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A New Control Scheme for Time-Delay Compensation for Structural Vibration ⋆

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Abstract: This paper addresses the vibration control of seismic-excited building structures in the presence of input time-delay. The control scheme is based on a prediction approach for input delay compensation and $\mathcal{H}_\infty$ theory. The prediction scheme, which relies on state observers, is tuned by means of the optimization of the smoothed spectral abscissa, that is a suitable robust stability measure since it provides a trade-off between the optimization of the spectral abscissa and the $\mathcal{H}_2$ norm of the system. The effectiveness of the proposed control scheme is illustrated with simulation results of a reduced scale two-storey building structure.

Keywords: Time-delay system; building structure; observer-predictor; smoothed spectral abscissa.

1. INTRODUCTION

During the last three decades, the use of control technologies aiming at the reduction of earthquake-induced vibrations of building structures, has received considerable attention. Several control techniques, such as $\mathcal{H}_2$ control (Yang et al., 2003), $\mathcal{H}_\infty$ control (Wang et al., 2009), neural network based control (Madan, 2005), sliding mode control (Guclu, 2006), fuzzy logic control (Guclu and Yazici, 2008), PD-PID control (Thenozhi and Yu, 2014), among others, have been proposed to attenuate structural vibrations in order to avoid structural damage. In spite of significant successes achieved in practical applications of the techniques, there are still some problems and challenges to solve. For instance, time-delay is a less studied issue of structural control, that appears in the entire control process due to the sum of different tasks, such as online measuring response, filtering, calculating control forces, transmitting of the data to actuators, etc. The existence of delay mainly affects the control performance and it may cause instability of the feedback loop.

This issue has been addressed by different approaches within the structural control community. A pioneering work that includes effects of time-delay can be found in Chung et al. (1988). The authors report satisfactory results when applying linear optimal feedback control algorithm experimentally considering small delays. Following this line, an optimal control method for seismic-excited linear structures with input delay is presented in Guoping and Jinzhi (2002). The bulk of research by Agrawal and Yang (1997) present a state-of-the-art literature survey on the effect of time-delay and also describe a numerical approach to determine the critical time-delay of multiple degree of freedom systems.

Other recent works have presented techniques for feedback control under input time-delay. For example, in Peng et al. (2018) model predictive control is used for vibration control in a large scale structure with multi-input time-delays. Design and implementation of a modified sliding mode controller for robust control of an Active Mass Damper system in the presence of model uncertainties and input time-delay is presented in Soleymani et al. (2018). In the same research direction, a compensation controller is achieved through Takagi-Sugeno fuzzy neural network method (Li et al., 2019), whereas vibration control are based on LQR. A modified two degree of freedom Smith control structure is described in Li-Ye and Qi-Bing (2020) to analyze the system robustness due to uncertainty plus time-delay. Also a time-delay compensation for vibrating system using Smith predictor approach is proposed in Araújo and Santos (2018).

In this paper, we propose a novel method for compensation of the delay in the problem of vibration control of building structures subjected to seismic excitation. It is based on the observer-predictor approach presented in Najafi et al. (2013), and the smoothed spectral abscissa, a new stability measure for time-delay systems, recently introduced in Gomez and Michiels (2019). More precisely, we tune the predictor scheme by optimizing the smoothed spectral abscissa. The predictor scheme is combined with classical $\mathcal{H}_\infty$ control in order to stabilize the closed-loop system while disturbances are attenuated. It is worth mentioning that the proposed methodology can be applied for a general class of linear time invariant systems in the presence of disturbances and input time-delay.

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The remainder of the paper is organized as follows. Section 2 describes the dynamic model of the building structure under study. The proposed method is then detailed in Section 3, and it is exemplified in Section 4. Finally, concluding remarks are provided in Section 5.

Notation: Throughout the paper the superscript 'T' stands for matrix transposition. The Euclidean norm is denoted by $||\cdot||$, the notation $Q > (\geq)\ 0$ means that Q is positive definite (positive semi-definite). The vectorization of a matrix $A \in \mathbb{R}^{n\times p}$ is denoted by $vec(A) \in \mathbb{R}^{np\times 1}$, obtained by stacking up the columns of A. The identity and null matrices of appropriate dimensions are represented by I and 0, respectively.

2. DYNAMIC MODEL OF A BUILDING STRUCTURE

Consider the n-degrees of freedom building structure depicted in Fig. 1, subjected to horizontal earthquake excitation with time-delay in the control input.

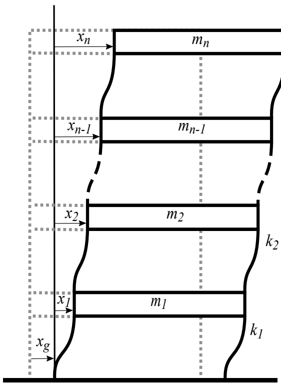

Fig. 1. Multi storey structure with n-degrees of freedom.

