A New Robust Command Shaping Method and Its Application in Quadrotor Slung System with Varying Parameters

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Abstract: Command shaping method is a mature feedforward control approach, and there exist many successful applications in the industrial fields. However, the traditional instruction shapers either are sensitive to the system parameters, or have some robustness but difficult to adjust the parameters, and meanwhile their conservations are increased unsurprisingly. To this end, a new robust command shaper is proposed in this paper, which is inspired by the two-mode ZV shaper and the EI shaper. The theoretical design procedure of the new shaper is presented. Besides, the robustness of the new shaper is analyzed and compared with other shapers based on the sensitivity curve. Finally, the shaper proposed in this paper is applied to the quadrotor slung system with varying parameters, and its effectiveness and superiority are proved by numerical simulations and comparative analyses.

Keywords: Command Shaper, Quadrotor Slung System, Vibration Suppression, Robustness, Modeling Error

1. INTRODUCTION

The quadrotor is a new type of air transport vehicle which has the potential to hover and take off, fly and land in small areas, cf.: Mellinger et al. (2011). Now, the quadrotor is widely used in military and civil applications, such as material delivery and logistics, etc. In modern logistic industry, the quadrotor trends to be used to transport some packages since transportation on road is almost saturated. Sometimes it is necessary to use quadrotor to share the transportation volume during the rush hours. Amazon, Google and S. F. Express have carried out UAV Express projects in remote areas such as Fig. 1. However, due to the lack of stability and reliability of transportation, the applications are limited. Therefore, it is necessary to solve these problems caused by payload vibration.

Different control methods have been proposed to control the quadrotor since the slung system significantly alters the flight characteristics of the quadrotor. These methods are divided into feedback and feed-forward control, cf.: Sadr et al. (2014), Haddadi et al. (2015). Feedback control methods use measurements and estimations of system states to reduce the vibration while feed-forward control methods change actuator commands for reducing the vibration. The feed-forward controller can improve the performance of feedback controller. Thus, proposing feed-forward algorithms can lead to more practical and accurate control of these systems, cf.: Njeri and Matsushita (2018). One effective feed-forward method is the command shaping theory.

Command shaper is realized by convoluting a sequence of impulses with desired instructions to generate an appropriate reference command. Creating special reference commands to move flexible systems without vibration is a traditional idea, cf.: Singhose (2009) and references therein. The early form of command shaper was posicast control, which is the prototype of Zero Vibration (ZV) shaper put forward by Smith (1957). It solves the problem of vibration suppression at natural frequency of system. Unfortunately, we will never know the system parameters perfectly, so we cannot assume that redesigning the command shaper is always an optimal option. Furthermore, the parameters are likely to change somewhat over time or during the motion. What is needed is a robust command shaper that works well even when there are modeling...
errors. This robustness problem was first solved by Singer and Seering (1990) by adding additional constraint on the derivative of residual vibration magnitudes, which results in Zero Vibration and Derivative (ZVD) shaper. On the basis of ZVD, Zero Vibration Double Derivative (ZVDD) was proposed by Singer and Seering (1990) to further enhance the robustness. Another shaper put forward by Ingram and Chiu (2002) of increasing robustness is to relax zero vibration constraints at natural frequency, such as Extra-Insensitive (EI) shaper, which has more robustness by sacrificing the accuracy at the natural frequency.

Although these command shapers are widely used in industry, cf.: Auernig and Troger (1987), Kim and Singhose (2010), Adams et al. (2015), they still have some disadvantages. The delay of ZV is the smallest, but its robustness is too poor to be used in practice. ZVDD has the best robustness, but its large delay is easy to cause stability problems. EI and ZV have the same appropriate delay, however, the robustness of ZVDD is not strong enough, while EI has poor design flexibility and high computational complexity. They more or less have some limitations. The purpose of this paper is to investigate a new command shaper that will cause the systems to complete desired moves robustly and accurately.

The rest of the paper is organized as follows. Section 2 reviews the basic definitions and typical command shaping methods. The main results are presented in Section 3, where the new robust command shaper is proposed, and its robustness and accuracy are analyzed and compared with the traditional ones. An example of quadrotor slung system with varying parameters is investigated in Section 4, where the effectiveness and the superiority in vibration suppression of the swing payload are illustrated by simulation comparison. Section 5 summaries the whole paper.

