

# On Weighing the Conflicting Cost Functions for Optimal Energy Management of Electrified Powertrain

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**Abstract:** Besides fuel economy, gear shifting and engine start-stop frequency in the optimal energy management for hybrid electric vehicles (HEVs) will also influence the overall performance and drivability. However, such drivability concerns will also impact the vehicle's energy efficiency. To solve this conflicting optimization problem, this paper aims to find the proper weighing of the conflicting costs to achieve a right balance based on sensitivity analysis. The problem is formulated by expanding the conventional cost function with additional penalty items for gear shifting and engine start-stop, and a range extended hybrid delivery truck is modeled as a case study. Dynamic programming (DP) algorithm is applied to guarantee all the comparisons are under the same benchmark, and a split-DP solution is carried out to accelerate the searching process. Analytical fitting and trend analysis methods are used to find the proper penalty factors. Eventually, a comprehensive comparison among optimized, experiential and none penalty factors is shown, indicating that such appropriate weighing can significantly improve drivability with only 0.2% more fuel cost.

**Keywords:** Hybrid electric vehicle, conflicting cost function, split dynamic programming

## 1. INTRODUCTION

In the context of the environmental deterioration and the fossil fuel shortage, the development of electric vehicles (EVs) is recognized as one of the most promising avenues for the daily transportation in cities due to its exploiting of renewable electricity and fewer emissions (Zhang *et al.*, 2019). Currently, plug-in hybrid electric vehicles (HEVs) are being regarded as a good alternative to conventional vehicles and has been widely used. The topology of the propulsion architectures of HEVs will influence the potentially achievable performance, as well as the overall energy management strategy (EMS) significantly. Considering a regular combination scenario of urban and freeway driving cycles, one of the most challenging problem is how to handle the trade-off between the capacity of battery and fuel economy. Hence, it seems that the extended range electric vehicles (REEVs) could perform as a suitable bridge between the battery electric vehicle and the conventional vehicles (Chen *et al.*, 2014).

In this paper, we will focus on a general extended range hybrid electric powertrain designed for a delivery truck as shown in Fig. 1. It is worth note that such powertrain mainly consists of an electrified driveline with a variable-speed transmission and a hybrid energy storage system. Here, a traction motor (TM) combined with a two-speed automated manual transmission (AMT) will produce the required propulsion force, and the electric power will be generated by the engine-generator set

(Genset). Therefore, the system control problem can be converted to an optimal energy management problem regarding the gear shift schedule and power split ratio.

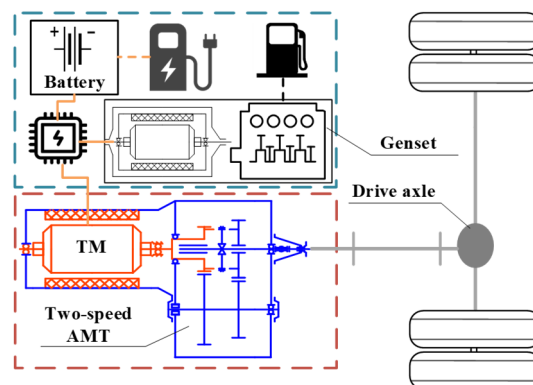


Fig. 1. Range extended hybrid electric powertrain.

As an important application of series hybrid electric vehicles, much research has been done on the topic of REEV energy management (Patil *et al.*, 2014). Strictly speaking, as constrained by complicated vehicle dynamics and uncertainties of driving conditions, such a problem is generally nonlinear and stochastic (Chen *et al.*, 2015). Considering online implementation, deterministic rule-based strategies could be a reliable solution. However, such a heuristic method depends on engineering experience greatly and cannot

