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Robust finite-time control of impulsive positive systems under $\mathcal{L}_1$-gain performance

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Abstract: This paper is concerned with robust finite-time $\mathcal{L}_1$-gain control problem for impulsive positive systems (IPSs). By adopting the average impulsive interval technique, sufficient conditions ensuring the finite-time boundedness of IPSs under $\mathcal{L}_1$-gain characterization are formulated. The design of a feedback controller is also addressed to make the closed-loop system be positive, finite-time bounded (FTB), and have $\mathcal{L}_1$-gain characterization. Results are presented in the form of linear programming (LP) inequalities. Finally, a numerical example is given to demonstrate the efficiency of the proposed design.

Keywords: Positive system, stability criterion, robustness, finite-time control, impulsive system.

1. INTRODUCTION

Positive systems are dynamic systems whose state variables and outputs stay in the positive orthant whenever their initial states and inputs are nonnegative. In fact, positive systems have wide applications in numerous areas such as chemical processes, mechanical systems, and biology (Johnson (2006); Shorten et al. (2006); Farina and Rinaldi (2011)). In the past decades, numerous significant contributions for positive systems have appeared, (see Zhao et al. (2014); Xiao et al. (2016); Zhang et al. (2016); Liu et al. (2018); Qi et al. (2019)). Note that there always exist instantaneous jump phenomena in the process of system dynamics, which can be described by impulsive systems (Ma et al. (2019)). Recently, impulsive positive systems (IPSs), which consist of positive continuous dynamics and positive discrete dynamics, have been noticed by some researchers, some important results have been reported in (Hu et al. (2017); Wang et al. (2014); Yang and Zhang (2019); Liu et al. (2015); Hu et al. (2019)).

When the system state doesn't overstep a settled bound within a specified time period if a prescribed bound on the initial conditions is given, the system is finite-time stable/ bounded (FTB) (Amato et al. (2011)). For positive systems, contributions can be found in (Chen and Yang (2014); Zhang et al. (2014b); Hu et al. (2019); Qi and Gao (2015)).

On the other hand, the input-output performance which can characterize the ability of anti-disturbance is a significant index for dynamic systems. Research works of $\mathcal{L}_1$-gain characterization of positive systems are reported in (Zhang et al. (2015); Wang and Zhao (2017); Xiang et al. (2017); Qi et al. (2017)). Combining with the importance and necessity of finite-time stability/ boundedness in practice, it is an important issue that designing a controller to achieve the finite-time stability/ boundedness and $\mathcal{L}_1$-gain characterization of the controlled system. However, as we know, there is few result available on the finite-time $\mathcal{L}_1$-gain controller design for IPSs, which motivates this study.

In this study, the robust finite-time $\mathcal{L}_1$-gain control problem for IPSs is studied. A criterion is established to make IPSs be FTB under $\mathcal{L}_1$-gain performance by applying the average impulsive interval method. Then, a feedback controller is proposed to guarantee that the closed-loop system is positive, FTB, and has $\mathcal{L}_1$-gain performance. The results are solved through the linear programming (LP) technique. A numerical example is presented to validate the efficiency of the obtained results.

Notations: $\mathbb{R}^n_+$ means the positive orthant of $\mathbb{R}^n$. 1-norm of a vector $z \in \mathbb{R}^n$ is $\|z\| = \sum_{k=1}^n |z_k|$. z_i refers to the ith element of z. $z \succ 0$, ($z \succeq 0$) means $z_i > 0$ ($z_i \geq 0$). $\max\{z\}$ and $\min\{z\}$ means the maximum and the minimum component of z, respectively. Matrix M is nonnegative (non-positive, positive and negative) refers to $M \succeq 0 (\preceq 0, \succ 0, \prec 0)$. Vector $\mathbf{1}$ refers to $[1, ..., 1]^T$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following impulsive systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + D\omega(t), t \neq t_k, k = 1, 2, ..., \\ x(t) = Hx(t^-), t = t_k, \\ z(t) = Cx(t) + F\omega(t), \\ \mathrm{x}(t_0) = x_0, t_0 = 0, \end{cases} \tag{1}$$

