Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

A Dual Splitting Method for Distributed Economic Dispatch in Multi-energy Systems

Zhibin Wang * **Jinming Xu** ** **Shanying Zhu** * **Cailian Chen** *

* *Department of Automation, Shanghai Jiao Tong University, and Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240, China*
** *College of Control Science and Engineering, Zhejiang University, Hangzhou 310027, China*

Abstract: Multi-energy systems which bring together different forms of energy, such as electricity, gas and heat, to coordinate in the process of supply, transmission and consumption provide much higher flexibility over traditional energy systems in energy utilization. This paper deals with the economic dispatch problem in multi-energy systems in a distributed manner. Each agent optimizes its local objective function with regard to local coupled limits and a global constraint via local communications only. A distributed algorithm is proposed based on duality analysis and splitting methods.This algorithm adopts a non-linear mapping method to linearize the nonlinearity arisen from coupling relationship among energy carriers. We show that the proposed algorithm converges at a nonergodic rate of $O(\frac{1}{k})$. Simulations are demonstrated to show the effectiveness of the algorithm.

Keywords: Multi-energy systems, economic dispatch, distributed algorithm, duality, splitting method

1. INTRODUCTION

Energy crisis and environmental pollution urge the reform of energy consumption patterns and extensive use of renewable energy. Multi-energy systems (MESs) in which different energy forms, e.g., eletricity, heat, cooling, fuels, interact with each other optimally at different levels (Mancarella, 2014) was proposed to overcome the challenge. MESs promote the overall efficiency of energy use and provide opportunities for most renewable energy sources that are needed to cope with the intrinsic intermittency and uncertainty (Mancarella, 2012; Krause et al., 2011; Huang et al., 2011).

A basic problem in MESs is the economic dispatch (ED) problem whose objective is to minimize some total operating cost while meeting some constraints caused by the supply-demand balance and physical limitations. Normally the concept of energy hub (EH) which represents an interface between energy producers and energy comsumers is used to model the problem since it can explicitly describe the coupling relationship among various energy carriers in the systems. As a fundamental issue of energy management for MESs, the ED based on EH has been receiving much attention recently. For example, an ED problem considering power loss was solved by a particle swarm optimization (PSO) algorithm in (Beigvand et al., 2017), while (Shi et al., 2017) used an enhanced PSO together with two stage stochastic linear programming method taking into account uncertain renewable energy resources.

Note that most existing studies used a centralized infrastructure. However, energy resources are normally managed by different entities in MESs which means privacy protection should be concerned and a center usually does not exist. A distributed formulation, where each agent optimizes its local objective function with regard to the constraints with local communications between its neighbors to obtain the global optimal solution, is more appropriate in reality. In fact, distributed ED has been widely studied in traditional power systems (Molzahn et al., 2017) which can be viewed as a special case of MESs where only electricity is involved in the system. Distributed ED of power systems is also termed as distributed resource allocation problem whose optimal solution is attained when the marginal costs are all equal (Lakshmanan and de Farias, 2008), which indicates that the consensus method can be employed to solve the problem. (Ziang Zhang et al., 2011; Zhang and Chow, 2011) proposed a distributed algorithm to solve ED in smart grid by viewing the incremental cost of each bus as a consensus variable. But a centralized leader was still needed to ensure the global constraint. (Kar and Hug, 2012) used a consensus + innovation framework to remove the center but a decaying stepsize was needed to guarantee the convergence. (Yang et al., 2013) proposed an algorithm based on standard Lambda-Iteration method to overcome this drawback. However, it could only deal with quadratic cost functions. (Xu et al., 2019) relaxed the cost function to be convex, differentiable and L_f-smooth, and proposed a dual splitting approach with a nonergodic convergence rate of $O(\frac{1}{k})$ based on primal-dual protocol.

This research is partially funded by National Key Research and Development Program of China (No. 2016YFB0901900), NSF of China under the grants 61922058, and 61521063, and NSF of Shanghai Municipality of China under grant 18ZR1419900.

It should be noted that various kinds of energy besides electricity are coupled together in MESs. This is quite different from traditional power systems and make distributed ED in MESs relatively new and difficult. The coupling relationship among energy carriers brings complex constraints to the optimization problem even with nonlinear factors which makes the feasible set non-convex. Distributed methods to solve linear coupling problems are mostly based on augmented Lagrangian method (ALM) (Zhang and Zavlanos, 2018) and alternating direction method of multipliers (ADMM) (Carli and Dotoli, 2020; Falsone et al., 2019). Nevertheless, none of these can deal with non-linear equality constraints directly while still guaranteeing the convergence rate.

In this paper, we consider a distributed ED problem in MESs. We attempt to eliminate the non-linear part in the constriants and propose a fully distributed dual splitting algorithm to solve the problem. The contributions of this paper can be summarized as follows:

1) First, we give the model of ED in MESs and utlize a non-linear mapping method to transform the original objective variables to a convex set, which converts the problem to linear constrained form. By a proper transformation of primal functions, we apply the dual splitting method and propose a fully distributed algorithm. The proposed algorithm only requires communication of dual variable.
2) Second, convergence properties are analyzed under proper assumptions. We show that the algorithm has a nonergodic convergence rate of $O(\frac{1}{k})$ for general convex cost functions. Moreover, we give the specific upper bound of stepsize for convergence.

