Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

A Fast Iterative Approach for Optimal Control of Nonlinear Systems

Jiaqi Liu * **Shiying Dong** ** **Jinlong Hong** * **Bingzhao Gao** *
Hong Chen ***,*

* *State Key Laboratory of Automotive Simulation and Control, Jilin University Nanling Campus, Changchun 130025, China*
** *Department of Control Science and Engineering, Jilin University, PR China*
*** *Clean Energy Automotive Engineering Center, Tongji University, Shanghai, 201804, China*

Abstract: To improve computational efficiency, a fast iterative solution method is proposed to solve optimization problems of nonlinear systems in this paper. The main idea is to extract the main linear part from the nonlinear system, and then regard the remaining nonlinear part as disturbance, so that the converted system can be solved by combining the optimal solution of the linear system with the fast iterative solution proposed. After the fast iterative solution method is introduced, its effectiveness is validated through a numerical example and an application which is engine energy-saving control while taking vehicle speed tracking into consideration. All of the simulation results show that the proposed iterative algorithm has the advantages of fast convergence speed and high accuracy.

Keywords: Nonlinear systems; Optimal control; Fast iterative solution; Vehicle control; High computation speed

1. INTRODUCTION

Optimal control has an important impact on control engineering, and has a wide range of applications in spacecraft, missiles, satellites and industrial production. Optimal control is to study and solve the problem of finding the optimal control scheme from all possible control schemes.

As we know, time-varying disturbance is very common in control systems, such as parameter perturbations and dynamic variations which may severely deteriorate control performance. In many cases, disturbance is first estimated because of their immeasurability so the controller can be designed according to the estimated disturbance. In the research and development of disturbance restraint, people have made a lot of contributions. In the 1990′s, Jingqing Han proposed a control framework considers the extended state observer as the fundamental part to estimate disturbances, see Gao et al. (2001), Han (2009). People have a good comprehension of stability analysis and implementation of active disturbance rejection controller, such as Zhao and Guo (2017) investigate a nonlinear extended state observer constructed from piece-wise smooth functions consisted of linear and fractional power functions, and Ahi and Nobakhti (2017) design an ADRC by utilizing an extended state observer for observing and suppressing the effects of external disturbances and internal parameter uncertainties, and Yang et al. (2017) investigate the attitude control for a quadrotor under gust wind via a dual closed-loop control framework. Furthermore, Liu and Svoboda (2005) introduced a nonlinear disturbance observer with a feedback-error-learning to control a class of time-varying nonlinear systems with unknown disturbances and improve the tracking performance. Moreover, parameter variation and disturbances, adaptive feedforward compensation (Bao et al. (2017)) and predictor-based disturbance rejection control (Liu et al. (2017)) are also used. We can get an overview of all the strategies through Chen et al. (2015).

In the paper Gao et al. (2019), an optimal control scheme for time-varying disturbances is proposed, which does not require specific forms of disturbance. To derive the control law, a novel form of multiplier function is introduced, where the objective function is adapted by multiplying the attenuation factor, guaranteeing the integrability of it. As a result, the control law deduced has the linear feedback form of system state, disturbance and its derivatives. It can guarantee the control performance under uncertainties and disturbances, and maintain the stability of the system.

Inspired by this algorithm, this paper transforms the nonlinear system into the linear part and the non-linear part, and regards the non-linear part as a disturbance of the system. Then we propose a new iterative optimal solution method according to the paper Gao et al. (2019). Like the method proposed by paper Tamimi and Li (2010), this method has the advantages of easy interpretation and high accuracy, and can solve the nonlinear system in a quick way.

The main contribution of this paper is to propose a fast iterative solution which has a good effect on improving

⋆ Corresponding author: Bingzhao Gao (gaobz@jlu.edu.cn).

efficiency for optimization problems of nonlinear systems. In the application of algorithm, as energy management is always the most important part of the control field shown in paper Zhang et al. (2019) and Sampathnarayanan et al. (2014), the fuel consumption control of automobiles is particularly important. Many strategies have been studied to improve fuel economy, such as designing dynamic programming algorithm (Hellström et al. (2009)) and using MPC (Kamal et al. (2012)). In this paper, the algorithm is applied to the optimal control of engine fuel consumption while taking vehicle speed tracking into consideration, and good results are obtained.

