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A Further Study on the Cooperative Control of Energy Storage Systems under Unreliable Communication Network ⋆

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Abstract: In Cai and Hu (2018), a dual objective control problem for an energy storage system was solved by a distributed control scheme which can achieve both state-of-energy balancing and power tracking. However, it relies on the assumption that the communication network is reliable and fixed. In this work, we further consider the same problem for the case that the communication network is unreliable and switched. It is proven that, under certain connectivity condition on the communication network, the same control law as in Cai and Hu (2018) can still achieve the dual control objective.

Keywords: Energy storage system, power tracking, state-of-energy balancing, unreliable communication network.

1. INTRODUCTION

Renewable energy, such as wind or solar energy, proves to be a promising source of electricity with demonstrated benefits. However, electricity generated from renewable sources can rarely provide immediate response to demands in that such sources have the characteristics of inherent variability and uncertainty (Zhao et al., 2015). In fact, the penetration of renewable sources has significantly increased the difficulties in stabilizing the power network (Ibrahim et al., 2008). Energy storage system (ESS) is recognized as a bridge over the gap between renewable energy and the grid. According to Bragard et al. (2010), Germany aims to install 40% of renewable energy by 2020, which, as pointed out by the authors, would not be possible without the employment of additional high-capacity ESS.

Most of the existing works on ESS pertain to specific systems, such as battery ESS, supercapacitor ESS or flywheel ESS (Chen et al., 2009; Dunn et al., 2011; Gyuk and Eckroad, 2004; Kondoh et al., 2000). In contrast, Cai and Hu (2018) considered a general ESS, which is expected to simultaneously achieve two control objectives. On one hand, the power output of the ESS is supposed to meet the reference determined by some upper level control. On the other hand, the power capacity of the ESS should be kept maximum in order for it to be fully functional. With that being said, the state-of-energy (SOE), defined as the ratio of the stored energy and the energy capacity, of all the energy storage units (ESUs) should be balanced since otherwise the ESUs reaching critical high or low energy level will be forced off-line for protection, which will adversely reduce the power capacity of the ESS.

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In this paper, we will further consider the same problem as in Cai and Hu (2018). However, unlike the study in Cai and Hu (2018) which only considered reliable and fixed communication network, we now take into account an unreliable and switched communication network. The result in Cai and Hu (2018) may not be applicable for certain circumstances in that attacks and network failures may occur from time to time. As indicated in Wei et al. (2010), while the electrical power industry is gaining all of the cost and performance of Information Technology (IT), existing IT security challenges are acquired as well. Amin (2011) introduced a switched PDE model, which can describe a class of fault and attack scenarios resulting from intermittent withdrawals through offtake nodes and compromise of sensor-control data, in order to better study the control systems. Su and Huang (2012b) elaborated that, in a practical scenario, the network graph should be switched so that the obtained results can handle unexpected changes on the information flow between any two sub-systems. In here, a switched communication network is employed to describe the unreliable communication environment. It is proven that if the communication network, though switches from time to time, satisfies certain connectivity condition, then the same control law as in Cai and Hu (2018) can still achieve the dual control objective.

The technical contributions of this paper are threefold. First, we construct a converse Lyapunov-like function for the switched linear sub-system consisting of the command generator and the distributed observer. Second, a time-invariant transformation is found which converts the time-varying closed-loop dynamics into two parts, one scalar part towards a free end and one vector part decaying to zero. Finally, thanks to a Barbalat's Lemma-like theorem, the proof is completed by combining together the distributed observer dynamics and the state balancing dynamics.

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2. PROBLEM FORMULATION

In this paper, we consider the same ESS as in Cai and Hu (2018) which consists of N ESUs with identical energy capacity, denoted by E_c. For the ith ESU, let E_i denote the stored energy, and $x_i = E_i/E_c$ denote SOE. Then we have

$$\dot{x}_i = -\gamma P_i \tag{1}$$

where P_i denotes the power output of the ith ESU and $\gamma = 1/E_c$. Moreover, let $P_{ESS} = \sum_{i=1}^{N} P_i$ denote the power output of the ESS, and $P_{REF} \in R$ denote the reference for P_{ESS}.