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where $x(t) \in \mathbb{R}^n$ is system state, $u(t) \in \mathbb{R}^{r\times n}$ is the input control. $z(t)$ is the system output. $\omega(t) \in \mathbb{R}^n_+$ denotes the exogenous disturbance vector and belongs to $\mathcal{L}_1[0, t_f)$. A, B, D, H, C, and F are known matrices with appropriate dimensions. $\sigma = \{t_k\} = \{t_1, t_2, t_3, ...\}$ is an impulsive sequence with $0 < t_1 < t_2 < t_3 < ... < t_{k-1} < t_k < ...$, where $\lim_{k\to\infty} t_k \to \infty$. $x(t_k^+) = \lim_{s\to 0^+} x(t_k + s)$ and $x(t_k^-) = \lim_{s\to 0^-} x(t_k + s)$. We assume that the system state is right continuous, then $x(t_k) = x(t_k^+)$.

The feedback controller is

$$u(t) = Kx(t), \tag{2}$$

where the gain matrix $K \in \mathbb{R}^{r\times n}$ is to be designed.

The closed-loop system of (1) and (2) is formulated as follows

$$\begin{cases} \dot{x}(t) = (A + BK)x(t) + D\omega(t), t \neq t_k, k = 1, 2, ... \\ x(t) = Hx(t^-), t = t_k, \\ z(t) = Cx(t) + F\omega(t), \\ \mathrm{x}(t_0) = x_0, t_0 = 0. \end{cases} \tag{3}$$

We will introduce some definitions and lemmas which are necessary for theoretical analysis.

Lemma 1. (Zhang et al. (2014a)). If and only if A is Metzler, $H \succeq 0$, $D \succeq 0$, $C \succeq 0$, and $F \succeq 0$, then impulsive system (1) without $u(t)$ is positive.

Lemma 2. (Horn (1994)). If and only if there exists a scalar ξ such that $A + \xi I \succeq 0$ holds, then A is Metzler.

Definition 1. (Lu et al. (2010)). If there exist $N_0 \in \mathbb{N}$ such that

$$\frac{t - t_0}{\tau_a} - N_0 \leq N_\sigma(t, t_0) \leq \frac{t - t_0}{\tau_a} + N_0, \forall t \geq t_0, \tag{4}$$

where $N_\sigma(t, t_0)$ is the number of impulsive jumps over (t_0, t), then the average impulsive interval of the impulsive sequence is τ_a.

Definition 2. (Zhang et al. (2015)). Given positive constants t_f, c_1, c_2 , where $c_1 < c_2$, an initial time t_0, and a vector $l \succ 0$, the IPS (3) is FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$ if

$$x(t_0)^T l \leqslant c_1 \Rightarrow x^T(t) l \leqslant c_2, \forall t \in [t_0, t_f]. \tag{5}$$

Definition 3. (Zhang et al. (2015)). System (3) with $u(t) = 0$ has a finite-time $\mathcal{L}_1$-gain γ, if

(1) System (3) is FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$;

(2) Under zero initial condition, there exist positive scalars δ, ς, and γ such that for nonzero $\omega(t)$,

$$\delta \int_{t_0}^{t_f} e^{-\varsigma t} \|z(t)\| dt \leqslant \gamma \int_{t_0}^{t_f} \|\omega(t)\| dt \tag{6}$$

holds.

3. MAIN RESULTS

In this section, the robust finite-time $\mathcal{L}_1$-gain control problem of IPSs is investigated. First, a sufficient criterion ensuring the $\mathcal{L}_1$-gain performance and finite-time boundedness of system (3) is proposed.

Here, in order to keep the positivity of impulsive system (1) with $u(t) = 0$, we assume that A is Metzler, $H \succeq 0$, $C \succeq 0$, $D \succeq 0$, and $F \succeq 0$.