2. PRELIMINARIES

In this section, some typical command shaping methods are reviewed. For the sake of brevity, we denote by $C$ and $S$ the cosine and sine functions throughout the paper.

2.1 Residual Vibration

A mathematical description of the residual vibration that results from an impulse sequence can be described as

$$V(\omega_n, \xi) = e^{-\xi \omega_n t_1} \sqrt{C(\omega_n, \xi)^2 + S(\omega_n, \xi)^2}$$

where

$$C(\omega_n, \xi) = \sum_{i=1}^{n} A_i e^{\xi \omega_n t_i} C(\omega_d t_i)$$

and

$$S(\omega_n, \xi) = \sum_{i=1}^{n} A_i e^{\xi \omega_n t_i} S(\omega_d t_i)$$

where $V \in [0, 1]$ is the residual vibration, $\xi \in (0, 1)$ is damping ratio, $\omega_n$ is undamped natural frequency of the system. $A_i$ and $t_i, i = 1, \ldots, n$ are the amplitudes and time locations of the impulses, $n$ is the number of impulses in the sequence, cf.: Singhose and Seering (2011). The damped natural frequency is

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

Without loss of generality, we can set the time location of the first impulse equals to zero, i.e., $t_1 = 0$. To generate an impulse sequence that causes no residual vibration, the following restrictions should be satisfied

$$V(\omega_n, \xi) = 0$$

and

$$\sum_{i=1}^{n} A_i = 1$$

where $A_i > 0, i = 1, \ldots, n$ aims to avoid the trivial solution of all zero-valued impulses and to obtain a normalized and bounded result.

2.2 Typical Command Shapers

If the accurate system model is available, the sequence of two impulses that leads to Zero Vibration (ZV) shaper can be stated in matrix form as

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{1 + K} \end{bmatrix} \begin{bmatrix} K \\ 0.5T_d \end{bmatrix}$$

where

$$K = e \sqrt{1 - \xi^2}$$

and

$$T_d = \frac{2\pi}{\omega_d}$$

A ZV shaper is based on the assumption that an accurate model of the plant dynamics exists. However, this assumption is not always the case in practice. The first command shaper designed to have robustness to modeling errors is Zero Vibration and Derivative (ZVD) shaper. This shaper is designed by requiring the partial derivative of the residual vibration equals to zero at the modeling frequency. Mathematically, this can be stated as

$$\frac{\partial V(\omega, \xi)}{\partial \omega} = 0$$

The ZVD shaper is obtained by

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1 + K)^2} & 2K \\ 0.5T_d & \frac{1}{(1 + K)^2} \end{bmatrix} \begin{bmatrix} K^2 \\ T_d \end{bmatrix}$$

Actually, forcing the command to produce exactly zero vibration at the modeling frequency is not a particularly useful design constraint in the case that there exists a fair amount of uncertainty. An Extra-Insensitive (EI) shaper is proposed which simply constrains the residual vibration at some tolerable level, so that more robustness can be obtained without incurring additional time delay. The impulse sequence of an EI shaper is

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1 + K)^2} & -V_{tol} \\ 0.5T_d & 1 + V_{tol} \end{bmatrix} \begin{bmatrix} K^2 \\ T_d \end{bmatrix}$$

where $V_{tol}$ is the tolerable limit on percentage residual vibration, cf.: Singhose and Seering (2011).

2.3 Sensitivity Curve

The robustness of a command shaper can be visualized by plotting its sensitivity curve, which shows the residual vibration after an impulse sequence is added to the system. The sensitivity curves for the ZV, ZVD and EI shapers are demonstrated in Fig. 2, where the horizontal axis is a normalized frequency, i.e., $\omega$ and $\omega_d$ denote the actual frequency and the modeling frequency, respectively, and
Fig. 2. Sensitivity curves of ZV, ZVD and EI shapers

3. MAIN RESULTS

Compared with the ZV shaper, a ZVD shaper is more robust which relaxes the necessary of the accurate system model. However, their robustness is not strong enough which results in other types of shapers such as the two-mode ZV and the EI shaper. A two-mode ZV shaper eliminates the vibration at two frequency points by convolving two ZV shapers, but the maximum vibration between the two frequency points is not considered. An EI shaper utilizes $V_{tol}$ to restrict the tolerable limit of the vibration, but its amplitudes and time locations of impulses are coupled with $V_{tol}$ and $\omega_d$, which causes a huge computational complexity, cf.: Singhose and Seering (2011).