The command generator (CG) proposed in Cai and Hu (2018) for the case of ESUs with identical energy capacity is given by

$$\dot{\zeta}_0 = 0,\ \zeta_0(0) = P_{REF}/N \tag{2}$$

where $\zeta_0 \in R$.

In contrast with the result of Cai and Hu (2018) which assumed a reliable and fixed communication network, we now proceed to an unreliable and switched communication network. The ESS (1) together with the CG (2) can be viewed as a multi-agent system associated with which we can define a switched graph[1] $\bar{G}_{\sigma(t)} = (\bar{V}, \bar{E}_{\sigma(t)})$ where $\bar{V} = \{0, 1, \ldots, N\}$, $\bar{E}_{\sigma(t)} \subseteq \bar{V} \times \bar{V}$ and the switching signal $\sigma(t)$ is defined as $\sigma(t) : [0, \infty) \to \Lambda = \{1, \ldots, k\}$ for some positive integer k. In here, the node 0 is associated with the CG and the node i, $i = 1, \ldots, N$, is associated with the ith ESU. For $i = 0, 1, \ldots, N$, $j = 1, \ldots, N$, $(i, j) \in \bar{E}_{\sigma(t)}$ if and only if the jth ESU can receive the information from the ith ESU or the CG at time instant t. Furthermore, we can define a subgraph $G_{\sigma(t)} = (V, E_{\sigma(t)})$ of $\bar{G}_{\sigma(t)}$ by letting $V = \{1, \ldots, N\}$ and $E_{\sigma(t)} = \{V \times V\} \cap \bar{E}_{\sigma(t)}$. In the following, let $\bar{\mathcal{A}}_{\sigma(t)} = [a_{ij}(t)] \in R^{(N+1)\times(N+1)}$ be the weighted adjacency matrix of $\bar{G}_{\sigma(t)}$, $L_{\sigma(t)} \in R^{N\times N}$ be the Laplacian of $G_{\sigma(t)}$ and $H_{\sigma(t)} = L_{\sigma(t)} + \mathrm{diag}\{a_{10}(t), \ldots, a_{N0}(t)\}$.

It is assumed that $\bar{G}_{\sigma(t)}$ satisfies the following assumption.

Assumption 1. For all $t \geq 0$, $\bar{G}_{\sigma(t)}$ contains a spanning tree with the node 0 as the root and $G_{\sigma(t)}$ is undirected and connected.

Now, the problem considered in this paper can be formulated as follows.

Problem 1. Given systems (1), (2) and the communication network $\bar{G}_{\sigma(t)}$, for $i = 1, \ldots, N$, find P_i such that

$$\lim_{t\to\infty} (P_{ESS}(t) - P_{REF}) = 0 \tag{3}$$

and

$$\lim_{t\to\infty} (x_i(t) - x_j(t)) = 0 \tag{4}$$

for $i, j = 1, \ldots, N$.

Remark 1. For the case where $\bar{G}_{\sigma(t)}$ reduces to a fixed graph, the following control law was proposed in Cai and Hu (2018) to solve Problem 1

$$\dot{\zeta}_i = \mu \sum_{j=0}^{N} a_{ij}(\zeta_j - \zeta_i) \tag{5a}$$

$$P_i = -\kappa \sum_{j=1}^{N} a_{ij}(x_j - x_i) + \zeta_i \tag{5b}$$

where $\zeta_i \in R$, $\mu, \kappa > 0$. While, under the situation of a switched graph $\bar{G}_{\sigma(t)}$, the control law (5) becomes

$$\dot{\zeta}_i = \mu \sum_{j=0}^{N} a_{ij}(t)(\zeta_j - \zeta_i) \tag{6a}$$

$$P_i = -\kappa \sum_{j=1}^{N} a_{ij}(t)(x_j - x_i) + \zeta_i. \tag{6b}$$

The main result of this paper, which will be detailed shortly in the next section, is to show that the control law (6) can still solve Problem 1 under Assumption 1.