Theorem 1. Consider system (3), for some positive constants t_f, α, $\mu \geqslant 1$, λ, h, $\underline{v}$ and $\bar{v}$, an integer $N_0 \in \mathbb{N}$, a scalar ξ, and a vector $l \in \mathbb{R}^n_+$, if there exist a positive scalar $\gamma \leq \lambda$, a vector $v \succ 0$, and a matrix K such that the following equalities hold,

$$A + BK + \xi I \succeq 0, \tag{7}$$

$$((A + BK)^T - \alpha I)v + C^T \mathbf{1}_n \preceq 0, \tag{8}$$

$$D^T v + F^T \mathbf{1}_n - \gamma \mathbf{1}_n \preceq 0, \tag{9}$$

$$(H^T - \mu I)v \preceq 0, \tag{10}$$

$$\underline{v}\mathbf{1}_n \preceq v \preceq \bar{v}\mathbf{1}_n, \tag{11}$$

and the average impulsive interval τ_a satisfies

$$\tau_a \geqslant \max\{\frac{\ln \mu}{\alpha}, \frac{t_f \ln \mu}{\ln \kappa - \ln \rho - \alpha t_f}\}, \tag{12}$$

where c_1 and c_2 are prescribed scalars with $c_1 < c_2$, $\rho = \frac{\bar{v}}{\underline{l}} c_1 + \lambda h$, $\kappa = \frac{\underline{v} c_2}{\bar{l}} \mu^{-N_0}$ with $\kappa > \rho e^{\alpha t_f}$, in which $\bar{l} = \max\{l\}$ and $\underline{l} = \min\{l\}$. Then, system (3) is positive, robust FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$, and has $\mathcal{L}_1$-gain γ over the impulsive sequence σ.

Proof. Proof can be carried out through three steps.

(1) The positivity of system (3)

According to Lemma 2, we have $A + BK$ is a Metzler matrix from inequality (7). In addition, $H \succeq 0$, $D \succeq 0$, $C \succeq 0$, and $F \succeq 0$, from Lemma 1, we have system (3) is positive.

(2) Finite-time boundedness of system (3)

Select a copositive Lyapunov function in what follows:

$$V(t) = x^T(t)v. \tag{13}$$

For the impulsive interval $t \in [t_{k-1}, t_k)$, $k = 1, 2, ...$, the derivative of $V(t)$ can be calculated:

$$\begin{aligned} &\dot{V}(t) - \alpha V(t) \\ &= x^T(t)((A + BK)^T - \alpha I)v + \omega^T(t) D^T v. \end{aligned} \tag{14}$$

By inequalities (8) and (9), we can obtain

$$V(t) \leqslant e^{\alpha(t - t_{k-1})} V(t_{k-1}) + \lambda \int_{t_{k-1}}^{t} e^{\alpha(t-s)} \|\omega(t)\| ds. \tag{15}$$

For the impulsive instants $t = t_k$, $k = 1, 2, ...$, from inequality (10), we have

$$\begin{aligned} V(t_k) &= x^T(t_k)v = x^T(t_k^-) H^T v \\ &\leqslant \mu x^T(t_k^-) v = \mu V(t_k^-). \end{aligned} \tag{16}$$

Combining (15) and (16) yields

$$\begin{aligned} V(t) \leqslant\ & \mu^{N_\sigma(t,t_0)} e^{\alpha(t-t_0)} V(t_0) \\ &+ \lambda \int_{t_0}^{t} e^{\alpha(t-s)} \mu^{N_\sigma(t,s)} \|\omega(s)\| ds. \end{aligned} \tag{17}$$

Since $\mu \geqslant 1$, together with (4), we have

$$V(t) \leqslant e^{(\alpha+\frac{ln\mu}{\tau_a})t_f}\mu^{N_0}(V(t_0)+\lambda h). \tag{18}$$

From the inequality (11), we have

$$V(t_0) = x^T(t_0)v \leqslant \frac{\bar{v}}{\underline{l}}x^T(t_0)l, \tag{19}$$

and

$$V(t) \geqslant \frac{\underline{v}}{\bar{l}}x^T(t)l. \tag{20}$$

Substituting (19) and (20) into (18), we can derive that

$$x^T(t)l \leqslant \frac{\bar{l}}{\underline{v}}\mu^{N_0}e^{(\alpha+\frac{\ln\mu}{\tau_a})t_f}(\frac{\bar{v}}{\underline{l}}x^T(t_0)l+\lambda h). \tag{21}$$

When $x^T(t_0)l \leqslant c_1$ holds, from inequality (12), we have

$$x^T(t)l \leqslant c_2, \tag{22}$$

which implies by Definition 2 that system (3) is FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$.

