Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Economic Dispatch Cost Reduction in Box-based Robust Unit Commitment ⋆

Youngchae Cho * **Takayuki Ishizaki** * **Nacim Ramdani** ** **Jun-ichi Imura** *

* *Department of Systems and Control Engineering, School of Engineering, Tokyo Institute of Technology, Tokyo, Japan*
{cho,ishizaki,imura}@cyb.sc.e.titech.ac.jp
** *Univ. Oreans, INSA-CVL, PRISME EA 4229, F45072, Orleans, France* nacim.ramdani@univ-orleans.fr

Abstract: This paper proposes a box expanding method to reduce the economic dispatch (ED) cost in the box-based robust unit commitment model (BUC). As a non-anticipative robust unit commitment model for a power system under demand uncertainty, BUC co-optimizes the commitment schedule and the feasible set of the ED problem to minimize the total operating cost for the worst-case realization in a set of possible demand scenarios; the feasible set of the ED problem is modeled as a box to enable the non-anticipative real-time dispatch. Meanwhile, as BUC considers the worst-case total operating cost, the actual total operating cost may be unnecessarily high. In this paper, the box feasible set of the ED problem in BUC is expanded to a larger one via multi-objective optimization with a no-preference method. The expanded box forms a new feasible set of the ED problem, which increases the chance of reducing the actual ED cost and thus the actual total operating cost. Simulation results using 5-, 14-, and 30-bus test systems demonstrate the effectiveness and generality of the proposed method.

Keywords: optimization problems, power systems, stochastic programming, uncertainty, unit commitment problem.

1. INTRODUCTION

In power system operations, power supply and demand always have to be balanced. To meet the actual power demand that may deviate from any predicted value, the current industry procures reserves of dispatchable generators (DGs). The minimum reserve requirement is set by a system operator, e.g., to some percentage of the peak load or to the capacity of the largest DG (Hirst and Kirby (1999)). Mathematical scheduling methods for DGs based on such reliability criteria include the unit commitment (UC), where their optimal commitment schedule is found. The UC problem usually incorporates the economic dispatch (ED) problem, where the most economical power output of each DG in operation is obtained for a given power demand scenario. Meanwhile, the growing penetration of renewable energy sources has increased uncertainty in forecasting power demand that has to be supplied by DGs, or the "net demand," imposing challenges on UC.

To address the increasing net-demand uncertainty in UC, various stochastic programming techniques have been applied; Zheng et al. (2015) present a comprehensive review. Among them, robust optimization is one of the most frequently used techniques. For instance, Jiang et al. (2011) and Bertsimas et al. (2013) proposes a "two-stage robust UC" model to obtain a commitment schedule that minimizes the total operating cost for worst-case net-demand realization in a set of possible net-demand scenarios. Based on this two-stage robust UC model, Moreira et al. (2015) pursue global optimality of the commitment schedule under correlation of nodal demands; Zhao and Guan (2013) minimize the weighted sum of the worst-case and expected costs to further consider more probable net-demand scenarios; Cho et al. (2019b) minimize exactly one of the expected or worst-case cost while putting a limit on the other via a hard constraint so that the upper limit of the expected or worst-case cost can be easily specified.

However, the two-stage robust UC models ignore the non-anticipativity constraint in the ED problem. That is, the ED solution at each timeslot is obtained using a net-demand scenario over the entire planning horizon as if it is known at that timeslot. This is not realistic given that what is known at each timeslot is only the past and current net-demand realizations. Thus, the feasibility of a two-stage robust UC model does not necessarily imply that of the ED problem that is practically solved at each timeslot.

Some of the robust-optimization-based UC models which explicitly consider the non-anticipativity constraint in ED, i.e., which integrate the ED problem that does not require the future net-demand realization, are studied by Lorca and Sun (2017); Cobos et al. (2018); Li and Zhai (2019); Cho et al. (2019a). Lorca and Sun (2017) model the ED solution at each timeslot as an affine function of the past and current net-demand realizations so that the worst-case total operating cost is minimized. Cobos et al. (2018)

⋆ This work was supported by JST MIRAI Grant Number 18077648, Japan.

and Li and Zhai (2019) select the power output of each DG at each timeslot within a predetermined range that satisfies the non-anticipativity constraint. The range is co-optimized with the commitment status to minimize the total operating cost for a base-case net-demand scenario.

From the same point of view, Cho et al. (2019a) develop the "box-based robust UC" model (BUC) for a power system including energy storage systems (ESSs). In BUC, which is based on the two-stage robust UC model, a box inside the set of feasible dispatch schedules of DGs and ESSs is found as their new feasible set in the ED problem. As there is no dynamic constraint in the reformulated ED problem, the ED solution at each timeslot depends only on the net demand at that timeslot. Consequently, if such a box exists that has an ED solution for any net-demand scenario in a prescribed set over the planning horizon, then the feasibility of the ED problem at each timeslot is ensured. The box as a new feasible set is determined so that the worst-case total operating cost is minimized. The advantage of BUC over the affine-policy-based non-anticipative robust UC model proposed by Lorca and Sun (2017) is that the former can incorporate the $n-K$ security criterion regarding the DG outage. Furthermore, BUC generalizes the concept of finding the dispatch ranges proposed by Cobos et al. (2018) and Li and Zhai (2019) so that ESSs as well as DGs can be incorporated in UC.