3. STABILITY ANALYSIS

Theorem 1. Given systems (1), (2) and the communication network $\bar{G}_{\sigma(t)}$, under Assumption 1, for any system initial condition and any $\mu, \kappa > 0$, Problem 1 is solvable by control law (6).

Proof: For $i = 1, \ldots, N$, let $\bar{\zeta}_i = \zeta_i - \zeta_0$ and $\bar{\zeta} = \mathrm{col}(\bar{\zeta}_1, \ldots, \bar{\zeta}_N)$[2]. Then by (2) and (6a), we have

$$\dot{\bar{\zeta}} = -\mu H_{\sigma(t)}\bar{\zeta} \triangleq \Pi_{\sigma(t)}\bar{\zeta}. \tag{7}$$

Let $\Phi(t, \tau)$ be the state transition matrix of system (7), i.e.,

$$\bar{\zeta}(t) = \Phi(t, \tau)\bar{\zeta}(\tau). \tag{8}$$

By definition (Chen, 1999), we have

$$\frac{\partial}{\partial t}\Phi(t, \tau) = \Pi_{\sigma(t)}\Phi(t, \tau) \tag{9}$$

and

$$\Phi(\tau, t)\Phi(t, \tau) = \Phi(\tau, \tau) = I. \tag{10}$$

Differentiating (10) with respect to t gives

$$\left[\frac{\partial}{\partial t}\Phi(\tau, t)\right]\Phi(t, \tau) + \Phi(\tau, t)\frac{\partial}{\partial t}\Phi(t, \tau) = 0. \tag{11}$$

Therefore

$$\frac{\partial}{\partial t}\Phi(\tau, t) = -\Phi(\tau, t)\Pi_{\sigma(t)}. \tag{12}$$

Note that under Assumption 1, by Lemma 4 of Su and Huang (2012b), the origin of system (7) is exponentially stable. Therefore,

$$||\Phi(t, \tau)|| \leq c_1 e^{-c_2(t-\tau)} \tag{13}$$

for some $c_1, c_2 > 0$.

Define

$$\Upsilon(t) = \int_t^{\infty} \Phi(s, t)^T \Phi(s, t) ds. \tag{14}$$

Since Λ is finite, there exists $l > 0$ such that $||\Pi_{\sigma(t)}|| \leq l$ for all $t \geq 0$. Then, similar to the proof of Theorem 4.12 of Khalil (2002), there exist $c_3, c_4 > 0$ such that

$$c_3||\bar{\zeta}||^2 \leq \bar{\zeta}^T\Upsilon(t)\bar{\zeta} \leq c_4||\bar{\zeta}||^2. \tag{15}$$

[1] See Su and Huang (2012a); Su and Huang (2012b) for a summary of graph notation.

[2] For $x_i \in R^{n_i}$, $i = 1, \ldots, m$, $\mathrm{col}(x_1, \ldots, x_m) = [x_1^T, \ldots, x_m^T]^T$. $1_n = \mathrm{col}(1, \ldots, 1) \in R^n$.

By (12), we have

$$\begin{aligned}\dot{\Upsilon}(t) &= -I + \int_t^\infty \left[\frac{\partial}{\partial t}\Phi(s,t)^T\right]\Phi(s,t)ds \\ &+ \int_t^\infty \Phi(s,t)^T \frac{\partial}{\partial t}\Phi(s,t)ds \\ &= -I - \Pi_{\sigma(t)}^T \int_t^\infty \Phi(s,t)^T\Phi(s,t)ds \\ &- \int_t^\infty \Phi(s,t)^T\Phi(s,t)ds \cdot \Pi_{\sigma(t)} \\ &= -I - \Pi_{\sigma(t)}^T \Upsilon(t) - \Upsilon(t)\Pi_{\sigma(t)}.\end{aligned} \quad (16)$$

Substituting (6b) into (1) yields

$$\dot{x}_i = \gamma\kappa \sum_{j=1}^N a_{ij}(t)(x_j - x_i) - \gamma\zeta_i. \quad (17)$$