(3) $\mathcal{L}_1$-gain characterization analysis of system (3)

For the impulsive interval $t \in [t_{k-1}, t_k)$, $k = 1, 2, ...,$ let $\psi(t) = \gamma\|\omega(t)\| - \|z(t)\|$, we have

$$\begin{aligned}&\dot{V}(t) - \alpha V(t) - \psi(t)\\ &= \begin{pmatrix} x^T(t) & \omega^T(t)\end{pmatrix}\begin{pmatrix}((A+BK)^T-\alpha I)v + C^T\mathbf{1}_n\\ D^Tv+F^T\mathbf{1}_n-\gamma\mathbf{1}_n\end{pmatrix}\end{aligned} \tag{23}$$

From (8) and (9), we can obtain when $t \in [t_{k-1}, t_k)$, $k = 1, 2, ...,$

$$V(t) \leqslant e^{\alpha(t-t_{k-1})}V(t_{k-1}) + \int_{t_{k-1}}^{t} e^{\alpha(t-s)}\psi(s)ds. \tag{24}$$

For $t = t_k$, $k = 1, 2, ...,$ by inequality (10), we can obtain

$$V(t_k) \leqslant \mu V(t_k^-), \tag{25}$$

together with (24), we have for $t \geqslant t_0$,

$$\begin{aligned}V(t) \leqslant\ & \mu^{N_\sigma(t,t_0)}e^{\alpha(t-t_0)}V(t_0)\\ &+\int_{t_0}^{t} e^{\alpha(t-s)}\mu^{N_\sigma(t,s)}\psi(s)ds.\end{aligned} \tag{26}$$

When the initial condition is set to be zero, since $\mu \geqslant 1$, by (4) and (12), we have

$$\mu^{-N_0}\int_{t_0}^{t} e^{-\alpha s}\|z(s)\|ds \leqslant \gamma\int_{t_0}^{t}\|\omega(s)\|ds. \tag{27}$$

Thus, inequality (6) holds. Therefore, by Definition 3, system (3) is positive, FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$, and has an $\mathcal{L}_1$-gain γ. This completes the proof. □

By applying Theorem 1, the following theorem will be proposed to design the finite-time $\mathcal{L}_1$-gain controller.

Theorem 2. Consider system (3) with $B \succ 0$, if for prescribed positive constants t_f, α, $\mu \geqslant 1$, λ, h, $\underline{v}$ and $\bar{v}$, integer $N_0 \in \mathbb{N}$, a scalar ξ, and vectors $l \in \mathbb{R}_+^n$ and $p \in \mathbb{R}_+^n$, there exist a positive scalar $\gamma \leq \lambda$, and vectors $v \succ 0$ and g such that the following equalities hold,

$$p^TB^TvA + Bpg^T + \xi I \succeq 0, \tag{28}$$
$$(A^T - \alpha I)v + C^T\mathbf{1}_n + g \preceq 0, \tag{29}$$
$$D^Tv + F^T\mathbf{1}_n - \gamma\mathbf{1}_n \preceq 0, \tag{30}$$
$$(H^T - \mu I)v \preceq 0, \tag{31}$$
$$\underline{v}\mathbf{1}_n \preceq v \preceq \bar{v}\mathbf{1}_n, \tag{32}$$

and the average impulsive interval τ_a satisfies

$$\tau_a \geqslant \max\{\frac{\ln\mu}{\alpha}, \frac{t_f\ln\mu}{\ln\kappa - \ln\rho - \alpha t_f}\}, \tag{33}$$

