Scalable PI Voltage Stabilization in DC Microgrids

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Abstract: This paper addresses the problem of decentralized PI-based voltage stabilization in islanded DC microgrids with DC-DC buck converters. We propose a voltage control approach with a decentralized PI control structure. The proposed voltage control design is scalable and does not rely on the global model of the microgrids and parameters of the distribution lines. Moreover, it guarantees the asymptotic stability of the DC microgrid systems. The scalability of the design and asymptotic stability are ensured by the use of a separable quadratic-type Lyapunov function, with a fixed-structure Lyapunov matrix, as well as the LaSalle’s invariance principle. The effectiveness of the proposed voltage control strategy is evaluated through simulation case studies.

Keywords: DC microgrids, Voltage control, Lyapunov theory, LaSalle’s invariance principle, PI control.

1. INTRODUCTION

Nowadays DC microgrids have drawn continually increasing attention due to their main advantages including higher efficiency, reduced losses, and easy and natural integration with renewable energy sources and energy storage resources (Dragicevic et al. (May 2016)). In the DC microgrid systems, the renewable energy sources with a DC-type output are normally interfaced to the microgrid through DC-DC power-electronic converters.

As the applicability of the DC microgrid systems is widely increasing, the design of appropriate control strategies which ensure microgrid’s stability and efficiency plays an important role. One of the main challenges in DC microgrids with multiple distribution generation (DG) units is about the design of scalable voltage stabilization of DG units. In the scalable design, the main objective is that the stable operation of microgrids and the voltage control design are not influenced by uncertainty sources affected the architecture of the microgrid systems (Nasirian et al. (Dec. 2014)). The examples of the uncertainty sources are plug-and-play operation of DG units, structural/electrical upgrades, and faults/failure in the distribution lines connecting different DG units.

The most common control approaches for the DC microgrids are based on droop control (e.g. J. M. Guerrero et al. (Jan. 2011); Guerrero et al. (Apr. 2013); Meng et al. (Sept. 2017)) consisting of three control levels named as primary, secondary, and tertiary control. The primary control level aims to stabilize the voltage of DC microgrids and facilitate an accurate power sharing. The second level with slower time scale compensates for the deviations in the voltage in the steady state (Guerrero et al. (Apr. 2013)). The tertiary level assigns the microgrid voltage and is responsible for an optimal operation and power management in DC microgrids (Guerrero et al. (Apr. 2013); Nasirian et al. (Dec. 2014)). From a control point of view, the primary level has a decentralized proportional (P) control framework whereas the secondary level has centralized/distributed integral (I) control structure which relies on communication networks and information exchange among DG units’ neighbors (Guerrero et al. (Apr. 2013)). In spite of the potential advantages of the droop-based control strategy, it comes at the expense of instability issues, slow dynamic responses, and dependency on the line impedance (Yazdanian and Mehrizi-Sani (Nov. 2014)).

Another category of voltage control strategies for DC microgrids is the so-called non-droop-based methods which combine primary and secondary control levels. Non-droop-based approaches are mainly based on decentralized advanced model-based control techniques (Sadabadi et al. (Nov. 2018)). Recently, several scalable voltage control approaches of DC microgrids have been proposed, e.g. Sadabadi et al. (Nov. 2018); Tucci et al. (Nov. 2016); M. Tucci et al. (May 2018); Sadabadi and Shafiee (Jan. 2020); Cucuzzella et al. (Jul. 2019); M. Cucuzzella et al. (Jul. 2019); Cucuzzella et al. (Mar. 2019). The proposed techniques which are mainly based on the Lyapunov theorem (e.g. Sadabadi et al. (Nov. 2018); Tucci et al. (Nov. 2016); M. Tucci et al. (May 2018)) and/or passivity theorem (e.g. Cucuzzella et al. (Jul. 2019); M. Cucuzzella et al. (Jul. 2019); Cucuzzella et al. (Mar. 2019)) provide a guarantee of voltage stability in the DC microgrids. However, they require information about the power lines connected to each distributed generation unit (Sadabadi et al. (Nov. 2018); Tucci et al. (Nov. 2016); Sadabadi and Shafiee (Jan. 2020); Cucuzzella et al. (Jul. 2019); M. Cucuzzella et al. (Jul. 2019); Cucuzzella et al. (Mar. 2019)). Furthermore, in these approaches, the proposed voltage control law, expressed in terms of a state-feedback control and/or some nonlinear control, requires solving optimization problems.