Let $\zeta = \mathrm{col}(\zeta_1, \ldots, \zeta_N)$. Then $\bar{\zeta} = \zeta - \zeta_0 1_N$. Let $x = \mathrm{col}(x_1, \ldots, x_N)$ and then we obtain

$$\begin{aligned}\dot{x} &= -\gamma\kappa L_{\sigma(t)}x - \gamma\zeta \\ &= -\gamma\kappa L_{\sigma(t)}x - \gamma\zeta_0 1_N - \gamma\bar{\zeta}.\end{aligned} \quad (18)$$

Define

$$U = (\, U_1 \;\, U_r \,) \quad (19)$$

where $U_1 = 1_N/\sqrt{N}$ and $U_r \in R^{N\times(N-1)}$ is an arbitrarily chosen constant matrix such that U is orthogonal. Then

$$U^{-1} = U^T = \begin{pmatrix} U_1^T \\ U_r^T \end{pmatrix}. \quad (20)$$

Under Assumption 1, $L_{\sigma(t)}$ is symmetric. Noting that $L_{\sigma(t)}1_N = 0$ gives

$$U^T L_{\sigma(t)} U = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & U_r^T L_{\sigma(t)} U_r & \\ 0 & & & \end{pmatrix}. \quad (21)$$

Let $J_{\sigma(t)} = U_r^T L_{\sigma(t)} U_r$. Under Assumption 1, by Lemma 1.3 of Ren and Cao (2011), for all $t \geq 0$, all the eigenvalues of $J_{\sigma(t)}$ are real and positive. Since Λ is finite, let $J_{\min}$ be the smallest eigenvalue of $J_{\sigma(t)}$ for all $t \geq 0$. Furthermore, since $U^{-1}U = I_N$, $U_r^T U_1 = 0$. Therefore,

$$U^{-1}1_N = \mathrm{col}(\sqrt{N}, 0, \ldots, 0) \in R^N.$$

Let $\bar{x} = U^{-1}x$. We have

$$\dot{\bar{x}} = -\gamma\kappa U^{-1} L_{\sigma(t)} U \bar{x} - \gamma\zeta_0 U^{-1} 1_N - \gamma U^{-1}\bar{\zeta}. \quad (22)$$

Divide $\bar{x}$ into $\bar{x} = \mathrm{col}(\bar{x}_s, \bar{x}_v)$ with $\bar{x}_s \in R$ and $\bar{x}_v \in R^{N-1}$. By (22), we have

$$\dot{\bar{x}}_s = -\gamma\zeta_0\sqrt{N} - \gamma U_1^T \bar{\zeta} \quad (23a)$$

$$\dot{\bar{x}}_v = -\gamma\kappa J_{\sigma(t)}\bar{x}_v - \gamma U_r^T \bar{\zeta}. \quad (23b)$$

Let

$$\alpha = 1 + \frac{\gamma||U_r||^2}{2\kappa J_{\min}}$$

and define

$$V(t) = \frac{1}{2}\bar{x}_v^T \bar{x}_v + \alpha\bar{\zeta}^T \Upsilon(t)\bar{\zeta}. \quad (24)$$

Then by (15), $V(t) \geq 0$. Along systems (7) and (23b), we have

$$\begin{aligned}\dot{V}(t) &= -\gamma\kappa\bar{x}_v^T J_{\sigma(t)}\bar{x}_v - \gamma\bar{x}_v^T U_r^T \bar{\zeta} \\ &+ \alpha\left(\bar{\zeta}^T \Pi_{\sigma(t)}^T \Upsilon(t)\bar{\zeta} + \bar{\zeta}^T\Upsilon(t)\Pi_{\sigma(t)}\bar{\zeta} + \bar{\zeta}^T\dot{\Upsilon}(t)\bar{\zeta}\right) \\ &= -\gamma\kappa\bar{x}_v^T J_{\sigma(t)}\bar{x}_v - \gamma\bar{x}_v^T U_r^T\bar{\zeta} - \alpha||\bar{\zeta}||^2 \\ &\leq -\gamma\kappa J_{\min}||\bar{x}_v||^2 + \frac{\gamma\kappa J_{\min}}{2}||\bar{x}_v||^2 \\ &+ \frac{\gamma||U_r||^2}{2\kappa J_{\min}}||\bar{\zeta}||^2 - \alpha||\bar{\zeta}||^2 \\ &= -\frac{\gamma\kappa J_{\min}}{2}||\bar{x}_v||^2 - ||\bar{\zeta}||^2 \leq 0.\end{aligned} \quad (25)$$