Meanwhile, BUC can be regarded practically as a method for calculating the reserve requirement of each power source with its availability ensured, as similarly done by Makarov et al. (2011); Dvorkin et al. (2014); Nosair and Bouffard (2015). Makarov et al. (2011) and Dvorkin et al. (2014) analyze the generation capacity, ramping capability and ramping duration requirements using a three-dimensional box that encloses a given percentage of data points representing deviations of the total demand. Nosair and Bouffard (2015) generalize this box enclosing the total demand deviation to a cone, reducing the employment of unnecessarily costly reserves. The difference of BUC and the methods studied by Makarov et al. (2011); Dvorkin et al. (2014); Nosair and Bouffard (2015) is that BUC employs a single mathematical programming problem to co-optimize the commitment status and the reserve requirement while explicitly integrating the transmission capacity constraint of the power system.

Notably, BUC adopts the worst-case analysis, implying that the actual total operating cost may be unnecessarily high. In the present paper, BUC is extended to increase the chance of reducing the actual ED cost. Specifically, the box obtained by using BUC is expanded inside the original feasible operation set of power sources. This corresponds to enlarging a feasible set of an optimization problem. Thus, solving the ED problem with the expanded box as a new feasible set ensures that the ED solution found in the expanded box is never less economical than that found in the box obtained by BUC for any net-demand scenario. The proposed box expanding method is first described as a multi-objective optimization problem, which is then reformulated as a quadratic programming problem with a no-preference method.

The remainder of this paper is organized as follows. Section 2 explains BUC and raises the issue regarding ED cost reduction. Section 3 describes the proposed box expanding method for BUC. Section 4 discusses the simulation results. Finally, Section 5 concludes the paper.

2. PRELIMINARIES

In this section, BUC and the issue on ED cost reduction are explained. To this end, a power system including DGs and ESSs is modeled and the two-stage robust UC model is described first. The DG outage contingency is not considered for the sake of simplicity.

2.1 Power System Modeling

A power system consisting of N nodes and multiple branches is considered, whose indices are denoted by i and l, respectively. Without loss of generality, it is assumed that a DG, an ESS, a load and a renewable energy generator, all with the index i, are connected to node i. As mentioned earlier, the load and renewable energy generation levels are dealt with together as the net demand. Moreover, the DC power flow representation is used to model the transmission network. The planning horizon is made up of T timeslots of unit length, whose index is denoted by t. The net demand at node i and timeslot t is denoted by d_{it}; $\mathbf{d}$ and $\mathcal{D}$ represent the vector of d_{it}, as a net-demand scenario, and the set of possible net-demand scenarios, respectively. In this study, $\mathcal{D}$ is assumed to be a box, i.e., $\mathcal{D}=[\underline{\mathbf{d}},\overline{\mathbf{d}}]$ where $\underline{\mathbf{d}}$ and $\overline{\mathbf{d}}$ are the vector of the lower limits of d_{it} and that of the upper limits of d_{it}, respectively.

To model the operational constraints of the power sources, let binary variables u_{it}, v_{it} and w_{it} denote the on/off, start-up and shut-down status, respectively, of DG i at timeslot t; the vector of u_{it}, v_{it} and w_{it} as the commitment schedule and its feasible set are denoted by $\mathbf{u}$ and $\mathcal{U}$, respectively. Any commitment schedule $\mathbf{u}\in\mathcal{U}$ satisfies the minimum up/down times of the DGs. Let also real variables x_{it}, x^{i}_{it} and x^{o}_{it} denote the power output of DG i, the power input of ESS i and the power output of ESS i, respectively, all at timeslot t. To simplify the notation, let

$$z_{jt} := \begin{cases} x_{jt} & \text{if } 1\le j\le N \\ x^{\mathrm{i}}_{(j-N)t} & \text{if } N+1\le j\le 2N \\ x^{\mathrm{o}}_{(j-2N)t} & \text{if } 2N+1\le j\le 3N \end{cases}$$

and denote the vector of z_{jt} by $\mathbf{z}\in\mathbb{R}^{3NT}$ where $\mathbb{R}$ represents the real number set. Then, the feasible set of $\mathbf{z}$ as a function over $\mathcal{U}$ is defined as

$$\mathcal{Z}(\mathbf{u}) := \big\{\mathbf{z}\in\mathbb{R}^{3NT} : \underline{P}_i u_{it}\le x_{it}\le \overline{P}_i u_{it}, \quad \forall i,\forall t, \tag{1a}$$

$$x_{it}-x_{i(t-1)}\le R^{\mathrm{u}}_i\left(1+u_{i(t-1)}-u_{it}\right) + R^{\mathrm{su}}_i\left(2-u_{it}-u_{i(t-1)}\right), \quad \forall i,\forall t, \tag{1b}$$

$$x_{i(t-1)}-x_{it}\le R^{\mathrm{d}}_i\left(1-u_{i(t-1)}+u_{it}\right) + R^{\mathrm{sd}}_i\left(2-u_{it}-u_{i(t-1)}\right), \quad \forall i,\forall t \tag{1c}$$

$$0\le x^{\mathrm{i}}_{it}\le P^{\mathrm{in}}_i,\ \forall i,\forall t, \tag{1d}$$

$$0\le x^{\mathrm{o}}_{it}\le P^{\mathrm{out}}_i,\ \forall i,\forall t, \tag{1e}$$

$$\left. 0\le S^0_i+\sum_{\tau=1}^{t}\left(E^{\mathrm{in}}_i x^{\mathrm{i}}_{i\tau}-\frac{1}{E^{\mathrm{out}}_i}x^{\mathrm{o}}_{i\tau}\right)\le S_i,\forall i,\forall t\right\} \tag{1f}$$

