(\hat{x}_j(t) - \operatorname{sgn}(a_{ij})\hat{x}_i(t)) |\hat{x}_j(t) - \operatorname{sgn}(a_{ij})\hat{x}_i(t)|^\alpha, t \in [t_k^i, t_{k+1}^i) \quad (3)$$

The state measurement error between the states of the last event instant and the current state is defined as

$$e_i(t) = (\hat{x}_i(t) - x_i(t))^{1/\alpha}, t \in [t_k^i, t_{k+1}^i) \quad (4)$$

Then we get

$$e_i^\alpha(t) = \hat{x}_i(t) - x_i(t) \quad (5)$$

Substituting (3) into (1), the closed-loop form of agent  $i$  can be written as

$$\dot{\hat{x}}_i(t) = \sum_{j=1}^n a_{ij} \operatorname{sgn}(\hat{x}_j(t) - \operatorname{sgn}(a_{ij})\hat{x}_i(t)) |\hat{x}_j(t) - \operatorname{sgn}(a_{ij})\hat{x}_i(t)|^\alpha, t \in [t_k^i, t_{k+1}^i) \quad (6)$$

## 3. MAIN RESULTS

For simplicity, intermediate variables are introduced as  $z_i(t) = \sigma_i x_i(t)$  and  $e_{zi}(t) = \hat{z}_i(t) - z_i(t)$ . Then, (6) is rewritten into

$$\dot{\hat{z}}_i(t) = \sum_{j=1}^n a_{ij} \operatorname{sgn}(\hat{z}_j(t) - \hat{z}_i(t)) |\hat{z}_j(t) - \hat{z}_i(t)|^\alpha, t \in [t_k^i, t_{k+1}^i) \quad (7)$$

Denote  $\bar{z}(t) = \frac{1}{n} \sum_{i=1}^n z_i(t)$ , since the Laplacian matrix  $L$  has a single zero eigenvalue and the corresponding eigenvector  $\mathbf{1}_n$ , we can get

$$\dot{\bar{z}}(t) = \frac{1}{n} \sum_{i=1}^n \dot{z}_i(t) = \frac{1}{n} \mathbf{1}_n^T \dot{z}(t) = \frac{1}{n} \mathbf{1}_n^T L_D \bar{z}(t) = 0 \quad (8)$$

where  $L_D = DLD$  and  $D$  is the same matrix as described Lemma 1.

Let  $\delta_i(t) = z_i(t) - \frac{1}{n} \sum_{i=1}^n z_i(t)$  and  $e_{zi}^\alpha(t) = \hat{\delta}_i(t) - \delta_i(t)$ , thus, (7) can be rearranged as

$$\dot{\delta}_i(t) = \sum_{j=1}^n a_{ij} \operatorname{sgn}(\hat{\delta}_j(t) - \hat{\delta}_i(t)) |\hat{\delta}_j(t) - \hat{\delta}_i(t)|^\alpha, \quad (9)$$

$$t \in [t_s, t_{s+1})$$

where  $\hat{\delta}_i(t) = \delta_i(t_s)$  and  $\hat{\delta}_j(t) = \delta_j(t_s)$ .

Considering  $e_{zi}^\alpha(t)$ , (9) is rearranged into

$$\dot{\delta}_i(t) = \sum_{j=1}^n a_{ij} \operatorname{sgn}(\delta_j(t) - \delta_i(t)) |\delta_j(t) - \delta_i(t)|^\alpha + e_{zi}^\alpha(t), t \in [t_s, t_{s+1}) \quad (10)$$

It is apparent that  $\dot{\delta}_i(t) = 0$  implies  $\lim_{t \rightarrow \infty} x_i(t) = \sigma_i \lim_{t \rightarrow \infty} \hat{z}_i(t) = \sigma_i \bar{z}(t)$ . For system (1) and the designed control protocol (3), the event-triggered circle formation control for the distributed MASs can be solved according to Theorem 1.

*Theorem 1.* Consider a MAS (1) and control law (3) over a structurally balanced signed graph  $\mathcal{G}$ , the finite-time bipartite consensus is achieved when the event-triggered condition is designed as

$$f_i(t, e_i(t), \delta_i(t)) = \|e_i(t)\| - \left( \frac{\mu(4\lambda_2(W))^{(1+\alpha)/2}}{2^{-(1-\alpha)/2}} \right)^{1/\alpha} \|\delta_i(t)\| \quad (11)$$

where  $W = [w_{ij}] \in \mathbb{R}^{n \times n}$ ,  $w_{ij} = (a_{ij})^{2/(1+\alpha)}$ , and  $0 < \mu < 1$ .