The rest of this paper is organized as follows. First, a distributed ED problem in MESs is formulated in Section 2. Then, a distributed algorithm is proposed in Section 3. Convergence analysis is given in Section 4. In Section 5, simulations are carried out to show the effectiveness of the algorithm. Finally, Section 6 concludes this paper.

Notations: We use $x = \left[x_1^T \ x_2^T \ ... \ x_m^T\right]^T$ to denote the collection of local variables x_i. Correspondingly, we denote by $x_{i,k}$ and x_k the generated iterates of x_i and x at time k and Δ is the difference between two consecutive vectors, e.g., $\Delta x_{k+1} = x_{k+1} - x_k$. In addition, we use $\mathbf{1}$ to denote all-ones column vector, e_i is the i-th column of the identity matrix I, $\partial g(x)$ denotes the subgradient of g at x, $f^*(u) = \sup_{v \in \mathrm{dom} f}\{\|u, v\| - f(v)\}$ denotes the convex conjugate of f and $\mathrm{prox}_{\tau\phi}(y) = \arg\min_{x\in\mathcal{H}}\{\phi(x) + \frac{1}{2\tau}\|x-y\|^2\}$ is the proximal operator.

2. PROBLEM FORMULATION

2.1 EH Model

In this work, we model the problem based on the EH which consists of a transformer, a combined heat and power (CHP) and a furnace. The coupling matrix between its inputs and outputs can be expressed as:

$$A = \begin{bmatrix} \eta^{e,e} & \eta^{chp,e}\alpha \\ 0 & \eta^{chp,h}\alpha + \eta^{g,h}(1-\alpha) \end{bmatrix}, \tag{1}$$

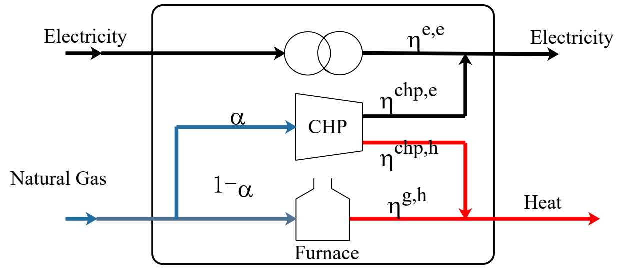

Fig. 1. The basic structure of an EH.

where $\eta^{e,e}$, $\eta^{chp,e}$, $\eta^{chp,h}$ and $\eta^{g,h}$ represent the conversion efficiency of transformer, CHP (gas-electricity), CHP (gas-heat) and gas furnace respectively, and $0 < \alpha < 1$ is the dispatch factor of natural gas to be allocated to the CHP.

2.2 Communication Network Model

Viewing each EH as a node, we can model the communication network by an undirected gragh $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Each edge $e_{ij} \in \mathcal{E}$ indicates that nodes i and j can communicate with each other. Denote the neighbors of node i by $\mathcal{N}_i = \{j | j \in \mathcal{V}, e_{ij} \in \mathcal{E}\} \cup \{i\}$. Define a weight matrix $W = (w_{ij})_{m\times m}$ of graph $\mathcal{G}$ with $w_{ij} > 0$ if $j \in \mathcal{N}_i$ and $w_{ij} = 0$ otherwise.

Assumption 1. Matrix W is symmetric positive definite and doubly stochastic, i.e., $W\mathbf{1} = \mathbf{1}$, $\mathbf{1}^T W = \mathbf{1}^T$. Moreover, the spectral radius $\rho(W - \frac{\mathbf{1}\mathbf{1}^T}{m})$ is less than one.

2.3 ED Problem in MESs

Consider the MES consisting of m EHs each of which is responsible for the energy supply of a certain area in collaboration with other EHs. Each EH connects to the electricity bus as well as the gas pipeline, some of which have local generators and can purchase natural gas from local gas companies. The extra electricity and gas can be transmitted to other EHs. In this paper, without considering the energy loss, we study the following economic dispatch problem of minimizing the cost function $C(s)$ that is the sum of all the electricity generation costs $C_i^e(s_i^e), \forall i$ and gas purchasing costs $C_i^g(s_i^g), \forall i$.

$$\begin{aligned} \min_{\{s,p,\alpha\}} C(s) &= \sum_{i=1}^{m} (C_i^e(s_i^e) + C_i^g(s_i^g)) \\ s.t. \sum_{i=1}^{m} s_i &= \sum_{i=1}^{m} p_i, \\ & A_i p_i = l_i, \forall i, \\ & \underline{s_i} \preceq s_i \preceq \overline{s_i}, \forall i, \\ & 0 \leq \alpha_i \leq 1, \forall i, \\ & p_i \succeq 0, \forall i, \end{aligned} \tag{2}$$

where, $s_i = \left[s_i^e \ s_i^g\right]^T, \forall i$ denotes the local power produced by the generator and local gas power bought from gas company, $\alpha_i, \forall i$ is the local dispatch factor, $p_i = \left[p_i^e \ p_i^g\right]^T, \forall i$ denotes the electricity and gas power that each EH consumes, $l_i = \left[l_i^e \ l_i^h\right]^T, \forall i$ is the local demand of electricity and heat power, $A_i, \forall i$ is the local coupling matrix defined by (1), $\underline{s_i} = \left[\underline{s_i^e} \ \underline{s_i^g}\right]^T, \forall i$ and $\overline{s_i} = \left[\overline{s_i^e} \ \overline{s_i^g}\right]^T, \forall i$ are lower and upper bounds of local generating capacity limit and local gas power limit.