where $\underline{P}_i$, $\overline{P}_i$, R^{u}_i, R^{su}_i, R^{d}_i, R^{sd}_i and x_{i0} are the minimum and maximum generation levels, ramp-up limit, start-up-ramp limit, ramp-down limit, shut-down-ramp limit and initial power output of DG i, respectively; P_i^{in}, P_i^{out}, S_i^0, E_i^{in}, E_i^{out}, and S_i are the maximum input power, maximum output power, initial stored energy, input efficiency, output efficiency and storage capacity of ESS i, respectively. The constraints (1a), (1b) and (1c) represent the feasible power output ranges, ramp-up limits and ramp-down limits, respectively; (1d), (1e) and (1f) represent the power input and output limits and storage limits, respectively. Although the set of feasible ESS operation schedules does not depend on $\mathbf{u}$, x_{it}^{i} and x_{it}^{o} are treated with x_{it} in the vector $\mathbf{z}$ for brevity.

Subsequently, to model the systemwide constraints depending on the net-demand realization, let $\mathcal{W}(\mathbf{d})$ denote the feasible set of dispatch schedules $\mathbf{z}$ in terms of the systemwide constraints for a net-demand scenario $\mathbf{d} \in \mathcal{D}$, defined as

$$\mathcal{W}(\mathbf{d}) := \Big\{\mathbf{z} \in \mathbb{R}^{3NT} :$$

$$-\overline{F}_l \leq \sum_i F_{il}\left(x_{it} - x_{it}^{\mathrm{i}} + x_{it}^{\mathrm{o}} - d_{it}\right) \leq \overline{F}_l,\ \forall l, \forall t, \tag{2a}$$

$$\sum_i \left(x_{it} - x_{it}^{\mathrm{i}} + x_{it}^{\mathrm{o}} - d_{it}\right) = 0,\ \forall t\Big\} \tag{2b}$$

where $\overline{F}_l$ and F_{il} are the maximum real power flow in branch l and DC power transfer distribution factor between node i and branch l, respectively. The constraint (2a) and (2b) represent the transmission capacity limits and the power balance equation, respectively.

2.2 Box-based Robust Unit Commitment Model

According to Bertsimas et al. (2013), the two-stage robust UC problem is formulated as

$$\min_{\mathbf{u}\in\mathcal{U}} \left\{\mathbf{c}_1^{\mathrm{T}}\mathbf{u} + \max_{\mathbf{d}\in\mathcal{D}} f\left(\mathcal{Z}(\mathbf{u}), \mathbf{d}\right)\right\} \tag{3}$$

where $\mathbf{c}_1$ is the cost coefficient vector associated with $\mathbf{u}$, including the no-load, start-up and shut-down costs of the DGs and

$$f(\mathcal{Z}, \mathbf{d}) := \min_{\mathbf{z}\in\mathcal{Z}\cap\mathcal{W}(\mathbf{d})} \mathbf{c}_2^{\mathrm{T}}\mathbf{z} \tag{4}$$

is the total ED cost for the feasible operation set $\mathcal{Z} \subset \mathbb{R}^{3NT}$ of the power sources and net-demand scenario $\mathbf{d} \in \mathcal{D}$ with $\mathbf{c}_2$ denoting the cost coefficient vector associated with $\mathbf{z}$ including the marginal generation costs of the DGs and the marginal input/outoput costs of the ESSs. In this paper, the problem (4) is assumed to be feasible for any $\mathcal{Z}$ and $\mathbf{d}$, which can be enforced by introducing penalty terms regarding the constraint (2). Then, implementing the solution $\mathbf{u}'$ to the problem (3) ensures that the actual ED cost is bounded by $\max_{\mathbf{d}\in\mathcal{D}} f\left(\mathcal{Z}(\mathbf{u}'), \mathbf{d}\right)$.

However, the non-anticipativity in ED is ignored in the problem (3). That is, the ED solution at each timeslot depends on the net-demand realization over the entire planning horizon, which includes unknown future information. This is because the ED problem (4) incorporated in (3) is a multi-period problem that have the dynamic constraints (1b), (1c) and (1f). If there is no dynamic constraints in (4), the ED solution at each timeslot depends only on the net demand at that timeslot. Furthermore, if $\mathcal{Z}$ is a box, then (4) has no dynamic constraints. Accordingly, in BUC, a box $\mathcal{Z}^{\mathrm{box}} = [\underline{\mathbf{z}}, \overline{\mathbf{z}}] \subseteq \mathcal{Z}(\mathbf{u})$ is obtained simultaneously with $\mathbf{u}$ to meet the non-ancitipativity constraint in ED, where $\underline{\mathbf{z}}$ and $\overline{\mathbf{z}}$ denote the vectors of the minimum and maximum allowable values $\underline{z}_{jt}$ and $\overline{z}_{jt}$ of z_{jt}, respectively, to be optimized. The box $\mathcal{Z}^{\mathrm{box}}$ then substitutes for $\mathcal{Z}(\mathbf{u})$ in (3) as follows:

$$f\left(\mathcal{Z}^{\mathrm{box}}, \mathbf{d}\right) = \min_{\mathbf{z}\in\mathcal{W}(\mathbf{d})} \mathbf{c}_2^{\mathrm{T}}\mathbf{z} \quad \text{s.t.} \quad \underline{\mathbf{z}} \leq \mathbf{z} \leq \overline{\mathbf{z}}.$$

This makes the ED problems at different timeslots temporally uncorrelated and ensures the non-anticipativity. Consequently, the BUC problem is written as

$$\min_{\mathbf{u}\in\mathcal{U}, \mathcal{Z}^{\mathrm{box}}\subseteq\mathcal{Z}(\mathbf{u})} \left\{\mathbf{c}_1^{\mathrm{T}}\mathbf{u} + \max_{\mathbf{d}\in\mathcal{D}} f\left(\mathcal{Z}^{\mathrm{box}}, \mathbf{d}\right)\right\}. \tag{5}$$

The problem (5) can be rewritten as an MILP problem and be solved via a cutting-plane algorithm (see, e.g., Zeng and Zhao (2013)). Let $\mathbf{u}^*$ and $\mathcal{Z}^{\mathrm{box}*}$ denote the solution to (5).