Considering the advantages and drawbacks of both droop-based and non-droop-based control strategies, we propose a voltage control framework which replaces the droop mechanism with a decentralized PI controller. The proposed control strategy enables us to take advantages of both droop-based and non-droop-based approaches. Similar to the droop control, the proposed approach is based on a PI control; however, it does not depend
on line parameters. Moreover, similar to the non-droop-base mechanism, the proposed method ensures the robust stability and reliable operation of DC microgrids. Explicit equalities on the proportional and integral terms of the stabilizing voltage controllers are provided. Therefore, the design procedure does not rely on a solution of optimization problems. Moreover, the local voltage control design is independent of the parameters of distribution lines. The proposed control stage guarantees the asymptotic stability of the DC microgrid system. The scalability of the design and asymptotic stability are ensured by utilizing a Lyapunov-based framework with a separable quadratic-type structured Lyapunov function as well as LaSalle’s invariance principle. The efficiency of the proposed voltage control strategy is evaluated via simulation case studies carried out in MATLAB/Simulink.

The rest of the paper is organized as follows. Section 2 presents a dynamical model of DC microgrids with DC-DC buck converters. The stabilizing PI voltage control framework and the proposed control design strategy are given in Section 3 and Section 4, respectively. Section 5 is devoted to simulation case studies. Section 6 concludes the paper.

The notation used in this paper is standard. In particular, matrix 0 is the zero matrix of appropriate dimensions. The symbols $A^T$ and $A[i,j]$ denote the transpose of matrix $A$ and the $ij$ element of matrix $A$. For symmetric matrices, $P \geq 0$ ($P < 0$) and $P \geq 0$ ($P \leq 0$) respectively indicate the positive-definiteness (negative-definiteness) and the positive semi-definiteness (negative semi-definiteness).

2. DYNAMICAL MODEL OF DC MICROGRIDS

We consider an islanded DC microgrid with $N$ DG units. The energy source of each DG is represented by a DC voltage source that supplies a local load through a DC-DC buck converter at the Point of Common Coupling (PCC). A schematic diagram of a buck-based DG is shown in Fig. 1. Different DG units are connected via distribution lines. DC microgrids are modeled as interconnected systems, where each DG forms a subsystem and the distribution lines represent the interactions among the subsystems. The symbols used in Fig. 1 are described in Table 1.

Graph Representation of DC Microgrids: The DC microgrid forms a network represented by a directed graph $G = (V', \delta')$, where $V'$ and $\delta'$ are the sets of vertices and edges, respectively. Each element in the vertex set $V' = \{1, \ldots, N\}$ represents a DG and each element in the edge set $\delta' = \{1, \ldots, m\}$ represents the distribution line between the corresponding DG units. The microgrid topology is described by the incidence matrix $D \in \mathbb{R}^{N \times m}$ of the directed graph $G$. The incidence matrix $D = [D_{ik}]$ defines the direction of the distribution line current. If the current $I_k$ leaves DG $i$ and enters DG $j$, $D_{ik} = 1$ and $D_{jk} = -1$ and if it enters the node $i$ and leaves DG $j$, $D_{ik} = -1$ and $D_{jk} = 1$. If the line $k$ does not connect DG $i$ and DG $j$, then $D_{ik} = 0$ and $D_{jk} = 0$. Therefore, $D_{ik}$ is formulated as follows:

$$D_{ik} = \begin{cases} 1 & \text{if line } k \text{ leaves DG } i, \\ -1 & \text{if line } k \text{ enters DG } i, \\ 0 & \text{otherwise}. \end{cases}$$

for $i = 1, \ldots, N$ and $k = 1, \ldots, m$.