This paper is organized as follows. In Section 2 the proposed approach algorithms and numerical examples will be presented and analyzed. Section 3 introduces an implementation to show the efficiency of the algorithm and in Section 4 the work is concluded.

2. THE NONLINEAR OPTIMAL SOLUTION ALGORITHM

2.1 Algorithmic Principle

The algorithm applied in this paper based on our previous published paper Gao et al. (2019), which is classified as a feasible method to solve nonlinear system problems. It can be used in common practical engineering problems such as the shortest time control problem and the minimum of fuel control problem. In that paper, to derive the control law, a novel form of multiplier function is introduced, while the control law is represented as a feedback form of the system states, the disturbances and their derivatives.

The algorithmic principle can be introduced from the real nonlinear system, which is

$$\begin{aligned} &\dot{x} = f(x(t), u(t)), \\ &y = Cx. \end{aligned} \tag{1}$$

We consider the non-linear part as a linear system with time-varying disturbance. The formula is turned to a linear system with time-varying disturbance $B_d d$ as

$$\begin{aligned} &\dot{x} = Ax + B_u u + B_d d, \\ &y = Cx, \end{aligned} \tag{2}$$

where $x \in R^{n\times 1}$ and $u \in R^{m\times 1}$ are the state and control variables, $d \in R^{l\times 1}$ is time-varying disturbance, $y \in R^{s\times 1}$ is output variable, $A \in R^{n\times n}$ and $B_u \in R^{n\times m}$ are the state and control matrices, and $B_d \in R^{n\times l}$ and $C \in R^{s\times n}$ are the disturbance and output matrices.

The control problem is to minimize the objective function :

$$\min_{u(t)} J = \frac{1}{2}\int_{t_0}^{\infty} (y^T Q y + u^T R u) e^{-\beta t} dt, \tag{3}$$

where $Q \in R^{s\times s}$ and $R \in R^{m\times m}$ are positive weighting matrices, t_0 is initial time of the receding horizon. The Hamiltonian approach is introduced as (4) to cope with the problem (3).

$$H = \frac{1}{2}(y^T Q y + u^T R u) + \lambda^T (Ax + B_u u + B_d d) \tag{4}$$

where we determine λ as the multiplier function. Then the regular equation can be derived:

$$\begin{aligned} &\dot{x} = \frac{\partial H}{\partial \lambda} \Rightarrow \dot{x} = Ax - B_u R^{-1} {B_u}^T \lambda e^{\beta t} + B_d d \\ &\dot{\lambda} = -\frac{\partial H}{\partial x} \Rightarrow \dot{\lambda} = -C^T Q C x e^{-\beta t} - A^T \lambda \\ &\frac{\partial H}{\partial u} = 0 \Rightarrow u = -R^{-1} {B_u}^T \lambda e^{\beta t} \end{aligned} \tag{5}$$

To solve the above nonlinear differential equations, we define:

$$x_1 = x e^{-\beta t}. \tag{6}$$

Then we get:

$$\dot{x}_1 = \dot{x} e^{-\beta t} - \beta x e^{-\beta t} \Rightarrow \dot{x} e^{-\beta t} = \dot{x}_1 + \beta x_1. \tag{7}$$

As the fact that the disturbances are time-varying, so the traditional choice of λ=$Px + hd$ can not be used. So we introduce a novel form of the multiplier function λ.

$$\lambda = Px + h_0 d + h_1 \dot{d} + ... + h_n d^{(n)} + ... \tag{8}$$

where P and h_i can be calculated in the following forms:

$$P(A - \beta I) - P B_u R^{-1} B_u^T P + C^T Q C + A^T P = 0, \tag{9}$$

$$\begin{aligned} &h_0 = (P B_u R^{-1} B_u^T - A^T)^{-1} P B_d e^{-\beta t}, \\ &h_i = (P B_u R^{-1} B_u^T - A^T)^{-1} h_{i-1}, \\ &i = 1, 2, \ldots \end{aligned} \tag{10}$$

