Intermittent Fault Detection for Nonlinear Stochastic Systems

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Abstract: In this paper, the problem of intermittent fault detection is investigated for nonlinear stochastic systems. The moving horizon estimation with dynamic weight matrices is proposed, where the weight matrices are adjusted by an unreliability index of prior estimate to avoid the smearing effects of intermittent faults. Based on the particle swarm optimization algorithm, the nonlinear optimization problem is solved and the approximate estimate is derived. Finally, the feasibility and effectiveness of the proposed algorithm are validated by a numerical example.

Keywords: Intermittent fault detection, Nonlinear stochastic systems, Moving horizon estimation, Dynamic weight matrices, Particle swarm optimization.

1. INTRODUCTION

For the sake of strengthening the reliability and safety of industrial processes, during the past several decades, tremendous effort has been devoted to the study of fault diagnosis techniques and a large number of research results have been effectively applied in various fields, such as chemical processes, aerospace systems, power systems and so on, see Fazai et al. (2019); Mandal et al. (2019); Shen et al. (2019). Nevertheless, it should be pointed out that most existing literature has concentrated on permanent faults, while little attention has been paid on another kind of common faults, intermittent faults (IFs). Different from permanent faults, a IF usually recurs by the same reason and lasts within a limited period of time. Since the appearing and disappearing times of IFs are nondeterministic, the system can recover without fault-tolerant operations (Rashid et al. (2015)). Nonetheless, if IFs are not treated properly and promptly, the destructiveness of IFs may become larger over time and finally lead to major accidents (Correcher et al. (2012)). In fact, in power systems, mechanical equipment, electrical industries and many other engineering applications with electronics, the occurrence frequency of IFs is much larger than permanent faults. Therefore, it is an urgent need to develop the fault diagnosis methods for IFs.

Generally speaking, the objective of fault diagnosis consists of fault detection, isolation and estimation, which respectively study the time, location and size of faults. It should be noted that the IF detection is more difficult than the permanent fault, since its aim is to detect all appearing and disappearing times of IFs. Especially for the detection of disappearing times, the residual is affected by previous IFs and then remains above the threshold for an uncertain period of time, which is the so-called smearing effects of IFs. Up to now, there have been some research results on the IF detection based on qualitative or quantitative analysis methods, see Constantinescu (2008); Correcher et al. (2012); Kim (2009); Yan et al. (2018, 2016). For example, in Yan et al. (2018) and Yan et al. (2016), the intermittent actuator and sensor fault detection problems for linear stochastic systems have been investigated, respectively.

On the other hand, it is well known that nonlinearity pervasively exists in almost all dynamic systems. In order to solve the fault detection for nonlinear systems, fruitful methods have been proposed by a variety of communities. These methods include, but are not limited to, the extended Kalman filter (EKF) method (Wang et al. (2019)), particle filter (PF) method (Daroogheh et al. (2018); Yin and Zhu (2015)), strong tracking filter (STF) method (Qin et al. (2016)). However, after a thorough literature search, it has been revealed that, for IFs in nonlinear systems, the corresponding research results on the fault detection are still in the blank.

In order to fill the research gap of existing literature, this paper studies the IF detection for nonlinear systems with stochastic noises. The main contributions are listed as follows. *1) This paper represents the first of few attempts to investigate the IF detection problem for nonlinear systems. 2) By means of the moving horizon estimation with dynamic weight matrices (MHEDWM), the smearing effects of IFs are properly suppressed.*

The rest of this paper is organized as follows. Section 2 gives the problem description about the IF detection for nonlinear systems and analyzes the deficiencies of existing methods for detecting IFs. Section 3 proposes the MHED-

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WM and then the effectiveness of such a algorithm is validated in Section 4. Finally, in Section 5, the conclusion is summarized and the future work is discussed.

Notations. $\mathbb{R}$ and $\mathbb{N}^+$ represent the space of all real numbers and positive integers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ stand for the n-dimensional Euclidean space and the set of all $m\times n$ real matrices, respectively. For any vector $x \in \mathbb{R}^n$, $\|x\|$ denotes the Euclidean norm of x. Additionally, for any nonnegative matrix $P \in \mathbb{R}^{n\times n}$, $\|x\|_P$ means the weighted norm of x (i.e., $\|x\|_P = \sqrt{x^T P x}$).