Model of a DG with a Buck Converter: Using Kirchhoff’s current and voltage laws, each DG unit is mathematically modeled as follows:

$$\dot{x}_i = A_i x_i + B_i u_i + B_{Li} I_i + B_{Lj} \sum_{k \in E_i} D_{ik} I_k$$

$$y_i = C_i x_i$$

(6)

where $x_i = [V_i, I_i]^T$ is the state, $u_i = V_{dc,i}$ is the input, $d_i$ is the duty cycle of the DC-DC converter, $y_i = V_i$ is the output, and $I_i$ is the load current. The term $B_{Li} \sum_{k \in E_i} D_{ik} I_k$ describes the interaction between DG $i$ and other DG units through the distribution lines $k$ with current $I_k$, $k \in E_i$. The set $E_i \subset \delta$ describes the set of the distribution lines connected to DG $i$.

The state space matrices are given as follows:

$$A_i = \begin{bmatrix} 0 & -\frac{1}{L_i} & -\frac{1}{L_i} \\ -\frac{1}{C_i} & 0 & -\frac{1}{L_i} \\ 0 & -\frac{1}{C_i} & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 \\ \frac{1}{L_i} \\ 0 \end{bmatrix}$$

$$C_i = \begin{bmatrix} 1 & 0 \\ \frac{1}{L_i} & 1 \end{bmatrix}$$

Dynamics of Distribution Lines: It is assumed that there are $m$ distribution lines in the DC microgrid system. The resistive-inductive distribution line $k$ with parameters $(R_k, L_k)$ and the line current $I_k$ from DG $i$ to DG $j$, is modeled as follows:

$$I_k = -\frac{R_k}{L_k} I_k + \frac{1}{L_k} (D_{ik} V_i + D_{jk} V_j)$$

(4)

Load Model: It is assumed that the loads are connected to the DG terminals at PCCs. The load current $I_{L_i}$ in (2) is modeled as follows:

$$I_{L_i} = G_i V_i + I_{L_i}$$

(5)

where $G_i$ is the load conductance and $I_{L_i}$ is a constant current load of DG $i$.

DC Microgrid Model: The overall microgrid system with the dynamics in (2) in presence of the loads in (5) can be presented as follows:

$$\dot{x}_i = A_i x_i + B_i u_i + B_{Li} I_i + B_{Lj} \sum_{k \in E_i} D_{ik} I_k$$

$$y_i = C_i x_i$$

(6)
where

\[
\begin{bmatrix}
-\frac{G_i}{C_{ti}} & \frac{1}{C_{ti}} \\
-\frac{1}{L_{ti}} & -\frac{R_{ti}}{L_{ti}}
\end{bmatrix}
\]

for \(i = 1, \ldots, N\) and \(k = 1, \ldots, m\).

3. PI-BASED VOLTAGE CONTROL FRAMEWORK

In this section, we formulate the control objectives aiming at the voltage stabilization of a DC microgrid system and voltage tracking by an appropriate design of PI controllers. The voltage controllers ensure that the voltage signals at PCCs track reference voltages \(V_{\text{ref},i}\) provided by a higher-level controller.

Control Structure: The control input of DG \(i, i = 1, \ldots, N\) can be structured as follows:

\[
u_i = K_{P_i}(V_i - V_{\text{ref},i}) + K_{I_i} \int (-V_i + V_{\text{ref},i})dt\]

where \(K_{P_i}\) and \(K_{I_i}\) respectively are the proportional and the integral parameters of the PI controllers. The term \(\int (-V_i + V_{\text{ref},i})dt\) is defined as a new variable \(v_i\) with the following dynamics:

\[
v_i = -V_i + V_{\text{ref},i}
\]

Change of Coordinates: For a given \(V_{\text{ref},i}\) and \(I_{L,i}\), the unique equilibrium point of (6)-(8) is given by:

\[
\begin{align*}
\bar{V}_i &= V_{\text{ref},i} \\
\bar{I}_k &= \frac{1}{R_k} \sum_{i=1}^{N} \partial_{k} V_{\text{ref},i} \\
\bar{I}_i &= V_{\text{ref},i} G_i + I_{L_i} + \sum_{k \in E_i} \partial_{k} I_k \\
\bar{u}_i &= V_{\text{ref},i} + R_i \bar{I}_i \\
\bar{v}_i &= \bar{u}_i \frac{1}{K_i}, \quad \bar{y}_i = V_{\text{ref},i}
\end{align*}
\]