The building structure is governed by the following equation of motion (Agrawal and Yang, 1997)

$$M\ddot{x}(t) + R\dot{x}(t) + Kx(t) = -\Gamma u(t-\tau) - Md\ddot{x}_g(t), \quad (1)$$

with

$$\begin{aligned} x(t) &= [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^{n\times 1},\\ \dot{x}(t) &= [\dot{x}_1(t), \dot{x}_2(t), \dots, \dot{x}_n(t)]^T \in \mathbb{R}^{n\times 1},\\ \ddot{x}(t) &= [\ddot{x}_1(t), \ddot{x}_2(t), \dots, \ddot{x}_n(t)]^T \in \mathbb{R}^{n\times 1},\\ d &= [1, 1, \dots, 1]^T \in \mathbb{R}^{n\times 1}, \end{aligned}$$

where $x_i(t),\ \dot{x}_i(t),\ \ddot{x}_i(t),\ i = 1, 2, \dots, n$, are respectively the displacement, velocity and acceleration of each floor relative to the ground. Signal $\ddot{x}_g(t) \in \mathbb{R}$ is the earthquake ground acceleration, that is distributed by the vector d; $u(t) \in \mathbb{R}^{n\times 1}$ is the control signal, $\tau \in \mathbb{R}^+$ is the time-delay. The matrix $\Gamma \in \mathbb{R}^{n\times n}$ determines the location of the controllers, defined as follows

$$\Gamma_{i,j} = \begin{cases} 1 \text{ if } i = j = \nu \\ 0 \text{ otherwise} \end{cases}, \forall\ i, j \in \{1, \dots, n\},\ \nu \subseteq \{1, \dots, n\}$$

where ν are the floors where actuators are installed.

Furthermore, $M,\ R$ and $K\ \in \mathbb{R}^{n\times n}$ are, respectively, the mass, damping, and stiffness matrices, defined as

$$M = diag\,[m_1\ m_2\ \dots\ m_n] > 0,$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & \dots & 0 & 0 \\ -k_2 & k_2 + k_3 & \dots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \dots & k_{n-1} + k_n & -k_n \\ 0 & 0 & \dots & -k_n & k_n \end{bmatrix} > 0,$$

and $R \geq 0$ has the same structure than matrix K. Here $m_i,\ r_i$, and $k_i,\ i = 1, 2, \dots, n$, are the ith mass coefficients, the inter-storey damping and stiffness, respectively.

Considering the change of variable $z(t) = [x(t)\ \dot{x}(t)]^T \in \mathbb{R}^{2n\times 1}$, the system (1) can be written in state-space form as

$$\dot{z}(t) = Az(t) + Bu(t-\tau) + D\ddot{x}_g(t), \quad (2)$$

where

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M^{-1}K & -M^{-1}R \end{bmatrix},\ B = \begin{bmatrix} 0_{n\times n} \\ -M^{-1}\Gamma \end{bmatrix},\ D = \begin{bmatrix} 0_n \\ -d \end{bmatrix}.$$

The following assumptions are made in this study.

Assumption 1. The building structure is initially at rest, i.e., $x(0) = 0_n$, $\dot{x}(0) = 0_n$ and $\ddot{x}(0) = 0_n$. Moreover, ground acceleration is zero before an earthquake perturbation.

Assumption 2. The disturbance signal $\ddot{x}_g(t) \in L_2[0, \infty)$ is bounded and has finite energy, i.e.,

$$||\ddot{x}_g(t)|| = \sqrt{\int_0^\infty \ddot{x}_g^T(t)\ddot{x}_g(t)dt} < \infty.$$

3. PROPOSED CONTROL SCHEME

In this section the observer-based control which allows to compensate small and large input delay of the perturbed system (2) is described. Due to the separation principle for observer-based control design, the stability conditions for the controller and the observer-based predictor are obtained separately.

3.1 Linear feedback control

We introduce an additional assumption for the delay-free system.