guarantee optimal results (Peng *et al.*, 2017). In contrast, with some acceptable hypotheses, such as the route is usually changeless, global optimization-based strategies can be applied on a specific driving cycle which can improve the fuel economy significantly, especially for commercial electric vehicles. Related algorithms have been developed and discussed for a long time, such as dynamic programming (DP) (Brahma, *et al.*, 2000), genetic algorithm (GA) (Herrera *et al.*, 2015) and particle swarm optimization (PSO) (Sabri, *et al.*, 2016). The core step is to formulate and solve the cost function subjected to the system dynamics and constraints. To tackle the online computational burden and unknown velocity profile prior, instantaneous optimization-based strategies have attracted attention recently. The basic framework is to make some predictions and using the feedback information to determine the control sequence. Stochastic model predictive control (SMPC) is widely used to solve the online problem by forecasting the future power requirement and rolling optimization in a certain horizon (Di Cairano *et al.*, 2013). Though it can take advantage of instantaneous information, the basic solution is also solving the cost function. In general, it is very important to set the problem formulation properly and take more related factors, which are usually conflicting, into account. Among the literature review, most of the researchers would only focus on the efficiency fraction of the vehicle performance. The other basic features, such as the gear shifting frequency and engine on/off times, have been ignored. These neglections may ruin the control effect, namely, it will violate the physical constraints though it may present great results in the simulation environment. Moreover, an empirical additional constraint is also unsatisfying when exploiting extreme performance. Therefore, it is worthy to discuss the balance between fuel economy and drivability performance.

In this paper, the selection of penalty factors for gear shifting frequency, engine on/off times and fuel cost for optimal energy management of HEVs is presented using analytical fitting and trend analysis methods. Split-DP algorithm is applied to explore all the potentials and a detailed comparison study is illustrated to show the effect of proper weighing for the optimal energy management of the electrified powertrains.

The rest of the paper is organized as follows. The vehicle modeling and problem formulation are expressed in section 2. The basic mechanism of DP and a split application is presented in section 3. In section 4, the trend and relationship between the key factors are shown from a statistical perspective. Simulation results and comparison study are given in section 5, and brief conclusions are summarized in the last section.

## 2. MODELING AND OPTIMAL CONTROL PROBLEM

### 2.1 Model of REEV powertrain

As shown in Fig. 1, the REEV powertrain mainly consists of the genset and electrified driveline. As the main purpose is to study the energy performance, a simplified longitudinal vehicle model can be used and the related power flow can be derived as shown in Fig. 2. The required wheel side power can be satisfied through the final drive (FD), gear box (GB) and

traction motor (TM). The electric energy can be provided by both the battery and internal combustion engine (ICE) with a proper generator (Gen). The methodology in this simulation is a backward simulator, namely, at each step the required power at the wheels is calculated firstly and then the specific power demands at the components are evaluated by proceeding back to the TM and the energy sources, neglecting internal powertrain dynamics. Thus, the mathematical description of the components is set to a quasi-static model.

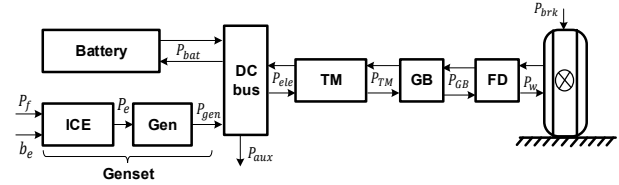


Fig. 2. Schematic diagram of the related power flow.

The wheel side power can be calculated by assuming the vehicle as a point mass model, which can be given by the following equations:

$$\begin{cases} (F_a + F_f + F_w + F_i) \cdot v = P_w \\ F_a = \delta_j m \cdot \frac{dv}{dt} (j = 1, 2) \\ F_f = \begin{cases} 0, & v = 0 \\ fmg \cos \alpha & v \neq 0 \end{cases} \\ F_w = \frac{1}{2} \rho C_d A v_a^2 \\ F_i = mg \sin \alpha \end{cases} \quad (1)$$

where  $v$  is the required velocity from a certain profile;  $F_a$ ,  $F_f$ ,  $F_w$ , and  $F_i$  are the acceleration resistance, the rolling resistance, the aerodynamic drag resistance, and the gradient resistance, respectively. The required torque and speed at each driveline component can be determined backward with a certain gear ratio and efficiency. The simplified dynamic relationship can be described as:

$$\begin{cases} T_w = T_{GB} \cdot i_0 = T_{TM} \cdot (i_g \cdot i_0) \\ \omega_w = \omega_{GB} / i_0 = \omega_{TM} / (i_g \cdot i_0) \end{cases} \quad (2)$$

where  $T_{GB}$  and  $\omega_{GB}$ ,  $T_w$  and  $\omega_w$ , are respectively the output torque and speed of transmission, required torque and speed at the wheel;  $i_g$  is the gear ratio and  $i_0$  is the final drive ratio. To describe the shifting process for further formulation properly, a discrete dynamic model with gear position and related control command can be expressed as Eq. (3) and Eq. (4).