2. PROBLEM FORMULATION

Consider the following nonlinear system with IFs

$$\begin{cases} x(k+1) = g(x(k)) + w(k) + Ff(k) \\ \quad y(k) = h(x(k)) + v(k) + Gf(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$, $y(k) \in \mathbb{R}^{n_y}$ and $f(k) \in \mathbb{R}^{n_f}$ are the state vector, measurement output and IF signal, respectively. $w(k) \in \mathbb{R}^{n_x}$ and $v(k) \in \mathbb{R}^{n_y}$ are mutually uncorrelated zero-mean Gaussian white noises with respective covariance matrices R_w and R_v. F and G are known matrices with appropriate dimensions. $g(\cdot)$ and $h(\cdot)$ are known nonlinear functions.

The intermittent fault $f(k)$ is assumed to satisfy the following form

$$f(k) = \sum_{s=1}^{\infty} (\Theta(k - k_{s,1}) - \Theta(k - k_{s,2}))m_s(k), \ s \in \mathbb{N}^+, \tag{2}$$

where $k_{s,1}$ and $k_{s,2}$ are the sth unknown appearing time and disappearing time of IF $f(k)$, respectively. $\Theta(\cdot)$ is a function satisfying $\Theta(i) = 1$ $(i \geq 0)$ and $\Theta(i) = 0$ $(i < 0)$. $m_s(k)$ is the sth unknown fault magnitude. Define $d_{s,1} = k_{s,2} - k_{s,1}$ and $d_{s,2} = k_{s+1,1} - k_{s,2}$ as the sth active duration time and inactive duration time of $f(k)$. In this paper, we suppose that there exist two known constants $\bar{d}_1 > 0$ and $\bar{d}_2 > 0$ satisfying $d_{s,1} \leq \bar{d}_1$ and $d_{s,2} \leq \bar{d}_2$ $(s \in \mathbb{N}^+)$, where $\bar{d}_1$ and $\bar{d}_2$ are respectively called the lower bounds of fault active duration and fault inactive duration.

If a residual $r(k)$ satisfies the following two conditions:

(1) there exists a constant $0 \leq \tau_1 < \bar{d}_1$ such that $r(k) \geq J_{\mathrm{th},1}$ holds for all $k \in [k_{s,1} + \tau_1, k_{s,2})$ $(s \in \mathbb{N}^+)$, where $J_{\mathrm{th},1}$ is the detection threshold for the appearing time and $k_{s,1} + \tau_1$ is the sth appearing time detected by the residual $r(k)$;
(2) there exists a constant $0 \leq \tau_2 < \bar{d}_2$ such that $r(k) < J_{\mathrm{th},2}$ holds for all $k \in [k_{s,2} + \tau_2, k_{s+1,1})$ $(s \in \mathbb{N}^+)$, where $J_{\mathrm{th},2}$ is the detection threshold for the disappearing time and $k_{s,2} + \tau_2$ is the sth disappearing time detected by the residual $r(k)$,

it is said that IF $f(k)$ is detectable by the residual $r(k)$.

Remark 1. The core task for IF detection is to detect all appearing and disappearing times of IFs. If condition (1) is fulfilled, the designed residual $r(k)$ must be larger than the threshold $J_{\mathrm{th},1}$ before fault $f(k)$ disappears, which means that there must exist a period of alarm time during $[k_{s,1}, k_{s,2})$ $(s \in \mathbb{N}^+)$. Condition (2) shows that $r(k)$ can decrease below the threshold $J_{\mathrm{th},2}$ before the next fault $f(k)$ appears, which ensures that the sth disappearing time and the $s+1$th appearing time of IF $f(k)$ can be clearly distinguished. Combining the two conditions, it is easy to deduce that all appearing and disappearing times of IF $f(k)$ can be detected by the residual $r(k)$.