By change of the coordinates as \(\bar{u}_i = u_i - \bar{u}_i, \quad \bar{y}_i = y_i - \bar{y}_i, \quad \bar{V}_i = V_i - \bar{V}_i, \quad \bar{I}_k = I_k - \bar{I}_k, \quad \bar{v}_i = v_i - \bar{v}_i, \quad \text{and} \quad \bar{I}_k = I_k - \bar{I}_k\), the system dynamics in (6) and (9) can be rewritten as follows:

\[
\begin{align*}
\dot{\bar{x}}_i &= \bar{A}_i \bar{x}_i + \bar{B}_i \bar{u}_i + \bar{B}_i \sum_{k \in E_i} \partial_{k} \bar{I}_k \\
\dot{\bar{I}}_k &= -\frac{R_k}{L_k} \bar{I}_k + \frac{1}{L_k} \sum_{i=1}^{N} \partial_{i} \bar{V}_i \\
\end{align*}
\]

where \(\bar{\bar{x}}_i = [\bar{V}_i \quad \bar{I}_i \quad \bar{v}_i]^T\) and

\[
\begin{align*}
\bar{A}_i &= \begin{bmatrix} A_i & 0 \\ -C_i & 0 \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix} \\
\bar{B}_i &= \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \bar{C}_i = \begin{bmatrix} C_i \\ 0 \end{bmatrix}
\end{align*}
\]

Closed-loop System of DG i: The closed-loop system of DG \(i\) with the PI-based control law in (8) is presented as follows:

\[
\begin{align*}
\dot{\bar{x}}_i &= \bar{A}_i \bar{x}_i + \bar{B}_i \bar{u}_i + \bar{B}_i \sum_{k \in E_i} \partial_{k} \bar{I}_k \\
\dot{\bar{I}}_k &= -\frac{R_k}{L_k} \bar{I}_k + \frac{1}{L_k} \sum_{i=1}^{N} \partial_{i} \bar{V}_i \\
\bar{u}_i &= K_{P_i} \bar{V}_i + K_{I_i} \bar{I}_i \\
\bar{y}_i &= \bar{C}_i \bar{x}_i 
\end{align*}
\]

The main objective is to design the proportional and integral gains \(K_{P_i}\) and \(K_{I_i}\) which guarantee the stability of the DC microgrid system.

4. PROPOSED CONTROL DESIGN APPROACH

In this section, a solution for the PI-based voltage stabilization of the DC microgrid system, based on the Lyapunov theorem and the LaSalle’s invariance principle (Khalil (2006)), is proposed.

4.1 VOLTAGE CONTROL DESIGN STRATEGY

Consider the following separable quadratic type Lyapunov function for the microgrid system in (13) consisting of \(N\) DG units and \(m\) distribution lines:

\[
\dot{V}(\bar{x}, \bar{I}) = V_{\text{line}}(\bar{I}) + \sum_{i=1}^{N} V_i(\bar{x}_i)
\]

where \(\bar{x} = [\bar{x}_1^T \ldots \bar{x}_N^T]^T, \bar{I} = [I_1 \ldots I_m]^T\), and

\[
V_{\text{line}}(\bar{I}) = \sum_{k=1}^{m} a_k \bar{I}_k
\]

\[
V_i(\bar{x}_i) = \bar{x}_i^T (A_i^T \bar{P}_i + P_i \bar{A}_i) \bar{x}_i + \bar{x}_i^T \bar{B}_i \bar{u}_i + \bar{u}_i^T \bar{B}_i^T P_i \bar{x}_i \\
+ \sum_{k \in E_i} \bar{I}_k \partial_{k} \bar{B}_i^T P_i \bar{x}_i + \bar{x}_i^T \bar{B}_i \bar{I}_k \sum_{k \in E_i} \partial_{k} \bar{I}_k
\]