$$i_{g,k} = \begin{cases} 1 & x_{gear,k} = 2 \\ 2.3 & x_{gear,k} = 1 \\ 0 & x_{gear,k} = 0 \end{cases} \quad (3)$$

$$x_{gear,k+1} = \begin{cases} 2 & x_{gear,k} + u_{shift,k} > 2 \\ 0 & x_{gear,k} + u_{shift,k} < 0 \\ x_{gear,k} + u_{shift,k} & \text{others} \end{cases} \quad (4)$$

where  $x_{gear}$  is operating gear status and  $u_{shift}$  is shift command which is constrained to be selected from the values of  $\{1, 0, -1\}$  denoting upshift, sustain and downshift command respectively; the index  $k$  indicates the instant step.

As for the traction motor (TM), since the transient responses can be ignored, here the total efficiency of the motor and converter can be modeled through a preset static map related to the output rotation speed and torque. Thus, the power-consuming of the TM can be described as:

$$\begin{cases} P_{ele} = T_{TM} \cdot \omega_{TM} \cdot \eta_{TM}^{-\text{sgn}(T_{TM})} / 9549 \\ \text{sgn}(T_{TM}) = \begin{cases} 1 & T_{TM} \geq 0 \\ -1 & T_{TM} < 0 \end{cases} \end{cases} \quad (5)$$

where  $P_{ele}$  presents the electric power of the motor,  $T_{TM}$  and  $\omega_{TM}$  are the torque and rotation speed of the motor, the overall efficiency  $\eta$  can be determined by a two-dimension lookup table according to the torque  $T_{TM}$  and rotation speed  $\omega_{TM}$ .

Table 1. Main parameters of the REEV

Specification	Value	Unit
Vehicle gross weight	8890	kg
Tire rolling radius	0.465	m
Frontal Area	5.4	m <sup>2</sup>
Coefficient of air drag	0.622	/
Coefficient of rolling resistance	0.0072	N/(m/s)
Coefficient of rotational inertia	1.05	/
Gear ratio	2.3, 1	/
Final drive ratio	4.17	/
Peak power of traction motor	185	kW
Peak power of battery	95	kW
Capacity of battery	74	Ah

An effective static equivalent circuit battery model with zero-order is used. A transformed discrete-time model of the battery state of charge (SoC) and power can be expressed as:

$$SoC_{k+1} = SoC_k - \frac{V_{oc,k} - \sqrt{V_{oc,k}^2 - 4(R_{int,k} + R_t) \cdot P_{bat,k}}}{2(R_{int,k} + R_t) \cdot C} \quad (6)$$

$$P_{bat,k} = P_{ele,k} + P_{gen,k} + P_{aux,k} \quad (7)$$

where  $V_{oc}$  is the open-circuit voltage,  $R_{int}$  is the internal resistance, and these two parameters are the function of  $SoC$  and can be determined through lookup tables;  $R_t$  is the terminal resistance;  $C$  is the battery capacity;  $P_{bat}$  is the power of battery which is the sum of the motor power  $P_{ele}$ , power from genset  $P_{gen}$  and the auxiliary consuming power  $P_{aux}$ .

Since both detailed dynamic response of the engine and generator is neglected, the genset is modeled based on the optimal operating line (OOL), namely, they would always work at the most combined efficient point at each power level. Using static map of power demand at the genset,  $P_{gen}$  to the corresponding fuel consumption can be determined by:

$$\dot{m}_f = f(P_{gen}) \quad (8)$$

It is worthwhile to note that though the required power from genset is zero, there is still a certain fuel cost due to the engine idle state unless the engine is totally shut down. And the engine on/off switch control can be modeled in a similar formulation as the ones of gear shifting. All the main parameters of the delivery truck can be found in Table 1.