Example 1: Consider the nonlinear system with the following parameters

$$\begin{aligned} &x(k) = [x_1(k), x_2(k)]^T, \ g(x(k)) = [g_1(x(k)), g_2(x(k))]^T, \\ &g_1(x(k)) = 0.89x_1(k) + 0.1x_2(k) - 0.11\sin(x_1(k)x_2(k)), \\ &g_2(x(k)) = 0.9x_2(k) - 0.2x_1(k) + 0.01\cos(x_2^2(k)), \\ &h(x(k)) = 0.5x_1(k) + x_2(k), \\ &F = [2, 0]^T, \ G = 0, \ R_w = 0.05^2 I, \ R_v = 0.05^2. \end{aligned}$$

The IF $f(k)$ is chosen as

$$f(k) = \begin{cases} f_a, \ k \in [50, 70] \cup [85, 105] \cup [120, 150] \\ \qquad \cup [165, 203] \cup [215, 243] \cup [255, 270], \\ 0, \ \text{otherwise}. \end{cases}$$

By means of EKF, the estimate $\hat{x}^\star(k)$ can be derived. Then the residual is defined as $r(k) = y(k) - h(\hat{x}^\star(k))$. The trajectories of $r(k)$ in the case of $f_a = 1$ and $f_a = 2$ are respectively depicted in Figs. 1 and 2.

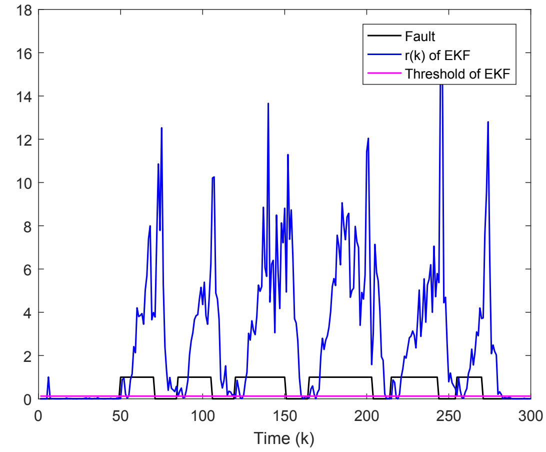

Fig. 1. IF and the residual of EKF in the case of $f_a = 1$

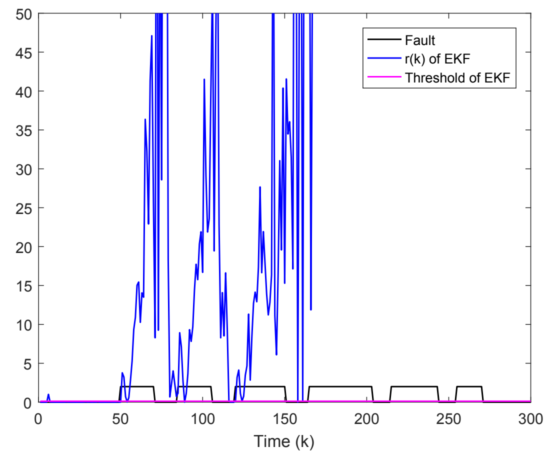

Fig. 2. IF and the residual of EKF in the case of $f_a = 2$

Then it can be found that there exist two following main difficulties for the IF detection of nonlinear systems.

(1) In traditional fault detection methods for nonlinear systems, such as EKF, PF, STF and so on, it can be only ensured that the designed residual is larger than the detection threshold after the fault appears However, when the fault disappears, owing to the smearing effects, it is hard to guarantee that the residual is smaller than the threshold, see Fig. 1.
(2) The model linearization method cannot be applied to the IF detection of nonlinear systems, due to the fact that the omitted high order terms of Taylor expansion maybe larger than the reserved lower order terms after the fault occurs. Thus, the existing IF detection methods for linear systems are unsuitable to be extended to nonlinear systems by the model linearization. Similarly, EKF, STF and other similar methods containing Taylor expansion approximation will also meet such a problem. By employing the imprecise approximation model, the estimation error may tend to diverge, see Fig. 2.