By replacing \(\bar{u}_i\) with \(\bar{u}_i - \bar{K}_i \bar{x}_i + \bar{K}_i \bar{x}_i, V_i(\bar{x}_i)\) is expressed as follows:

\[
V_i(\bar{x}_i) = \bar{x}_i^T ((A_i + \bar{K}_i \bar{x}_i)^T \bar{P}_i + P_i (A_i + \bar{K}_i \bar{x}_i)) \bar{x}_i \\
+ (-\bar{K}_i \bar{x}_i + \bar{u}_i)^T \bar{B}_i^T P_i \bar{x}_i + \bar{x}_i^T \bar{B}_i \bar{I}_k (-\bar{K}_i \bar{x}_i + \bar{u}_i)
\]

Therefore,

\[
\dot{V}(\bar{x}, \bar{I}) = -2\alpha \sum_{k=1}^{m} R_k \bar{I}_k + \alpha (V^T \bar{D} \bar{I} + \bar{I}^T \bar{D} \bar{V}) + \sum_{i=1}^{N} \bar{x}_i^T \bar{Q}_i \bar{x}_i \\
+ \sum_{i=1}^{N} ((-\bar{K}_i \bar{x}_i + \bar{u}_i)^T \bar{B}_i^T P_i \bar{x}_i + \bar{x}_i^T \bar{B}_i \bar{I}_k (-\bar{K}_i \bar{x}_i + \bar{u}_i))
\]

\[
\bar{x}^T \bar{Q}_i (\bar{A}_i + \bar{B}_i \bar{x}_i)^T \bar{P}_i + P_i (\bar{A}_i + \bar{B}_i \bar{x}_i)
\]

In the next theorem, we show that \(\dot{V}(\bar{x}, \bar{I}) \leq 0\) for all \((\bar{x}, \bar{I})\) by an appropriate choice of \(\bar{u}_i, i = 1, \ldots, N\).
Theorem 1. Consider the closed-loop microgrid system in (13). The following control law makes \( \dot{V}(\tilde{x}, \tilde{I}) \leq 0 \) for all \( (\tilde{x}, \tilde{I}) \).

where
\[
\tilde{K}_i = \tilde{K}_i - \beta_i \tilde{P}_i^T P_i
\]
(22)

where \( \beta_i \) is a real positive scalar and \( P_i > 0 \) has the following structure:
\[
P_i = \alpha \begin{bmatrix}
C_i & 0 & 0 \\
0 & \rho_2 & -\rho_1 \rho_2 \\
-\rho_1 \rho_2 & \rho_1 (\rho_1 \rho_2 + 1)
\end{bmatrix}
\]
(23)

and \( \rho_1 > 0 \) and \( 0 < \rho_2 < \alpha^{-1} \beta_i^{-1} R_i L_i \).

Moreover, the gain matrix \( \tilde{K}_i = [k_{1,i}, k_{2,i}, k_{3,i}] \) is obtained as follows:
\[
k_{1,i} = 1 - L_i (\rho_2^{-1} + \rho_1)
\]
\[
k_{2,i} = \frac{\alpha \beta_i}{L_i} \rho_2
\]
\[
k_{3,i} = \rho_1 (R_i - k_{2,i})
\]
(24)

\[\text{Proof 1.} \] The proof of Theorem 1 is as follows:

**Neutral interactions \( \tilde{I}_i \).** By employing \( P_i \) in (23), it can be shown that
\[
\sum_{i=1}^{N} \left( \tilde{x}_i^T P_i \tilde{B}_i \sum_{k=1}^{\infty} \tilde{G}_{dk} \tilde{I}_k + \tilde{x}_i^T P_i \tilde{B}_i \tilde{P}_i \tilde{x}_i \right) + \alpha \left( \tilde{V}^T \tilde{G} + \tilde{F}^T \tilde{G}^T \tilde{V} \right) = 0
\]
(25)

The above equation indicates that the interactions within the DC microgrid systems are neutral due to the structured Lyapunov matrix \( P_i \) in (23). As a result, (19) simplifies to:
\[
\dot{V}(\tilde{x}) = -2 \alpha \sum_{i=1}^{N} R_i \tilde{x}_i^2 + \sum_{i=1}^{N} \tilde{x}_i^T P_i \tilde{x}_i - 2 \alpha \sum_{i=1}^{N} \tilde{G}_{di} \tilde{I}_k \tilde{B}_i \tilde{P}_i \tilde{x}_i + \alpha \left( \tilde{V}^T \tilde{G} + \tilde{F}^T \tilde{G}^T \tilde{V} \right)
\]
(26)

The first term in (26), i.e. \(-2 \alpha \sum_{i=1}^{N} R_i \tilde{x}_i^2\), is non-positive. In the next step, we show that \( Q_i \leq 0 \) for \( i = 1, \ldots, N \).

