

A Computationally Efficient Predictive Cruise Control for Automated Electric Vehicles

Shiyong Dong^{*,**} Bingzhao Gao^{*} Qifang Liu^{**} Jiaqi Liu^{*}
Hong Chen^{*,**,***}

^{*} State Key Laboratory of Automotive Simulation and Control, Jilin University Nanling Campus, Changchun 130025, China

^{**} Department of Control Science and Engineering, Jilin University, PR China

^{***} Clean Energy Automotive Engineering Center, Tongji University, Shanghai, 201804, China

Abstract: This paper proposes an energy-efficient predictive cruise control (PCC) system to cope with range anxiety of automated electric vehicles. The proposed approach is formulated as an optimal control problem to realize better energy efficiency and ensure safe inter-vehicle distance. To improve computational efficiency, a fast algorithm combining Gauss pseudospectral method (GPM) and moving horizon control (MHC) is introduced to solve this nonlinear optimal problem. The comparative simulation results reveal that the energy economy of the PCC system is improved about 4.1%, and its computation time is reduced compared with the Euler method while ensuring the same accuracy.

Keywords: Predictive cruise control, Range anxiety, Automated electric vehicles, Gauss pseudospectral method, Moving horizon control.

1. INTRODUCTION

In recent years, the greenhouse gas emission and global warming are becoming increasingly serious problems. There have been a number of approaches to reduce fuel consumption and carbon dioxide emission for saving global environment Barkenbus (2010). With the remarkable development in eco-driving, the energy efficient control of electric vehicles is becoming a hot research field.

However, the limited cruising range presents the major customer and market concern impeding trends towards electric mobility. To improve energy efficiency of electric vehicles, the vehicle connectivity and driving automation are becoming the important solutions in the field of range extension in EVs. Several studies have shown that the eco-driving can be beneficial for reducing energy consumption of vehicles (Vahidi and Sciarretta (2018) Martinez et al. (2016)). In the paper Dib et al. (2014), the analytical energy-optimal speed trajectory is obtained based on the constraints from the road speed limit and surrounding traffic information. Eco-driving techniques are discussed and formulated as an optimal control problem in order to improve energy efficiency over a time and distance horizon. The authors in Bertoni et al. (2017) proposed an energy-saving cooperative adaptive cruise control (eco-CACC) that consists of the minimization of the autonomous electric vehicles consumption over a time horizon. With an objective of reducing computation time, a novel method using linear MPC for computationally efficient velocity control

for EVs is formulated (Morlock et al. (2019)). With the same purpose, the paper Hamednia et al. (2018) presented an approach for computation of energy-efficient velocity of vehicles driving in a hilly terrain using the information about upcoming topography over a long horizon.

Due to the challenging problem of high computational effort with long-horizon traffic information usage, the traditional optimization algorithm such as dynamic programming (DP) (Bertsekas et al. (1995)) is difficult to meet the real-time implementation. The work (Schwickart et al. (2016)) proposed a real-time capable model-predictive cruise controller for EVs aiming at reducing charge consumption while tracking the given reference speed. A fast algorithm combining the Gauss pseudospectral method and model predictive control is proposed to hybrid electric vehicles (HEVs) energy management problem (Guo et al. (2017)). The same computational accuracy can be ensured using fewer discrete points.

In this paper, for the automated EVs car-following problem, a predictive cruise control is presented to find optimal motor torque and brake force while ensuring safety and energy economy, as shown in Fig. 1. Furthermore, to reduce computational burden, a fast algorithm is proposed combining Gauss pseudospectral method and model horizon control strategy, which aiming at finding the optimal solution while taking into account the preceding vehicle constraints.

The reminder of this paper is organized as follows. In Section 2, a system model and problem formulation is developed. Section 3 describes the computationally effec-

* Corresponding author: Hong Chen (chenh@jlu.edu.cn).