2.3 Issue on the Actual ED Cost

While bounded by $\max_{\mathbf{d}\in\mathcal{D}} f\left(\mathcal{Z}^{\mathrm{box}*}, \mathbf{d}\right)$, the actual ED cost in BUC may be unnecessarily high. This is because, regarding any net-demand scenario other than the worst-case ones, only the feasibility of the ED problem is ensured without the optimality of the associated solution evaluated. However, evaluating it explicitly when determining $\mathcal{Z}^{\mathrm{box}}$ through a single optimization problem may increase the worst-case total operating cost. One possible way to increase the chance of reducing the actual ED cost with the worst-case total operating cost still minimized is to adopt as a feasible set in the ED problem (4) a larger box $\mathcal{Z}^{\mathrm{exp}} \subseteq \mathcal{Z}(\mathbf{u}^*)$ containing $\mathcal{Z}^{\mathrm{box}*}$. This corresponds to enlarging a feasible set of an optimization problem, whose mathematical description is given in the following section.

3. BOX EXPANDING METHOD

3.1 Box Expanding Problem

The optimal value of a constrained minimization problem may be reduced when its feasible set is expanded. Based on this idea, the proposed method increases the chance of reducing the actual ED cost assumed in BUC by obtaining another box $\mathcal{Z}^{\mathrm{exp}} = [\underline{\mathbf{z}}^{\mathrm{exp}}, \overline{\mathbf{z}}^{\mathrm{exp}}]$ including $\mathcal{Z}^{\mathrm{box}*}$ and adopting it as a feasible set of the ED problem instead of $\mathcal{Z}^{\mathrm{box}*}$; $\underline{\mathbf{z}}^{\mathrm{exp}}$ and $\overline{\mathbf{z}}^{\mathrm{exp}}$ denote the vectors of the revised minimum and maximum allowable values $\underline{z}_{jt}^{\mathrm{exp}}$ and $\overline{z}_{jt}^{\mathrm{exp}}$ of z_{jt}, respectively, to be optimized. The following proposition summarizes how this box expanding method contributes to the ED cost reduction.

Proposition 1. For any $\mathbf{u} \in \mathcal{U}$, consider the problem (4) and a box $\mathcal{Z}^{\mathrm{exp}}$ such that $\mathcal{Z}^{\mathrm{box}*} \subseteq \mathcal{Z}^{\mathrm{exp}} \subseteq \mathcal{Z}(\mathbf{u})$. Then, it holds that

$$f\left(\mathcal{Z}^{\mathrm{exp}}, \mathbf{d}\right) \leq f\left(\mathcal{Z}^{\mathrm{box}*}, \mathbf{d}\right), \quad \forall \mathbf{d} \in \mathcal{D}.$$

In the meantime, the ED cost reduction for any $\mathbf{d} \in \mathcal{D}$, i.e., $f\left(\mathcal{Z}^{\mathrm{exp}}, \mathbf{d}\right) - f\left(\mathcal{Z}^{\mathrm{box}*}, \mathbf{d}\right)$, is likely to increase with the size of the interval $[\underline{z}_{jt}^{\mathrm{exp}}, \overline{z}_{jt}^{\mathrm{exp}}]$ for any j and t. Thus, the problem to optimize $\mathcal{Z}^{\mathrm{exp}}$ can be formulated as a multi-objective optimization problem

$$\max_{\underline{\mathbf{z}}^{\mathrm{exp}}, \overline{\mathbf{z}}^{\mathrm{exp}}} \overline{\mathbf{z}}^{\mathrm{exp}} - \underline{\mathbf{z}}^{\mathrm{exp}} \tag{6a}$$

$$\text{s.t.} \quad \mathcal{Z}^{\mathrm{box}*} \subseteq \mathcal{Z}^{\mathrm{exp}} \subseteq \mathcal{Z}(\mathbf{u}^*) \tag{6b}$$

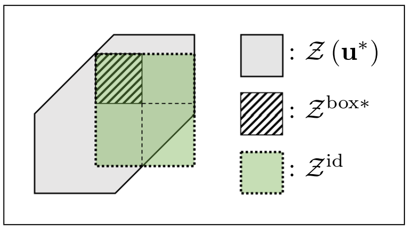

Fig. 1. Inclusion relation of $\mathcal{Z}(\mathbf{u}^*)$, $\mathcal{Z}^{\text{box}*}$, and $\mathcal{Z}^{\text{id}}$.

where the objective function (6a) indicates the vector of the widths of allowable dispatch ranges and the constraint (6b) ensure that the worst-case ED cost is minimized. Since it is impossible to maximize simultaneously all the entries in (6a) unless $\mathcal{Z}(\mathbf{u}^*)$ is a box, the box expanding problem belongs to multi-objective optimization.