Inspired by the two-mode ZV and EI shaper, a new command shaping method is proposed in this section. It is called Virtual Insensitive (VI) shaper, which causes systems to complete desired moves robustly and accurately.

3.1 Configuration of the New Shaper

Assume $\omega_1$ and $\omega_2$ are the virtual frequency points which locates on the left and right side of $\omega_d$, respectively. Without loss of generality, assume $\omega_1 \leq \omega_d \leq \omega_2$. Based on the two-mode ZV shaper, it is necessary to combine the qualities of the two shapers by convolving them, which leads to an impulse sequence with four impulses. In the sequel, the amplitude and time location of the impulses should be determined, respectively.

(1) Amplitudes of the impulses

The amplitudes of the impulses should suppress vibrations at $\omega_1$ and $\omega_2$, so that

$$V(w_1, \xi) = V(w_2, \xi) = 0$$

which means

$$\sum_{i=1}^{4} A_i e^{i\omega_1 t_i} C(\omega_1, t_i) = 0 \quad \text{(14)}$$

where $j = 1, 2$. By solving (14) and (15), the amplitudes of the impulses can be derived as

$$K^2 A_1 = K A_2 = K A_3 = A_4 \quad \text{(16)}$$

If damping is ignored, i.e., let $\xi = 0$, the amplitudes of the impulses can be obtained as

$$A_i = 0.25, \; i = 1, 2, 3, 4 \quad \text{(17)}$$

(2) Time locations of the impulses

According to (1) and (17), the relation between the residual vibration and the natural frequency is represented as

$$V^2 = \sum_{i=1}^{4} A_i^2 + 2A_1 A_2 C(\omega t_2) + 2A_1 A_3 C(\omega t_3) + 2A_1 A_4 C(\omega t_4) + 2A_2 A_3 C(\omega t_3) + 2A_2 A_4 C(\omega (t_4 - t_2)) \quad \text{(18)}$$

which can be simplified as follows,

$$V = |C(\frac{\omega t_2}{2}) \cdot C(\frac{\omega t_3}{2})| \quad \text{(19)}$$

Based on the EI shaper, $V$ reaches its maximum $V_{max}$ when $\omega = \omega_d$. If $V_{tol}$ is given, the system could meet the requirements of the maximum vibration by guaranteeing $V_{max} \leq V_{tol}$,

$$V_{max} = |C(\frac{\omega_d t_2}{2}) \cdot C(\frac{\omega_d t_3}{2})| \leq V_{tol} \quad \text{(20)}$$

The design idea of the new VI shaper is that, if $\omega_d$ and $V_{tol}$ are given, an appropriate virtual frequency $\omega_1$ on the left side of $\omega_d$ can be selected, then the range of $\omega_d$ can be obtained by (20) on the right side of $\omega_d$, and vice versa. Thus, the time locations of impulses can be expressed as

$$t_1 = 0, \; t_2 = 0.5 T_{d2}, \; t_3 = 0.5 T_{d1}, \; t_4 = t_2 + t_3 \quad \text{(21)}$$

where $T_{d1} = \frac{\pi}{\omega_1}$ and $T_{d2} = \frac{\pi}{\omega_2}$ are the virtual vibration periods, respectively. Finally, the matrix form of the impulse sequence with a VI shaper can be formulated as

$$A_1 = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & 0.5 T_{d2} & 0.5 T_{d1} & 0.5(T_{d1} + T_{d2}) \end{bmatrix} \quad \text{(22)}$$

3.2 Robustness Analysis

The sensitive curve of the new VI shaper is shown in Fig. 3, together with the well-developed ZV, ZVD and EI shapers. Obviously, it has a wider width of its sensitivity curve at 5% tolerable vibration level, which demonstrates a significant robustness compared with the traditional shaper. It should be noted that the robustness of a VI shaper is at the same level compared with the EI shaper in the case that they have the same $V_{tol}$ on $\omega_d$.