## 2.2 Problem formulation

Before employing DP method, the state variables  $x$  and control variables  $u$  should be determined. Here a typical driving cycle including velocity profile is adopted, so only SoC, engine status and gear position are selected as the state variables. To simplify the control variables as possible, only gear shifting command, engine start command and power split ratio (PSR) of the energy sources are selected. Hence, the state equation of REEV can be expressed by the following equations:

$$\begin{cases} x_{k+1} = f(x_k, u_k) \\ x_k = [SoC_k \ x_{eng,k} \ x_{gear,k}] \\ u_k = [u_{shift,k} \ u_{eng,k} \ u_{PSR,k}] \end{cases} \quad (9)$$

Besides the fuel cost performance, the drivability representing the gear shift and engine on/off frequency should also be considered. To unify the evaluation standard, the switching impact would also be expressed as an additional energy loss. Therefore, the optimization goal is to minimize:

$$J = \sum_{k=0}^{N-1} L(x_k, u_k) = \sum_{k=0}^{N-1} (Q_{lhw} \dot{m}_{f,k} + \alpha Q_{lhw} \times |u_{gear,k}| + \beta Q_{lhw} \times |u_{eng,k}|) \quad (10)$$

where  $N$  is the duration of driving cycle;  $L$  is the instantaneous cost;  $Q_{lhw}$  denotes the lower heating value of the fuel consumption;  $\alpha$  is set as a penalty factor of engine start and  $\beta$  is set as a penalty factor of shift frequency. It should be mentioned that the cost function is conflicting since the exploiting of efficient operation is limited due to narrow down the freedom of control variables. Profitably, switching frequency of the gear and engine status would be optimized as well, which could make it adapt to real drivability demands. Thus, the weighing of the factors is very important to achieve a better trade-off between the conflicting performances.

At each step, all the state and control variables in the dynamic model should not violate the constraints below, which arise from physical limitations and problem definition:

$$\begin{cases} SoC_{\min} < SoC < SoC_{\max} \\ T_{TM,\min} < T_{TM} < T_{TM,\max} \\ 0 \leq \omega_{TM} < \omega_{TM,\max} \\ i_{bat,\min} < i_{bat} < i_{bat,\max} \\ 0 \leq P_{gen} < P_{gen,\max} \end{cases} \quad (11)$$

where  $SoC_{\min}$ ,  $SoC_{\max}$  are the pre-established minimum and maximum SoC limits of the battery. In this case study, the SoC will start at 95% and will end at 15%. Since the vehicle is a plug-in REEV, the battery will drain to a low level in a certain driving cycle. The limitations on the TM are given by a curve representing its continuous output power with  $T_{TM,\min}$ ,  $T_{TM,\max}$

and  $\omega_{TM,max}$ . Here the torque of TM could be negative since it may work in regeneration mode. The battery current  $i_{bat}$  is limited by the charging and discharging limitation. And the output power of the genset cannot exceed the smaller power of the engine and generator, which is the peak power  $P_{gen,max}$ .

### 3. SPLIT DYNAMIC PROGRAMMING SOLUTION

Dynamic programming method is based on Bellman's principle of optimality. It can divide the global optimization problem into a sequence of secondary problems backward from the terminal stage, which would always numerically lead to the optimal results according to the preset resolution of state and control variables. Thus, the results from DP algorithm can be regarded as the benchmark, which can make sure the comparison is under the same conditions. The optimization problem above can be described by the recursive equations as:

$$J_k^*(x_k) = \min_{u_k} [L(x_k, u_k) + J_{k+1}^*(x_{k+1})] \quad (12)$$

where  $J_k^*(x_k)$  is the optimal cost-to-go function at stage  $x_k$  from step  $k$  to the end of the given velocity profile, and  $x_{k+1}$  is the  $(k+1)$  step after the control variable at step  $k$  is applied to state  $x_k$  according to Eq. (9).

Though DP is a powerful tool to solve such a numerical optimization problem, it is very sensitive to the number of state variables. Namely, the computational burden will increase exponentially due to the additional state variables (Sundström *et al.*, 2010). Besides, to analyze the potential relationship between the conflicting fuel cost and drivability performance, a lot of repeated calculations would be carried out to estimate the intrinsic trends. It is obvious that applying the basic DP algorithm into this problem directly is very time-consuming.