Therefore, $\bar{x}_v$ is bounded and so is $\dot{\bar{x}}_v$. Let the switching time instants of the switching signal be given by $\{t_0, t_1, t_2, \ldots,\}$ with $\lim_{k\to\infty} t_k = \infty$. Hence, there exists $K > 0$ such that

$$\sup_{t_i \leq t < t_{i+1}, i=0,1,2,\ldots} |\ddot{V}(t)| \leq K. \quad (26)$$

Then, by Corollary 1 of Su and Huang (2012a),

$$\lim_{t\to\infty} \dot{V}(t) = 0,$$

which in turns implies that

$$\lim_{t\to\infty} \bar{x}_v(t) = 0.$$

As a result, noting that

$$x = U\bar{x} = U_1\bar{x}_s + U_r\bar{x}_v$$

gives

$$\lim_{t\to\infty}(x(t) - U_1\bar{x}_s) = 0,$$

which indicates, for $i, j = 1, \ldots, N$, that

$$\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0. \quad (27)$$

On the other hand, we have

$$\begin{aligned}&P_{ESS} - P_{REF} \\ &= \textstyle\sum_{i=1}^N P_i - N\zeta_0 \\ &= -\kappa\textstyle\sum_{i=1}^N\sum_{j=1}^N a_{ij}(t)(x_j - x_i) + \sum_{i=1}^N \zeta_i - N\zeta_0 \\ &= -\kappa\textstyle\sum_{i=1}^N\sum_{j=1}^N a_{ij}(t)(x_j - x_i) + \sum_{i=1}^N \bar{\zeta}_i\end{aligned} \quad (28)$$

and thus

$$\lim_{t\to\infty}(P_{ESS}(t) - P_{REF}) = 0.$$

□

4. SIMULATION

In this section, we consider an ESS consisting of eighteen ESUs. The energy capacity of the ESUs is given by E_c = 10kwh. The control gains are selected to be $\kappa = 10^5, \mu = 100$. The system initial states are given by $x_i(0) = 0.9 - 0.01 * (i-1)$, $\zeta_i(0) = 0$, $i = 1, \ldots, 18$ Suppose at t = 0h, P_{REF} = 30kw, and at t = 4h, P_{REF} = −30kw. The four subgraphs of the switched undirected communication network considered in this case are shown by Fig. 1. Suppose the graph associated with the switched communication network switches in the cyclic order of *abcda...* with the dwelling time in each of the subgraphs denoted by τ. The simulation results subject to different τ are shown by Figs. 2, 3,4.