As usual in many multi-objective optimization problems, there exist multiple approaches of different criteria to the problem (6). In this paper, it is assumed that decision makers do not show any preference information and (6) is solved via a no-preference method (Miettinen (1998)). Specifically, (6) is converted to the single-objective optimization problem

$$\min_{\underline{\mathbf{z}}^{\text{exp}},\overline{\mathbf{z}}^{\text{exp}}} \delta\left(\mathcal{Z}^{\text{id}}, \mathcal{Z}^{\text{exp}}\right) \text{ s.t. } \mathcal{Z}^{\text{box}*} \subseteq \mathcal{Z}^{\text{exp}} \subseteq \mathcal{Z}(\mathbf{u}^*) \quad (7)$$

where $\delta(\mathcal{A},\mathcal{B})$ is the distance of two K-dimensional boxes $\mathcal{A} = \prod_{k=1}^{K}[\underline{a}_k, \overline{a}_k]$ and $\mathcal{B} = \prod_{k=1}^{K}[\underline{b}_k, \overline{b}_k]$ for an arbitrary natural number K, defined in this paper as

$$\delta(\mathcal{A},\mathcal{B}) := \sum_{k=1}^{K}\left\{\left(\overline{a}_k - \overline{b}_k\right)^2 + \left(\underline{a}_k - \underline{b}_k\right)^2\right\}.$$

Also, $\mathcal{Z}^{\text{id}} = [\underline{\mathbf{z}}^{\text{id}}, \overline{\mathbf{z}}^{\text{id}}]$ represents the box as the ideal maximizer of (6) where $\underline{\mathbf{z}}^{\text{id}}$ and $\overline{\mathbf{z}}^{\text{id}}$ are the vectors of $\underline{z}_{jt}^{\text{id}}$ and $\overline{z}_{jt}^{\text{id}}$ that are associated with the entries $\underline{z}_{jt}^{\text{exp}}$ and $\overline{z}_{jt}^{\text{exp}}$ of $\underline{\mathbf{z}}_{jt}^{\text{exp}}$ and $\overline{\mathbf{z}}_{jt}^{\text{exp}}$ in (6). That is, for each pair of j and t, $\underline{z}_{jt}^{\text{id}}$ and $\overline{z}_{jt}^{\text{id}}$ are obtained by solving the single-objective optimization problem

$$\max_{\underline{\mathbf{z}}^{\text{exp}},\overline{\mathbf{z}}^{\text{exp}}} \overline{z}_{jt}^{\text{exp}} - \underline{z}_{jt}^{\text{exp}} \quad \text{s.t.} \quad \mathcal{Z}^{\text{box}*} \subseteq \mathcal{Z}^{\text{exp}} \subseteq \mathcal{Z}(\mathbf{u}^*).$$

The inclusion relation of the sets $\mathcal{Z}(\mathbf{u})$, $\mathcal{Z}^{\text{box}*}$ and $\mathcal{Z}^{\text{id}}$ is illustrated in Fig. 1. Let $\mathcal{Z}^{\text{exp}*}$ denote the solution of the problem (7). According to Proposition 1, the actual ED cost can be reduced by solving the ED problem over $\mathcal{Z}^{\text{exp}*}$ instead of $\mathcal{Z}^{\text{box}*}$.

3.2 Solution Method

The problem (7) is a typical quadratic programming problem since all the constraints can be expressed as linear inequalities. Specifically, suppose that

$$\mathcal{Z}(\mathbf{u}^*) = \left\{\mathbf{z} \in \mathbb{R}^{3NT} : A\mathbf{z} \leq B\right\}$$

where A and B are a given matrix and a given vector, respectively. Then, the constraint $\mathcal{Z}^{\text{exp}} \subseteq \mathcal{Z}(\mathbf{u}^*)$ in (7) can be represented as

$$A\underline{\mathbf{z}}^{\text{exp}} + A^{+}\left(\overline{\mathbf{z}}^{\text{exp}} - \underline{\mathbf{z}}^{\text{exp}}\right) \leq B,$$

where A^{+} is the matrix any element of which is equal to the corresponding element of A, if the latter is positive, or zero, otherwise (Bemporad et al. (2004)). Therefore, (7) can be solved with any quadratic programming algorithm.

In summary, the proposed method is used to expand $\mathcal{Z}^{\text{box}*}$ to $\mathcal{Z}^{\text{exp}*}$, which corresponds to solving (7), after BUC is used to obtain $\mathbf{u}^*$ and $\mathcal{Z}^{\text{box}*}$, which corresponds to solving (5). The performance of the proposed method is discussed in the following section.

Remark. The net-demand scenarios whose associated ED cost can be reduced by the proposed method are unknown a priori. However, the proposed method is still necessary for increasing the chance of reducing the actual ED cost even when another box optimization method targeting a particular scenario set, e.g., a specific quantile in a given probability distribution, is applied. This is because the number of net-demand scenarios modeled in the ED problem has to be limited to make it practically solvable, i.e., it is difficult to consider each and every net-demand scenario in a continuous set. The actual ED cost for the rest of the net-demand scenarios that are not explicitly modeled in the ED problem can be reduced by the proposed method. Thus, the proposed method can be regarded as an essential technique for refining any box as a feasible set of the ED problem under the net-demand uncertainty.