To accelerate the searching process, one of the reliable solutions is to separate the optimization problem described in Eq. (10) into cascaded subproblems. As shown in Fig. 2, the whole model can be cut off between the DC bus and the TM components, where the right-hand part is the propulsion subsystem influenced by the gear status  $x_{gear}$  and the left-hand part is the energy source subsystem which will follow the power split command  $u_{PSR}$  according to SoC and engine on/off status. Because of such decoupled features, the previous complex problem with higher state variables can be downsized to two subproblems with fewer state variables as:

$$\begin{cases} J_1 = \sum_{k=0}^{N-1} (P_{ele} + \alpha Q_{lthv} \times |u_{gear,k}|), P_{ele} = f(v, u_{shift,k}) \\ J_2 = \sum_{k=0}^{N-1} (Q_{lthv} \dot{m}_f + \beta Q_{lthv} \times |u_{eng,k}|), \dot{m}_f = f(P_{ele}^*, u_{eng,k}, u_{PSR,k}) \end{cases} \quad (13)$$

After the calculation of the propulsion part under DP is completed, we can get the optimized required power at the DC bus, and then apply it to the energy part to get the final results. In this way, the penalty factors can be analyzed individually and the calculation speed would improve significantly. The result deviation is as small as about 0.8% (Oruganti *et al.*, 2018), which is acceptable considering the time cost.

### 4. IMPACT OF PENALTY FACTORS

#### 4.1 Analysis of Gear shift penalty factor

To analyze the impact of gear shift penalty factor, a series of candidates ranging from 0 to 2 are applied in the simulation above. For each penalty factor, the power loss  $P_{loss}$  of the driveline and the total gear shift numbers  $\varphi_{shift}$  are recorded, and the relationship is illustrated in Fig. 3. We can see that the power loss has an approximate positive correlation with the penalty factor, and the total gear shift numbers reduced sharply at the very beginning. Such trend is caused by the limitation of gear shifting, which may relocate the TM operating points to the area of lower efficiency and remain the current gear position. These points can be further fitted to the analytical curves as shown below, with an R-square as high as 0.995 and 0.977, respectively:

$$P_{loss} = 2.107\alpha + 129.6 \quad (14)$$

$$\begin{cases} \varphi_{shift} = 96\alpha^{-0.346} \\ \frac{d\varphi_{shift}}{d\alpha} = -33.216\alpha^{-1.346} \end{cases} \quad (15)$$

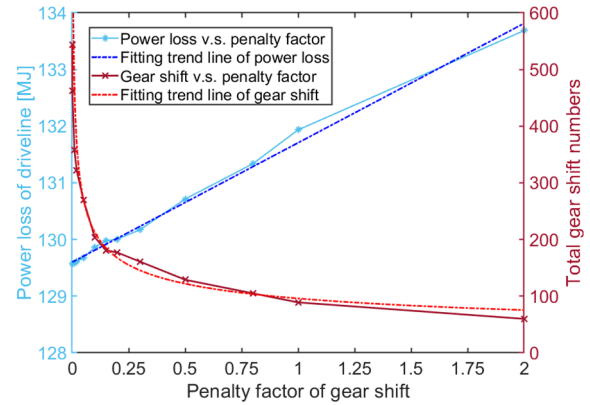


Fig. 3. Trends of power loss and gear shift numbers with the increase of  $\alpha$ .

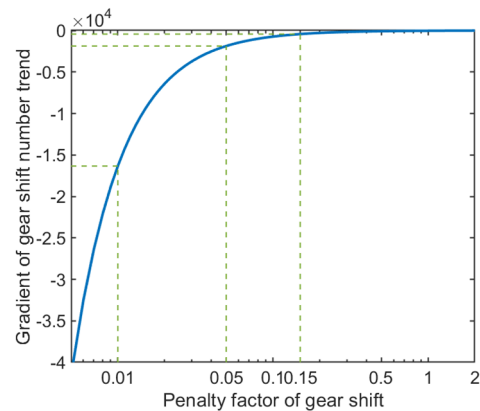


Fig. 4. The gradient of gear shift number trend.