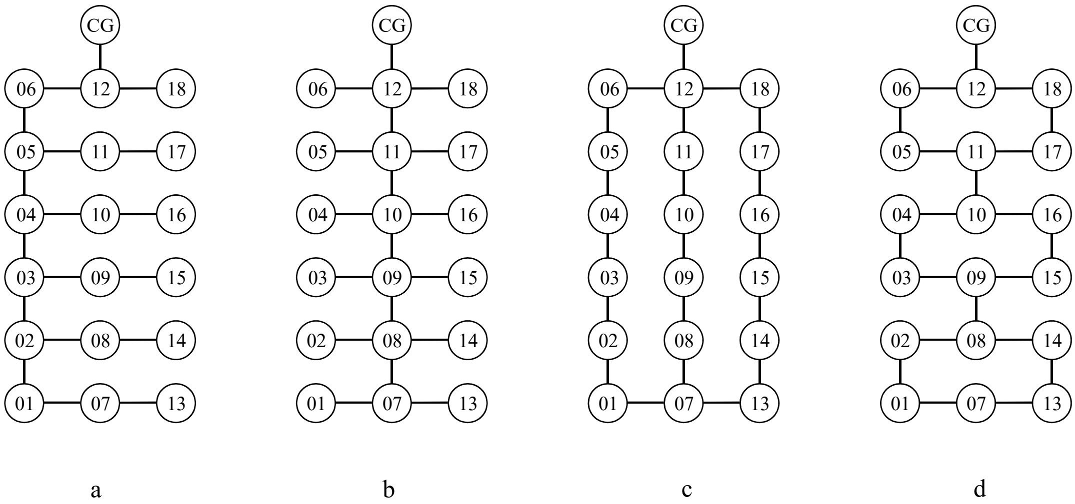

Fig. 1. The four subgraphs of the switched undirected communication network.

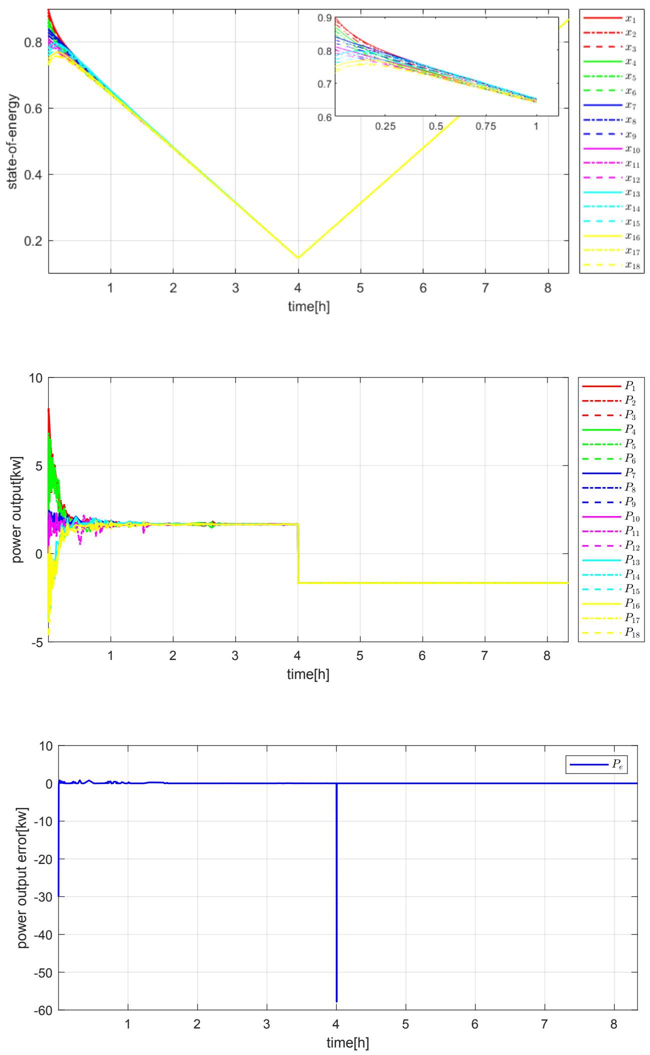

Fig. 2. System profiles for $\tau = 0.1$s.