We define:

$$\begin{aligned} &{h_0}' = (P B_u R^{-1} B_u^T - A^T)^{-1} P B_d, \\ &{h_i}' = (P B_u R^{-1} B_u^T - A^T)^{-1} {h_{i-1}}', \\ &i = 1, 2, \ldots \end{aligned} \tag{11}$$

So equation (10) can be rewritten as

$$\begin{aligned} &h_0 = {h_0}' e^{-\beta t}, \\ &h_i = {h_i}' e^{-\beta t}, \\ &i = 1, 2, ... \end{aligned} \tag{12}$$

By the utilization of the novel multiplier function (8), substitute (8) and (12) into (5):

$$\begin{aligned} &u = -R^{-1} {B_u}^T \lambda e^{\beta t} \\ &= -R^{-1} {B_u}^T P x_1 e^{\beta t} - R^{-1} {B_u}^T h_0 d e^{\beta t} \\ &-R^{-1} {B_u}^T h_1 \dot{d} e^{\beta t} - ... - R^{-1} {B_u}^T h_n d^{(n)} e^{\beta t} - ... \\ &= -R^{-1} {B_u}^T P x - R^{-1} {B_u}^T {h_0}' d - \\ &R^{-1} {B_u}^T {h_1}' \dot{d} - ... - R^{-1} {B_u}^T {h_n}' d^{(n)} - ... \end{aligned} \tag{13}$$

The control law can be represented as a feedback form of the system states, the disturbances and their derivatives as

$$u = K_x x + K_d d + \sum_{i=1}^{\infty} K_{di} d^{(i)}. \tag{14}$$

where

$$\begin{aligned} &K_x = -R^{-1} {B_u}^T P, \\ &K_d = -R^{-1} {B_u}^T {h_0}', \\ &K_{di} = -R^{-1} {B_u}^T {h_i}'. \\ &i = 1, 2, \ldots \end{aligned} \tag{15}$$

At this moment, we can obtain the optimal solution of the objective function by establishing an iteration model in the optimization process. In addition, from Hong (2019) we konw the convergence rate of output under the control law $u = K_x x + K_d d + K_{d1} \dot{d}$ is faster than it under the control law $u = K_x x + K_d d$, and the output under the control law $u = K_x x + K_d d + K_{d1} \dot{d}$ and control law $u = K_x x +$

Table 1. Algorithm 1

Algorithm 1

Main Steps:

1: In the first iteration, we shall remove the disturbance, then the nonlinear system (17) can be turned into linear system $\dot{x}_1 = Ax_1 + B_u u_1$. According to Gao et al. (2019), we chose the control law as $u_1 = K_x x_1$, so we can obtain the the optimal solution of the control variable u_1 and sate variable x_1;
2: In the second iteration, by substituting the control law u_1 and x_1 into the original system (16), the disturbance of the first iteration can be obtained as $B_d d_1 = f(x_1, u_1) - Ax_1 - B_u u_1$, and then the nonlinear system is turned into $\dot{x}_2 = Ax_2 + B_u u_2 + B_d d_1$;
3: Combine the updated system with the control law $u_2 = K_x x_2 + K_d d_1 + K_{d1}\dot{d}_1$, the optimal control law u_2 and x_2 of the second iteration can be accessed;
4: By constantly updating the system, we can get the results of several iterations and finally obtain the optimal control laws of iterations and find the rules.

$K_d d + K_{d1}\dot{d} + K_{d2}\ddot{d}$ are almost overlap. On the one hand, derivative information of the disturbance are of great help for the disturbance rejection performance improvement, on the other hand also shows that the disturbance derivative of higher order term is becoming more and more limited to the promotion of disturbance rejection performance. Hence we chose $u = K_x x + K_d d + K_{d1}\dot{d}$ as the control law we use in simulations. The process that we build the iterative solution is as follows.

For a real non-linear system:

$$\begin{aligned} \dot{x} &= f(x(t), u(t)), \\ y &= Cx, \end{aligned} \tag{16}$$

transform it to a linear system with time-varing disturbance as

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_d d, \\ y &= Cx. \end{aligned} \tag{17}$$

The minimum of the objective function J is given in

$$\min_{u(t)} J = \frac{1}{2}\int_{t_0}^{\infty} (y^T Q y + u^T R u) e^{-\beta t} dt. \tag{18}$$

Then main steps of the proposed approach are shown in Table 1.