The objective of this paper is to solve problem (2) in a distributed approach.

3. DISTRIBUTED ED ALGORITHM VIA DUAL SPLITTING

In this section, we first use a non-linear mapping method to convert the problem to a linear constrained form and then propose a distributed algorithm based on dual splitting method.

3.1 Linearization of Constraints

Note that $A_i p_i = l_i$ of problem (2) is actually a non-linear equality constraint for each i, since the variables p_i and α_i are coupled by referring to (1). This makes the problem (2) non-convex.

To eliminate the non-linear part, we define a set of non-linear mappings $\sigma_i : \mathbf{R} \to \mathbf{R}$ as $\sigma_i(x) = \frac{l_i^h(\eta_i^{chp,h} - \eta_i^{g,h})x}{\eta_i^{chp,h}x + \eta_i^{g,h}(1-x)}$, $\forall i$. Letting $\beta_i = \sigma_i(\alpha_i), \forall i$, we can rewrite the local equality constraints in (2) as

$$p_i = d_i - \beta_i u_i, \ \forall i, \tag{3}$$

where

$$d_i = \begin{bmatrix} \frac{l_i^e}{\eta_i^{e,e}} & \frac{l_i^h}{\eta_i^{g,h}} \end{bmatrix}^T, \ u_i = \begin{bmatrix} \frac{\eta_i^{chp,e}}{\eta_i^{ee}(\eta_i^{chp,h} - \eta_i^{g,h})} & \frac{1}{\eta_i^{g,h}} \end{bmatrix}^T.$$

Let $D_i = [e_1 \ e_2 \ u_i]$ and $x_i = \begin{bmatrix} s_i^e & s_i^g & \beta_i \end{bmatrix}^T$. We can merge the local coupling equalities into the global one and conclude the constraints as an overall equality term

$$\sum_{i=1}^m D_i x_i = \sum_{i=1}^m d_i \tag{4}$$

with the local limits $\underline{x_i} \preceq x_i \preceq \overline{x_i}$, where $\underline{x_i} = \begin{bmatrix} \underline{s_i^e} & \underline{s_i^g} & \overline{\beta_i} \end{bmatrix}^T$ and $\overline{x_i} = \begin{bmatrix} \overline{s_i^e} & \overline{s_i^g} & \overline{\beta_i} \end{bmatrix}^T$. From the definition of β_i, we have

$$\underline{\beta_i} = \max\left\{ \frac{\eta_i^{chp,h} - \eta_i^{g,h}}{\eta_i^{chp,h}} l_i^h, \frac{\eta_i^{chp,h} - \eta_i^{g,h}}{\eta_i^{chp,e}} l_i^e \right\}, \ \overline{\beta_i} = 0.$$

The problem is turned into a convex one since the equality constraint (4) together with these new inequality constraints forms a convex feasible set and the objective function has nothing changed.

Encoding d_i in the initial value of x_i, and using an indicator function g_i to denote the regularization term encoding certain feasible set $\{x_i = (s_i^e, s_i^g, \beta_i) | \underline{s_i^e} \leq s_i^e \leq \overline{s_i^e}, \ \underline{s_i^g} \leq s_i^g \leq \overline{s_i^g}, \ \underline{\beta_i} \leq \beta_i \leq \overline{\beta_i}\}, \forall i$, the ED problem (2) can be rewritten as

$$\min \ \phi(x) = \sum_{i=1}^m (f_i(x_i) + g_i(x_i))$$
$$s.t. \ \sum_{i=1}^m D_i x_i = 0, \tag{5}$$

where $f_i(x_i) = C_i^e(s_i^e) + C_i^g(s_i^g), \forall i$.

Assumption 2. The function g_i is proper, closed and convex, and f_i is convex, differentiable and $L_{f_i} - smooth$, i.e.,

$$\|\nabla f_i(x_i') - \nabla f_i(x_i)\| \leq L_{f_i} \|x_i' - x_i\|, \forall i.$$

3.2 Dual Decomposition

Minimizing $\phi(x)$ is equivalent to maximizing $-\phi(x)$. The global Lagrangian for (5) is given by

$$L(\lambda, x) = -\phi(x) + \lambda^T \sum_{i=1}^m D_i x_i.$$

Let $y_i, \forall i$ be the duplicates of the dual variable shared by all agents with conditions $y_i = y_j, \forall i, j \in \mathcal{V}$. The dual problem of (5) can be viewed as a consensus problem as follows

$$\min \ \varphi(y) = \sum_{i=1}^m \varphi_i(y_i)$$
$$s.t. \ y_i = y_j, \forall i, j \in \mathcal{V}, \tag{6}$$

where $\varphi_i(y_i) = \sup_{x_i}\{-\phi_i(x_i) + y_i^T D_i x_i\}, \forall i$ denotes the local dual funcion and $\phi_i(x_i) = f_i(x_i) + g_i(x_i), \forall i$.