Based on the above analysis, it can be seen that the IF detection for nonlinear systems is a quite challenging problem, which cannot be properly solved by the existing methods. Hence, the main objective of this paper is to design a new algorithm to deal with such a problem.

3. IF DETECTION ALGORITHM

In this section, the following algorithm of MHEDWM is designed, where for each time $k \geq N$ ($N < \min\{\bar{d}_1 - 1, \bar{d}_2 - 1\}$), the system state $x(k-N)$ is estimated depending on the past measurement outputs $\{y(k-i)\}_{0\leq i\leq N}$. For facilitating the understanding, we respectively define $\hat{x}(k-N)$ and $\bar{x}(k-N|k) = g(\hat{x}(k-1-N))$ as the posteriori estimate and prior estimate of $x(k-N)$. Construct the following quadratic cost function (QCF)

$$\mathscr{J}(k, \hat{x}(k-N|k)) = \|\hat{x}(k-N|k) - \bar{x}(k-N|k)\|^2_{P(k)} + \sum_{i=0}^{N} \|y(k-i) - \hat{y}(k-i|k)\|^2_{Q(k)}, \tag{3}$$

where $P(k) \geq 0$ and $Q(k) \geq 0$ is a set of weight matrices to be designed, $\hat{y}(k-i|k) = h(\hat{x}(k-i|k))$ $(0 \leq i \leq N)$ and $\hat{x}(k-i+1|k) = g(\hat{x}(k-i|k))$ $(1 \leq i \leq N)$. Therefore, the desired estimate $\hat{x}(k-N)$ can be derived by solving the following optimization problem (OP)

$$\hat{x}(k-N) = \arg \min_{\hat{x}(k-N|k)} \mathscr{J}(k, \hat{x}(k-N|k)). \tag{4}$$

In this paper, an unreliability index of prior estimate $\bar{x}(k-N|k)$ is designed as follows

$$\rho(k) = \|\sigma(k)\|^2, \tag{5}$$

where

$$\begin{aligned}
&\sigma(k) = Y(k) - \bar{Y}(k|k),\\
&Y(k) = \left[y^T(k-N), \cdots, y^T(k)\right]^T,\\
&\bar{Y}(k|k) = \left[\bar{y}^T(k-N|k), \cdots, \bar{y}^T(k|k)\right]^T,\\
&\bar{y}(k-i|k) = h(\bar{x}(k-i|k)),\ 0 \leq i \leq N,\\
&\bar{x}(k-i+1|k) = g(\bar{x}(k-i|k)),\ 1 \leq i \leq N.
\end{aligned}$$

In order to avoid the smearing effects of IFs, the prior estimate $\bar{x}(k-N|k)$ should be properly discarded during the estimation process, which can be achieved by regulating the weight matrices $P(k)$ and $Q(k)$. Then the following rules are established

1) If $\rho(k) \leq \underline{\rho}$, let $P(k) = I$ and $Q(k) = 0$;
2) If $\rho(k) \geq \bar{\rho}$, let $P(k) = 0$ and $Q(k) = I$;
3) If $\underline{\rho} < \rho(k) < \bar{\rho}$, let $P(k) = \beta(k)I$ and $Q(k) = (1 - \beta(k))I$, where $\beta(k) = (\bar{\rho} - \rho(k))/(\bar{\rho} - \underline{\rho})$,

where $\bar{\rho} \geq \underline{\rho} \geq 0$ are given scalars related to the stochastic noises.

Defining $g^{(i)}(x) = g(g^{(i-1)}(x))$ $(i \in \mathbb{N}^+)$ and $g^{(0)}(x) = x$, one has

$$\hat{x}(k-N+i|k) = g^{(i)}(\hat{x}(k-N|k)). \tag{6}$$

Then the QCF (3) can be rewritten as the following form

$$\mathscr{J}(k, \hat{x}(k-N|k)) = \|\hat{x}(k-N|k) - \bar{x}(k-N|k)\|^2_{P(k)} + \left\|Y(k) - \hat{Y}(k|k)\right\|^2_{\tilde{Q}(k)}, \tag{7}$$

where

$$\begin{aligned}
&\tilde{Q}(k) = \mathrm{diag}\{\underbrace{Q(k), \cdots, Q(k)}_{N+1}\},\\
&\hat{Y}(k|k) = \left[\hat{y}^T(k-N|k), \cdots, \hat{y}^T(k|k)\right]^T\\
&\hat{y}(k-N+i|k) = h\left(g^{(i)}(\hat{x}(k-N|k))\right),\ i = 0, \cdots, N.
\end{aligned}$$