Assumption 3. The linear system

$$\dot{z}(t) = Az(t) + Bu(t) + D\ddot{x}_g(t), \quad (3)$$

can be stabilized by a feedback control of the form

$$u(t) = Fz(t) = [F_p\ F_d]z(t), \quad (4)$$

where

$$\begin{aligned} F_p &= diag\,[f_{p_1}\ f_{p_2}\ \dots\ f_{p_n}] \in \mathbb{R}^{n\times n},\\ F_d &= diag\,[f_{d_1}\ f_{d_2}\ \dots\ f_{d_n}] \in \mathbb{R}^{n\times n}. \end{aligned}$$

It is worthy of mention that the closed-loop system (3), (4) given by

$$\begin{aligned} \dot{z}(t) &= (A + BF)z(t) + D\ddot{x}_g(t) \quad (5)\\ &= \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M^{-1}(K + \Gamma F_p) & -M^{-1}(R + \Gamma F_d) \end{bmatrix} z(t) + \begin{bmatrix} 0_n \\ -d \end{bmatrix} \ddot{x}_g(t) \end{aligned}$$

has been extensively studied in literature, see among others, Chen et al. (2010) and Rubió-Massegú et al. (2016). In this research, the well known $\mathcal{H}_\infty$ control is considered. Classical $\mathcal{H}_\infty$ control problem aims at designing a controller to guarantee the asymptotic stability of a closed-loop system and the attenuation of a disturbance signal.

Theorem 1. If there exist matrices $P = P^T > 0$ and J of appropriate dimensions such that the following inequality is fulfilled

$$\begin{bmatrix} AP + BJ + J^T B^T + PA^T & D & P \\ D^T & -\beta^2 I & 0 \\ P & 0 & -I \end{bmatrix} \leq 0,$$

then system (3) with the feedback control (4) and $F = JP^{-1}$ satisfies the $\mathcal{H}_\infty$ criterion for disturbance rejection with attenuation level β.

Proof 1. The proof is a special case of the results given in Gahinet (1996).

3.2 Input delay compensation by a single observer

In order to compensate the input delay τ of system (2), let us consider the following observer-based controller (Najafi et al., 2013)

$$\begin{cases} \dot{\hat{z}}(t) = A\hat{z}(t) + Bu(t) + L(\hat{z}(t-\tau) - z(t)) \\ u(t) = F\hat{z}(t), \end{cases} \tag{6}$$

where $\hat{z}(t) \in \mathbb{R}^{2n}$ is the predicted state and $L \in \mathbb{R}^{2n\times 2n}$ is the observer gain matrix.

Here, the prediction error is defined as

$$e(t) = \hat{z}(t-\tau) - z(t), \tag{7}$$

therefore, the dynamic observer error is

$$\dot{e}(t) = Ae(t) + Le(t-\tau) - D\ddot{x}_g(t). \tag{8}$$

As pointed out in (Najafi et al., 2013), the gain L must be tuned in such a way that system (8) has a fast convergence. This can be formulated as the minimization over L of the spectral abscissa of system (8), defined as

$$\alpha(l) := \sup\{Re(s) : \det\left(sI - A - Le^{-s\tau}\right) = 0,\ s \in \mathbb{C}\}, \tag{9}$$

with $l := vec(L)$. However, since the spectral abscissa of the system is a non-smooth function of the system parameters (Michiels and Niculescu, 2014), its minimization cannot be carried out by standard optimization techniques. Here, in order to tune L, we use the smoothed spectral abscissa, a stability measure recently introduced in (Gomez and Michiels, 2019). It can be interpreted as a smooth approximation of the spectral abscissa, whose optimization provides a trade-off between the optimization of the $\mathcal{H}_2$ norm and the spectral abscissa of time-delay systems.

The smoothed spectral abscissa of system (8) associated with a given smoothing parameter $\epsilon > 0$ is denoted by $\tilde{\alpha}_\epsilon(l)$, and it is defined as the mapping $l \mapsto \tilde{\alpha}_\epsilon(l)$ that uniquely solves the equation

$$f(l, \tilde{\alpha}_\epsilon(l)) = \frac{1}{\epsilon},$$

where

$$f(l, s_\gamma) := \| - (sI - (A - s_\gamma I) - Le^{-s\tau}e^{-s_\gamma \tau})^{-1} D\|^2_{\mathcal{H}_2},$$

with $s_\gamma > \alpha(l)$. Its optimization can be formulated as an unconstrained minimization problem of the form

$$\min_{l=vec(L)} \tilde{\alpha}_\epsilon(l), \tag{10}$$

which can be solved by using standard gradient-based techniques as $l \mapsto \tilde{\alpha}_\epsilon(l)$ is smooth. The computation and optimization of the smoothed spectral abscissa is carried out by using the so-called delay Lyapunov matrix and its sensitivity; see (Gomez and Michiels, 2019) for details.

In order to obtain a stabilizing matrix L, we solve problem (10) for a given $\epsilon = \hat{\epsilon}_1 > 0$. If the obtained $l = l^\star(\hat{\epsilon}_1)$ is such that $\tilde{\alpha}_{\hat{\epsilon}_1}(l^\star(\hat{\epsilon}_1)) < 0$, then system (8) is exponentially stable, by the property $\tilde{\alpha}_\epsilon(l) > \alpha(l)$ for any $\epsilon > 0$. Otherwise, since $\epsilon \mapsto \tilde{\alpha}_\epsilon(l)$ is increasing, we can set $\epsilon = \hat{\epsilon}_2 < \hat{\epsilon}_1$ and solve (10). Since $\tilde{\alpha}_\epsilon(l) \to \alpha(l)$ as $\epsilon \to 0$, the minimizer $l^\star(\hat{\epsilon}_1)$ approximates a minimizer of the spectral abscissa for values of $\hat{\epsilon}_1$ close to zero, ensuring fast convergence of system (8).