2.2 Numerical Example

In this work, we consider following example to demonstrate the performance of the proposed algorithm.

Example 1.

The nonlinear system is

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 + u - 0.1{x_1}^2, \\ \dot{x}_2 &= 2x_1 + x_2 + 0.1\sin x_2, \\ y &= x, \end{aligned} \tag{19}$$

and the objective function is

$$\min_{u(t)} J = \frac{1}{2}\int_{0}^{t_f} (y^2 + u^2) e^{-0.4t} dt. \tag{20}$$

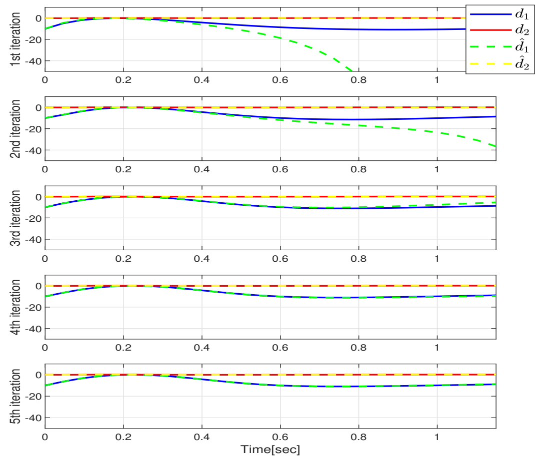

Fig. 1. The results of the first to fifth iterations of the changes of actual system disturbances d_1 and d_2 as well as iterative model disturbances $\hat{d}_1$ and $\hat{d}_2$

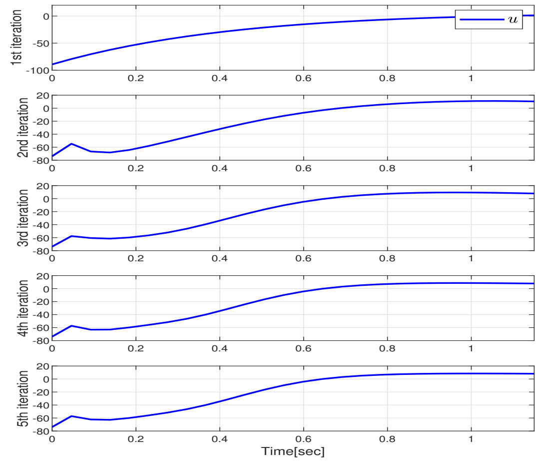

Fig. 2. The results of the first to fifth iterations of the changes of iterative model control inputs u

We discretize the time horizon $t_f = 1.15$s into 50 subintervals, by using Table 1, we can achieve the desired results. In each iteration simulation, the changes of actual system disturbances and iterative model disturbances over time can be represented by the Fig. 1, and the changes of iterative model control inputs over time can be represented by the Fig. 2.

Although we only list one example here due to page constraints, we actually did a lot of examples. According to the numerical examples, we can draw the conclusion till now: within a certain period of time, the results of the iterative model can gradually keep up with the results of the real model. With the increase of the number of iterations, the results obtained are closer to those obtained by the real model and tend to be stable.

3. ALGORITHMIC APPLICATION

The fuel consumption rate of a car has always been the most important issue in optimization problems Schmied et al. (2015), and people have been trying to solve this problem in various ways, such as Guo et al. (2019) proposed a method considering the influence of gearshift control and combines the velocity optimization and the

Table 2. Vehicle Parameters

Symbol	Description	Value [Unit]
C_d	Coefficient of air resistance	0.373
A_f	Face area	2.58 $[\mathrm{m}^2]$
ρ	Air density	1.29
M	Vehicle mass	1658 [kg]
η_t	Drive train total efficiency	1
g	Gravity acceleration	9.8 $[\mathrm{m/s}^2]$
f	Coefficient of rolling resistance	0.02
i_g	Drive speed ratio multiply the gear ratio	4
r_w	Dynamic tire radius	0.3 [m]

Table 3. Values of Coefficients

Coefficient	Value
h_{20}	6.7×10^{-8}
h_{11}	5.25×10^{-6}
h_{02}	2.635×10^{-5}
h_{10}	0.00019
h_{01}	-0.0024

powertrain control to improve fuel economy. In this section, we apply the previous algorithm to optimize the fuel consumption efficiency of the car. And the accuracy and the computation speed of this approach will be shown in comparison with that from the optimization software GPOPS.