Since the power loss is a linear function to the penalty factor, a reasonable selection could be determined from the gradient of the gear shifting number curve, as shown in Fig. 4. Here we can see that even a small additional penalty (e.g. from 0.01 to 0.05) can result in a prominent decrease of the shifting

numbers. And when the factor is greater than 0.15, the reducing speed is getting slower indicating that the next sequentially increasing penalty cannot improve the gear hunting issues obviously. It should be mentioned that the reducing effect of the gear shifting numbers will tend to a certain level in a rational range, as shown at the end of the trend curve in Fig. 3, which is a meaningful denotation to show the extreme gear shifting performance for the given configuration. Namely, the impact of gear shifting penalty factor is limited. If the performance is still unsatisfying with a big penalty, then the size of the components may need to be redesigned.

#### 4.2 Analysis of engine start penalty factor

Comparing to the gear shifting, the engine on/off switching frequency is much more complex in the REEV system. It would also influence the cold-start problem and related emissions. A larger searching range from 0 to 10 is adopted here for trend analysis which can be found in Fig. 5. In general, with the increase of penalty factor  $\beta$ , the total engine on/off times decreases dramatically and the power loss raises up gradually because of the potential power-consuming from additional engine idle operation.

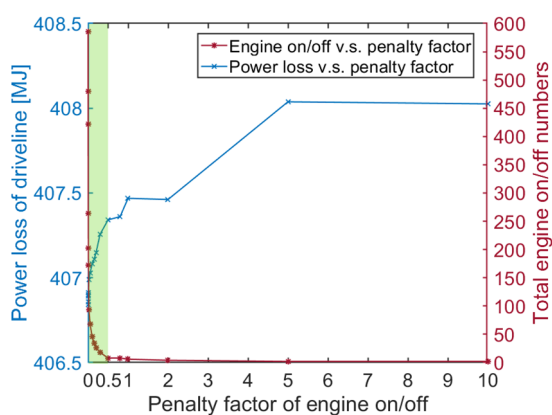


Fig. 5. Trends of power loss and engine on/off numbers with the increase of  $\beta$ .

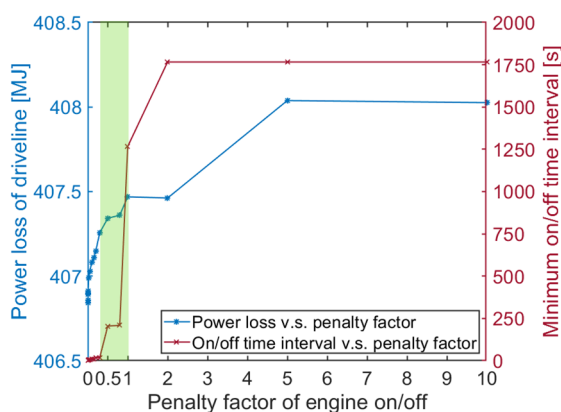


Fig. 6. Trends of power loss and engine on/off minimum time interval with the increase of  $\beta$ .

Besides, considering the thermal issues of the catalytic converter, the engine should also try to avoid the intermittent

operation. Thus, the minimum time interval between each switching of engine status is also taken into account as shown in Fig. 6. We can see that the interval time increases slightly at the beginning and jumps up to a remarkable higher level at some specific points (e.g.  $\beta = 0.5, 1$ ), showing a strong nonlinear feature. Also, if the penalty factor is set to a very large number, it seems all of the power loss, total switching time and the minimum time interval will not change a lot, which indicates the extreme conditions. According to the trends, we can find a distinct area covering the optimal balanced performance. It is also worth note that a larger penalty may also result in lower power loss. This phenomenon implies that each engine on/off segment will influence the others, and may result in a quite different global performance, especially the switching frequency is very small.

### 5. SIMULATION RESULTS

As analyzed in the sections above, in this case, the sweet spots of  $\alpha = 0.15$  and  $\beta = 0.5$  are employed. Fig. 7 shows a comparison of the SoC variation trend of different penalty settings. Generally, there is a very close match between these trend curves. The result of experiential factors adheres more closely to that without any penalty, but all of them would converge to the same line in the end. The whole gear and engine switching performances are illustrated in Fig. 8, where the red area means the engine is on. It is obvious that the experiential penalty factors can hardly improve the drivability performance. And the optimized ones could not only eliminate most of the gear huntings but also avoid all the transitory engine on/off operation. It seems that the gear position will remain constant properly and the scattered engine on/off pieces are concentrated to several blocks to reduce the total switching frequency. Such optimized result is consistent with the real driving demands, which has a promising online implementation potential.