In view of (7), the closed-loop system (2), (6) is

$$\begin{aligned} \dot{z}(t) =& Az(t) + BF\hat{z}(t-\tau) + D\ddot{x}_g(t) \\ =& (A + BF)z(t) + BFe(t) + D\ddot{x}_g(t), \end{aligned}$$

and when $e(t) \longrightarrow 0$, a delay-free system is obtained

$$\dot{z}(t) = (A + BF)z(t) + D\ddot{x}_g(t).$$

Thus, under Assumption 3, the closed-loop system consisting of (2) and (6) is stable.

3.3 Input delay compensation by sequential sub-predictors

When the input delay τ is large, the observer-based controller (6) may fail to stabilize system (2). To solve this problem, the sequential structure of sub-predictors proposed in Najafi et al. (2013) is considered. The main idea of this approach is to split the time-delay τ and apply a set of sub-predictors to predict the states for the entire time-delay. Here, we use a set of N coupled sub-predictors, each of them predicts the sates for Δ_τ seconds ahead, where

$$\Delta_\tau = \frac{\tau}{N},\ N \in \mathbb{Z}^+.$$

Next, we establish the stability of the closed-loop control scheme.

Theorem 2. Let Assumption 3 be satisfied. Consider the following observer-based controller

$$\begin{cases} \dot{\hat{z}}_i(t) = A\hat{z}_i(t) + L_i(\hat{z}_i(t-\Delta_\tau) - \hat{z}_{i+1}(t)) + Bu(t-(i-1)\Delta_\tau) \\ \dot{\hat{z}}_N(t) = A\hat{z}_N(t) + L_N(\hat{z}_N(t-\Delta_\tau) - z(t)) + Bu(t-(N-1)\Delta_\tau) \\ u(t) = F\hat{z}_1(t), \end{cases} \tag{11}$$

where $i = \overline{1, N-1}$. Then the closed-loop system consisting of (2) and (11) is stable if the following systems are stable

$$\begin{aligned} \dot{e}_i(t) &= Ae_i(t) + L_i e_i(t-\Delta_\tau),\ i = \overline{1, N-1} \\ \dot{e}_N(t) &= Ae_N(t) + L_N e_N(t-\Delta_\tau) - D\ddot{x}_g(t). \end{aligned} \tag{12}$$

Proof 2. The proof is a particular case of the work presented in Zhou et al. (2017). For the sake of completeness, we present it here. Defining the prediction errors

$$\begin{aligned} e_i(t) =& \hat{z}_i(t-(N-i+1)\Delta_\tau) \\ & - \hat{z}_{i+1}(t-(N-i)\Delta_\tau),\ i = \overline{1, N-1} \\ e_N(t) =& \hat{z}_N(t-\Delta_\tau) - z(t), \end{aligned}$$

we can see that

$$\hat{z}_1(t - N\Delta_\tau) = z(t) + e_1(t) + e_2(t) + \ldots + e_N(t),$$

for $t \geq 0$. And the closed-loop system (2), (11) results

$$\begin{aligned} \dot{z}(t) =& Az(t) + BF\hat{z}_1(t-N\Delta_\tau) + D\ddot{x}_g(t) \\ =& (A + BF)z(t) + D\ddot{x}_g(t) \\ & + BF(e_1(t) + e_2(t) + \ldots + e_N(t)). \end{aligned} \tag{13}$$

By differentiating the prediction errors one obtains that

$$\begin{aligned}\dot{e}_i(t) =& Ae_i(t) + L_i e_i(t-\Delta_\tau) \\ &- L_{i+1}e_{i+1}(t-\Delta_\tau),\ i=\overline{1,N-1} \\ \dot{e}_N(t) =& Ae_N(t) + L_N e_N(t-\Delta_\tau) - D\ddot{x}_g(t). \end{aligned} \quad (14)$$

Because of the upper triangular structure of the closed-loop system (13), (14), the closed-loop system (2), (11) is stable if the following systems

$$\begin{aligned}\dot{\eta}_0(t) &= (A+BF)\eta_0(t) + D\ddot{x}_g(t),\\ \dot{\eta}_i(t) &= A\eta_i(t) + L_i\eta_i(t-\Delta_\tau),\ i=\overline{1,N-1}\\ \dot{\eta}_N(t) &= A\eta_N(t) + L_N\eta_N(t-\Delta_\tau) - D\ddot{x}_g(t)\end{aligned}$$

are stable. In view of Assumption 3, the stability of the last set of equations is equivalent to the stability of

$$\begin{aligned}\dot{e}_i(t) &= Ae_i(t) + L_ie_i(t-\Delta_\tau),\ i=\overline{1,N-1}\\ \dot{e}_N(t) &= Ae_N(t) + L_Ne_N(t-\Delta_\tau) - D\ddot{x}_g(t).\end{aligned}$$

This ends the proof.