\( \tilde{Q}_i \leq 0 \). The negative semi-definiteness of \( Q_i \) is equivalent to the negative semi-definiteness of matrix \( \tilde{Q}_i = P_i^{-1} Q_i P_i^{-1} \). We define new matrices \( Y_i = P_i^{-1} \) and \( g_i = \tilde{K}_i P_i^{-1} \). As a result, \( \tilde{Q}_i \) is presented as follows:
\[
\tilde{Q}_i = Y_i \tilde{A}_i^T + \tilde{A}_i Y_i + \tilde{B}_i g_i + g_i^T \tilde{B}_i^T
\]
(27)

Matrices \( Y_i > 0 \) and \( g_i \) are parametrized as follows:
\[
Y_i = \begin{bmatrix}
\alpha^{-1} C_i^{-1} & 0 & 0 \\
0 & y_{22} & y_{23} \\
0 & y_{23} & y_{33}
\end{bmatrix}
\]
(28)

\[
g_i = \begin{bmatrix}
g_{11} & g_{22} & g_{23} \\
\frac{k_{1,i}}{\alpha} & k_{2,i} & k_{3,i} \\
k_{2,i} y_{22} + k_{3,i} y_{23} + k_{3,i} y_{33}
\end{bmatrix}
\]
(29)

By replacing \( Y_i \) and \( G_i \) in (27), \( \tilde{Q}_i \) is rewritten as follows:
\[
\tilde{Q}_i = \begin{bmatrix}
\frac{g_{11}}{\alpha} & \tilde{Q}_{12} & \tilde{Q}_{13} \\
\tilde{Q}_{12} & \tilde{Q}_{22} & \tilde{Q}_{23} \\
\tilde{Q}_{13} & \tilde{Q}_{23} & 0
\end{bmatrix}
\]
(30)

where
\[
\tilde{Q}_{12} = \frac{1}{C_i} y_{22} - \frac{1}{\alpha C_i L_i} + \frac{1}{L_i} g_{11},
\]
\[
\tilde{Q}_{13} = \frac{1}{C_i} y_{23} - \frac{1}{\alpha C_i L_i},
\]
\[
\tilde{Q}_{22} = -\frac{R_i}{L_i} y_{22} - \frac{1}{L_i} g_{22},
\]
(31)

\[
\tilde{Q}_{23} = -\frac{R_i}{L_i} y_{23} + \frac{1}{L_i} g_{33},
\]

The non-positiveness of \( \tilde{Q}_i \leq 0 \) implies that the third column and row of \( \tilde{Q} \) must be equal to zero. We also consider \( \tilde{Q}_{12} = 0 \).

As a result,
\[
y_{22} = \frac{1}{\alpha} R_i
\]
(32)

\[
y_{33} = \frac{1}{\alpha} R_i (R_i - k_{2,i})
\]
\[
y_{23} = \frac{1}{\alpha} R_i
\]
(33)

As \( Y_i = P_i^{-1} \), it can be easily shown that \(-\alpha y_{33} y_{23} + \frac{1}{\alpha} = -\alpha y_{33} y_{23} \), where \( \rho_2 = P_i[2, 2] \). Due to the positive definiteness of \( P_i, \rho_2 \) is a positive scalar. Therefore, \( Y_i \) and \( \tilde{Q}_i \) are presented as follows:
\[
\tilde{Q}_i = \begin{bmatrix}
\alpha^{-1} C_i^{-1} & 0 & 0 \\
0 & \alpha^{-2} y_{33}^{-1} + \rho_2^{-1} \alpha^{-1} & 0 \\
0 & 0 & \alpha^{-1}
\end{bmatrix}
\]
(34)

If we assume that \( \tilde{k}_{3,i} > 0, \tilde{Q}_i \leq 0 \); therefore, \( Q_i \leq 0 \). Therefore, if \( Q_i \leq 0 \), \( \tilde{Q}_i \leq 0 \); therefore, \( Q_i \leq 0 \).