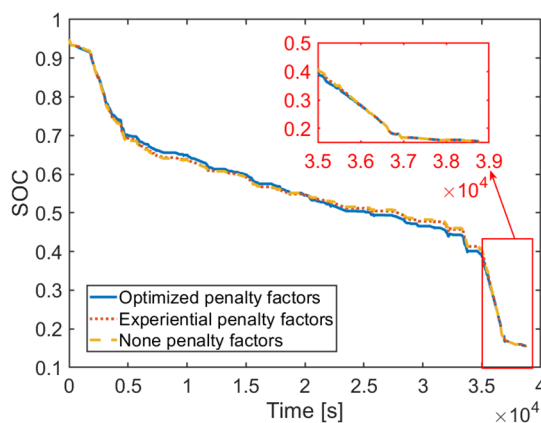


Fig. 7. SoC changing trends with different penalty factors.

The detailed numerical results are shown in Table 2. The gear shift times and engine turn-on times are reduced significantly with a small sacrifice of the fuel economy of only 0.2% in the whole driving cycle. Also, the engine charging duration is quite similar in all of the results, which means the optimized penalty only redistributed the operating opportunity but not affected the energy allocation management.

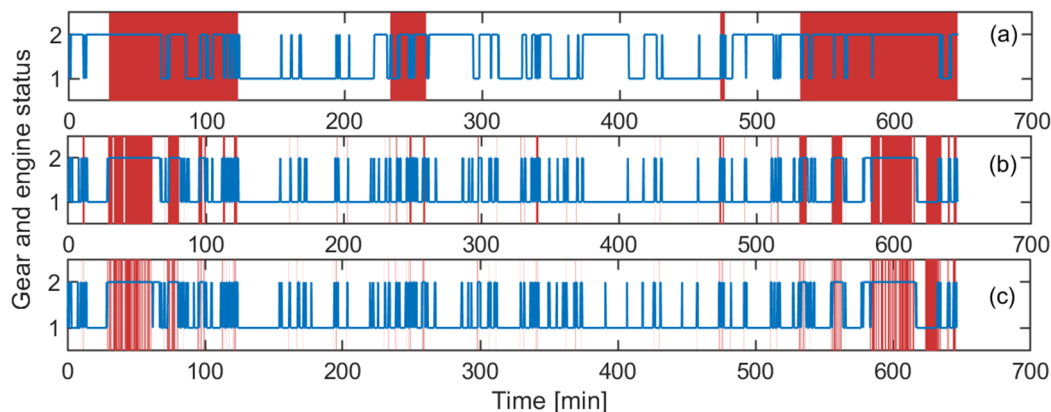


Fig. 8. Gear position and engine on/off variation with (a) optimized (b) experiential (c) none penalty factors.

Table 2. Results of different penalty factors.

Penalty factor	Performance	Value
Optimized Engine on/off 0.5 Gear shift 0.15	Fuel consumption	19.59 L
	Final SOC	15.48%
	Engine turn on times	4
	Engine charging duration	61.92 min
Experiential Engine on/off 0.02 Gear shift 0.02	Fuel consumption	19.55 L
	Final SOC	15.48%
	Engine turn on times	47
	Engine charging duration	62.02 min
None Engine on/off 0 Gear shift 0	Fuel consumption	19.54 L
	Final SOC	15.48%
	Engine turn on times	291
	Engine charging duration	61.78 min

## 6. CONCLUSIONS

In this paper, a conflicting cost function with economy and drivability is optimized by sensitivity analysis for split-DP based energy management of HEV's. The economy and drivability performance are not simply linear with the penalty factors. There is usually a nonlinear trend in the curves indicating a better balance between the power loss and switching frequency. An experiential number with manual tuning may not guarantee reasonable results. As shown in the delivery truck case, proper penalty factors will reduce the gear shifting and engine start-stop frequency by 43.8% and 91.5% respectively with only 0.2% more fuel cost, comparing to the empirical settings.

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