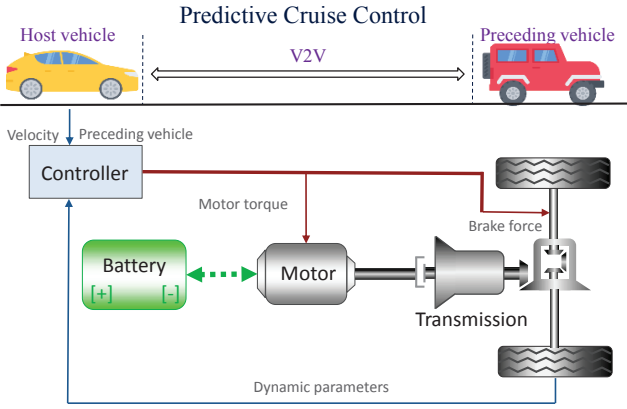


Fig. 1. Structure of the proposed energy-efficient PCC for EV.

tive algorithm based on the Gauss pseudospectral method and model horizon control for finding optimal solution. In Section 4, the effectiveness of the energy-efficient PCC is evaluated through the comparative simulations. Finally, conclusion and future work are given in Section 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

This section describes the formulation of the predictive cruise control problem. The car motion used in this paper is given by an EV model with a single speed transmission described below in detail.

2.1 Vehicle Model

According to the Newton's second law, the longitudinal dynamic motion of a vehicle can be given as

$$\begin{cases} \frac{ds}{dt} = v \\ M_{\text{veh}} \frac{dv}{dt} = T_m \frac{i_g \eta_t}{r_w} - F_b - F_r - F_{\text{aero}}, \end{cases} \quad (1)$$

here s is the traveling distance, i_g is the gear ratio, v is the longitudinal vehicle speed, T_m is the motor torque, F_b is brake force, η_t is the drive train total efficiency, M_{veh} is the total mass of the vehicle. The rolling resistance and the gradient resistance, F_r is determined by the road slope α , can be described as

$$F_r = M_{\text{veh}}(gf \cos(\alpha) + g \sin(\alpha)), \quad (2)$$

where g is the gravitational constant, f is the rolling resistance coefficient. The aerodynamic drag resistance, F_{aero} , which is the other resistance at high velocities

$$F_{\text{aero}} = \frac{1}{2} \rho c_d A_f v^2, \quad (3)$$

where ρ is the air density, c_d is the drag coefficient, A_f is the frontal area of the vehicle.

2.2 Charge Consumption Model

For analytical formulation of the optimal control problem, a simplified approach is introduced to model the total demanding power. The power is modeled as a function

of the motor torque T_m , the motor speed n_m , and the machine efficiency η_m of the motor, given by

$$P_m(t) = T_m(t)n_m(t)\eta_m(T_m, n_m)^{-\text{sign}(T_m(t))}, \quad (4)$$

where $T_m(t)n_m(t)$ denotes the mechanical power, the machine efficiency η_m of the motor is a function of both T_m and n_m , as shown in Fig. 2. For online implementation, the approximate closed-form expressions are used instead of equation (4). In this paper, a two-dimensional polynomial expression is adopted, as

$$P_m = a_1 n_m + a_2 n_m T_m + a_3 T_m + a_4 n_m^2 + a_5 n_m T_m^2, \quad (5)$$

where $a_i, i = 1, 2, \dots, 5$ are the fitting coefficients, and the values are $\{-2.892 \times 10^{-5}, 1.044 \times 10^{-4}, 2.059 \times 10^{-3}, 1.315 \times 10^{-8}, 8.437 \times 10^{-8}\}$. The motor speed n_m (r/min) is determined by the gear ratio i_g and the vehicle speed v

$$n_m = \frac{30i_g v}{\pi r_w}. \quad (6)$$

It follows that the power can be formulated as the T_m and the v as follows

$$P_m = b_1 v + b_2 v T_m + b_3 T_m + b_4 v^2 + b_5 v T_m^2, \quad (7)$$

where $b_i, i = 1, 2, \dots, 5$ are determined by $a_i, i = 1, 2, \dots, 5$ and the equation (6).