It can be found that for time instant $k > N$, the QCF $\mathscr{J}(k, \hat{x}(k-N|k))$ is a nonlinear function of $\hat{x}(k-N|k)$, which is related to functions $g(\cdot)$ and $h(\cdot)$. For general nonlinear functions $g(\cdot)$ and $h(\cdot)$, it is hardly possible to give a precise analytical solution of the nonlinear OP (4). Hence, in this paper, the particle swarm optimization (P-SO) algorithm is introduced to search for an approximate solution $\hat{x}^{\circ}(k-N)$ of OP (4). Defining the residual

$$r(k) = \hat{x}^{\circ}(k-N) - g(\hat{x}^{\circ}(k-1-N)), \tag{8}$$

the evaluation function $J(k)$ and threshold J_{th} can be given as follows

$$J(k) = \sum_{l=0}^{L-1} \|r(k-l)\|^2, \tag{9}$$

$$J_{\mathrm{th}} = \sup_{f(k)=0} J(k), \tag{10}$$

where L is a given positive integer satisfying $N + L < \max\{\bar{d}_1 - 1, \bar{d}_2 - 1\}$. The IF $f(k)$ can be detected by the following test

$$\begin{cases} J(k) \geq J_{\mathrm{th}} \Longrightarrow \text{faults occur} \Longrightarrow \text{alarm}\\ J(k) < J_{\mathrm{th}} \Longrightarrow \text{no faults.}\end{cases}$$

Remark 2. As is known to all, the prior estimate derived by the previous estimates plays an important role in traditional estimation methods, such as Kalman filter, PF, Luenberger observer and so on. When the fault disappears, the posteriori estimate is still affected by IFs existing in the prior estimate. Therefore, the key point for detecting the disappearing times of IFs is to appropriately discard the prior estimate. In this paper, according to the index $\rho(k)$, the unreliability degree of prior estimate can be clearly reflected. Then the weight matrices $P(k)$ and $Q(k)$ are dynamically regulated, which can guarantee the performance of estimator, in the meantime suppress the smearing effects.

Based on the above analysis, the following IF detection algorithm is obtained.

IF Detection Algorithm for Nonlinear Systems

(1) Set $\bar{\rho}$ and $\underline{\rho}$ according to R_w and R_v. Select N and L depending on $\bar{d}_1$ and $\bar{d}_2$. The threshold J_{th} is obtained by 100 Monte Carlo simulations without faults.
(2) If $k > N + L$, calculate the index $\rho(k)$ and the weight matrices $P(k)$ and $Q(k)$. Otherwise, jump to Step (6).
(3) Compute the QCF (7) by means of $P(k)$, $Q(k)$, $\bar{x}(k-N|k)$ and $y(k-i)$ $(i = 0, \cdots, N)$.
(4) By utilizing the PSO algorithm, solve the suboptimal estimate $\hat{x}^\circ(k-N)$ of OP (4).
(5) Calculate the residual $r(k)$ and evaluation function $J(k)$. If $J(k) \geq J_{\text{th}}$, faults occur. Otherwise, no fault occurs.
(6) Let $k = k + 1$. Return to Step (2).