3.3 Algorithm Development

Consider the functions

$$h_i(\xi_i) = \inf_{x_i, D_i x_i = \xi_i} \{\phi_i(x_i)\}, \forall i.$$

We have

$$\begin{aligned} h_i^*(y_i) &= -\inf_{y_i}\{\inf_{x_i, D_i x_i = \xi_i}\{\phi_i(x_i)\} - y_i^T \xi_i\} \\ &= -\inf_{y_i} \inf_{x_i, D_i x_i = \xi_i}\{\phi_i(x_i) - y_i^T \xi_i\} \\ &= -\inf_{x_i}\{\phi_i(x_i) - y_i^T D_i x_i\}, \forall i. \end{aligned}$$

Hence h_i and φ_i are conjugates of each other, and it is easy to see that problem (5) is equivalent to

$$\min \ H(\xi) = \sum_{i=1}^m h_i(\xi_i)$$
$$s.t. \ \sum_{i=1}^m \xi_i = 0. \tag{7}$$

To solve the primal-dual problem, inspired by the work (Xu et al., 2018), we propose the following algorithm:

$$y_{i,k+1} = \operatorname{prox}_{\tau\varphi_i}\left(\sum_{j\in\mathcal{N}_i} w_{ij} y_{i,k} + \tau\xi_{i,k}\right)$$
$$\xi_{i,k} = \xi_{i,k} - \frac{1}{\tau}\left(y_{i,k+1} - \sum_{j\in\mathcal{N}_i} w_{ij} y_{i,k+1}\right), \forall i, \tag{8}$$

where $\tau > 0$ is a certain tuning parameter. Note that the first update $y_{i,k+1}$ involves $\operatorname{prox}_{\tau\varphi_i}(\cdot)$, which is computationally forbidden for nodes. To amend it, we introduce auxiliary variables $z_i, \forall i$ as in (Xu et al., 2019). Let $z_{i,k} = \operatorname{prox}_{h_i/\tau}((\sum_{j\in\mathcal{N}_i} w_{ij} y_{i,k} + \tau\xi_k)/\tau)$. By Moreau identity (Bauschke and Combettes, 2011, Th. 14.3) $\operatorname{prox}_{\tau h^*}(s) = s - \tau \operatorname{prox}_{h/\tau}(s/\tau)$, we have $y_{i,k+1} = \sum_{j\in\mathcal{N}_i} w_{ij} y_{i,k} + \tau(\xi_{i,k} - z_{i,k})$. Thus $y_{i,k+1} \in \partial h(z_{i,k})$. We further relax it to $y_{i,k+1} \in \partial h(z_{i,k+1})$ to ease the computation burden. It follows that

$$h_i(z_i) - h_i(z_{i,k+1}) \geq y_{i,k+1}^T (z_i - z_{i,k+1}),$$

which indicates that

$$z_{i,k+1} \in \arg\min_{z_i}\{h_i(z_i) - y_{i,k+1}^T z_i\}. \tag{9}$$

Algorithm 1: Distributed ED algorithm in MESs

Initialization: set $\xi_{i,0} = d_i$, and $y_{i,0},\ z_{i,0}$ arbitrarily assigned.

Step 1: Node i updates

$$y_{i,k+1} = \sum_{j\in\mathcal{N}_i} w_{ij} y_{j,k} + \tau(\xi_{i,k} - D_i x_{i,k}).$$

Step 2: Node i updates

$$\xi_{i,k+1} = \xi_{i,k} - \tfrac{1}{\tau}(y_{i,k+1} - \sum_{j\in\mathcal{N}_i} w_{ij} y_{j,k+1}).$$

Step 3: Node i updates

$$x_{i,k+1} = \mathrm{prox}_{\gamma g_i}(x_{i,k} - \gamma(\nabla f_i(x_{i,k}) - D_i^T(2y_{i,k+1} - y_{i,k}))).$$

Step 4: Node i computes

$$(s^e_{i,k+1}, s^g_{i,k+1}, \beta_{i,k+1}) = x_{i,k+1},$$

$$\alpha_{i,k+1} = \frac{\eta_i^{g,h}\beta_{i,k+1}}{(\eta_i^{g,h} - \eta_i^{chp,h})(\beta_{i,k+1} - l_i^h)},$$

and obtains $p_{i,k+1}$ through (3).

Step 5: Set k→ k+1 and go to Step 1 until certain stopping criterion is satisfied.