The battery dynamics in the electric vehicle are formulated through an internal resistance model. According to the Ohm's law, the battery current I can be calculated as

$$I = \frac{V_{oc}}{2R_b} - \sqrt{\left(\frac{V_{oc}}{2R_b}\right)^2 - \frac{P_m}{R_b}}, \quad (8)$$

here V_{oc} is the open circuit voltage, R_b is the internal resistance, P_m is the motor power. Then, the state of charge SOC is determined by

$$\frac{dSOC}{dt} = -\frac{I}{Q_{bat}}, \quad (9)$$

where Q_{bat} is the battery nominal capacity. In this paper, the efficiency of the battery η_b is assumed equal to 1.

2.3 Problem Formulation

The objective of predictive cruise control system is to minimize the energy consumption as much as possible while maintaining a safe inter-vehicle distance. Therefore, the energy consumption over the prediction horizon with a terminal penalty is the cost function, as

$$J = \sum_{k=1}^N C(x(k), u(k)) \Delta t + \varphi(v(N+1) - v_f)^2. \quad (10)$$

Here $C(x(k), u(k))$ is defined as

$$C(x(k), u(k)) = w_f P_m(k), \quad (11)$$

where state variable $x := \{s, v\}$, control input $u := \{T_m, F_b\}$, w_f is the weight coefficient. The terminal penalty is used to guarantee the safe inter-vehicle distance in the car-following scenario.

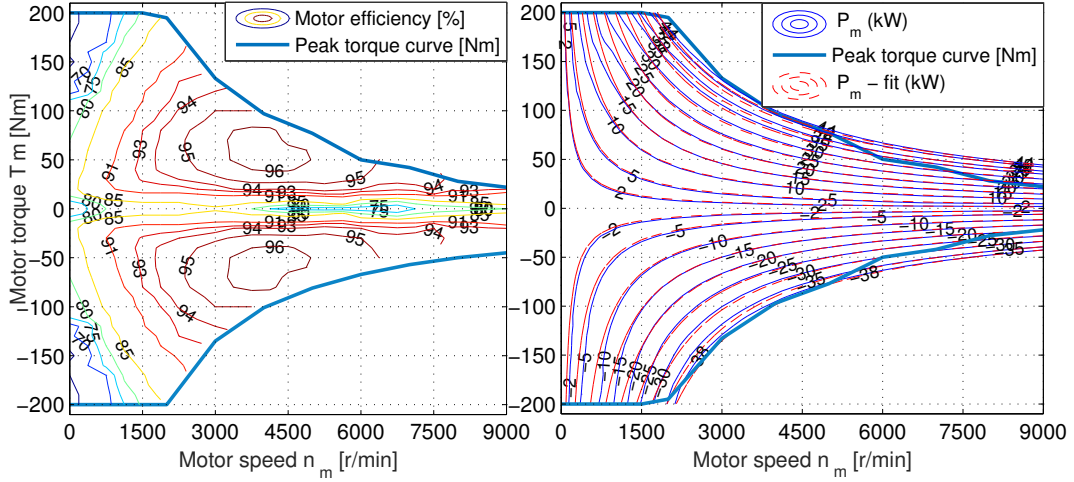


Fig. 2. The motor efficiency η_m and the equivalent energy consumption rate P_m , The blue lines of the right figure are experimental data and the red lines represent the results based on the dimensional polynomial by fitting, with correlation coefficient $R^2 = 99.91\%$.

In the time domain, a safe spacing policy with constant time headway is performed by determining a final speed \tilde{v}_f in the terminal prediction horizon (Chen et al. (2018))

$$\tilde{v}_f T_r + s(N_p + 1) \leq s_p(N_p + 1) + s_{p0}, \quad (12)$$

where s_{p0} is the current inter-vehicle distance, $s_p(N_p + 1)$ is running distance of preceding vehicle in the prediction horizon, $s(N_p + 1)$ is the running distance of host vehicle in the prediction horizon, T_r is the safe time headway. We assume that the host vehicle approximately accelerates linearly from the current speed $v(1) = v_1^h$ to the terminal speed \tilde{v}_f . Therefore, the $s(N_p + 1)$ is calculated by