4. A NUMERICAL EXAMPLE

Consider Example 1 in Section 2 and the parameters of MHEDWM are selected as follows

$$N = 4,\ L = 3,\ \bar{\rho} = 0.1,\ \underline{\rho} = 0.001,\ x(0) = [-1, 1]^T.$$

The simulation results are shown in Figs. 3-8. Figs. 3 and 4 depict the trajectories of system state and state estimates in the absence of faults, where $x_i(k)$, $\hat{x}_i^\circ(k)$, $\hat{x}_i^\bullet(k)$ and $\hat{x}_i^\star(k)$ $(i = 1, 2)$ respectively represent the system state, the estimate of MHEDWM, the estimate of moving horizon estimation with constant weight matrices (MHECWM) $P(k) = 0.5I$, $Q(k) = 0.5I$, and the estimate of EKF. Figs. 5 and 7 respectively describe the trajectories of IF, the evaluation functions of MHEDWM and MHECWM, and the corresponding thresholds in the case of $f_a = 1$ and $f_a = 2$. The alarm times detected by MHEDWM and MHECWM in the case of $f_a = 1$ and $f_a = 2$ are respectively shown in Figs. 6 and 8.

Define $\delta = 1/Z \sum_{z=1}^{Z} 1/k_f \sum_{k=1}^{k_f} e_z^T(k) e_z(k)$ as the mean square estimate error (MSEE), where Z is the number of simulation tests, k_f is the step number of each simulation test, and $e_z(k)$ is the estimate error in the zth simulation test. In the case of no fault and after 100 simulations tests, the corresponding MSEEs of MHEDWM, MHECWM and EKF are derived as follows

$$\delta_{\text{MHEDWM}} = 0.0152,\ \delta_{\text{MHECWM}} = 0.0148,\ \delta_{\text{EKF}} = 0.0196.$$

From the simulation results, it can be seen that 1) MHECWM is a hysteretic estimation algorithm with the highest estimation accuracy and the smearing effects; 2) EKF is a real-time estimation algorithm with the worst smearing effects; 3) MHEDWM is a hysteretic estimation algorithm with the second-highest estimation accuracy and without the smearing effects, which can detect all appearing and disappearing times of IF $f(k)$ accurately and timely by properly selecting N and L. Therefore, MHEDWM is superior to MHECWM and EKF in the problem of IF detection for nonlinear systems.