Moreover, the convergence time  $T$  satisfies

$$T \leq \frac{4V(0)^{(1-\alpha)/2}}{(1-\mu)(4\lambda_2(W))^{(1+\alpha)/2}(1-\alpha)} \quad (12)$$

At the same time, the positive lower bound of the inter-event times  $t_{k+1} - t_k$  is given as

$$\tau_{min} = \frac{\mu^{1/(\alpha)\theta}}{\|L_D\|(1+\mu^{1/\alpha}\theta)} \quad (13)$$

where  $\theta = \left( \frac{(4\lambda_2(W))^{(1+\alpha)/2}}{2^{-(1+\alpha)/2}} \right)^{1/\alpha}$ .

### Proof.

Consider a candidate Lyapunov function as

$$V(t) = \frac{1}{2} \delta^T(t) \delta(t) \quad (14)$$

Then, the derivative of the Lyapunov function (14) along of the trajectories of the system yields to

$$\begin{aligned}
 \frac{dV(t)}{dt} &= \sum_{i=1}^n \delta_i \dot{\delta}_i \\
 &= \sum_{i=1}^n \delta_i \left( \sum_{j=1}^n a_{ij} \operatorname{sgn}(\delta_j(t) - \delta_i(t)) |\delta_j(t) - \delta_i(t)|^\alpha + e_{zi}^\alpha \right) \\
 &= \sum_{i=1}^n \delta_i \sum_{j=1}^n a_{ij} \operatorname{sgn}(\delta_j(t) - \delta_i(t)) |\delta_j(t) - \delta_i(t)|^\alpha + \sum_{i=1}^n \delta_i e_{zi}^\alpha \\
 &= \frac{1}{2} \sum_{i,j=1}^n a_{ij} (\delta_i \operatorname{sgn}(\delta_j(t) - \delta_i(t)) |\delta_j(t) - \delta_i(t)|^\alpha \\
 &\quad + \delta_j \operatorname{sgn}(\delta_i(t) - \delta_j(t)) |\delta_i(t) - \delta_j(t)|^\alpha) + \sum_{i=1}^n \delta_i e_{zi}^\alpha \\
 &= -\frac{1}{2} \sum_{i,j=1}^n a_{ij} |\delta_j(t) - \delta_i(t)|^{1+\alpha} + \sum_{i=1}^n \delta_i e_{zi}^\alpha \\
 &= -\frac{1}{2} \sum_{i,j=1}^n ((a_{ij})^{2/(1+\alpha)} (\delta_j(t) - \delta_i(t))^2)^{(1+\alpha)/2} + \sum_{i=1}^n \delta_i e_{zi}^\alpha
 \end{aligned} \tag{15}$$

From  $0 < \alpha < 1$ , we can get  $0.5 < (1 + \alpha)/2 < 1$ . For clarity, (15) is separated into two parts

$$S_1(t) = -\frac{1}{2} \sum_{i,j=1}^n ((a_{ij})^{2/(1+\alpha)} (\delta_j(t) - \delta_i(t))^2)^{(1+\alpha)/2} \tag{16}$$

and

$$S_2(t) = \sum_{i=1}^n \delta_i e_{zi}^\alpha \tag{17}$$

By Lemma 3, (16) is calculated as

$$\begin{aligned}
 S_1(t) &\leq -\frac{1}{2} \left( \sum_{i,j=1}^n (a_{ij})^{2/(1+\alpha)} (\delta_j(t) - \delta_i(t))^2 \right)^{(1+\alpha)/2} \\
 &= -\frac{1}{2} \left( \frac{\sum_{i,j=1}^n (a_{ij})^{2/(1+\alpha)} (\delta_j(t) - \delta_i(t))^2}{\frac{1}{2} \sum_{i=1}^n \delta_i^2(t)} V(t) \right)^{(1+\alpha)/2} \\
 &= -\frac{1}{2} \left( \frac{4\delta^T L(W) \delta}{\delta^T \delta} V(t) \right)^{(1+\alpha)/2} \\
 &\leq -\frac{1}{2} (4\lambda_2(L(W)))^{(1+\alpha)/2} V^{(1+\alpha)/2}(t)
 \end{aligned} \tag{18}$$