4. NUMERICAL SIMULATIONS

In this section, the effectiveness of the proposed method is demonstrated via 30 numerical simulation sets from S1 to S30, where modified versions of the MATPOWER 5-bus test system (S1-S10), IEEE 14-bus test system (S11-S20), IEEE 30-bus test system (S21-S30) are used (Zimmerman et al. (2011)). The 5-bus test system has three loads, five DGs from G_1 to G_5 and ESS E_1. The 14-bus test system has 11 loads and five DGs from G_6 to G_{10} and ESS E_2, where the transmission line capacities are set to 100 MW. The 30-bus test system has 16 loads and six DGs from G_{11} to G_{16} and ESS E_3. The operational parameters of the DGs and ESSs are listed in TABLE 1 and TABLE 2, respectively, with the node indices omitted. In TABLE 1, SU, SD and C represent the start-up, shut-down and marginal generation costs, respectively. Any DG is initially turned off. In TABLE 2, C^{in} and C^{out} represent the marginal power input and output costs, respectively.

In each simulation set, to construct the net-demand scenario set, nominal load values are randomly generated over the planning horizon of 24 timeslots at intervals of an hour. Subsequently, the maximum and minimum values of each load at each timeslot are set to $(1 \pm \alpha)$ times the nominal values, which form the net-demand scenario set. For this scenario set, the feasible set $\mathcal{Z}^{\text{box}*}$ is obtained using BUC, which is then expanded to the feasible set $\mathcal{Z}^{\text{exp}*}$ by solving (7). Subsequently, the ED cost over $\mathcal{Z}^{\text{box}*}$ and that over $\mathcal{Z}^{\text{exp}*}$ are calculated for 100 randomly generated load scenarios, whose differences are the cost reductions achieved by the proposed method. In the first five out of the 10 simulations for each test system, different nominal load scenarios are used with α set commonly to 0.2. In the remaining five, a single nominal load scenario is used with different values of $\alpha = 0, 1, 0.15, \ldots, 0.3$.

TABLE 3 shows the average ED cost reduction and the average ED cost reduction ratio achieved by applying the proposed method in each simulation set. The proposed method is implemented, i.e., the problem (7) is solved, in less than a second for any case. From TABLE 3, it is observed that the average ED costs are reduced in all the simulation sets. This is because the feasible sets $\mathcal{Z}^{\exp*}$ of the ED problem contain more economical dispatch schedules than $\mathcal{Z}^{\text{box}*}$. For example, in simulation S4 using the 5-bus test system, the power outputs of G_2 with marginal cost of 15 decreases averagely, while those of G_5 with marginal cost of 10 increases averagely after their feasible sets are expanded, thus reducing the ED cost (Fig. 2). The operations of the other power sources do not change remarkably in simulation S4.

In the meantime, no clear relationship is observed between the ED cost reduction and the net-demand scenario set parameters. For instance, the ED cost reduction and the total nominal load value are not positively nor negatively related in any case. The ED cost reduction and the maximum relative prediction error α are related positively only in simulations S16-S20 using the 14-bus test system, which is hardly generalized. This implies that the "shape" of the box obtained in BUC is highly arbitrary and thus the efficiency of the proposed method depends on the system parameters and the net-demand scenario set. Nonetheless, the proposed method is still necessary for reducing the actual ED cost otherwise incurred by BUC as it is difficult to consider the ED costs for all the net-demand scenarios in a continuous set simultaneously in BUC.

5. CONCLUSION

While BUC ensures power system reliability under the net-demand uncertainty with the worst-case total operating cost minimized, the actual ED cost may be unnecessarily high. By using the proposed box expanding method based on multi-objective optimization, the actual ED cost can be reduced. The proposed method is applicable to any box as an ED feasible set that is determined to minimize the ED cost for a set of net-demand scenarios. The numerical simulation results demonstrated the effectiveness and generality of the proposed method. In this paper, the no-preference method was used as a multi-objective optimization technique. To improve the proposed method, various preference-based multi-objective optimization techniques will be tested in future research, e.g., considering the size of net-demand uncertainty at each node and timeslot.

Table 1. Dispatchable generator parameters.

DG	SU ($\times 10^3$)	SD	$\underline{P}$ (MW)	$\overline{P}$ (MW)	$R^{\text{d}}, R^{\text{sd}}$ (MW/h)	$R^{\text{u}}, R^{\text{su}}$ (MW/h)	C (/MWh)
G_1	14	12.6	4	40	23.4	23.4	14
G_2	15	13.5	17	170	99.45	99.45	15
G_3	30	27	52	520	304.2	304.2	30
G_4	40	36	20	200	117	117	40
G_5	10	9	60	600	351	351	10
G_6	20	18	33.24	332.4	194.45	194.45	20
G_7	20	18	14	140	81.9	81.9	20
G_8	40	36	10	100	58.5	58.5	40
G_9	40	36	10	100	58.5	58.5	40
G_{10}	40	36	10	100	58.5	58.5	40
G_{11}	2	1.8	0	80	64	64	2
G_{12}	1.75	1.6	0	80	64	64	1.75
G_{13}	1	0.9	0	50	40	40	1
G_{14}	3.25	2.9	0	55	44	44	3.25
G_{15}	3	2.7	0	30	24	24	3
G_{16}	3	2.7	0	40	32	32	3

Table 2. Energy storage system parameters.

ESS	P^{in}, P^{out} (MW)	E^{in}, E^{out}	S^0 (MWh)	S (MWh)	C^{in} (/MWh)	C^{out} (/MWh)
E_1	6	0.8	0	60	4	4.8
E_2	3.324	0.8	0	33.24	4	4.8
E_3	0.8	0.8	0	8	0.325	0.39

REFERENCES

Bemporad, A., Filippi, C., and Torrisi, F.D. (2004). Inner and outer approximations of polytopes using boxes. *Computational Geometry*, 27(2), 151–178.