3.1 Fitting the Form of Objective Function With the Original Data

The research object parameters of this engine fuel consumption problem are in the Table 2. As for the establishment of engine fuel consumption model, engine fuel consumption is mainly related to engine speed and torque, so the engine fuel consumption can be described by (21), which includes the second and first terms of speed and torque:

$$\begin{aligned} \min_{u(t)} J = h_{20}{n_e}^2 + h_{11}n_eT_e + \\ h_{02}{T_e}^2 + h_{10}n_e + h_{01}T_e, \end{aligned} \tag{21}$$

where n_e is engine speed [r/min], T_e is engine torque [Nm].

The experimental data were fitted by 1stOpt software. As shown in the Fig. 3, the fitting effect is pretty good, which means that the formula we set is close to the real system, while the values of coefficients in the equation can be obtained in Table 3.

3.2 Minimizing Engine Energy Consumption While Taking Vehicle Speed Tracking Into Consideration

For convenience, the rotational speed is converted into speed, meanwhile in order to continue to improve the objective function, we add velocity tracking to the objective function and introduce an ideal velocity as a part of the objective function, w_r is the velocity tracking coefficient, v_r is the expected velocity. So as to obtain new formula (22), and coefficients are in Table 4.

$$\begin{aligned} \min_{u(t)} J = p_{20}v^2 + p_{11}vT_e + p_{02}{T_e}^2 + \\ p_{10}v + p_{01}T_e + w_r{v_r}^2, \end{aligned} \tag{22}$$

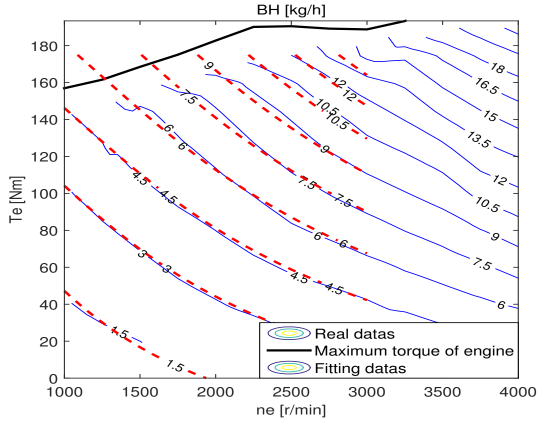

Fig. 3. Comparison of engine fuel consumption rate fitting

Table 4. Values of Coefficients

Coefficient	Value
k	$1/2pai \times 60 \times i_g/r_w$
p_{20}	$6.7 \times 10^{-8} \times k^2 \times \frac{1000}{3600} + w_r$
p_{11}	$5.25 \times 10^{-6} \times k \times \frac{1000}{3600}$
p_{02}	$2.635 \times 10^{-5} \times \frac{1000}{3600}$
p_{10}	$0.00019 \times k \times \frac{1000}{3600} - 2 \times w_r \times v_r$
p_{01}	$-\frac{2}{3} \times 10^{-3}$

where v is vehicle speed [m/s], T_e is engine torque [Nm].

In order to facilitate the deformation and representation later, we choose the matrix forms. We set

$$x = \begin{bmatrix} v \\ T_e \end{bmatrix}, Q = \begin{bmatrix} p_{20} & \frac{1}{2}p_{11} \\ \frac{1}{2}p_{11} & p_{02} \end{bmatrix}, c = \begin{bmatrix} p_{10} \\ p_{01} \end{bmatrix}. \tag{23}$$