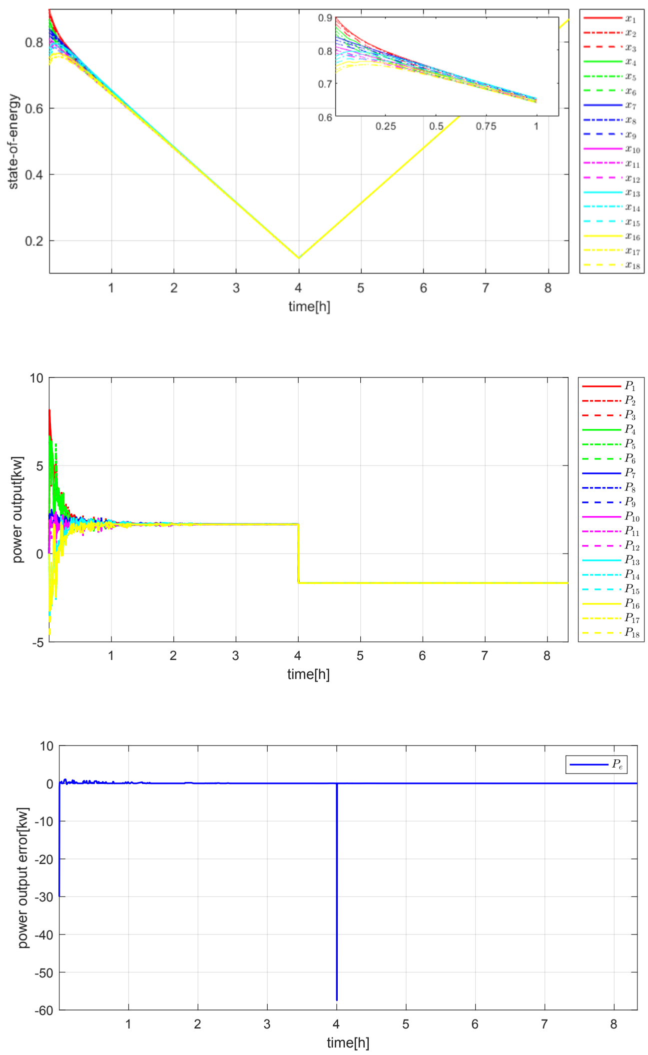

Fig. 3. System profiles for $\tau = 1$s.

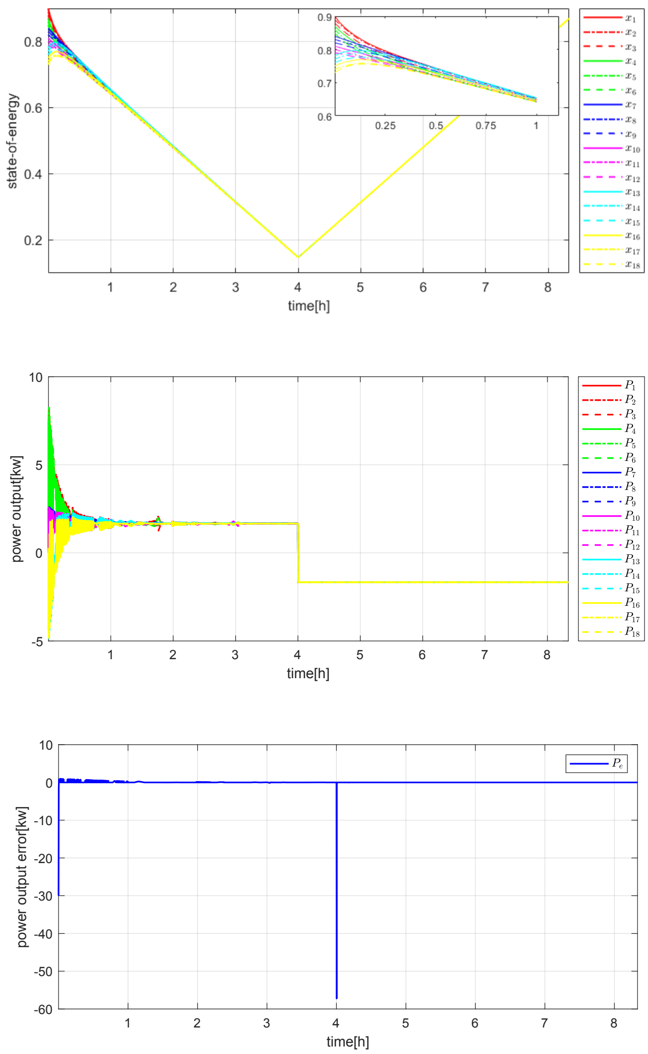

Fig. 4. System profiles for $\tau = 10$s.

5. CONCLUSION

This paper presents a further study on the cooperative control of ESSs extending the results of Cai and Hu (2018) to an unreliable and switched communication network. It has been proven that, under certain connectivity condition, the same control law as in Cai and Hu (2018) can still achieve the dual control objective. Moreover, we have shown by simulation that the all-time and undirected connectivity condition might further be relaxed to jointly and directed connectivity condition, which has shed light on our future work.

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