Similarly to the case with a single observer, the dynamic observer errors (12) can be stabilized by means of the smoothed spectral abscissa optimization.

3.4 Summary of the observer-based controller

The proposed stabilization method can be summarized as follows:

(1) Obtain the dynamic model of the building structure according to (2).
(2) Consider the delay-free system (3), obtain a stabilizing gain F using the result in Theorem 1 for the closed-loop system (5) and compute its decay rate γ_f given by its spectral abscissa.
(3) Define a set of N observers-predictors and compute the observers gain matrices L_i, $i=\overline{1,N}$ by means of the smoothed spectral abscissa optimization approach described in subsection 3.2.
(4) Compute the spectral abscissa $\alpha(l)$ of the dynamic observer errors with the gains L_i, $i=\overline{1,N}$ obtained in the previous step.
(5) Check if $\alpha(l) < \gamma_f$ holds. If it does not hold, then increase N in step 3 until $\alpha(l) < \gamma_f$ is satisfied.
(6) Finally, apply the observer-based control (11) to system (2).

4. ILLUSTRATIVE EXAMPLE

Let us consider the case of a two-storey building prototype with an Active Mass Damper located in the second storey. For this case, equation (2) reduces to

$$\dot{z}(t) = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\dfrac{k_1+k_2}{m_1} & \dfrac{k_2}{m_1} & -\dfrac{r_1+r_2}{m_1} & \dfrac{r_2}{m_1}\\ \dfrac{k_2}{m_2} & -\dfrac{k_2}{m_2} & \dfrac{r_2}{m_2} & -\dfrac{r_2}{m_2}\end{bmatrix} z(t) + \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & -\dfrac{1}{m_2}\end{bmatrix} u(t-\tau) - \begin{bmatrix}0\\0\\1\\1\end{bmatrix}\ddot{x}_g(t). \quad (15)$$

In this example, the system parameters are set as follows: $m_1 = 3.17\ kg$, $m_2 = 4.609\ kg$, $r_1 = 7.388\ N\cdot s/m$, $r_2 = 6.834\ N\cdot s/m$, $k_1 = 9199.834\ N/m$ and $k_2 = 7531.628\ N/m$.

4.1 Delay-free case

The stabilization of the system (15) when $\tau = 0$ can be achieved with the feedback control

$$u(t) = Fz(t) = [F_p\ F_d]z(t), \quad (16)$$

where $F_p = diag\,[f_{p_1}\ f_{p_2}]$ and $F_d = diag\,[f_{d_1}\ f_{d_2}]$. These parameters are set as $f_{p_2} = -4044.786$ and $f_{d_2} = 30.025$ obtained from the $\mathcal{H}_\infty$ procedure. Since there is only one actuator installed in the second storey of the building, it follows that $f_{p_1} = f_{d_1} = 0$. For the delay-free case, the spectral abscissa of the closed-loop system (15) and (16) is found to be $\gamma_f = -3.0431$.

It is well known that, when $\tau > 0$, the feedback control (16) with the previously obtained gains cannot guarantee the stability of system (15), for example when $\tau = 0.1$, the closed-loop system is unstable, see Fig. 3.

4.2 Time-delay compensation by a single observer

In order to compensate an input delay $\tau = 0.1$ for the system (15), we use the observer-based control (6)

$$\begin{cases}\dot{\hat{z}}(t) = A\hat{z}(t) + Bu(t) + L(\hat{z}(t-\tau) - z(t))\\ u(t) = F\hat{z}(t),\end{cases}$$

where F is the previously obtained feedback control gain, and L is the observer gain matrix obtained from the smoothed spectral abscissa optimization of the dynamic observer error

$$\dot{e}(t) = Ae(t) + Le(t-\tau) - D\ddot{x}_g(t), \quad (17)$$

where L is set as

$$L = diag\,[l_1\ l_2\ l_3\ l_4].$$

By optimizing the smoothed spectral abscissa over L with $\epsilon = e^{-6}$ and starting point $l_0 = 0 \in \mathbb{R}^{16\times 1}$ (for which $\alpha(l_0) = -0.3260$), we obtain

$$L = diag\,[20.0918\ 7.1784\ -14.633\ 5.8074]. \quad (18)$$

With these parameters, the smoothed spectral abscissa of the dynamic observer error (17) is $\tilde{\alpha}_\epsilon(l) = -3.7366$, whereas the spectral abscissa is $\alpha(l) = -3.7383$. It is worth emphasizing that $\alpha(l)$ is obtained evaluating the minimizer (18) in (9). Note that $\tilde{\alpha}_\epsilon(l)$ and $\alpha(l)$ are very close, due to $\tilde{\alpha}_\epsilon(l) \to \alpha(l)$ as $\epsilon \to 0$.