We use the splitting method mentioned in (Bauschke and Combettes, 2011) and seek the optimum of $z_{i,k+1}$ by solving the augmented counterpart of (9) as

$$\min_{z_i} \left\{ h_i(z_i) - y_{i,k+1}^T z_i + \frac{\epsilon}{2} \left\| \tilde{\xi}_{i,k} - z_i \right\|^2 \right\}, \tag{10}$$

where $\tilde{\xi}_{i,k} = 2\xi_{i,k} - \xi_{i,k-1}$ and ϵ is the penalty parameter. We use the definition of h_i to write (10) as

$$\begin{aligned}
&\inf_{z_i}\{ \inf_{x_i, D_i x_i = z_i} \{\phi_i(x_i)\} - y_{i,k+1}^T z_i + \frac{\epsilon}{2}\left\|\tilde{\xi}_{i,k} - z_i\right\|^2 \} \\
&= \inf_{z_i} \inf_{x_i, D_i x_i = z_i} \{\phi_i(x_i) - y_{i,k+1}^T z_i + \frac{\epsilon}{2}\left\|\tilde{\xi}_{i,k} - z_i\right\|^2 \} \\
&= \inf_{x_i}\{\phi_i(x_i) - y_{i,k+1}^T D_i x_i + \frac{\epsilon}{2}\left\|\tilde{\xi}_{i,k} - D_i x_i\right\|^2 \},
\end{aligned}$$

which indicates that (10) is equivalent to finding the optimum of $x_{i,k+1}$ by solving

$$\min_{x_i}\{\phi_i(x_i) - y_{i,k+1}^T D_i x_i + \frac{\epsilon}{2}\left\|\tilde{\xi}_{i,k} - D_i x_i\right\|^2 \}. \tag{11}$$

We employ the proximal gradient descent method and obtain

$$x_{i,k+1} = \mathrm{prox}_{\gamma g_i}(x_{i,k} - \gamma(\nabla f_i(x_{i,k}) - D_i^T(2y_{i,k+1} - y_{i,k}))).$$

Together with the updates of y_{k+1} and ξ_{k+1}, the proposed algorithm is summarized in Algorithm 1.

Remark 3. Although we use an approximation term to minimize x_i in (11) which requires infinite times to converge to the solution, we will show that the algorithm still converges when Step 3 runs once per iteration. Step 4 converts variable x to original variables in problem (2).

Remark 4. It is easy to see that the proposed algorithm can be carried out in a distributed manner in that each node only requires information from its neighbors at each iteration. Besides, since the only information communicated directly is the dual variable, the algorithm is privacy preserved.

4. CONVERGENCE ANALYSIS

In this section, we analyze the convergence properties of the proposed algorithm.

At each iteration k, let $q_k = \begin{bmatrix} y_k^T & \xi_k^T & x_k^T \end{bmatrix}^T$ and $\{q_k\}_{k\geq 0}$ denote the sequence generated by the proposed algorithm.

Lemma 5. Consider the iterates $\{\xi_k\}_{k\geq 0}$ generated by the algorithm. Suppose that Assumption 1 holds, then we have $\mathbf{1}^T\xi_{k+1} = \mathbf{1}^T\xi_k$ for all $k \geq 0$.

Proof. We write Step 2 in Algorithm 1 as

$$\xi_{k+1} = \xi_k - \frac{1}{\tau}(I - \overline{W})y_{k+1},$$

where $\overline{W} = W \otimes I_d$ and d is the dimension of y_i. By Assumption 1, we have $\mathbf{1}^T\xi_{k+1} = \mathbf{1}^T\xi_k - \frac{1}{\tau}(\mathbf{1}^T - \mathbf{1}^T\overline{W})y_{k+1} = \mathbf{1}^T\xi_k$.

Next, we establish two important lemmas for further analysis. The proofs can be shown following similar arguements in (Xu et al., 2019). So we omit the details of the proofs.

Lemma 6. If $\gamma < \frac{2}{2\tau\rho + L_f}$, then

$$\|\Delta q_{k+1}\|_G^2 < \|\Delta q_k\|_G^2, \forall k \geq 0,$$

where $L_f = \max\{L_{f_i}\}$, $G = \begin{bmatrix} \overline{W} & 0 & -\tau D \\ 0 & \tau^2 Q^{-1} & 0 \\ -\tau D^T & 0 & \frac{\tau}{\gamma} I \end{bmatrix}$, $Q = (I - \overline{W} + \frac{\mathbf{1}\mathbf{1}^T}{m})$, $\rho = \rho(D^T\overline{W}^{-1}D)$, $D = \mathrm{diag}(D_1, D_2, ..., D_m)$ and $\|\cdot\|_G^2 = \langle G\cdot, \cdot\rangle$.

Lemma 7. Let $\mathcal{S}$ be the optimal set and $(y^*, b^*, z^*) \in \mathcal{S}$ be an optimal solution pair to the primal-dual problem. Given that $\gamma < \frac{1}{\tau\rho}$, we have

$$\begin{aligned}
&\|q_{k+1} - q^*\|_G^2 - \|q_k - q^*\|_G^2 + \|\Delta q_{k+1}\|_G^2 \\
&\leq -2\tau \langle \nabla f(x_k) - \nabla f(x^*), x_{k+1} - x^* \rangle, \forall k \geq 0.
\end{aligned} \tag{12}$$

Theorem 8. Suppose Assumptions 1 and 2 hold. If $\gamma < \frac{1}{\tau\rho + 2L_f}$, then the sequence $\{(y_k, x_k)\}_{k\geq 0}$ generated by Algorithm 1 will converge to an optimal set $\{(y^*, x^*)\}$ at a nonergodic rate of $O(\frac{1}{k})$.