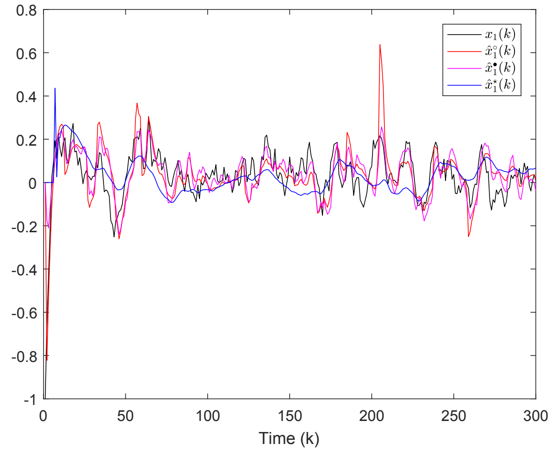

Fig. 3. System state $x_1(k)$ and its estimates

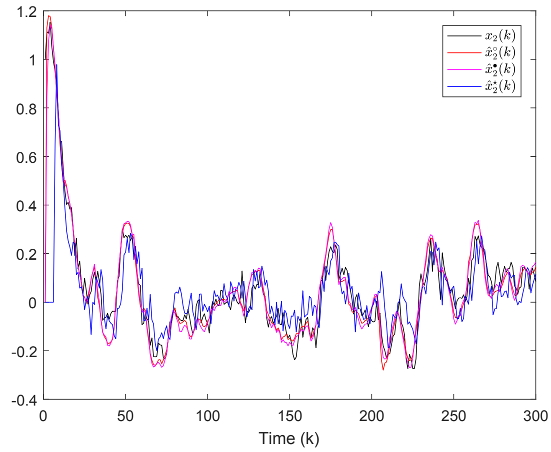

Fig. 4. System state $x_2(k)$ and its estimates

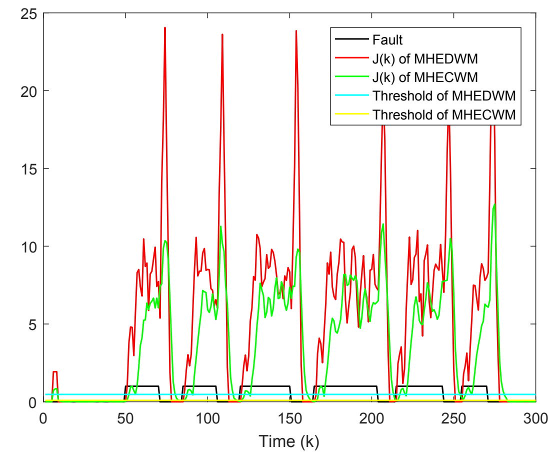

Fig. 5. IF and residuals in the case of $f_a = 1$

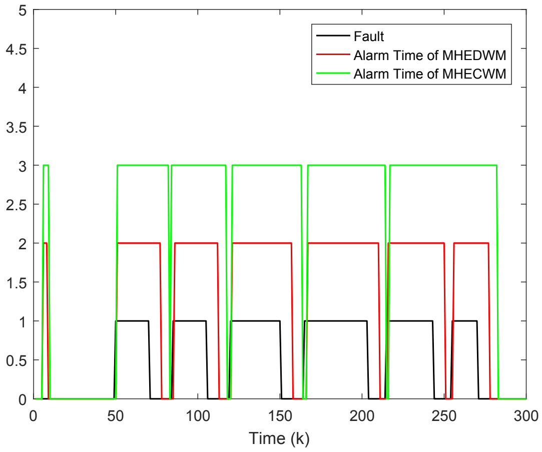

Fig. 6. Alarm times of IF in the case of $f_a = 1$

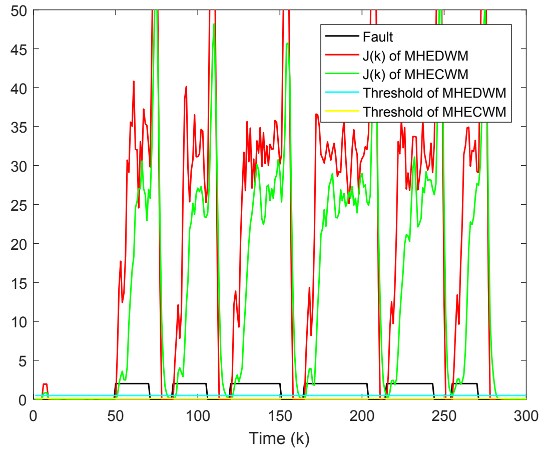

Fig. 7. IF and residuals in the case of $f_a = 2$

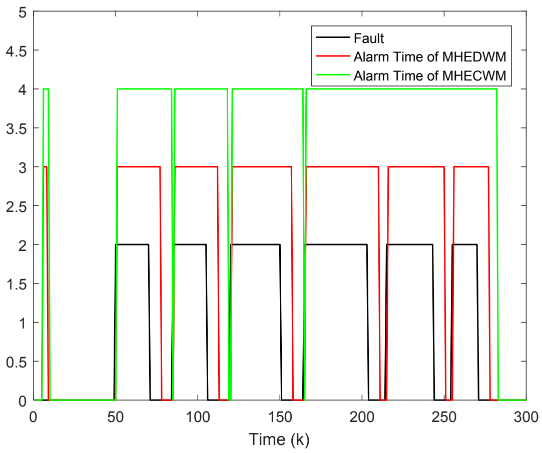

Fig. 8. Alarm times of IF in the case of $f_a = 2$

5. CONCLUSIONS AND PERSPECTIVES

In this paper, the IF detection problem for nonlinear stochastic systems has been investigated based on the moving horizon estimation (MHE) algorithm. By introducing the unreliability index of prior estimate, the weight matrices in MHE has been dynamically adjusted, which can avoid the smearing effects of IFs. The simulation has shown the proposed MHEDWM can guarantee the accuracy of estimator, in the meantime detect all appearing and disappearing times of IFs.

Further research topics include 1) the convergence analysis for the estimation error of MHEDWM; 2) the reduction of the calculation load for nonlinear OP; 3) the simplification of QCF.

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