Then (24) can be easily obtained:

$$\min_{u(t)} J = x^TQx + c^Tx + w_r{v_r}^2. \tag{24}$$

3.3 Vehicle Dynamics Modeling

On the base of vehicle dynamics models, we know that

$$\dot{v} = \frac{\eta_t i_g}{Mr_w} * T_e - \frac{C_dA_f\rho}{2M} * v^2 - gf, \tag{25}$$

we set

$$a = -\frac{C_dA_f\rho}{2M}, b = \frac{\eta_t i_g}{Mr_w}, c = -gf, \tag{26}$$

then we have

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{v} \\ \dot{T}_e \end{bmatrix} = \begin{bmatrix} a{x_1}^2 + bx_2 + c \\ u \end{bmatrix}. \tag{27}$$

In order to ensure that the objective function remains positive definite, we introduce a constant matrix Δx through the transformation of matrix operation.

$$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 + \Delta x_1 \\ x_2 + \Delta x_2 \end{bmatrix} = \begin{bmatrix} v + \Delta x_1 \\ T_e + \Delta x_2 \end{bmatrix} \tag{28}$$

$$\min_{u(t)} J = x^TQx + c^Tx + \delta = \bar{x}^TQ\bar{x} \tag{29}$$

Ultimately, we can describe the problem in the following way:

equation of system state is

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_d d, \\ y &= Cx. \end{aligned} \tag{30}$$

objective function is

$$\min_{u(t)} J = \frac{1}{2} \int_{t_0}^{\infty} (y^T Q y + u^T R u) e^{-\beta t} dt, \tag{31}$$

each of them are as follows:

$u = \dot{T}_e$;

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v + \Delta x1 \\ T_e + \Delta x2 \end{bmatrix}$;

$\Delta x = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = (\frac{1}{2} c^T Q^{-1})^T$;

$A = \begin{bmatrix} aa & bb \\ 0 & 0 \end{bmatrix}$; $B_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$;

$B_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $C = [1\ 0]$;

$Q = \begin{bmatrix} q_1 & q_{12} \\ q_{12} & q_2 \end{bmatrix}$; $w_r = 0.1$;

$q_1 = p_{20}$; $q_{12} = \frac{1}{2} p_{11}$;

$q_2 = p_{02}$; $c_{10} = p_{10}$;

$c_{01} = p_{01}$; $R = 0.01$;

$\beta = 0.01$; $d = a{x_1}^2 + cc$;

$aa = -2a\Delta x_1$; $bb = b$;

$cc = a{\Delta x_1}^2 - b\Delta x_2 + c$,

we assume the initial values as $x_0 = \begin{bmatrix} 0 + \Delta x_1 \\ 150 + \Delta x_2 \end{bmatrix}$, the expected velocity $v_r = 4m/s$, and we discretize $t_f = 180s$ into 1800 subintervals.

3.4 Simulation Results

According to the control algorithm in Section 2, with the standard state equation and objective function, the optimal solution of the objective function can be obtained step by step. We can use MATLAB to establish the iteration model we want. The number of iterations is chosen based on a criterion, we assume when the misalignment of the Nth iteration system disturbances curve with the N-1th iteration system disturbances curve is less than e-8, then we satisfy the stopping criterium. At this point, we use residual sum of squares to describe the misalignment of the disturbances curve, we found that residual sum of squares of the 1st iterative disturbances curve with the 2nd iteration is 0.1391, while the residual sum of squares of the 2nd iterative disturbances curve with the 3rd iteration disturbances curve is 7.7109e-09, so we iterate 3 times. And draw the state and input variables as Fig. 4.

From the Fig. 4 we can make it clear that as time goes by, the vehicle speed increases gradually to the maximum, then decreases gradually to the expected speed and maintains stability. The engine torque decreases gradually at first, then rises slightly and remains unchanged. The change rate of engine torque increases gradually to zero at the highest point and maintains ability. It can be seen that after three iterations, the results are basically unchanged, while results of the real model coincide with the iterative models, and variables also maintain the results of the last iteration and can maintain stability after a period of time,