$$s(N_p + 1) = \frac{(v_1^h + \tilde{v}_f)}{2} t_f. \quad (13)$$

To predict the preceding vehicle speed accurately, the acceleration of the preceding vehicle is described by a tuning parameter (Chen et al. (2018) Kamal et al. (2012))

$$a_p = a_{p,0} \kappa(v_p), \quad (14)$$

where $a_{p,0}$ is the current acceleration of the preceding vehicle, v_p is the speed. The $\kappa(v_p)$ is defined as

$$\kappa(v_p) = \begin{cases} \frac{1}{1 + e^{-\beta_1(v_p - \gamma_1)}}, & \text{if } a_{p,0} \leq 0 \\ \frac{1}{1 + e^{\beta_2(v_p - \gamma_2)}}, & \text{else} \end{cases}, \quad (15)$$

where $\beta_1 > 0, \beta_2 > 0$ are the sharpness of function, γ_1, γ_2 define the range of the function. Then, we can obtain the speed of preceding vehicle in the prediction horizon. The above function is shown in Fig. 3.

According to the above analysis, the $s_p(N_p + 1)$ can be obtained. By substituting (13) into (12), the expression of \tilde{v}_f is deduced:

$$\tilde{v}_f \leq \frac{s_p(N_p + 1) + s_{p0} - v_1^h t_f / 2}{T_r + t_f / 2}. \quad (16)$$

Besides, the other constraints in the energy-efficient PCC system can be formulated as

$$\begin{aligned} T_{m,\min}(n_m(k)) &\leq T_m(k) \leq T_{m,\max}(n_m(k)), \\ n_{m,\min} &\leq n_m(k) \leq n_{m,\max}, \\ F_b(k) &\leq F_{b,\max}. \end{aligned} \quad (17)$$

3. GAUSS PSEUDOSPECTRAL METHOD

3.1 Preliminaries

In this section, we introduce a general nonlinear model predictive control problem in time domain. A nonlinear control system is developed by the following state equation

$$\dot{x}(t) = f(x(t), u(t), p(t)), \quad (18)$$

where $x(t), u(t), p(t)$ are the states, the input variables and the system parameters, respectively. The control inputs at each time t are determined to minimize the following function:

$$\min J = \varphi(x(t_0), x(t_f)) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt, \quad (19)$$

where the prediction horizon is $t \in [t_0, t_f]$, $\varphi(x(t_0), x(t_f))$ is the terminal penalty, $L(x(t), u(t), t)$ is the cost function. The system constraints are summarized as

$$\begin{aligned} h(x(t), u(t), p(t), t) &\leq 0 \\ \Phi(x(t_0), x(t_f)) &= 0. \end{aligned} \quad (20)$$

Without considering the loss of generality, the nonlinear model predictive control problem can be reformulated as follows:

$$J = \varphi(x(-1), x(1)) + \frac{t_f - t_0}{2} \int_{-1}^1 L(x(\tau), u(\tau), \tau) d\tau, \quad (21)$$

with constraints as

$$\begin{aligned} \dot{x}(\tau) &= \frac{t_f - t_0}{2} f(x(\tau), u(\tau), p(\tau)), \\ h(x(\tau), u(\tau), p(\tau)) &\leq 0, \\ \Phi(x(-1), x(1)) &= 0, \end{aligned} \quad (22)$$

where the time interval is transformed from $[t_0, t_f]$ to the time interval $[-1, 1]$ via the affine transformation

$$t = \frac{t_f - t_0}{2} \tau + \frac{t_f + t_0}{2}, \quad \tau \in [-1, 1]. \quad (23)$$