Then, (17) is written as

$$\begin{aligned}
 S_2(t) &= \sum_{i=1}^n \delta_i e_{zi}^\alpha \\
 &= \delta^T(t) e_{zi}^\alpha(t) \leq \|\delta(t)\| \|e_{zi}^\alpha(t)\| \\
 &= \frac{\|\delta(t)\| \|e_z(t)\|^\alpha V^{(1+\alpha)/2}(t)}{V^{(1+\alpha)/2}(t)} \\
 &= \frac{\|\delta(t)\| \|e_z(t)\|^\alpha V^{(1+\alpha)/2}(t)}{2^{-(1+\alpha)/2} \|\delta(t)\|^{1+\alpha}} \\
 &= \frac{\|\delta(t)\|^{-\alpha} \|e_z(t)\|^\alpha V^{(1+\alpha)/2}(t)}{2^{-(1+\alpha)/2}}
 \end{aligned} \tag{19}$$

Combined with (18) and (19), (15) can be rearranged as

$$\begin{aligned}
 \frac{dV(t)}{dt} &\leq -\frac{(4\lambda_2(W))^{(1+\alpha)/2} V^{(1+\alpha)/2}(t)}{2} \\
 &\quad + \frac{\|\delta(t)\|^{-\alpha} \|e_z(t)\|^\alpha V^{(1+\alpha)/2}(t)}{2^{-(1+\alpha)/2}} \\
 &\leq \left( \frac{\|\delta(t)\|^{-\alpha} \|e_z(t)\|^\alpha}{2^{-(1+\alpha)/2}} - \frac{(4\lambda_2(W))^{(1+\alpha)/2}}{2} \right) V^{(1+\alpha)/2}
 \end{aligned}$$

According to the triggered condition (11), we obtain

$$\|e_{zi}(t)\|^\alpha \leq \frac{\mu(4\lambda_2(W))^{(1+\alpha)/2} \|\delta_i(t)\|^\alpha}{2^{-(1+\alpha)/2}} \tag{20}$$

Therefore,

$$\frac{dV(t)}{dt} \leq \frac{1}{2} (\mu - 1) (4\lambda_2(W))^{(1+\alpha)/2} V^{(1+\alpha)/2}(t) \tag{21}$$

By Lemma 2,  $V(t)$  reaches zero in a finite time. Besides, the settling time  $T$  meets

$$T \leq \frac{4V(0)^{(1-\alpha)/2}}{(1-\mu)(4\lambda_2(W))^{(1+\alpha)/2}(1-\alpha)}$$

Consequently, the MAS can achieve distributed finite-time bipartite consensus via the event-triggered condition (11).

To rule out Zeno behaviour, we further prove that there exists a positive lower bound at the inter-event times  $t_{k+1} - t_k$ . Furthermore, the time derivative of  $s_i(t) = \|(e_{zi}(t)\|/\|\delta_i(t)\|)$  is

$$\begin{aligned}
 &\frac{d(\|(e_{zi}(t)\|/\|\delta_i(t)\|))}{dt} \\
 &= -\frac{e_{zi}^T(t) \dot{\delta}_i(t)}{\|(e_{zi}(t)\| \|\delta_i(t)\|)} - \frac{\delta_i^T(t) \dot{\delta}_i(t) \|e_{zi}(t)\|}{\|\delta_i(t)\|^2 \|\delta_i(t)\|} \\
 &\leq \frac{\|e_{zi}(t)\| \|\dot{\delta}_i(t)\|}{\|(e_{zi}(t)\| \|\delta_i(t)\|)} + \frac{\|\dot{\delta}_i(t)\| \|e_{zi}(t)\|}{\|\delta_i(t)\| \|\delta_i(t)\|} \\
 &= \left( 1 + \frac{\|e_{zi}(t)\|}{\|\delta_i(t)\|} \right) \frac{\|\dot{\delta}_i(t)\|}{\|\delta_i(t)\|} \\
 &\leq \left( 1 + \frac{\|e_{zi}(t)\|}{\|\delta_i(t)\|} \right) \frac{\|L_D\| (\|e_{zi}(t)\| + \|\delta_i(t)\|)}{\|\delta_i(t)\|} \\
 &= \|L_D\| \left( 1 + \frac{\|e_{zi}(t)\|}{\|\delta_i(t)\|} \right)^2
 \end{aligned} \tag{22}$$

From (22), we derive

$$\dot{s}_i(t) \leq \|L_D\| (1 + s_i(t))^2 \tag{23}$$

Solving the difference equation (3), we get

$$\psi(\tau_i, 0) = \frac{\tau_i \|L_D\|}{1 - \tau_i \|L_D\|}$$

From (20), we have

$$\psi(\tau_i, 0) \leq \left( \frac{\mu(4\lambda_2(W))^{(1+\alpha)/2}}{2^{-(1+\alpha)/2}} \right)^{1/\alpha} \tag{24}$$

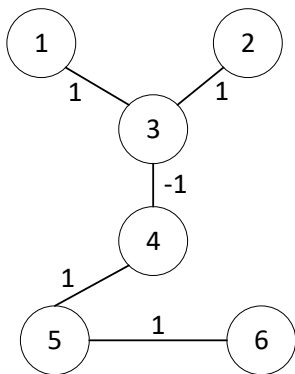


Fig. 1. Communication topology.