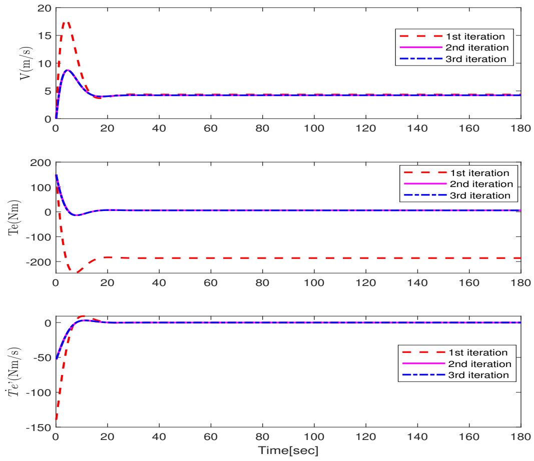

Fig. 4. The results of the first to third iterations

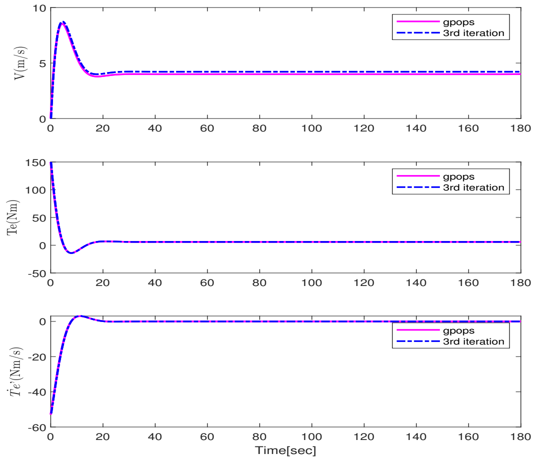

Fig. 5. Comparisons between GPOPS results and iteration results

which shows that the approximate optimal solution can be obtained by iterating several times to find the desired results.

After solving the practical problem by algorithm, we solve the same problem using GPOPS, a MATLAB software for solving complex optimal control problems to find out the differences. In this section, we use GPOPS to verify the correctness of the previous results and describe the accuracy and limitations of the iterative algorithm in application by observing the consistency of the results achieved by the two methods. GPOPS is organized as follows and we write MATLAB functions that define the following functions of the problem:

consider the optimal control problem, minimize the cost functional

$$J = \int_{t_0}^{t_f} (q_1 {x_1}^2 + q_{12} x_1 x_2 + q_2 {x_2}^2 + Ru^2) e^{-\beta t} dt, \tag{32}$$

subject to the dynamic constraint

$$\begin{aligned} \dot{x}_1 &= a(x_1 - \Delta x_1)^2 + b(x_2 - \Delta x_2) + c, \\ \dot{x}_2 &= u, \end{aligned} \tag{33}$$

and the boundary conditions subject to the dynamic constraint

$$
\begin{aligned}
x_1(0) &= 0 + \Delta x_1 \\
x_2(0) &= 150 + \Delta x_2 \\
x_1(t_f) &= 4 + \Delta x_1
\end{aligned}
\tag{34}
$$

with tf= 180.

Each of them are as follows:
$x_1 = v + \Delta x_1$; $x_2 = T_e + \Delta x_2$; $u = \dot{T}_e$; $q_1 = p_{20}$; $q_{12} = \frac{1}{2}p_{11}$; $q_2 = p_{02}$; $R = 0.01$; $\beta = 0.01$.

We compare the results with those obtained by the iteration model as Fig. 5. At the same time, with the CPU of AMD Ryzen 5 3550H with Radeon Vega Mobile Gfx and the main frequency 2.10Ghz, time to get the result by the fast iterative algorithm is 0.064895s, while when using GPOPS it takes 2.406618s. Based on the observation of the comparative results, we can draw the conclusion that comparing the result of gpops module with the simulation result of iteration, it is found that the accuracy of the proposed optimal iterative algorithm is high enough while computation time is reduced significantly.

4. CONCLUSION

This paper solves the optimal solution of the nonlinear equation based on the iteration model, and applies it to the problem of engine fuel consumption. Good results have been obtained and compared with the results from optimization software. In addition, the numerical example is taken as case study to indicate the algorithm. The simulation and experimental results demonstrate that the algorithm is robust enough and has strong ability in rejecting time-varying disturbances. From all the work we can tell that this method is reasonable, feasible and efficient when a nonlinear system's optimal control is to be solved. The implementation of parallel computing and stability analysis of this algorithm will be considered in our future work.