It is clear that $\alpha(l) < \gamma_f$ holds. Thus, the observer-based control (6) stabilizes system (15) for $\tau = 0.1$.

4.3 Time-delay compensation by sequential sub-predictors

Now let us consider an input delay $\tau = 0.4$. When the smoothed spectral abscissa optimization is carried out for a single observer-based control, we found that $l = vec\,(diag\,[9.7683\ -16.255\ 20.158\ 6.3307])$, $\tilde{\alpha}_\epsilon(l) = -0.9343$ and $\alpha(l) = -0.9691$. However, the condition $\alpha(l) < \gamma_f$ is not satisfied, thus the closed-loop system (6) and (15) is unstable. In order to compensate this delay we apply a set of $N = 4$ sub-predictors, therefore

$$\Delta_\tau = \frac{\tau}{N} = \frac{0.4}{4} = 0.1.$$

Considering the observer-based controller (11) with $N = 4$ it follows that

$$\begin{cases} \dot{\hat{z}}_1(t) = A\hat{z}_1(t) + L_1(\hat{z}_1(t-\Delta_\tau) - \hat{z}_2(t)) + Bu(t) \\ \dot{\hat{z}}_2(t) = A\hat{z}_2(t) + L_2(\hat{z}_2(t-\Delta_\tau) - \hat{z}_3(t)) + Bu(t-\Delta_\tau) \\ \dot{\hat{z}}_3(t) = A\hat{z}_3(t) + L_3(\hat{z}_3(t-\Delta_\tau) - \hat{z}_4(t)) + Bu(t-2\Delta_\tau) \\ \dot{\hat{z}}_4(t) = A\hat{z}_4(t) + L_4(\hat{z}_4(t-\Delta_\tau) - z(t)) + Bu(t-3\Delta_\tau) \\ u(t) = F\hat{z}_1(t). \end{cases} \tag{19}$$

Then, under Assumption 3, the closed-loop system consisting of (15) and (19) is asymptotically stable if the following systems are asymptotically stable

$$\begin{aligned} \dot{e}_1(t) &= Ae_1(t) + L_1e_1(t-\Delta_\tau) \\ \dot{e}_2(t) &= Ae_2(t) + L_2e_2(t-\Delta_\tau) \\ \dot{e}_3(t) &= Ae_3(t) + L_3e_3(t-\Delta_\tau) \\ \dot{e}_4(t) &= Ae_4(t) + L_4e_4(t-\Delta_\tau) - D\ddot{x}_g(t). \end{aligned}$$

Since the last expressions have the same structure as equation (17), the gains are chosen as $L_1 = L_2 = L_3 = L_4 = L$, with L given in (18).

In order to verify the effectiveness of the proposed control method, numerical simulations are presented in the following subsection.

4.4 Simulation results

The simulation results correspond to a reduced scale two-storey building prototype constructed of aluminum. The experiments are carried out using the 1940 El Centro earthquake as a seismic excitation, which is fitted to match with the structure, as depicted in Fig. 2.

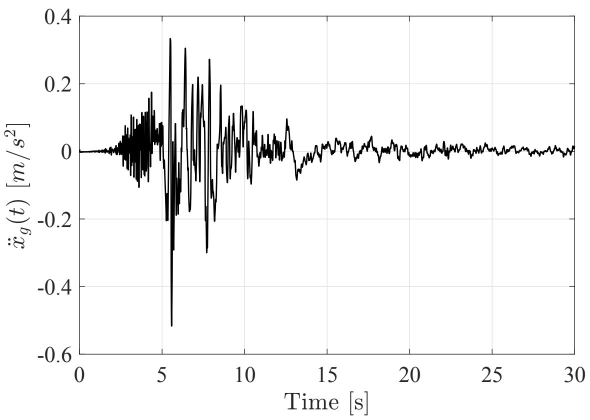

Fig. 2. The 1940 El Centro earthquake record.

Assuming that the building is equipped with an Active Mass Damper on the roof, an $\mathcal{H}_\infty$ controller is applied for vibration attenuation. However, when a time-delay is considered in the control process, for example $\tau = 0.1$, the control performance is deteriorated producing instability of the system even if the $\mathcal{H}_\infty$ controller is used. This behavior can be observed in Fig. 3, where the displacements corresponding to the first and second storey increase in an unbounded way with time.