**PI control law \( u_i \).** We consider the control law as \( \tilde{u}_i = \tilde{K}_i \tilde{x}_i - \beta_i \tilde{B}_i^T P_i \tilde{x}_i \) with a real positive scalar \( \beta_i \). Therefore, \( \dot{V}(\tilde{x}) \) is expressed as follows:
\[
\dot{V}(\tilde{x}, \tilde{I}) = -2 \alpha \sum_{i=1}^{N} R_i \tilde{x}_i^2 + \sum_{i=1}^{N} \tilde{x}_i^T Q_i \tilde{x}_i - 2 \sum_{i=1}^{N} \tilde{G}_{di} \tilde{I}_k \tilde{B}_i \tilde{P}_i \tilde{x}_i + \alpha \left( \tilde{V}^T \tilde{G} + \tilde{F}^T \tilde{G}^T \tilde{V} \right)
\]
(35)

Since \(-\beta_i \tilde{x}_i^T P_i \tilde{B}_i^T P_i \tilde{x}_i \leq 0, \dot{V}(\tilde{x}, \tilde{I}) \leq 0 \) for all \( (\tilde{x}, \tilde{I}) \). Due to the structure of \( \tilde{u}_i = \tilde{K}_i \tilde{V}_i + \tilde{K}_i \tilde{v}_i \), the term \( \tilde{k}_{2,i} - \tilde{B}_i \rho_2 \), \( \tilde{k}_{2,i} \) must be equal to zero. Therefore, there is the following equality constraint on \( \tilde{k}_{2,i} \), and \( \rho_2 \):
\[
\tilde{k}_{2,i} = \frac{\beta_i}{L_i} \rho_2
\]
(36)

**Constraints on \( \tilde{K}_i \).** From (32), (33), and (36), the gain matrix \( \tilde{K}_i \) is subject to the following constraints:

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\[ y_{22i} > 0 \Rightarrow \tilde{k}_{1,i} < 1 \]
\[ y_{33i} > 0 \Rightarrow \tilde{k}_{2,i} < R_t \]
\[ \tilde{k}_{2,i} = \frac{\beta_i}{L_i} \rho_{2i} \]
\[ \tilde{Q}_{22i} \leq 0 \Rightarrow \tilde{k}_{3,i} > 0 \]
(37)

Structure of the Lyapunov matrix \( P_i \). Since \( P_i = Y_i^{-1} \), the matrix \( P_i \) is structured as (23), where \( \rho_{1i} = \frac{1}{\alpha y_{33i}} > 0 \) and \( \rho_{2i} > 0 \). Moreover, based on (36) and the inequality constraint on \( \tilde{k}_{2,i} \) in (37), the choice of \( \rho_{2i} \) must satisfy the following constraint:
\[ 0 < \rho_{2i} < \alpha^{-1} \beta_i^{-1} R_t L_i \]
(38)

Explicit equalities on gain matrix \( \tilde{K}_i \). From (32), (36), and the relationship between \( Y_i \) and \( P_i \), the gain matrix \( \tilde{K}_i \) is obtained as (24).

Proportional and integral gains (\( K_P, K_I \)). The proportional and integral terms of the PI controllers are obtained as follows:
\[ K_P = \tilde{k}_{i,1} = 1 - L_i \left( \rho_{2i}^{-1} + \rho_{1i} \right) \]
\[ K_I = \tilde{k}_{i,3} + \frac{\alpha \beta_i}{\tilde{T}_i} \rho_{1i} \rho_{2i} \]
(39)

for \( i = 1, \ldots, N \).

Theorem 2. The PI control law given in (21) and (22) with the coefficients in (39) guarantees the stability of the closed-loop microgrid system.