Remark 1 Analogous to the ZVD and EI shaper, the VI shaper has one vibration period, which is twice of the ZV shaper. However, a substantial amount of robustness is obtained for this small increase in time delay.

3.3 Accuracy Comparison with EI Shaper

For an EI shaper, when $\omega_d$ and $V_{tol}$ are given, the impulse sequence can be designed by (12). However, when $\omega_d$ changes, the time locations can be redesigned according to the change, while the amplitudes cannot be changed easily since $V_{tol}$ after change is cannot be known, which means $V_{max} = V_{tol}$ remaining unchanged after $\omega_d$ changed.

For a VI shaper, $\omega_1$ or $\omega_2$ is set roughly on the left and right side of $\omega_d$ for a VI shaper. If one of them is fixed, the other
Fig. 3. Sensitivity curves of ZV, ZVD, EI and VI shapers can be calculated by solving (20). Therefore, the turning of the VI shaper is more convenient than the EI shaper. More importantly, $V_{\text{max}}$ of a VI shaper will decrease after turning when the parameters of the system change, which is illustrated below.

**Theorem 1.** For a VI shaper, the maximum vibration $V_{\text{max}}$ will decrease from the initial tolerable vibration level $V_0$ if the virtual left-side frequency $\omega_1$ increases or the virtual right-side frequency $\omega_2$ decreases within their limits.

**Proof.** If $\omega_1 \leq \omega \leq \omega_2$, $V$ in (19) is negative, which can be expressed as

$$V = -C\left(\frac{\omega t_3}{2}\right) \cdot C\left(\frac{\omega t_2}{2}\right)$$

(23)

The partial derivative of $V$ about $\omega$ is

$$\frac{\partial V}{\partial \omega} = \frac{1}{2} t_3 S\left(\frac{\omega t_3}{2}\right) \cdot C\left(\frac{\omega t_2}{2}\right) + t_2 S\left(\frac{\omega t_2}{2}\right) \cdot C\left(\frac{\omega t_3}{2}\right)$$

(24)

Let $\frac{\partial V}{\partial \omega} = 0$, $V$ reaches its maximum value which satisfies

$$t_3 \tan\left(\frac{\omega t_3}{2}\right) + t_2 \tan\left(\frac{\omega t_2}{2}\right) = 0$$

(25)

Thus $V_{\text{max}}$ can be expressed as

$$V_{\text{max}} = -C\left(\frac{\omega^* t_3}{2}\right) \cdot C\left(\frac{\omega^* t_2}{2}\right)$$

(26)

where $\omega^*$ is the extreme point which makes $V = V_{\text{max}}$.

Considering the relationship between $V_{\text{max}}$ and virtual frequency $\omega_1$ and $\omega_2$, it is necessary to treat $V_{\text{max}}$ as a variable, and to obtain partial derivative for $t_2$ and $t_3$, which is related with $\omega_1$ and $\omega_2$, that is

$$\frac{\partial V_{\text{max}}}{\partial t_3} = \frac{\omega^*}{2} S\left(\frac{\omega^* t_3}{2}\right) \cdot C\left(\frac{\omega^* t_2}{2}\right)$$

(27)

$$\frac{\partial V_{\text{max}}}{\partial t_2} = \frac{\omega^*}{2} C\left(\frac{\omega^* t_2}{2}\right) \cdot S\left(\frac{\omega^* t_2}{2}\right)$$

(28)

Considering that

$$C\left(\frac{\omega^* t_2}{2}\right) > 0, S\left(\frac{\omega^* t_3}{2}\right) > 0$$

(29)

$$C\left(\frac{\omega^* t_2}{2}\right) < 0, S\left(\frac{\omega^* t_3}{2}\right) > 0$$

(30)

According to (29) and (30), it is obvious that if $\omega_1$ increases, $V_{\text{max}}$ will decrease. Analogously, if $\omega_2$ decreases, $V_{\text{max}}$ will decrease too. Since we started our design at certain tolerable vibration level $V_0$, thus we derive that the maximum vibration $V_{\text{max}} \leq V_0$.