In order to overcome this effect, the time-delay compensation algorithm developed in subsections 3.2 and 3.3 are applied. Figs. 4 and 5 show results about the relative displacements of system (15), when the observer-based controllers (6) and (19) are implemented. In both cases, the time-delay effect has been reduced, and the system is now controlled.

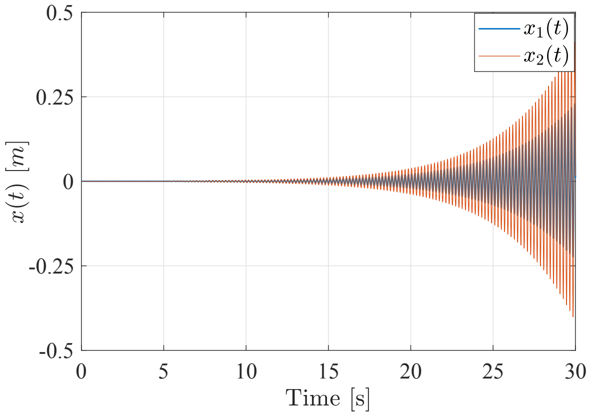

Fig. 3. Relative displacements of the building without time-delay compensation, here $\tau = 0.1$.

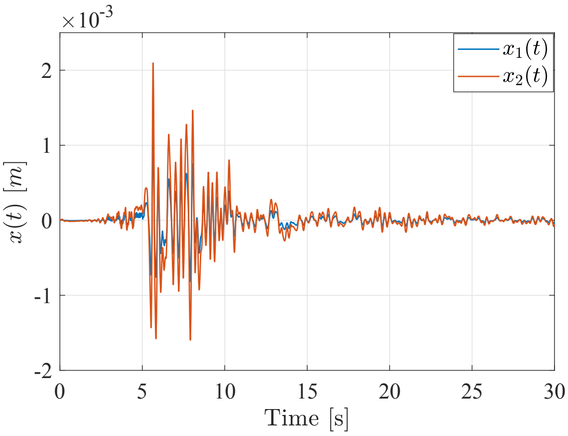

Fig. 4. Relative displacement of the building with input delay $\tau = 0.1$, plus time-delay compensation by a single observer.

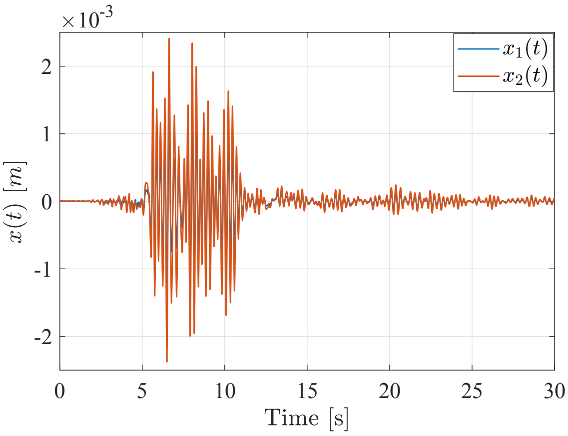

Fig. 5. Relative displacement of the building with input delay $\tau = 0.4$, plus time-delay compensation by multiple observers.

Fig. 6 shows the performance of the closed-loop system given by (15) and (19) evaluated by means of the Integral of Absolute Error (IAE)

$$IAE = \int_0^{t_f} ||e(t)||dt,$$

where $e(t) := x_d(t) - x(t)$, and $x_d(t) = 0 \in \mathbb{R}^{4\times 1}$ is the desired state vector.

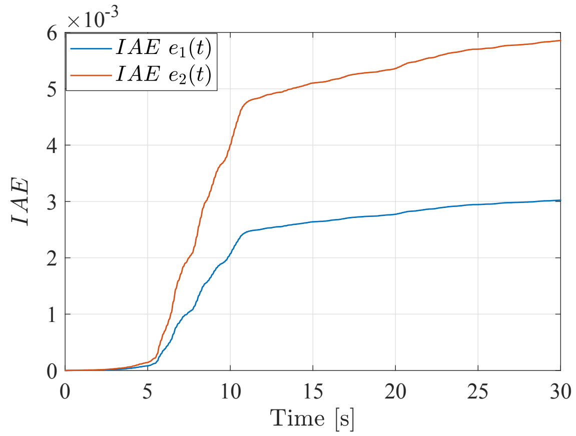

Fig. 6. IAE performance index of $e(t)$ obtained with the closed-loop system (15) and (19) when $\tau = 0.4$.