ACKNOWLEDGEMENTS

This work was supported in part by the China Automobile Industry Innovation and Development Joint Fund under Grant U1664257, in part by the National Nature Science Foundation of China under Grant No.61790564, in part by the National Natural Science Foundation of China under Grant 61520106008, and in part by the Jilin Provincial Science Foundation of China under Grant 20190103047JH.

REFERENCES

Ahi, B. and Nobakhti, A. (2017). Hardware implementation of an ADRC controller on a gimbal mechanism. *IEEE Transactions on Control Systems Technology*, 26(6), 2268–2275.

Bao, Y., Wang, L.Y., Wang, C., Jiang, J., Jiang, C., and Duan, C. (2017). Adaptive feedforward compensation for voltage source disturbance rejection in dc–dc converters. *IEEE Transactions on Control Systems Technology*, 26(1), 344–351.

Chen, W.H., Yang, J., Guo, L., and Li, S. (2015). Disturbance-observer-based control and related methodsan overview. *IEEE Transactions on Industrial Electronics*, 63(2), 1083–1095.

Gao, B., Hong, J., Qu, T., Yu, S., and Chen, H. (2019). An output regulator with rejection of time-varying disturbance: Experimental validation on clutch slip control. *IEEE Transactions on Control Systems Technology*.

Gao, Z., Huang, Y., and Han, J. (2001). An alternative paradigm for control system design. In *Proceedings of the 40th IEEE conference on decision and control (Cat. No. 01CH37228)*, volume 5, 4578–4585. IEEE.

Guo, L., Chen, H., Gao, B., and Liu, Q. (2019). Energy management of HEVs based on velocity profile optimization. *Science China Information Sciences*, 8(62), 1–3.

Han, J. (2009). From PID to active disturbance rejection control. *IEEE transactions on Industrial Electronics*, 56(3), 900–906.

Hellström, E., Ivarsson, M., Åslund, J., and Nielsen, L. (2009). Look-ahead control for heavy trucks to minimize trip time and fuel consumption. *Control Engineering Practice*, 17(2), 245–254.

Hong, J. (2019). *AMT Drivability Optimization Control During Launch and Gear Shift Process*. Ph.D. thesis, Jilin University.

Kamal, M.A.S., Mukai, M., Murata, J., and Kawabe, T. (2012). Model predictive control of vehicles on urban roads for improved fuel economy. *IEEE Transactions on control systems technology*, 21(3), 831–841.

Liu, T., Hao, S., Li, D., Chen, W.H., and Wang, Q.G. (2017). Predictor-based disturbance rejection control for sampled systems with input delay. *IEEE Transactions on Control Systems Technology*, 27(2), 772–780.

Liu, Z.L. and Svoboda, J. (2005). A new control scheme for nonlinear systems with disturbances. *IEEE Transactions on Control Systems Technology*, 14(1), 176–181.

Sampathnarayanan, B., Onori, S., and Yurkovich, S. (2014). An optimal regulation strategy with disturbance rejection for energy management of hybrid electric vehicles. *Automatica*, 50(1), 128–140.

Schmied, R., Waschl, H., Quirynen, R., Diehl, M., and del Re, L. (2015). Nonlinear MPC for emission efficient cooperative adaptive cruise control. *IFAC-papersonline*, 48(23), 160–165.

Tamimi, J. and Li, P. (2010). A combined approach to nonlinear model predictive control of fast systems. *Journal of Process Control*, 20(9), 1092–1102.

Yang, H., Cheng, L., Xia, Y., and Yuan, Y. (2017). Active disturbance rejection attitude control for a dual closed-loop quadrotor under gust wind. *IEEE Transactions on control systems technology*, 26(4), 1400–1405.

Zhang, F., Hu, X., Langari, R., and Cao, D. (2019). Energy management strategies of connected HEVs and PHEVs: Recent progress and outlook. *Progress in Energy and Combustion Science*, 73, 235–256.

Zhao, Z.L. and Guo, B.Z. (2017). A nonlinear extended state observer based on fractional power functions. *Automatica*, 81, 286–296.