Thus, we can obtain the positive lower bound of interval between two event instants

$$\tau_{min} = \frac{\mu^{1/\alpha\theta}}{\|L_D\|(1 + \mu^{1/\alpha\theta})} \quad (25)$$

The proof is complete.

#### 4. SIMULATION EXAMPLE

Consider a MAS with six agents, the communication topology of which is shown as Figure 1.

From Figure 1, the corresponding Laplacian matrix  $L$  is given by

$$L = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

The initial values of the MAS are randomly generated as  $x(0) = [-2 \ 0 \ 3 \ 5 \ -3 \ 1]^T$ . To ensure the condition (11) holds in real-time control, the permitted range  $\alpha$  and  $\mu$  are set to 0.3 and 0.2, respectively.

Simulation results are shown in Figure 2. Figure 2 (a) shows the trajectories of event-triggered state  $x_i(t_k^i)$  for  $i = 1, 2, \dots, N$ , Figure 2 (b) reveals  $V(t)$  is exponentially to 0 when  $h = 0.01s$ . The results indicate that the finite-time bipartite consensus can be asymptotically achieved in  $T = 2s$  under the event-triggered control protocol. Since the controller of each agent updates at the triggered instant, the resource utilization is higher.

#### 5. CONCLUSION

In this paper, the finite-time bipartite consensus problem of MASs was studied via the event-triggered control method. Comparing with the continuous-time control methods, the proposed event-triggered control scheme was proven capable of reducing the frequency of control updates. Moreover, a link between the convergence time and the event-triggering condition was derived, which shows that the event threshold brings a tradeoff between the control updates cost and the time performance. Moreover, Zeno behavior can be ruled out. Simulation results show

that the designed controller has better disturbance rejection properties and can achieve bipartite consensus faster compared to an asymptotic controller.

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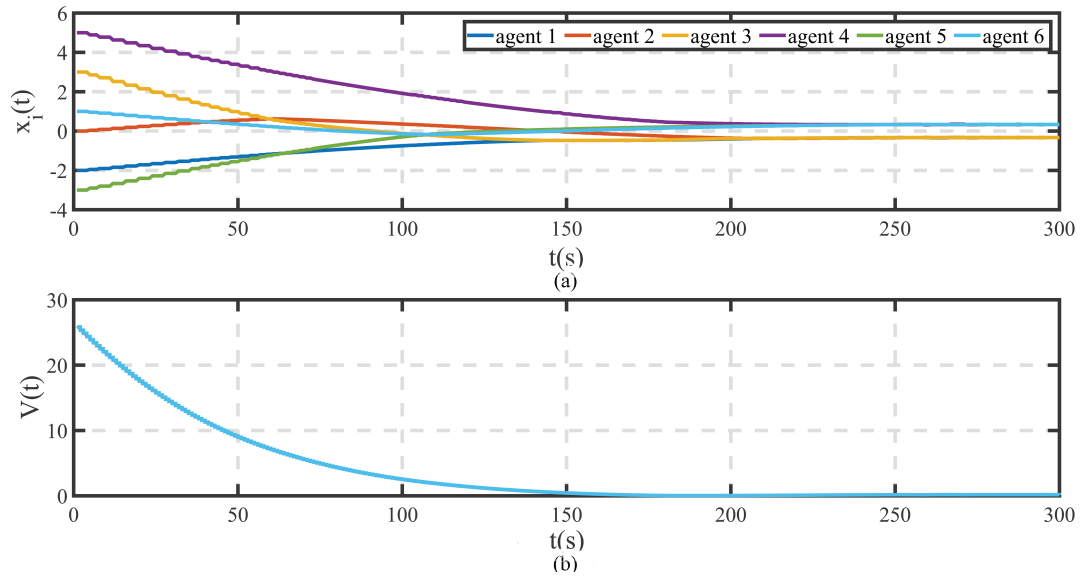


Fig. 2. Simulation results of bipartite consensus for MASs: (a) The evolution of  $x_i(t_k^i)$ . (b) The evolution of the Lyapunov function  $V(t)$  in Theorem 1 for  $i = 1, 2, \dots, N$  when  $h = 0.01s$ .

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