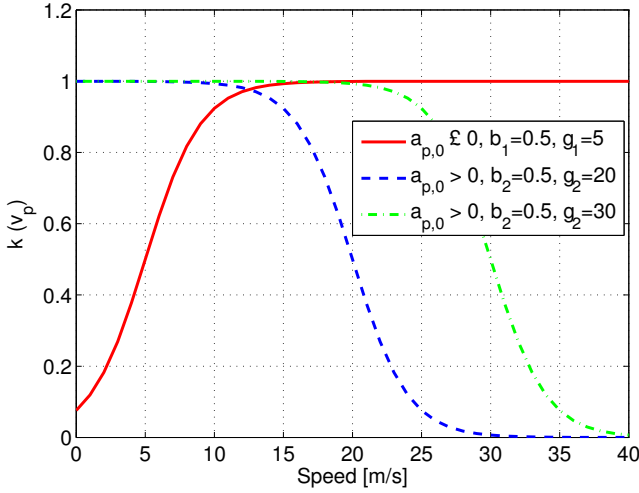


Fig. 3. The $\kappa(v_p)$ in the prediction function (15).

3.2 Gauss Pseudospectral Discretization

Taking into consideration of the collocation at the N Legendre-Gauss points noted as $\tau_1, \tau_2, \dots, \tau_N$ and the τ_0 corresponding to the initial time t_0 . The state variables $x(\tau)$ in equation (1) are discretized as (Benson et al. (2006)):

$$X = [x(\tau_0), x(\tau_1), x(\tau_2), \dots, x(\tau_N)]. \quad (24)$$

Additionally, the control inputs $U(\tau)$ are also discretized as:

$$U = [u(\tau_0), u(\tau_1), u(\tau_2), \dots, u(\tau_N)]. \quad (25)$$

The state variables and control inputs are approximated by the Lagrange interpolation polynomials at collocation points

$$x(\tau) \approx XL(\tau), u(\tau) \approx UL(\tau), \quad (26)$$

here $L(\tau)$ is the basis of Lagrange polynomials given by

$$L(\tau) = [L_0(\tau), L_1(\tau), \dots, L_N(\tau)]^T, \quad (27)$$

$$L_i(\tau) = \prod_{j=0, j \neq i}^N \frac{\tau - \tau_j}{\tau_i - \tau_j} \quad (i = 0, 1, \dots, N).$$

Differentiating the expression (26), we can obtain the differential operation on the Lagrange bases:

$$\dot{x}(\tau) \approx X\dot{L}(\tau). \quad (28)$$

The derivative of each Lagrange polynomial at the LG points can be represented in a differential approximation matrix, $D = [D_1, D_2, \dots, D_N]$. The elements of the differential approximation matrix are determined offline as follows:

$$D_k = [\dot{L}_0(\tau_k), \dot{L}_1(\tau_k), \dots, \dot{L}_N(\tau_k)]^T, \quad (29)$$

where $k = 1, 2, \dots, N$. The dynamic constraints are reformulated as algebraic constraints via the differential approximation matrix as follows:

$$XD_k - \frac{t_f - t_0}{2} f(X_k, U_k) = 0, k = 1, 2, \dots, N, \quad (30)$$

where $X_k \equiv x(\tau_k)$, $U_k \equiv u(\tau_k)$.

As regards the terminal penalty in the model predictive control, the additional variables in the discretization are performed as:

$$x(-1) = x(\tau_0), \quad (31)$$

$$x(1) = x(\tau_N) + \frac{t_f - t_0}{2} (1 - \tau_N) f(X_N, U_N).$$

According to the Legendre-Gauss quadrature

$$\int_{-1}^1 f(x, \tau) d\tau \approx \sum_{i=0}^N w_i f(x(\tau_i)), \quad (32)$$

where w_i are the Gauss weights, thus the nonlinear optimal problem can be reformulated as an nonlinear programming problem that find optimal solution $Q = [X^T, U^T]^T$, such that

$$\min J = \varphi(x(-1), x(1)) + \frac{t_f - t_0}{2} \sum_{k=0}^N w_k g(X_k, U_k). \quad (33)$$

Furthermore, the system constraints can be summarized as

$$XD_k - \frac{t_f - t_0}{2} f(X_k, U_k) = 0, \quad (34)$$

$$h(X_k, U_k) \leq 0,$$

$$\Phi(x(-1), x(1)) = 0, k = 1, 2, \dots, N.$$

The above NLP problem is solved by the Sequential Quadratic Programming (SQP) algorithm in this paper.