5. CONCLUDING REMARKS

This paper provides a novel method for compensation of the delay in the problem of vibration control building structures. The control scheme that we propose is based on predictor-based control, and unlike previous work, it is tuned by minimizing the smoothed spectral abscissa of the dynamic observer errors. As shown in the numerical simulations, the proposed design provides a controller such that ensures the stability of the closed-loop and attenuates the disturbance effect. Future work includes extending this result to the case of multiple input delays, and evaluate the performance of delay-based controllers such as the proportional retarded control and the proportional integral retarded control to this class of system.

REFERENCES

Agrawal, A. and Yang, J. (1997). Effect of fixed time delay on stability and performance of actively controlled civil engineering structures. *Earthquake Engineering & Structural Dynamics*, 26(11), 1169–1185.

Araújo, J.M. and Santos, T.L.M. (2018). Control of a class of second-order linear vibrating systems with time-delay: Smith predictor approach. *Mechanical Systems and Signal Processing*, 108, 173–187.

Chen, Y., Zhang, W., and Gao, H. (2010). Finite frequency H_∞ control for building under earthquake excitation. *Mechatronics*, 20(1), 128–142.

Chung, L., Reinhorn, A., and Soong, T. (1988). Experiments on active control of seismic structures. *Journal of Engineering Mechanics*, 114(2), 241–256.

Gahinet, P. (1996). Explicit controller formulas for LMI-based H_∞ synthesis. *Automatica*, 32(7), 1007–1014.

Gomez, M.A. and Michiels, W. (2019). Characterization and optimization of the smoothed spectral abscissa for time-delay systems. *International Journal of Robust and Nonlinear Control*, 29(13), 4402–4418.

Guclu, R. (2006). Sliding mode and PID control of a structural system against earthquake. *Mathematical and Computer Modelling*, 44(1-2), 210–217.

Guclu, R. and Yazici, H. (2008). Vibration control of a structure with ATMD against earthquake using fuzzy logic controllers. *Journal of Sound and Vibration*, 318(1-2), 36–49.

Guoping, C. and Jinzhi, H. (2002). Optimal control method for seismically excited building structures with time-delay in control. *Journal of Engineering Mechanics*, 128(6), 602–612.

Li, Z., Chen, C., and Teng, J. (2019). A multi-time-delay compensation controller using a Takagi-Sugeno fuzzy neural network method for high-rise buildings with an active mass damper/driver system. *The Structural Design of Tall and Special Buildings*, 28(13), 1051–1064.

Li-Ye, L. and Qi-Bing, J. (2020). Analytical optimization of IMC-PID design based on performance/robustness tradeoff tuning strategy for the modified Smith structure. *Journal of Low Frequency Noise, Vibration and Active Control*, 39(1), 158–173.

Madan, A. (2005). Vibration control of building structures using self-organizing and self-learning neural networks. *Journal of sound and vibration*, 287(4-5), 759–784.

Michiels, W. and Niculescu, S.I. (2014). *Stability, Control, and Computation for Time-delay Systems: An Eigenvalue-based Approach*, volume 27. SIAM.

Najafi, M., Hosseinnia, S., Sheikholeslam, F., and Karimadini, M. (2013). Closed-loop control of dead time systems via sequential sub-predictors. *International Journal of Control*, 86(4), 599–609.

Peng, H., Chen, Y., Li, E., Zhang, S., and Chen, B. (2018). Explicit expression-based practical model predictive control implementation for large-scale structures with multi-input delays. *Journal of Vibration and Control*, 24(12), 2605–2620.

Rubió-Masségú, J., Rossell, J.M., Karimi, H.R., et al. (2016). Vibration control strategy for large-scale structures with incomplete multi-actuator system and neighbouring state information. *IET Control Theory & Applications*, 10(4), 407–416.

Soleymani, M., Abolmasoumi, A.H., Bahrami, H., Khalatbari-S, A., Khoshbin, E., and Sayahi, S. (2018). Modified sliding mode control of a seismic active mass damper system considering model uncertainties and input time delay. *Journal of Vibration and Control*, 24(6), 1051–1064.

Thenozhi, S. and Yu, W. (2014). Stability analysis of active vibration control of building structures using PD/PID control. *Engineering Structures*, 81, 208–218.

Wang, Y., Lynch, J.P., and Law, K.H. (2009). Decentralized H_∞ controller design for large-scale civil structures. *Earthquake Engineering & Structural Dynamics*, 38(3), 377–401.

Yang, J.N., Lin, S., and Jabbari, F. (2003). H_2-based control strategies for civil engineering structures. *Journal of Structural Control*, 10(3-4), 205–230.

Zhou, B., Liu, Q., and Mazenc, F. (2017). Stabilization of linear systems with both input and state delays by observer–predictors. *Automatica*, 83, 368–377.