Proof. We define the Lyapunov function as

$$V_k = \|q_k - q^*\|_G^2 - 2\tau D_f(x^*, x_k), \tag{13}$$

where $D_f(x, y) = f(x) - f(y) - \langle\nabla f(y), x - y\rangle$. We first state the decrease of the Lyapunov function at every iteration k.

Reranging the terms of (12) with the convexity of f taken into consideration, we can obtain

$$V_k - V_{k+1} \geq \|\Delta q_{k+1}\|_G^2 - 2\tau D_f(x_{k+1}, x_k). \tag{14}$$

By Assumption 2, we have

$$D_f(x_{k+1}, x_k) \leq L_f \|\Delta q_{k+1}\|_{I'}^2. \tag{15}$$

If $\gamma < \frac{1}{\tau\rho + 2L_f}$, we have $H - 2\tau L_f I' > 0$ by Schur complement lemma. Then (14) can be rewritten as

$$V_k - V_{k+1} \geq \|\Delta q_{k+1}\|_{G - 2\tau L_f I'}^2. \tag{16}$$

Letting $\varsigma = \frac{\gamma}{1 - \tau\gamma\rho}$, we have $\varsigma < \frac{1}{2L_f}$. Let $c \in (\varsigma, \frac{1}{2L_f})$. Then $G - \frac{\tau}{c}I' > 0$, i.e., $2\tau L_f I' < 2cL_f G$, which, combined with (16) indicates that $V_k - V_{k+1} > 0$. Similarly we have

$$V_k \geq \|q_k - q^*\|_{G - 2\tau L_f I'}^2 > 0.$$

Summing (16) from 1 to k over t we obtain

$$(1 - 2cL_f)\sum_{t=1}^{k} \|\Delta q_{t+1}\|_G^2 < V_0 - V_{k+1} \leq V_0.$$

Since $c < \frac{1}{2L_f}$, we obtain

$$\sum_{t=1}^{k} \|\Delta q_{t+1}\|_G^2 < \frac{V_0}{1 - 2cL_f}. \tag{17}$$

Combining (17) with Lemma 6 yields

$$\|\Delta q_{k+1}\|_G^2 < \frac{1}{k}\sum_{t=1}^{k}\|\Delta q_{t+1}\|_G^2 < \frac{V_0}{k(1-2cL_f)} \tag{18}$$

and that $\lim_{k\to\infty}\|\Delta q_{k+1}\|_G = 0$. From (16) we know that x_k and y_k are bounded. The sequence $\{(y_k, x_k)\}_{k\geq 0}$ thus has at least a cluster point (y_∞, x_∞) and the corresponding subsequence $\{(y_{k_i}, x_{k_i})\}_{i\geq 0}$. Taking the limit along $k_i \to \infty$ we obtain that the cluster point is a saddle point of the primal-dual problem. Since $\{V_k\}_{k\geq 0}$ converges and is contractive with respect to the optimal set $\mathcal{S}$, (y_∞, z_∞) is the unique cluster point (Bauschke and Combettes, 2011, Th. 5.5) and (He and Yuan, 2012, Th. 3.7). It then follows that $\{(y_k, x_k)\}$ will converge to a unique point of the optimal set of $\mathcal{S}$. From (18) we know that $\|\Delta q_{k+1}\|_G^2$ will decrease to 0 at a rate of $O(\frac{1}{k})$. Thus, we can conclude that the sequence $\{(y_k, x_k)\}$ will converge to a solution pair (y^*, x^*) at a nonergodic rate of $O(\frac{1}{k})$

5. SIMULATION

In this section, we report simulation results to show the effectiveness of the proposed algorithm.

5.1 Parameter Settings

We condider a system based on the modified IEEE 14-bus network. A pipeline network as well as an EH are added at each bus to the system. The structure of the system is shown in Fig 2. There are five generators at nodes 1, 2, 3, 6 and 8 and five local gas companies at nodes 1, 2, 4, 6, 8, respectively. The energy conversion coefficients $\eta^{e,e}$, $\eta^{chp,e}$, $\eta^{chp,h}$ and $\eta^{g,h}$ are selected as 0.98, 0.35, 0.4 and 0.9. In this paper, the cost functions are approximately modeled as quadratic functions, i.e., $C_i(s_i^e) = a_i^e(s_i^e)^2 + b_i^e s_i^e + c_i^e, \forall i$ and $C_i(s_i^g) = a_i^g(s_i^g)^2 + b_i^g s_i^g + c_i^g, \forall i$. The cost parameters and local limits of each EH are listed in Table 1 and Table 2. We choose the weight matrix W as $I - L/m$, where L denotes the Laplacian matrix of the graph.