Proof 2. In Theorem 1, it is shown that \( V(\tilde{x}, \tilde{l}) \leq 0 \) for all \( (\tilde{x}, \tilde{l}) \). We use the LaSalle’s invariance principle to show that the closed-loop microgrid system with control law given in (21) is asymptotically stable. To this end, we need to show that the only solution of \( V(\tilde{x}, \tilde{l}) = 0 \) is \( (\tilde{x}, \tilde{l}) = 0 \), \( \forall t > 0 \). To see this, note that \( V(\tilde{x}, \tilde{l}) = 0 \) implies that
\[ \dot{\tilde{k}}_i = 0 \]
\[ \tilde{x}_i^T \tilde{Q}_i \tilde{x}_i = 0 \]
\[ \tilde{B}_i^T P_i \tilde{x}_i = 0 \]
(40)

for \( k = 1, \ldots, m \) and \( i = 1, \ldots, N \). It is required to show that the only state trajectory of the system which satisfies all the constraints in (40) is origin. Let’s compute the set \( \chi \) as follows:
\[ \chi = \{ \tilde{l}_k = 0 \} \cap \{ \tilde{x}_i : \tilde{x}_i^T \tilde{Q}_i \tilde{x}_i = 0 \} \cap \{ \tilde{x}_i : \tilde{x}_i^T \tilde{P}_i \tilde{B}_i \tilde{P}_i \tilde{x}_i = 0 \} \]
(41)

From the set \( \chi_1 \), it is concluded that
\[ \tilde{l}_k = 0 \Rightarrow \tilde{l}_k = 0 \Rightarrow \tilde{V}_i = \tilde{V}_j; \quad i, j \in \gamma \]
(42)

Based on the set \( \chi_2 \), we should find a state trajectory \( \tilde{x}_i^* \) which maximizes the term \( f(\tilde{x}_i^*) = \tilde{x}_i^T \tilde{Q}_i \tilde{x}_i \). Note that the maximum value of \( f(\tilde{x}_i^*) \) is zero. Therefore,
\[ \frac{df(\tilde{x}_i^*)}{d\tilde{x}_i} \bigg|_{\tilde{x}_i^*} = 2 \tilde{Q}_i \tilde{x}_i^* = 0 \]
(43)

By premultiplying the above equation by \( Y_i = P_i^{-1} \) and defining a new variable \( \tilde{y}_i^* = Y_i^{-1} \tilde{x}_i^* \), we have
\[ \tilde{Q} \tilde{y}_i^* = 0 \]
(44)

5. SIMULATION RESULTS

In order to evaluate the proposed voltage control scheme, simulation studies are performed on an islanded DC microgrid with DC-DC buck converters, borrowed from Sadabad et al. (Nov. 2018). The microgrid consists of \( N = 6 \) DG units and \( m = 7 \) distribution lines as graphically shown in Fig. 2. The parameters of each DG and the distribution lines are given in Sadabad et al. (Nov. 2018).

The stabilizing PI voltage controller of each DG is designed by using the proposed algorithm in Subsection 4.2. The voltage controllers are applied to the DC microgrid in Fig. 2 implemented in MATLAB/Simulink Electrical.
The performance and transient behavior of DG1 in voltage tracking scenario is assessed in this study. The voltage reference of DG1 is initially set at 52V. Then, the reference is stepped down to 47V at $t = 0.5 s$ and stepped up to 52V at $t = 0.9 s$. Fig. 3 shows the dynamic responses of DG1. The results indicate that the proposed voltage control technique is able to regulate the voltage at PCCs with zero steady state error and small transient time.

The voltage signals at PCC2, PCC3, and PCC6 as well as the control inputs of DG2, DG3, and DG6 are depicted in Fig. 4. As one can observe from Fig. 4, due to the line-independent voltage control design, the voltage changes at PCC1 have negligible effects on the voltage signals of the neighbors of DG1.

6. CONCLUSION

This paper studies the scalable voltage stabilization for DC microgrids with DC-DC buck converters. Using a Lyapunov-based framework together with the LaSalle’s invariance principle, a decentralized PI-based voltage controller is proposed, which relies on only the local electrical parameters of each DG unit. A set of parameters of stabilizing PI-based voltage controllers is presented. The control design strategy is scalable and allows the plug-and-play operation of DG units. The proposed voltage control scheme guarantees the asymptotic stability of the DC microgrid systems. Future works include the extension of our results to boost converters and DC microgrids with constant power loads.

REFERENCES