3.3 Moving Horizon Control Strategy

As discussed in above, the prediction horizon is discretized with a nonuniform subdivision approach, which is different from the Euler method. According to the control process of MPC, at each sampling instant, the first control input in the obtained sequence is applied to the controlled plant.

Assuming the sampling time Δt in real-time application is fixed, if the discrete points at the end of the horizon are dense, which means $\Delta\tau < \Delta t$. After obtaining the discrete optimal control variable $U(\tau)$, it's appropriate to apply the first several elements of the control variable to the plant, such as

$$u(t) = u^*(t), t \in [t_0, t_0 + \Delta t]. \quad (35)$$

However, if $\Delta\tau \geq \Delta t$ the moving control instruction $u(t)$ is derived as

$$u(t) = u^*(t_0). \quad (36)$$

The details of the influence of the interval $\Delta\tau$ in control performance are discussed in the study Guo et al. (2017).

4. SIMULATION ANALYSIS

In this section, several comparative simulations are presented to observe the energy saving capacity of the proposed energy-efficient PCC, and compare the computation time with the baseline method. The parameters of the chosen EV model are shown in Table. 1.

Table 1. Vehicle parameters

Description	Value
Vehicle Mass M_{veh}	1500kg
Air Density ρ	1.205kg/m ³
Frontal Area of the Vehicle A_f	2.1m ²
Air Resistance Coefficient c_d	0.36
Rolling Resistance Coefficient f	0.011
Gravitational Constant g	9.8m/s ²
Road Grade α	0
η_t	0.96
Gear ratio i_g	13.396
Maximum motor torque $T_{m,max}$	200Nm
Maximum motor speed $n_{m,max}$	9000r/min
Battery capacity Q_{bat}	70Ah
Safe time headway T_r	1.8s

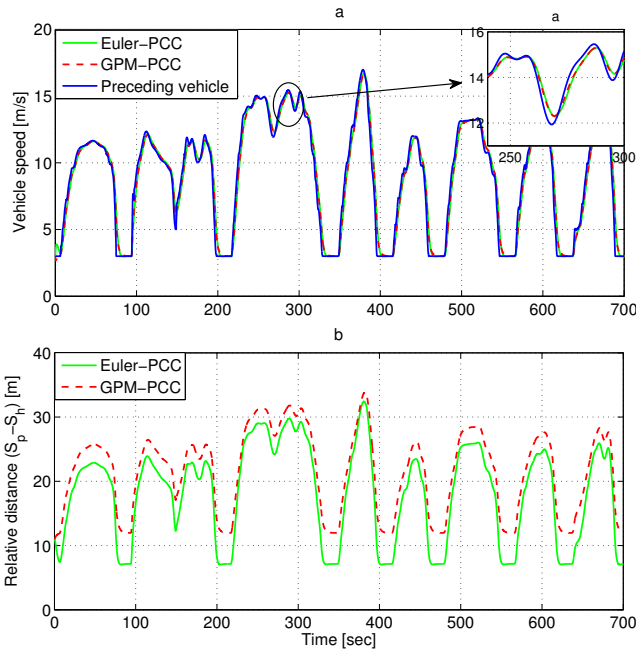


Fig. 4. Results comparison in vehicle speed and relative distance.

4.1 Comparison of Energy Consumption

Firstly, for clear description, the method using MHC with GPM proposed in this paper is marked as GPM-PCC, that using Euler method is marked as Euler-PCC. The sampling time and discrete point number are chosen as $\Delta t = 0.05s$ and $N_{gpm} = 10, N_{euler} = 20$.

The simulation results are shown in Fig. 4. From top to bottom, the first figure shows the velocity trajectories of the preceding vehicle and host vehicle, the second figure shows the relative distance between the host and the preceding vehicle. From the results, we can see that the GPM-PCC with half discrete point number can achieve the same performance in the car-following scenario. As it can be observed in the bottom figure, the inter-distance is not constant, which also indicates that the vehicle controlled by the PCC can accelerate gradually to follow the preceding vehicle.

Fig. 5 indicates that the energy efficiency obtained using GPM-PCC is the same as that using Euler-PCC. And the energy economy is found to be improved 4.1% on average.