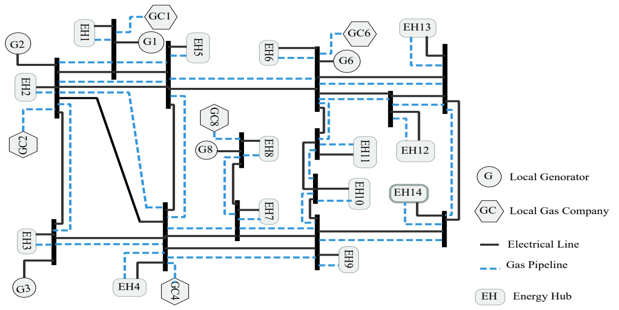

Fig. 2. Structure of the MES.

Table 1. Parameter settings of generators

G	G1	G2	G3	G6	G8
a^e(mu/pu^2)	0.12	0.08	0.09	0.05	0.13
b^e(mu/pu)	12.0	13.0	12.5	13.2	11.5
c^e	0	0	0	0	0
$[\underline{s^e}, \overline{s^e}]$(pu)	[0,200]	[0,150]	[0,175]	[0,210]	[0,200]

[1] [mu] denotes monetary unit and [pu] denotes power unit.

Table 2. Parameter settings of natural gas companies

GC	GC1	GC2	GC4	GC6	GC8
a^g(mu/pu^2)	0.033	0.023	0.042	0.033	0.012
b^g(mu/pu)	5.8	6.0	5.5	6.3	6.6
c^g	0	0	0	0	0
$[\underline{s^g}, \overline{s^g}]$(pu)	[0,200]	[0,275]	[0,150]	[0,175]	[0,375]

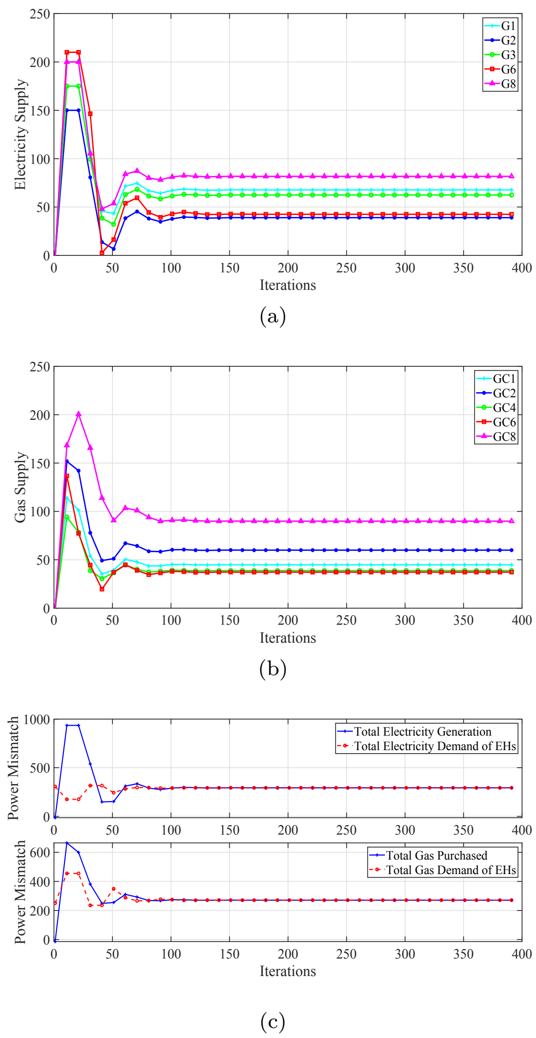

Fig. 3. Simulation results:(a)Electricity produced by generators; (b)Gas purchased from companies; (c)The supply-demand power mismatches at the input side of EHs.

5.2 Simulation Results

We initialize the electricity and heat demands at each bus as [60,10], [21.7,25.2], [66.2,15], [47.8,25.3], [17.6,5], [11.2,25], [4,3], [5,36.8], [29.5,20], [9,5], [3.5,7], [6.1,14], [13.5,10.5] and [14.9,10], respectively. The historical evolution of the energy supply and mismatches between energy supply and demand of EHs are shown in Fig. 3. It can be observed that the energy supply converges to a fixed point in several iterations. A balance between supply and demand is reached when all the objective variables reach to a steady state. We compare the results of our proposed

Table 3. Simulation results

G/GC		G1/N1	G2/N2	G3/N4	G6/N6	G8/N8
DA	s^e	67.7100	39.0650	62.5022	42.5040	81.7323
	s^g	44.8326	59.7337	38.7971	37.2569	89.9564
CA	s^e	67.7040	39.0608	62.5158	42.5115	81.7236
	s^g	44.8321	59.9773	38.7968	37.2567	89.9542

[2] [DA] denotes the proposed distributed algorithm and [CA] denotes the centralized algorithm.

algorithm with a centralized algorithm as is shown in Table 3. We can see that the proposed algorithm can exactly find an optimal solution.