**Remark 2.** Compared with the EI shaper, the impulses sequence of a VI shaper is no longer coupled with $V_{\text{tol}}$ and $\omega_0$, so that the parameters of the shaper have good design flexibility. Furthermore, $V_{\text{max}}$ is less than the initial $V_{\text{tol}}$ after turning the shaper due to the change of parameters, which means the VI shaper has a higher accuracy.

Fig. 4 shows the change of $V_{\text{max}}$ when $\omega_0$ is changing from 3.5 rad/s to 3.1 rad/s. In this process, $\omega_1 = 3$ rad/s remains unchanged while $\omega_2$ is changed at specific values.

4. CASE STUDIES AND COMPARISONS

In this section, an example of quadrotor slung system with varying parameters will be investigated to test and verify the robustness and accuracy of the VI shaper. The following assumptions are considered for the sake of brevity: (1) The quadrotor is rigid and symmetrical; (2) Distribution of mass is uniform; (3) The suspension point of the payload is the geometric center of the quadrotor, which is also the point of gravity. The definition of coordinate system is shown in Fig. 5, and see Huo et al. (2019) for the coordinate transformation and the detailed information.

4.1 Quadrotor Slung System

The Lagrangian Equations can be described as

$$\frac{d}{dt} \left( \frac{\partial \Lambda}{\partial \dot{\rho}} \right) - \frac{\partial \Lambda}{\partial \rho} = 0$$

(31)

where $\rho$ is the variable that we concern, $\Lambda$ is Lagrangian operator which equals to kinetic energy minus potential energy.

Considering the influence of the payload on the quadrotor body, the translation model of the quadrotor body can be expressed as follows,

$$\begin{align*}
\dot{x} &= \frac{2M + m}{M + m} F_1 - \frac{m}{M + m} p^2 + \frac{m}{M + m} q^2 + \frac{m}{M + m} r^2 + mg \frac{z}{M + m} \\
\dot{y} &= \frac{2M + m}{M + m} F_1 - \frac{m}{M + m} \dot{p} + \frac{m}{M + m} \dot{q} + \frac{m}{M + m} \dot{r} \\
\dot{z} &= \frac{2M + m}{M + m} F_1 - \frac{m}{M + m} \dot{p} + \frac{m}{M + m} \dot{q} + \frac{m}{M + m} r^2 + mg \frac{z}{M + m}
\end{align*}$$

(32)

The rotation model can be expressed as

$$\begin{align*}
\phi &= \frac{I_x}{I_z} - \frac{I_y}{I_z} \dot{\psi} + \frac{I_y}{I_z} (F_1 - F_2) \\
\dot{\theta} &= \frac{I_x}{I_z} \dot{\psi} + \frac{I_y}{I_z} (F_1 - F_3) \\
\ddot{\psi} &= \frac{I_x}{I_z} \dot{\phi} + \frac{I_y}{I_z} (F_1 + F_3 - F_4 - F_2)
\end{align*}$$

(33)

where $x, y, z$ are the positions to the quadrotor, $p, q, r$ are the relative positions of the payload to the quadrotor. $\varphi,$
In this paper, inspired from the two-mode ZV and EI shaper, a new robust VI shaper is proposed. Compared with the existing shapers, the new shaper has the superiorities of good robustness and improved accuracy. In addition, it is more convenient considering the parameter...
Fig. 7. Comparison results under accurate model

Fig. 8. Results when $L$ is changed from 1 m to 1.5 m

Fig. 9. Comparison result of EI and VI when $L = 1$ m

Fig. 10. Comparison result of EI and VI when $L$ is changed to 1.5 m

Fig. 11. Comparison result of EI and VI when $L$ is changed to 0.6 m

Table 2. Vibration Comparison

<table>
<thead>
<tr>
<th>Parameter (m)</th>
<th>uncorrective EI</th>
<th>corrective EI</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>1.5</td>
<td>0.26</td>
<td>0.05</td>
<td>0.025</td>
</tr>
<tr>
<td>0.6</td>
<td>0.005</td>
<td>0.005</td>
<td>0.002</td>
</tr>
</tbody>
</table>

... tuning. Finally, the new command shaper is used in a quadrotor slung system, and its effectiveness and flexibility in design are illustrated by numerical simulations compared with other shapers. In future, this control method will be verified by experimental studies.

REFERENCES