where constants c_1, c_2 with $c_1 < c_2$, $\kappa = \frac{\underline{v}c_2}{\bar{l}}\mu^{-N_0}$, $\rho = \frac{\bar{v}}{\underline{l}}c_1 + \lambda h$ with $\kappa > \rho e^{\alpha t_f}$, in which $\bar{l} = \max\{l\}$ and $\underline{l} = \min\{l\}$, then system (3) with controller

$$u(t) = Kx(t) = \frac{pg^T}{p^TB^Tv}x(t) \tag{34}$$

is positive, robust FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$, and has an $\mathcal{L}_1$-gain γ over the impulsive sequence σ.

4. ILLUSTRATIVE EXAMPLES

In this section, an example is given to demonstrate the efficiency of designed finite-time $\mathcal{L}_1$-gain controller.

Example 1: Consider system (3) with:

$$A = \begin{pmatrix}-0.16 & 0.5\\ 0.44 & -0.2\end{pmatrix}, H = \begin{pmatrix}1.05 & 0.2\\ 0.05 & 0.85\end{pmatrix}, B = \begin{pmatrix}0.3\\ 0.3\end{pmatrix},$$
$$D = \begin{pmatrix}0.026 & 0.03\\ 0.032 & 0.026\end{pmatrix}, C = \begin{pmatrix}0.01 & 0.02\\ 0.003 & 0.03\end{pmatrix},$$
$$F = \begin{pmatrix}0.006 & 0.005\\ 0.008 & 0.009\end{pmatrix},$$

and the exogenous disturbance is $\omega(t) = \begin{pmatrix}0.1e^{-t} & 0.1e^{-t}\end{pmatrix}^T$, then $h = 0.2$.
Let parameters be $c_1 = 0.09$, $c_2 = 1$, the finial time of finite-time interval be $t_f = 8$, $l = \begin{pmatrix}0.6 & 0.6\end{pmatrix}^T$, and $\tau_a = 1.1$.
The initial state is selected as $[\,0.1\ 0.05\,]^T$, which satisfies the initial constraint $x(t_0)l \leq c_1$. When the controller $u(t) = 0$, the state simulation is presented in Fig. 1. Fig. 2 displays the simulation of $x^T(t)l$. We can observe that $x^T(t)l$ is greater than c_2 when $t \geq 6.427s$, which suggests that the system without control is not FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$.

According to Theorem 2, we select $\alpha = 0.09$, $\lambda = 0.1$, $\bar{v} = 1.6$, $\underline{v} = 0.642$, $\mu = 1.1$ and $N_0 = 0$ satisfying the condition (33). Choose parameters $\gamma = 0.1$, $\xi = 1$ and $p = \begin{pmatrix}1 & 1\end{pmatrix}^T$. Other parameters are the same as those in

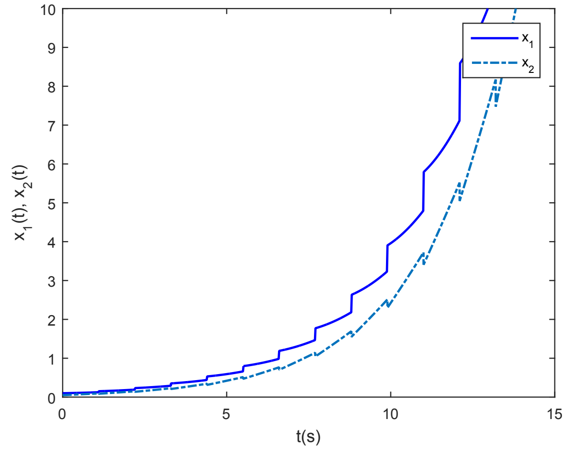

Fig. 1. State evolution of the open-loop system.