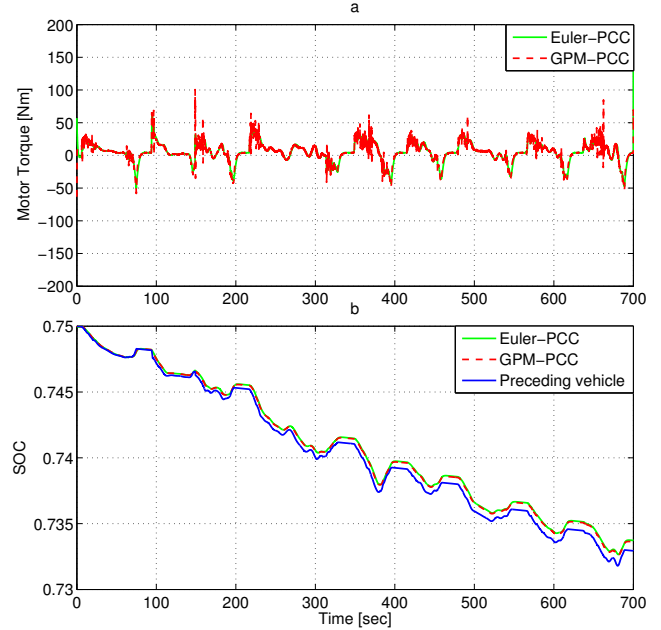


Fig. 5. Results comparison in energy efficiency.

4.2 Computational Efficiency

In this section, the simulation was run on an Intel (R) Core (TM) i7-7700 CPU (3.60GHz), and an estimate of CPU computation time was obtained using the CPU command in MATLAB 2015b. The computational time results are shown in Fig. 6. The second figure shows the computation time in time horizon $t \in [390s, 440s]$. The results show that the proposed GPM-PCC is more computationally efficient than the baseline Euler-PCC while ensuring the same accuracy.

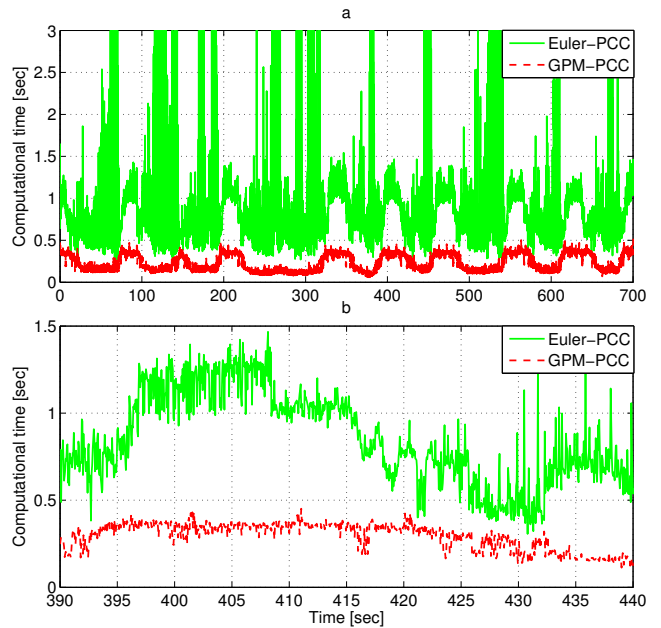


Fig. 6. Computation time of GPM-PCC and Euler-PCC.

5. CONCLUSION

In this paper, a predictive cruise control for minimising charge consumption of the automated electric vehicles has been proposed as an optimal control problem. The optimal velocity is obtained by the optimal trade off between the energy economy and the inter-vehicle distance. To reduce the computational burden, a fast algorithm combining Gauss pseudospectral method and moving horizon control is introduced to solve this nonlinear optimal problem. Results of several comparative simulations indicate that the average energy-saving can reach about 4.1%, and the computational speed is improved compared with the Euler method while ensuring the same accuracy. In the future work, the real vehicle implementation is our main focus. We will also take the more traffic information into account.

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