Bertsimas, D., Litvinov, E., Sun, X.A., Zhao, J., and Zheng, T. (2013). Adaptive robust optimization for the security constrained unit commitment problem. *IEEE Transactions on Power Systems*, 28(1), 52–63.

Cho, Y., Ishizaki, T., Ramdani, N., and Imura, J. (2019a). Box-based temporal decomposition of multi-period economic dispatch for two-stage robust unit commitment. *IEEE Transactions on Power Systems*, 34(4), 3109–3118.

Cho, Y., Ishizaki, T., and Imura, J.I. (2019b). Hybrid method of two-stage stochastic and robust unit commitment. In *2019 18th European Control Conference (ECC)*, 922–927. IEEE.

Cobos, N.G., Arroyo, J.M., Alguacil, N., and Street, A. (2018). Network-constrained unit commitment under significant wind penetration: A multistage robust approach with non-fixed recourse. *Applied Energy*, 232, 489–503.

Dvorkin, Y., Kirschen, D.S., and Ortega-Vazquez, M.A. (2014). Assessing flexibility requirements in power systems. *IET Generation, Transmission & Distribution*, 8(11), 1820–1830.

Hirst, E. and Kirby, B. (1999). Technical and market issues for operating reserves. *The Electricity Journal*, 12(2), 36–48.

Jiang, R., Wang, J., and Guan, Y. (2011). Robust unit commitment with wind power and pumped storage hydro. *IEEE Transactions on Power Systems*, 27(2), 800–810.

Li, X. and Zhai, Q. (2019). Multi-stage robust transmission constrained unit commitment: A decomposition framework with implicit decision rules. *International Journal of Electrical Power & Energy Systems*, 108, 372–381.

Lorca, A. and Sun, X.A. (2017). Multistage robust unit commitment with dynamic uncertainty sets and energy storage. *IEEE Transactions on Power Systems*, 32(3), 1678–1688.

Makarov, Y.V., Etingov, P.V., Ma, J., Huang, Z., and Subbarao, K. (2011). Incorporating uncertainty of wind power generation forecast into power system operation, dispatch, and unit commitment procedures. *IEEE Transactions on Sustainable Energy*, 2(4), 433–442.

Miettinen, K. (1998). No-preference methods. In *Nonlinear Multiobjective Optimization*, 67–76. Springer.

Moreira, A., Street, A., and Arroyo, J.M. (2015). Energy and reserve scheduling under correlated nodal demand uncertainty: An adjustable robust optimization

Table 3. Average economic dispatch cost reduction achieved by the proposed method.

Simulation (5 bus)	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Total nominal load ($\times 10^4$ MW)	1.7538	1.8972	1.8522	1.9008	1.8978	1.9903	1.9903	1.9903	1.9903	1.9903
α	0.2	0.2	0.2	0.2	0.2	0.1	0.15	0.2	0.25	0.3
Avg. cost ($\mathcal{Z}^{\text{box}*}$) ($\times 10^5$)	2.3893	2.6599	2.5764	2.8971	2.6897	2.8291	2.8118	2.8713	2.9472	3.0192
Avg. cost ($\mathcal{Z}^{\text{exp}*}$) ($\times 10^5$)	2.3739	2.6407	2.5218	2.8348	2.6722	2.7828	2.8019	2.8517	2.9409	3.0001
Avg. cost reduction ($\times 10^3$)	1.4653	1.9204	5.4587	6.2276	1.7579	4.6314	0.9898	1.9572	0.6356	1.9130
Avg. reduction ratio (%)	0.6195	0.7227	2.1192	2.1509	0.6548	1.6371	0.3521	0.6826	0.2160	0.6345
Simulation (14 bus)	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20
Total nominal load ($\times 10^3$ MW)	5.6319	5.6091	5.5257	5.6195	5.5340	7.0455	7.0455	7.0455	7.0455	7.0455
α	0.2	0.2	0.2	0.2	0.2	0.1	0.15	0.2	0.25	0.3
Avg. cost ($\mathcal{Z}^{\text{box}*}$) ($\times 10^5$)	1.1532	1.1518	1.1374	1.1608	1.1324	1.5343	1.5675	1.5888	1.6099	1.6791
Avg. cost ($\mathcal{Z}^{\text{exp}*}$) ($\times 10^5$)	1.1482	1.1411	1.1277	1.1459	1.1263	1.4837	1.4985	1.5239	1.5376	1.5508
Avg. cost reduction ($\times 10^3$)	0.4969	1.0701	0.9718	1.4979	0.6112	5.0606	6.9052	6.4937	7.2254	12.8276
Avg. reduction ratio (%)	0.4310	0.9293	0.8545	1.2905	0.5397	3.2991	4.4067	4.0886	4.4900	7.6446
Simulation (30 bus)	S21	S22	S23	S24	S25	S26	S27	S28	S29	S30
Total nominal load ($\times 10^3$ MW)	2.4182	2.8960	2.9096	2.9018	2.8965	2.3891	2.3891	2.3891	2.3891	2.3891
α	0.2	0.2	0.2	0.2	0.2	0.1	0.15	0.2	0.25	0.3
Avg. cost ($\mathcal{Z}^{\text{box}*}$) ($\times 10^3$)	3.3590	4.2059	4.2363	4.2271	4.2273	3.2906	3.3024	3.2966	3.2958	3.2845
Avg. cost ($\mathcal{Z}^{\text{exp}*}$) ($\times 10^3$)	3.3446	4.2048	4.2342	4.2236	4.2090	3.2852	3.2841	3.2828	3.2809	3.2824
Avg. cost reduction	14.3557	1.1140	2.1922	3.5302	18.3650	5.4196	18.2689	13.7883	14.9056	2.1756
Avg. reduction ratio (%)	0.4274	0.0265	0.0517	0.0836	0.4345	0.1647	0.5534	0.4182	0.4524	0.0662