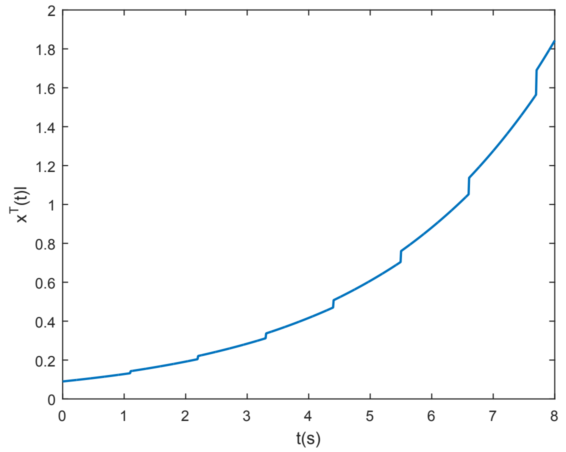

Fig. 2. Simulation of $x^T(t)l$ for the open-loop system.

the open-loop simulation. By solving the inequalities (28)-(32) by the LP approach, we obtain one feasible solution as follows

$$g = (\,-0.2748\ \ -0.4377\,)^T, v = (\,1.5074\ \ 1.4180\,)^T.$$

Thus, by (34), the matrix K is calculated as:

$$K = (\,-0.3131\ \ -0.4988\,)\,.$$

Choose the same initial state as that in the open-loop simulation. The state response of the closed-loop system is illustrated in Fig. 3. Fig. 4 shows the simulation of $x^T(t)l$. We can find that with the controller designed in this paper, the system is positive, robust FTB w.r.t. $(c_1, c_2, t_f, l, \sigma)$, and has the $\mathcal{L}_1$-gain lever $\gamma = 0.1$, which validate the efficiency of the present design.

5. CONCLUSION

This study has considered the robust finite-time $\mathcal{L}_1$-gain control design problem for IPSs. By utilizing the average impulsive interval method, a sufficient criterion has been formulated to make IPSs be FTB with $\mathcal{L}_1$-gain characterization. Then, a controller design problem has been studied, and the closed-loop system can be positive, FTB, and has $\mathcal{L}_1$-gain characterization. The results have been solved through the LP technique. Finally, a numerical example has been provided to validate the efficiency of the controller.

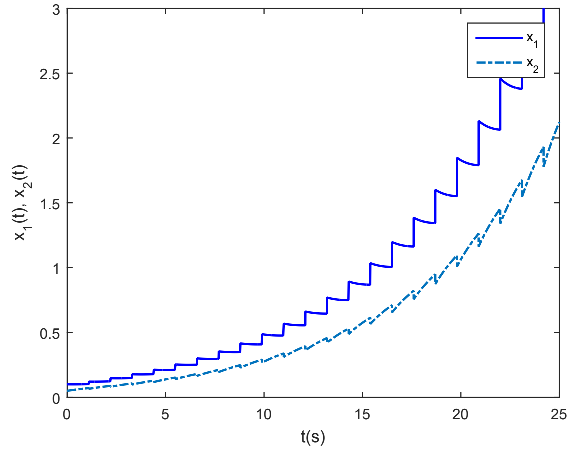

Fig. 3. State evolution of the closed-loop system.

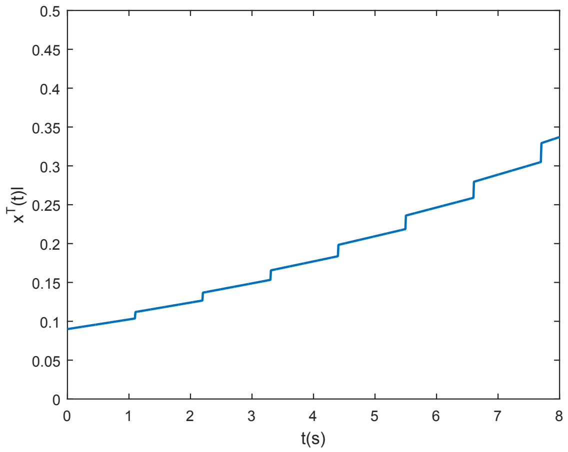

Fig. 4. Simulation of $x^T(t)l$ for the closed-loop system.

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