After it converges to an optimal point, we increase the demand by 20% for each load to test the perfomance in

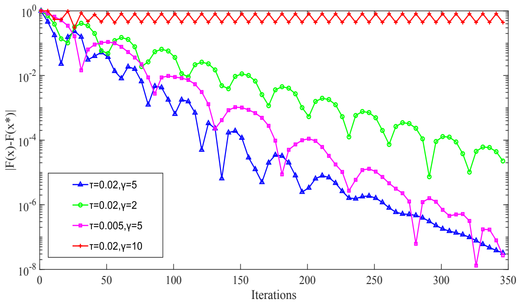

Fig. 4. Convergence of the proposed algorithm with different τ, γ.

response to the demand change under different parameter settings. Fig 4 depicts the convergence results. It can be seen that we can increase the stepsize to accelerate the iteration but the convergece will be broken when the stepsize exceeds a certain upper bound.

6. CONCLUSION

In this paper, we proposed a distributed algorithm to solve the ED problem in MESs based on non-linear mapping methods and dual splitting techniques. We have showed that the algorithm is able to find the optimal solution of the problem for general convex cost functions with a nonergodic convergence rate of $O(\frac{1}{k})$ when the stepsize is within a specific range. In addition, the alogrithm works well for large-scale systems and allows for generators plug-and-play. Since we modelled our communication network as an undirected fixed graph, more general situations such as directed and time-varying networks can be taken into account for future work.

REFERENCES

Bauschke, H.H. and Combettes, P.L. (2011). *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*. Springer, New York, NY, USA.

Beigvand, S.D., Abdi, H., and Scala, M.L. (2017). Economic dispatch of multiple energy carriers. *Energy*, 138, 861 – 872.

Carli, R. and Dotoli, M. (2020). Distributed alternating direction method of multipliers for linearly constrained optimization over a network. *IEEE Control Systems Letters*, 4(1), 247–252.

Falsone, A., Notarnicola, I., Notarstefano, G., and Prandini, M. (2019). Tracking-ADMM for Distributed Constraint-Coupled Optimization. *arXiv e-prints*, arXiv:1907.10860.

He, B.S. and Yuan, X. (2012). Convergence analysis of primal-dual algorithms for a saddle-point problem: From contraction perspective. *SIAM J. Imaging Sciences*, 5, 119–149.

Huang, A.Q., Crow, M.L., Heydt, G.T., Zheng, J.P., and Dale, S.J. (2011). The future renewable electric energy delivery and management (freedm) system: The energy internet. *Proceedings of the IEEE*, 99(1), 133–148.

Kar, S. and Hug, G. (2012). Distributed robust economic dispatch in power systems: A consensus + innovations approach. In *2012 IEEE Power and Energy Society General Meeting*, 1–8.

Krause, T., Andersson, G., Froehlich, K., and Vaccaro, A. (2011). Multiple-energy carriers: Modeling of production, delivery, and consumption. *Proceedings of the IEEE*, 99, 15 – 27.

Lakshmanan, H. and de Farias, D.P. (2008). Decentralized resource allocation in dynamic networks of agents. *SIAM Journal on Optimization*, 19(2), 911–940.

Mancarella, P. (2012). Smart multi-energy grids: Concepts, benefits and challenges. In *2012 IEEE Power and Energy Society General Meeting*, 1–2.

Mancarella, P. (2014). Mes (multi-energy systems): An overview of concepts and evaluation models. *Energy*, 65, 1 – 17.

Molzahn, D.K., Dörfler, F., Sandberg, H., Low, S.H., Chakrabarti, S., Baldick, R., and Lavaei, J. (2017). A survey of distributed optimization and control algorithms for electric power systems. *IEEE Transactions on Smart Grid*, 8(6), 2941–2962.

Shi, L., Wang, R., and Yao, L. (2017). Modelling and solutions of coordinated economic dispatch with wind–hydro–thermal complex power source structure. *IET Renewable Power Generation*, 11(3), 262–270.

Xu, J., Zhu, S., Soh, Y.C., and Xie, L. (2018). A bregman splitting scheme for distributed optimization over networks. *IEEE Transactions on Automatic Control*, 63(11), 3809–3824.

Xu, J., Zhu, S., Soh, Y.C., and Xie, L. (2019). A dual splitting approach for distributed resource allocation with regularization. *IEEE Transactions on Control of Network Systems*, 6(1), 403–414.

Yang, S., Tan, S., and Xu, J. (2013). Consensus based approach for economic dispatch problem in a smart grid. *IEEE Transactions on Power Systems*, 28(4), 4416–4426.

Zhang, Y. and Zavlanos, M.M. (2018). A consensus-based distributed augmented lagrangian method. In *2018 IEEE Conference on Decision and Control (CDC)*, 1763–1768.

Zhang, Z. and Chow, M. (2011). Incremental cost consensus algorithm in a smart grid environment. In *2011 IEEE Power and Energy Society General Meeting*, 1–6.

Ziang Zhang, Xichun Ying, and Mo-Yuen Chow (2011). Decentralizing the economic dispatch problem using a two-level incremental cost consensus algorithm in a smart grid environment. In *2011 North American Power Symposium*, 1–7.