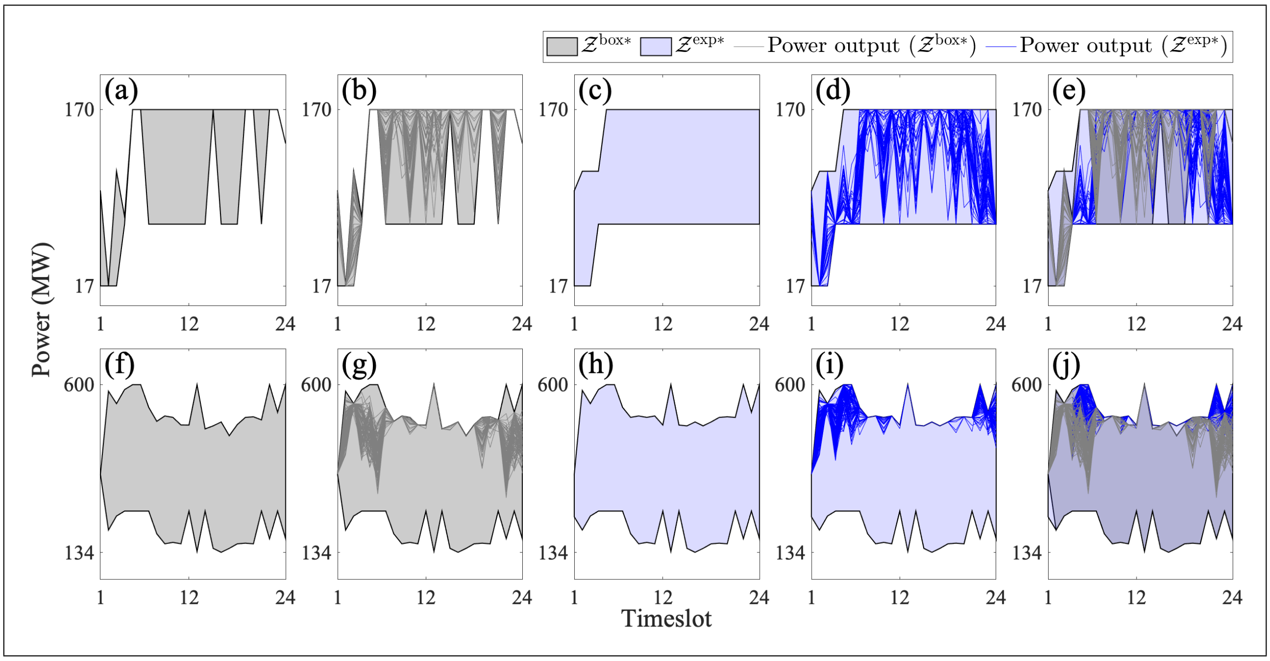

Fig. 2. The feasible sets $\mathcal{Z}^{\text{box}*}$ and $\mathcal{Z}^{\text{exp}*}$ of power outputs, and the actual power outputs as ED solution for the 100 randomly generated net-demand scenarios. (a) $\mathcal{Z}^{\text{box}*}$ of G_2. (b) The power outputs (grey lines) of G_2 in $\mathcal{Z}^{\text{box}*}$. (c) $\mathcal{Z}^{\text{exp}*}$ of G_2. (d) The power outputs (blue lines) of G_2 in $\mathcal{Z}^{\text{exp}*}$. (e) The overlap of (b) and (d). (f) $\mathcal{Z}^{\text{box}*}$ of G_5. (g) The power outputs (grey lines) of G_5 in $\mathcal{Z}^{\text{box}*}$. (h) $\mathcal{Z}^{\text{exp}*}$ of G_5. (i) The power outputs (blue lines) of G_5 in $\mathcal{Z}^{\text{exp}*}$. (j) The overlap of (g) and (i). As can be seen from (e) and (j), the feasible operation set is expanded by the proposed method and the more economical dispatch schedules are selected.

approach. *International Journal of Electrical Power & Energy Systems*, 72, 91–98.

Nosair, H. and Bouffard, F. (2015). Flexibility envelopes for power system operational planning. *IEEE Transactions on Sustainable Energy*, 6(3), 800–809.

Zeng, B. and Zhao, L. (2013). Solving two-stage robust optimization problems using a column-and-constraint generation method. *Operations Research Letters*, 41(5), 457–461.

Zhao, C. and Guan, Y. (2013). Unified stochastic and robust unit commitment. *IEEE Transactions on Power Systems*, 28(3), 3353–3361.

Zheng, Q.P., Wang, J., and Liu, A.L. (2015). Stochastic optimization for unit commitment—a review. *IEEE Transactions on Power Systems*, 30(4), 1913–1924.

Zimmerman, R.D., Murillo-Sánchez, C.E., Thomas, R.J., et al. (2011). Matpower: Steady-state operations, planning, and analysis tools for power systems research and education. *IEEE Transactions on power systems*, 26(1), 12–19.